$0 \le P(A) \le 1$

"Law of Large Numbers"

event ⇒ subset of sample Event

Sample Space and Complement

Sample space (S): all possible outcomes
Complement (A^c): all outcomes other than A in S

- Coin flip: S = {Heads, Tails}
  - Heads^c = Tails
  - Tails^c =Heads
- Die roll: S = {1, 2, 3, 4, 5, 6}
  - $\{3\}^c = \{1, 2, 4, 5, 6\}$
  - $\{2, 5\}^c = \{1, 3, 4, 6\}$

$P(A) + P(A^c) = 1$

Disjoint vs. Not Disjoint

Disjoint: cannot happen at the same time

- A coin can't be heads and tails
- A student can't pass and fail a single class
- A playing card can't be an Ace and a 10

Not disjoint: can happen at the same time

- A student can pass one class and fail another
- A playing card can be an Ace and a Heart

Disjoint ⇒ Mutually Exclusive

$\boxed{\text{Disjoint} \underset{\not\Rightarrow}{\Leftarrow} \text{Complementary}}$

cannot happen at same time

$P(A) + P(A^c) = 1$

Probability Distributions

Lists all possible outcomes & probability of each

Example:

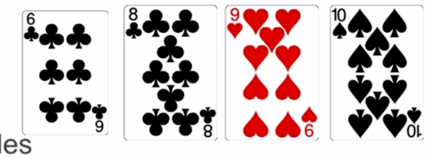
Probability of Rh+/- blood type	
+	-
0.15	0.85

Rules:

- Outcomes in list must be mutually exclusive
- Probabilities must be between 0 and 1
- Probabilities must sum to 1

Outcomes vs. Events

- Often we are not concerned with a single outcome, but rather collections of outcomes ("events")



- Examples
  - Rolling a die and getting an even number
  - Choosing a song at random and getting a Queen song

Addition Rules

Addition Rule for disjoint outcomes:
P(A or B) = P(A) + P(B)

General Addition Rule:
P(A or B) = P(A) + P(B) - P(A and B)
Note: also works for disjoint events

$P(A \text{ or } B) = P(A) + P(B) - P(A \& B)$

Independence

Processes are independent if knowing the outcome of one provides no useful information about the outcome of the other

- Flipping a coin twice. First time: Heads
  - P(Heads) for second coin?
  - P(Heads) if we hadn't seen a Heads?
- Drawing from a deck of cards. First time: Ace
  - P(Ace) for second card?
  - P(Ace) if we hadn't seen an Ace?
- "Gambler's fallacy:" believing that independent processes such as dice or roulette wheels have "hot streaks"

Product Rule

$P(A \& B) = P(A) \cdot P(B)$

The Z-score

The Z-score of an observation is the number of standard deviations it falls above or below the mean. We compute the Z-score for an observation x that follows a distribution with mean μ and standard deviation σ using

$$Z = \frac{x - \mu}{\sigma}$$

$Z * sd + mean = x$

1 standard deviation below the mean

$Z = -1$

Conditional

Three different probabilities

If A and B are two events:

- Joint probability: P(A and B)
- Marginal probability: P(A) or P(B)
- Conditional probability: P(A | B) (read: 'probability of A given B')

How do each of these arise in an experimental context?

Marginal Probability

Marginal probabilities come from the margins of the table: the totals

		relapse		
		No	Yes	Total
treatment	desipramine	14	10	24
	lithium	6	18	24
	placebo	4	20	24
	Total	24	48	72

Conditional Probability

P(A|B) = P(A and B) / P(B)

P(relapse|desipramine)

= P(relapse and desipramine) / P(desipramine)

Joint Probability

Joint probabilities

		relapse		
		No	Yes	Total
treatment	desipramine	14	$\boxed{10}$	24
	lithium	6	18	24
	placebo	4	20	24
	Total	24	48	$\boxed{72}$

P(desipramine and relapse) = 10/72 ≈ .19

Conditional Probability

Probability of relapse if we know the patient took desipramine?

		relapse		
		No	Yes	Total
treatment	desipramine	14	$\boxed{10}$	$\boxed{24}$
	lithium	6	18	24
	placebo	4	20	24
	Total	24	48	72

General Multiplication Rule

If A and B are random processes:

P(A and B) = P(A | B) x P(B)

Note that A and B do not have to be independent

P(draw 2 Aces) =

P(1st Ace) x P(2nd Ace | 1st Ace) =

4/52 x 3/51 ≈ 0.0045

$$P(A|B) = \frac{P(A\&B)}{P(B)}$$

Independence

Independence

If A and B are independent:

P(A | B) = P(A)

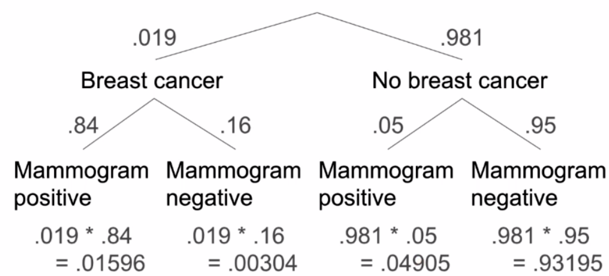
Conceptually:

B tells us nothing about A

Mathematically:

$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)} = \frac{P(A) \times P(B)}{P(B)} = P(A)$$

Tree Diagram



P(BC|+) = P(BC and +) / P(+)

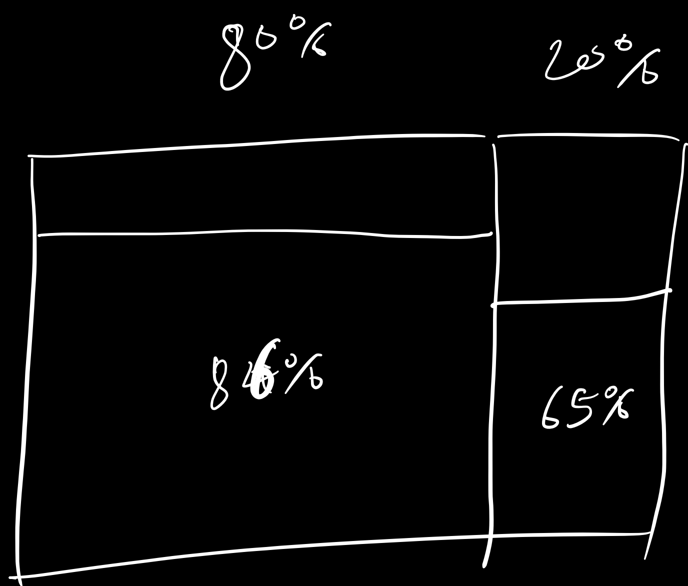
= .01596 / (.01596 + .04905) ≈ .25

Bayes' Theorem

Given:

- One variable with outcomes $A_1, A_2, \ldots, A_k$
- Another variable with an outcome B

$$P(A_1|B) = \frac{P(B|A_1)P(A_1)}{P(B)}$$
$$= \frac{P(B|A_1)P(A_1)}{P(B|A_1)P(A_1) + \ldots + P(B|A_k)P(A_k)}$$



can draw

pass course

$$P(A|B) = \frac{P(A \& B)}{P(B)}$$

$$\frac{0.86 * 0.8}{0.86 * 0.8 + 0.65 * 0.2}$$

0.688

0.1961

$$P(A|B) = \frac{0.53 * 0.37}{0.53 * 0.37 + 0.47 * 0.44}$$

0.4029

0.2068