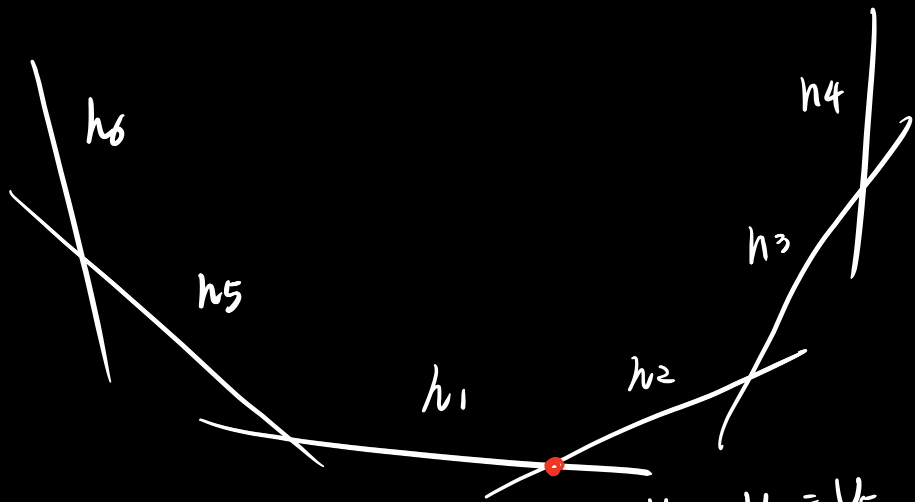
$$P(n,r) = \frac{n!}{(n-r)!}$$

$A[\,] = \{3, 17, 90\}$

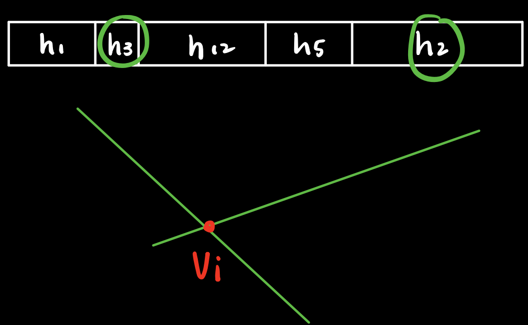
permutation $= 3! = 6$




unique optimal $V_2 = V_3 = V_4 = V_5$

What is the probability that $V_{i-1} \neq V_i$,

Which is $V_{i-1} \notin h_i$.



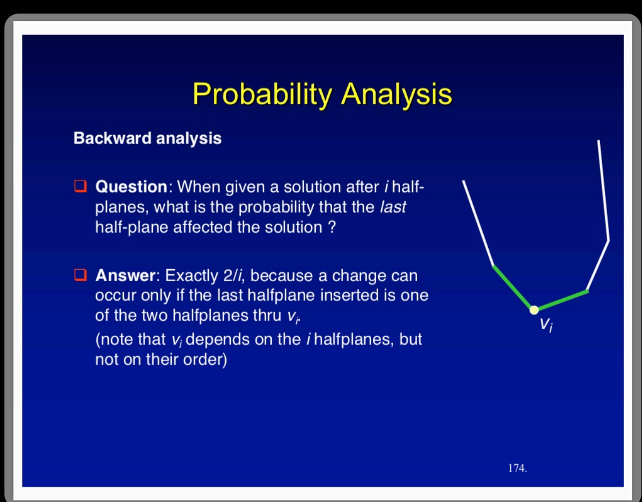

Expected works at the i's step.

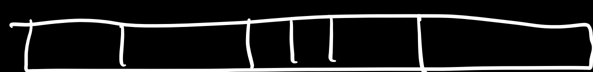
$$P(\text{work}) = \frac{i-2}{i} \cdot \text{work} + \frac{2}{i} \cdot i\,\text{work}$$

$$= \left(\frac{i-2}{i} + 2\right) \text{works}$$

$$= O(1)$$

$O(n)$ for n steps



$A[1 \dots n] \to$ set of keys

output $B[1 \dots n]$ contains same keys (dif order)

