Example

Two scientists want to know if a certain drug is effective against high blood pressure. The first scientist wants to give the drug to 1000 people with high blood pressure and see how many of them experience lower blood pressure levels. The second scientist wants to give the drug to 500 people with high blood pressure, and not give the drug to another 500 people with high blood pressure, and see how many in both groups experience lower blood pressure levels. Which is the better way to test this drug?

A. All 1000 get the drug
B. 500 get the drug, 500 don't

Parameter and point estimate

Parameter of interest

proportion of **all** Americans who have good intuition about experimental design

p a population proportion

Point estimate

proportion of **sampled** Americans who have good intuition about experimental design

$\hat{p}$ a sample proportion

Central Limit for Proportions

$$\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$$

That is, sample proportions will be approximately normally distributed

Data must meet the following conditions:

- Observations are independent
- Observations include at least 10 "successes" and 10 "failures" (10 of each outcome)

GSS confidence interval

GSS found 952 of 1167 (82%) of Americans answered the question on experimental design correctly. What's the 95% confidence interval?

Check conditions: Independence
Sample is random
1167 < 10% of all Americans
Check conditions: Success-failure
952 (>10) successes
215 (>10) failures

Results from the GSS

The General Social Survey (GSS) asks the same question and found:

All 1000 get the drug	215
500 get the drug, 500 don't	952
Total	**1167**

What percent of all Americans have good intuition about experimental design, i.e. would answer "500 get the drug 500 don't"?

Proportion confidence interval

point estimate ± critical value × standard error

How do we calculate the standard error?

How do we calculate the critical value?

Central Limit for Proportions

$$\hat{p} \sim N\left(p, \sqrt{\frac{p(1-p)}{n}}\right)$$

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Data must meet the following conditions:

- Observations are independent
- Observations include at least 10 "successes" and 10 "failures" (10 of each outcome)

GSS confidence interval

GSS found 952 of 1167 (82%) of Americans answered the question on experimental design correctly. What's the 95% confidence interval?

$$SE = \sqrt{\frac{\frac{952}{1167}\left(1-\frac{952}{1167}\right)}{1167}} \approx 0.011$$

$$CI = 0.82 \pm 1.96 \times 0.011 \approx (0.80, 0.84)$$

95% of True proportion of Americans who have the right answer

Confidence Interval

Hypothesis test

GSS found 952 of 1167 (82%) of Americans answered the question on experimental design correctly. Is this convincing evidence that more than 80% of Americans have a good intuition about experimental design?

H_0:
$p = 0.80$
H_A:
$p > 0.80$

Central Limit for Proportions

$$\hat{p} \sim N\left(p, \sqrt{\frac{p\,(1-p)}{n}}\right)$$

Standard error depends on the population proportion

Population proportion is unknown, so we must use an estimate

Calculating the standard error

When we computed a confidence interval:

$$SE = \sqrt{\frac{\frac{952}{1167}\left(1 - \frac{952}{1167}\right)}{1167}}$$

When performing a hypothesis test:

$$SE = \sqrt{\frac{0.8\,(1-0.8)}{1167}}$$

Why: assuming null is true

GSS hypothesis test

952 of 1167 of Americans answered the question on experimental design correctly

H_0: $p = 0.80$ H_A: $p > 0.80$

Check conditions: Independence
- Sample is random
- 1167 < 10% of all Americans

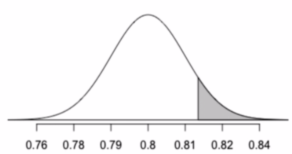
Check conditions: Success-failure
- $0.8 \times 1167 \approx 934$ (>10) successes
- $0.2 \times 1167 \approx 233$ (>10) failures

GSS hypothesis test

952 of 1167 of Americans answered the question on experimental design correctly

H_0: $p = 0.80$ H_A: $p > 0.80$

```
> n <- 1167
> se <- sqrt(0.8 * 0.2 / n)
> pnorm(952 / n, 0.8, se,
+       lower.tail=FALSE)
[1] 0.08906264
```



Can we reject H_0?

No; $0.089 \not< 0.05$

Difference of 2 Proportions

Survey: Melting Ice Cap

If the northern ice cap completely melted, would this bother you:

A. A great deal
B. Some
C. A little
D. Not at all

The General Social Survey asked this question of a sample of US residents; a separate survey sampled Duke University students.

