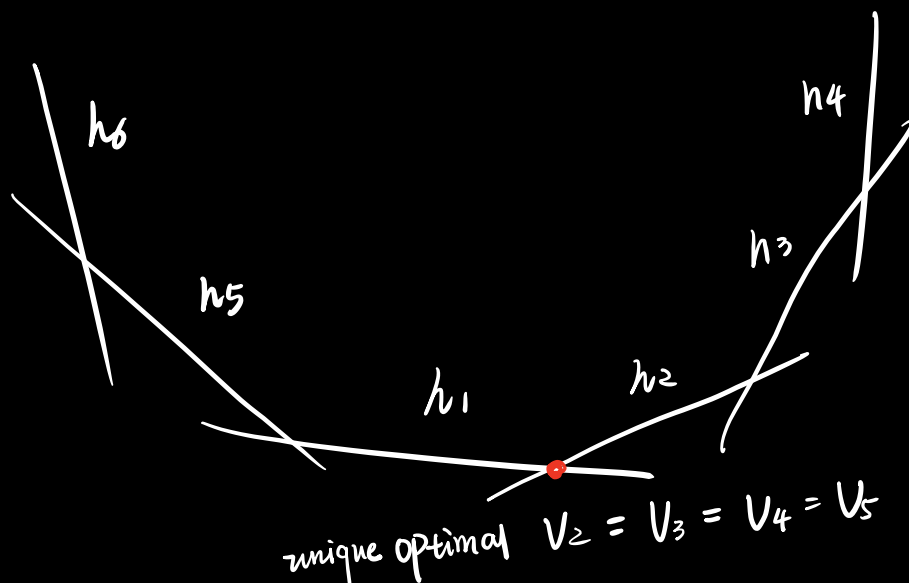


$$P(n, r) = \frac{n!}{(n-r)!}$$

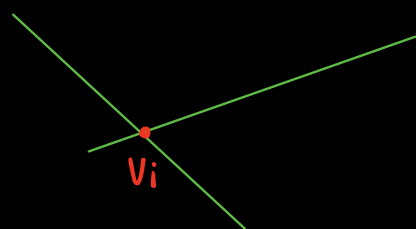
$$A[3] = \{3, 17, 90\}$$

$$\text{permutation} = 3! = 6$$



What is the probability that $V_{i-1} \neq V_i$,

which is $V_{i-1} \notin h_i$.



Expected works at the i 's step.

$$P(\text{work}) = \frac{i-2}{i} \cdot \text{work} + \frac{2}{i} \cdot i \cdot \text{work}$$

$$= \left(\frac{i-2}{i} + 2 \right) \text{works}$$

$$= O(1)$$

$$O(n) \text{ for } n \text{ steps}$$

Probability Analysis

Backward analysis

- Question:** When given a solution after i half-planes, what is the probability that the *last* half-plane affected the solution?
- Answer:** Exactly $2/i$, because a change can occur only if the last halfplane inserted is one of the two halfplanes thru v_i (note that v_i depends on the i halfplanes, but not on their order)

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$A[1 \dots n] \rightarrow$ set of keys

output $B[1 \dots n]$ contains same keys (diff order)

repeat

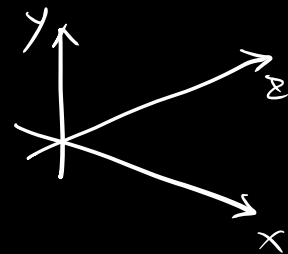
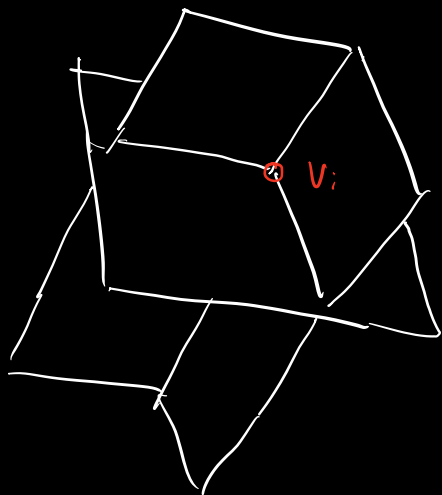
$i = \text{rand}() * n$

swap $(A[i], A[n])$

$n--$

Linear Time

3-dimensions



$$y \geq ax + b \in 2D$$

$$z \geq 3x + 2y$$

LP in 3D

Now the input is a collection of **half-spaces** $\{h_1, \dots, h_n\}$.

Now l_i is the plane bounding h_i . (notations are analogous to the 2D case).

We will define v_3 as the intersection of the **planes** l_1, l_2 and l_3 .

