Linear Transform preserves vector addition & scalar multiplication

$$f(x) + f(y) = f(x+y)$$
$$k f(x) = f(kx)$$

△ $f(x) = x + (a, b, c)$ is not linear, as

$$f(x+y) = x + y + 2(a, b, c)$$

Affine Transform performs a scalar transform & translation, by a 4x4 matrix.

Homogeneous Notation direction $v = (v_x, v_y, v_z, 0)$ point $= (v_x, v_y, v_z, 1)$

Orthogonal Matrix inverse = transpose

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} x_0 & x_1 \\ y_0 & y_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$ interchanging rows and columns

Notation	Name	Characteristics
$\mathbf{T}(\mathbf{t})$	translation matrix	Moves a point. Affine.
$\mathbf{R}_x(\rho)$	rotation matrix	Rotates ρ radians around the x-axis. Similar notation for the y- and z-axes. Orthogonal & affine.
$\mathbf{R}$	rotation matrix	Any rotation matrix. Orthogonal & affine.
$\mathbf{S}(\mathbf{s})$	scaling matrix	Scales along all x-, y-, and z-axes according to $\mathbf{s}$. Affine.
$\mathbf{H}_{ij}(s)$	shear matrix	Shears component i by a factor s, with respect to component j. $i, j \in \{x, y, z\}$. Affine.
$\mathbf{E}(h, p, r)$	Euler transform	Orientation matrix given by the Euler angles head (yaw), pitch, roll. Orthogonal & affine.
$\mathbf{P}_o(s)$	orthographic projection	Parallel projects onto some plane or to a volume. Affine.
$\mathbf{P}_p(s)$	perspective projection	Projects with perspective onto a plane or to a volume.
$\mathrm{slerp}(\hat{\mathbf{q}}, \hat{\mathbf{r}}, t)$	slerp transform	Creates an interpolated quaternion with respect to the quaternions $\hat{\mathbf{q}}$ and $\hat{\mathbf{r}}$, and the parameter t.

Rotation Matrix $X = T(p) R_z(\phi) T(-p)$

Scaling Matrix

$$\begin{bmatrix} S_x & 0 & 0 & 0 \\ 0 & S_y & 0 & 0 \\ 0 & 0 & S_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$S_x = S_y = S_z$ uniform / isotropic

$S_x \neq S_y \neq S_z$ nonuniform anisotropic

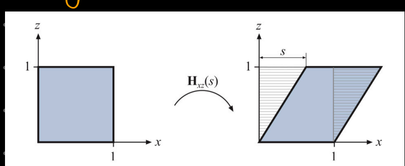
can also be done by manipulating this

0 Homogenization is needed

scaling on any axis $F = \begin{pmatrix} f_x, & f_y, & f_z, & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$$X = F S(s) F^T$$

Shearing Matrix



$$H_{xz}(S) = \begin{array}{c} \text{moving } x \text{ index} \quad \langle 0, 2 \rangle \quad \text{base } z \text{ index} \\ \begin{bmatrix} 1 & 0 & s & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Matrix Concatenation TRS

Rigidbody Transform consists of only translation & rotation

$$X = T(t)R = \begin{bmatrix} r_{00} & r_{01} & r_{02} & t_x \\ r_{10} & r_{11} & r_{12} & t_y \\ r_{20} & r_{12} & r_{13} & t_z \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

inverse of X

$$X^{-1} = (T(t)R)^{-1} = R^{-1}T(t)^{-1}$$
$$= R^T T(-t)$$

△ **Orienting the camera**

$$v = \frac{(c - l)}{\| c - l \|} \quad r = \frac{-(v \times u')}{\| v \times u' \|} \quad u = v \times r$$

$$\mathbf{M} = \underbrace{\begin{pmatrix} r_x & r_y & r_z & 0 \\ u_x & u_y & u_z & 0 \\ v_x & v_y & v_z & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{change of basis}} \underbrace{\begin{pmatrix} 1 & 0 & 0 & -t_x \\ 0 & 1 & 0 & -t_y \\ 0 & 0 & 1 & -t_z \\ 0 & 0 & 0 & 1 \end{pmatrix}}_{\text{translation}} = \begin{pmatrix} r_x & r_y & r_z & -\mathbf{t} \cdot \mathbf{r} \\ u_x & u_y & u_z & -\mathbf{t} \cdot \mathbf{u} \\ v_x & v_y & v_z & -\mathbf{t} \cdot \mathbf{v} \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$r \longrightarrow \langle 1, 0, 0 \rangle$

$u \longrightarrow \langle 0, 1, 0 \rangle$

$v \longrightarrow \langle 0, 0, 1 \rangle$

align to

Normal Transformation

surface normal cannot use the same matrix as points. lines. ...

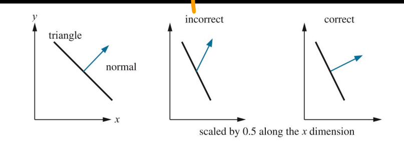


Figure 4.6. On the left is the original geometry, a triangle and its normal shown from the side. The middle illustration shows what happens if the model is scaled along the x- axis by 0.5 and the normal uses the same matrix. The right figure shows the proper transform of the normal.