Survey: Melting Ice Cap

If the northern ice cap completely melted, this would bother you:

Do Duke students differ from the general US population in this regard?

	GSS	Duke
A great deal	454	69
Some	124	30
A little	52	4
Not at all	50	2
Total	**680**	**105**

Parameter and point estimate

Parameter of interest:

Difference between proportions of **all** Duke students and **all** US residents bothered a great deal by ice cap melting.

$p_{Duke} - p_{US}$

Point estimate:

Difference between proportions of **sampled** Duke students and **sampled** US residents bothered a great deal by ice cap melting

$\hat{p}_{Duke} - \hat{p}_{US}$

Proportion difference inference

Confidence interval:

point estimate ± critical value × standard error

Hypothesis test:

Test statistic is $\dfrac{\text{point estimate} - \text{null value}}{\text{standard error}}$

Standard error?

population proportions

$$SE_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1(1-p_1)}{n_1} + \frac{p_2(1-p_2)}{n_2}}$$

Melting Ice: 95% confidence int.

$$SE_{\hat{p}_{Duke} - \hat{p}_{US}} = \sqrt{\frac{\frac{69}{105}\left(1-\frac{69}{105}\right)}{105} + \frac{\frac{454}{680}\left(1-\frac{454}{680}\right)}{680}} \approx 0.0497$$

$$(\hat{p}_{Duke} - \hat{p}_{US}) \quad \pm z^* \times SE_{\hat{p}_{Duke} - \hat{p}_{US}}$$

$$\approx \frac{69}{105} - \frac{454}{680} \quad \pm 1.96 \times 0.0497$$

$$\approx -0.0105 \quad \pm 1.96 \times 0.0497$$

$$\approx (-0.108, 0.087)$$

0 lies in the center

	GSS	Duke
A great deal	454	69
Some	124	30
A little	52	4
Not at all	50	2
Total	**680**	**105**

Melting ice: hypothesis conditions

1. Independence within groups
   a. Each group was randomly sampled
   b. 105 < 10% of all Duke students
   c. 680 < 10% of all US residents
2. Independence between groups
   a. Duke students independent of US residents
3. Success-failure condition?

Need to use expected counts -- but how to calculate?

Pooled proportion success-failure

Check conditions: Success-failure

Estimated n = pooled p̂ × observed n

Successes

$0.666 \times 105 \approx 70$ ✓

$0.666 \times 680 \approx 453$ ✓

Failures

$0.334 \times 105 \approx 35$ ✓

$0.334 \times 680 \approx 227$ ✓

	GSS	Duke
A great deal	454	69
Some	124	30
A little	52	4
Not at all	50	2
Total	**680**	**105**

Melting Ice: conditions

1. Independence within groups
   a. Each group was randomly sampled
   b. 105 < 10% of all Duke students
   c. 680 < 10% of all US residents
2. Independence between groups
   a. Duke students independent of US residents
3. Success-failure
   a. 69 (>10) and 454 (>10) observed successes
   b. 36 (>10) and 226 (>10) observed failures

Hypothesis Test

Hypothesis test for difference

Null hypothesis: no difference between Duke students and general US residents

H_0: $p_{US} - p_{Duke} = 0$

Alternative hypothesis: difference between Duke students and general US residents

H_A: $p_{US} - p_{Duke} \neq 0$

Pooled proportion estimate

When H_0 is $p_1 - p_2 = 0$:

$$\hat{p} = \frac{\text{total successes}}{\text{total observations}} = \frac{\hat{p}_1 n_1 + \hat{p}_2 n_2}{n_1 + n_2}$$

For the melting ice hypothesis:

$$\hat{p} = \frac{69 + 454}{105 + 680} \approx 0.666$$

Melting ice: hypothesis test

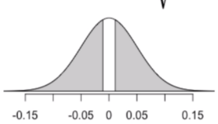
H_0: $p_{Duke} - p_{US} = 0$

H_A: $p_{Duke} - p_{US} \neq 0$ Can we reject H_0?

pooled $\hat{p} \approx 0.666$

$$SE_{\hat{p}_{Duke} - \hat{p}_{US}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n_{Duke}} + \frac{\hat{p}(1-\hat{p})}{n_{US}}}$$

$$\approx \sqrt{\frac{0.666 \times 0.334}{105} + \frac{0.666 \times 0.334}{680}} \approx 0.0494$$




```
> 2 * pnorm(69/105 - 454/680, 0, 0.0494)
[1] 0.8316112
```