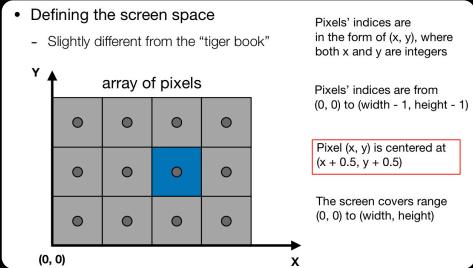


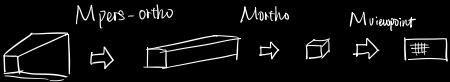
Orthographic  
Perspective  $\Rightarrow$  Canonical cube

? 多边形转化为屏幕上的像素值

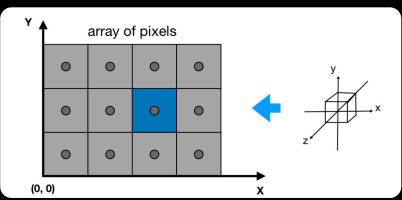
- What is a screen?
  - An array of pixels
  - Size of the array: resolution
  - A typical kind of raster display
- Raster == screen in German
  - Rasterize == drawing onto the screen
- Pixel (FYI, short for "picture element")
  - For now: A pixel is a little square with uniform color
  - Color is a mixture of (red, green, blue)



## Defining Screen Space



## Canonical Cube to Screen

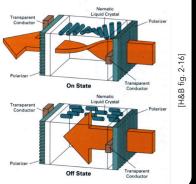


- Irrelevant to  $z$
  - Transform in  $xy$  plane:  $[x, y, z]$  to  $[x, width \cdot x, height \cdot y]$
  - Viewport transform matrix
- $$M_{\text{viewport}} = \begin{pmatrix} \frac{\text{width}}{2} & 0 & 0 \\ 0 & \frac{\text{height}}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
- 

## LCD

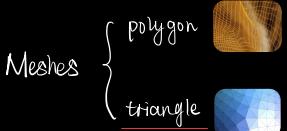
### LCD (Liquid Crystal Display) Pixel

Principle: block or transmit light by twisting polarization  
Illumination from backlight (e.g. fluorescent or LED)  
Intermediate intensity levels by partial twist



## Rasterization

Drawing to raster display

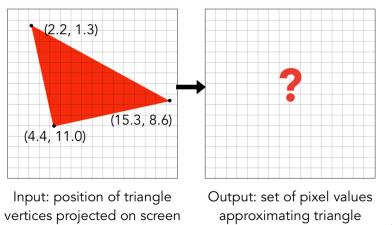


Why triangles?

- Most basic polygon  $\rightarrow$  can break up other shapes
- Unique properties
  - $\rightarrow$  guaranteed to be planar
  - $\rightarrow$  well-defined interior
  - $\rightarrow$  well-defined method for interpolating values at vertices over triangle barycentric interpolation

## Triangle

### What Pixel Values Approximate a Triangle?



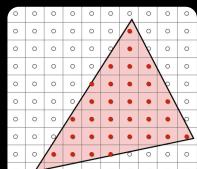
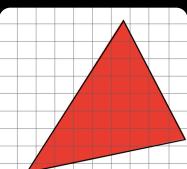
## Sampling

a simple approach 利用像素中心对屏幕空间采样

Evaluating a function at a point is sampling.  
We can discretize a function by sampling.

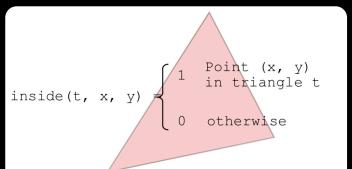
```
for (int x = 0; x < xmax; ++x)
    output[x] = f(x);
```

Sampling is a core idea in graphics.  
We sample time (1D), area (2D), direction (2D), volume (3D) ...

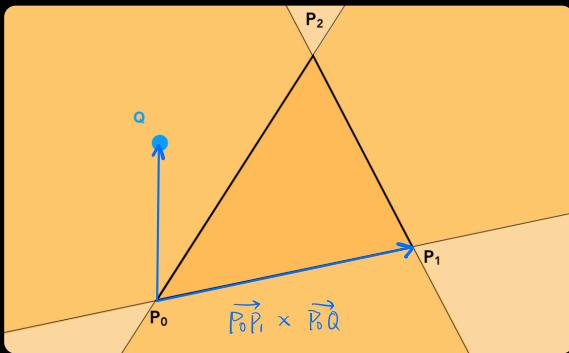


交叉采样

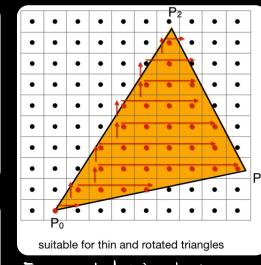
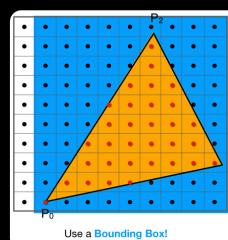
判断像素中心  
是否在图形内



```
for (int x = 0; x < xmax; ++x)
    for (int y = 0; y < ymax; ++y)
        image[x][y] = inside(tri,
            x + 0.5,
            y + 0.5);
```

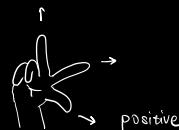


本课选择忽略  
Edge Cases (Literally)  
Is this sample point covered by triangle 1, triangle 2, or both?