repeat

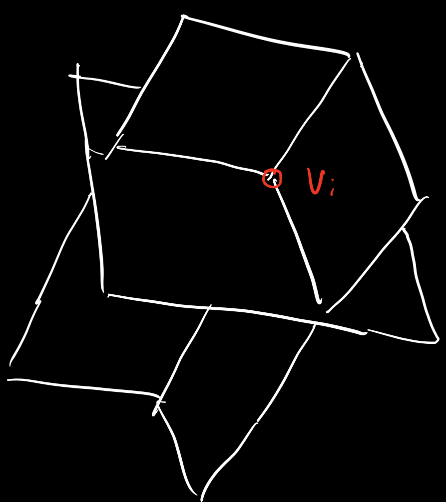
$i = \text{rand}() * n$

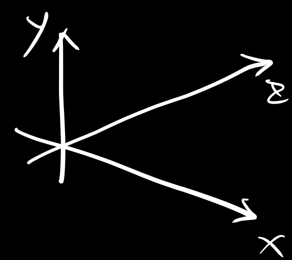
swap $(A[i], A[n])$

$n--$

Linear Time

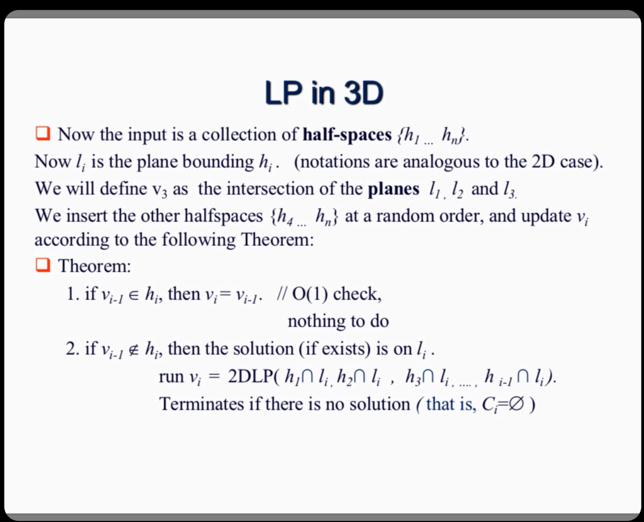
# 3-dimensions



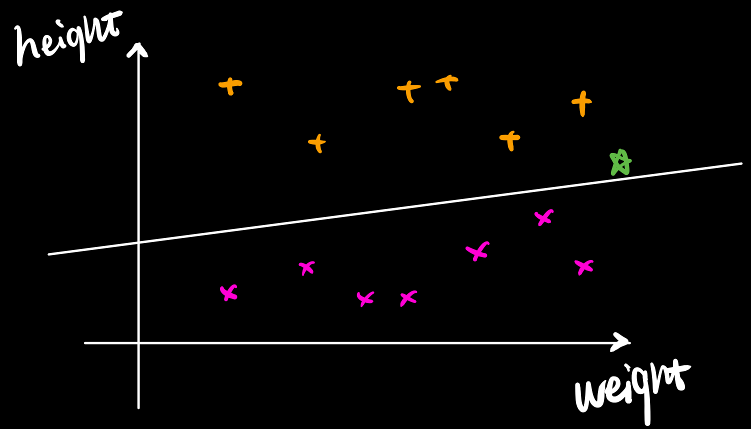




$y \geqslant ax + b \in 2D$

$z \geqslant 3x + 2y$




3 special planes $\left(\frac{3}{i}\right)$




minimize linear
mis-classify

# vector machine

Fitting a line into a set of points.

Input $P\{p_1 \ldots p_n\}$

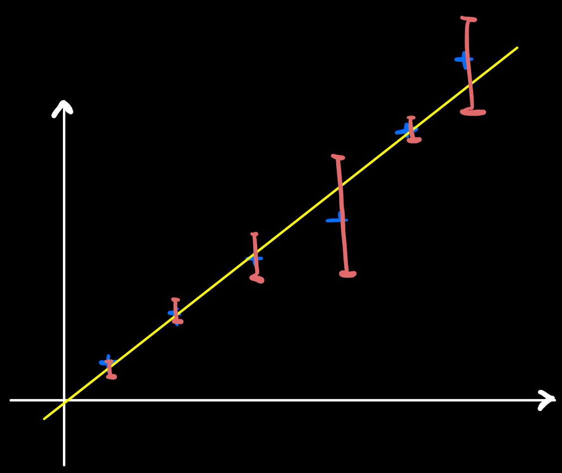
error term 0.1 tolerance

$p_i = (x_i, y_i)$ given

Assume $l = \alpha x + b$

$y_i = \alpha x_i + b$ -- go through

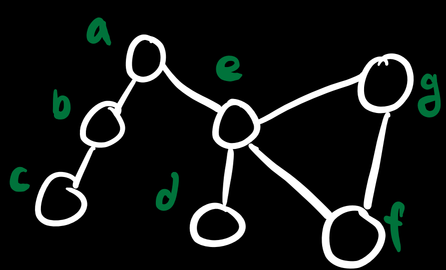
$$\begin{cases} y_i \le \alpha x_i + b + 0.1 \\ y_i \ge \alpha x_i + b - 0.1 \end{cases}$$



Output a line that is vertically at most 0.1 away from each $p_i$

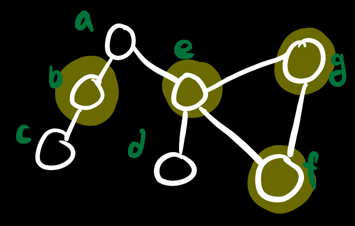
ILP min cordinality

# Vertex Cover




find a subset of vertices $U \subseteq V$ that "stab" every edge of $E$

That is every edge $(u, v) \in E$ has at least one of its endpoint at $U$




$U = \{b, e, f, g\}$

$U' = \{b, e, g\}$ optimal

---

Exaustivity

穷举

For every subset $U \subseteq V$, check if $U$ hits every edge

$U = \{v_0, v_1, \dots, v_n\}$

let $x_i = 1$, if $v_i \in U$
$= 0$ else

$\Rightarrow (x_0, x_1, x_2 \dots x_n)$
$\phantom{\Rightarrow (} 0 \quad 1 \quad 1 \quad\quad 0$

combination of binary bits

$2^n$

---

Phrase the problem as an ILP

① Phrase as LP

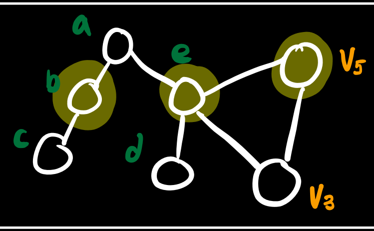
② Constrain that each var is an integer.

Answer $x_1, x_2, \dots, x_n$ $(x_i \geq 0, \forall i)$

if $x_i == 0$ then $v_i \notin U$

if $x_i == 1$ then $v_i \in U$

for every edge $(v_i, v_j) \in U$




$x_3 + x_5 \geq 1$