We insert the other halfspaces $\{h_4, \dots, h_n\}$ at a random order, and update v_i according to the following Theorem:

□ Theorem:

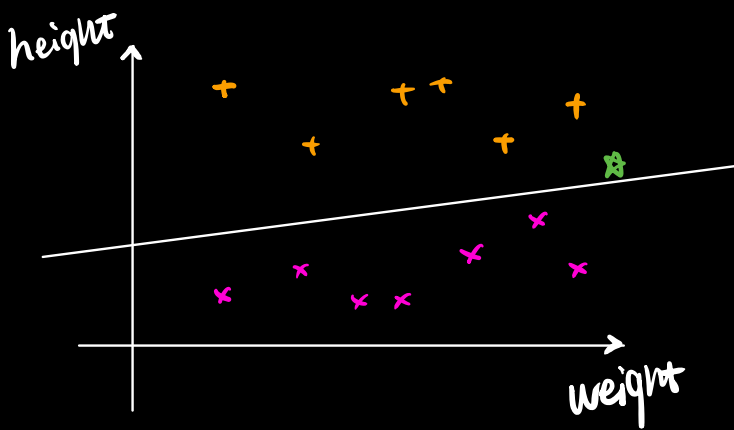
1. if $v_{i-1} \in h_i$, then $v_i = v_{i-1}$. // O(1) check,
nothing to do

2. if $v_{i-1} \notin h_i$, then the solution (if exists) is on l_i .

run $v_i = \text{2DLP}(h_i \cap l_i, h_2 \cap l_i, h_3 \cap l_i, \dots, h_{i-1} \cap l_i)$.

Terminates if there is no solution (that is, $C_i = \emptyset$)

3 special planes $(\frac{3}{2})$



minimize linear
mis-classify

Vector machine

Fitting a line into a set of points.

Input $P \in P_1 \dots P_n$

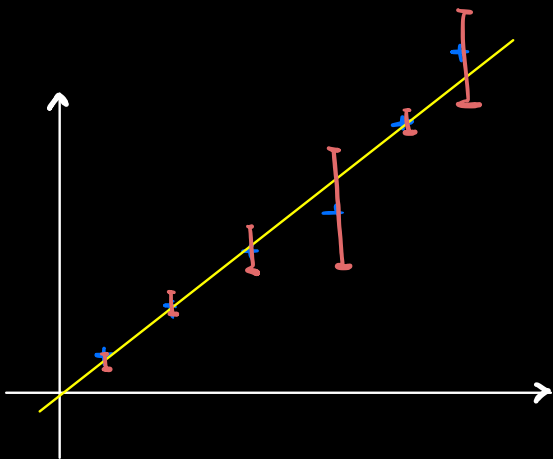
error term 0.1 tolerance

$P_i = (x_i, y_i)$ given

Assume $d = ax + b$

$y_i = ax_i + b$ -- go through

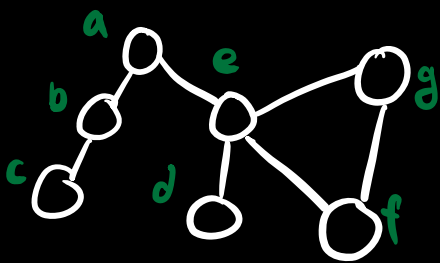
$$\begin{cases} y_i \leq ax_i + b + 0.1 \\ y_i \geq ax_i + b - 0.1 \end{cases}$$



Output a line that is vertically
at most 0.1 away from
each p_i

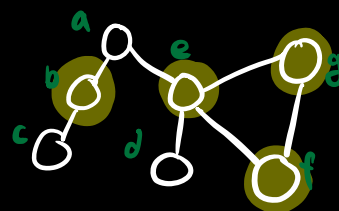
ILP min cardinality

Vertex Cover



find a subset of vertices $U \subseteq V$ that "stab" every edge of E

That is every edge $(u, v) \in E$ has at least one of its endpoint in U



$U = \{b, e, f, g\}$

$U' = \{b, e, g\}$ optimal

For every subset $U \subseteq V$, check if U hits every edge

Exhaustivity

穷举

$U = \{v_0, v_1, \dots, v_n\}$

let $x_i = 1$ if $v_i \in U$
 $= 0$ else

$\Rightarrow (x_0, x_1, x_2, \dots, x_n)$
 $\quad \quad \quad 0 \quad \quad 1 \quad \quad 1 \quad \quad 0$

combination of binary bits

2^n

Phrase the problem as an ILP

① Phrase as LP

② Constrain that each var is an integer.

Answer x_1, x_2, \dots, x_n ($x_i \geq 0, \forall i$)

if $x_i = 0$ then $v_i \notin U$

if $x_i = 1$ then $v_i \in U$

for every edge $(v_i, v_j) \in E$