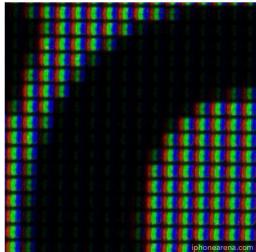


suitable for thin and rotated triangles  
Incremental triangle traversal

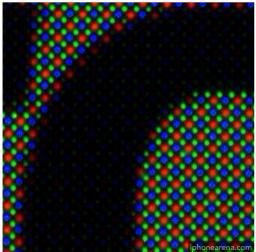
Axis Aligned Bounding Box  
AABB



## Real LCD Screen Pixels (Closeup)



iPhone 6S



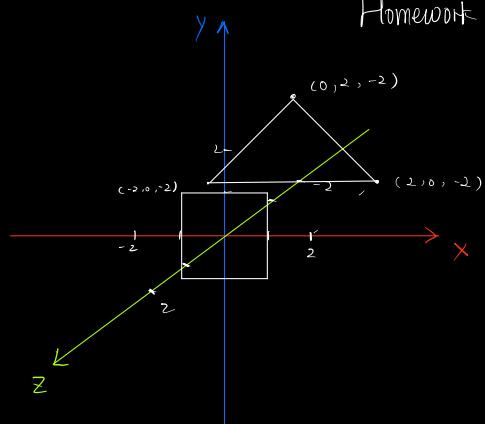
Galaxy S5

Notice R,G,B pixel geometry! But in this class, we will assume a colored square full-color pixel.

绿色密度最大 → 绿色过于扎眼 导致抑制

RGBG

Homework 1 Note



## Antialiasing

抗锯齿 (反走样)

Sampling is ubiquitous in computer graphics

Sampling artifact < error / mistakes / inaccuracies >



Artifacts due to sampling - "Aliasing"

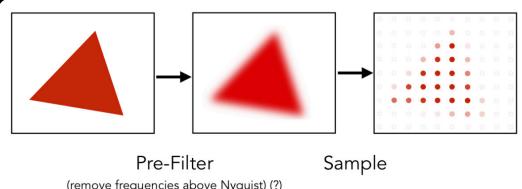
- Jaggies – sampling in space
- Moiré – undersampling images
- Wagon wheel effect – sampling in time
- [Many more] ...

Behind the Aliasing Artifacts

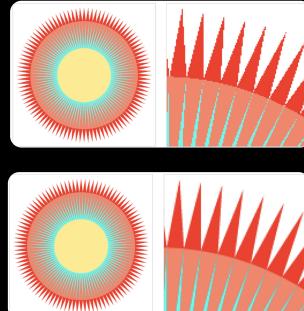
- Signals are changing too fast (high frequency), but sampled too slowly



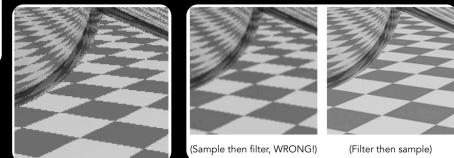
Antialiasing idea: blurring < pre-filtering > before sampling



Note antialiased edges in rasterized triangle where pixel values take intermediate values



BLURRED ALIASING



# Fourier Transform

傅里叶级数展开与傅里叶变换

频率与频域

$$f = \frac{1}{T} \int_{-\infty}^{\infty} f(x) e^{-j\omega x} d\omega$$

周期 =  $\frac{1}{\Delta \omega}$

## Fourier Transform

Represent a function as a weighted sum of sines and cosines



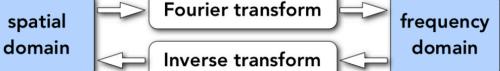
Jean-Baptiste Joseph Fourier 1768 - 1830

$$f(x) = \frac{A}{2} + \frac{2A \cos(t\omega)}{\pi} + \frac{2A \cos(3t\omega)}{3\pi} + \frac{2A \cos(5t\omega)}{5\pi} - \frac{2A \cos(7t\omega)}{7\pi} + \dots$$

频率递增 ↗

## Fourier Transform Decomposes A Signal Into Frequencies

$$f(x) \quad F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i \omega x} dx \quad F(\omega)$$

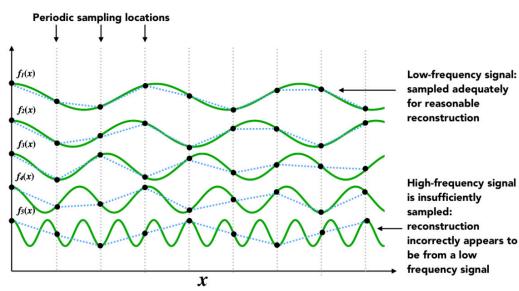


$$f(x) = \int_{-\infty}^{\infty} F(\omega) e^{2\pi i \omega x} d\omega$$

$$\text{Recall } e^{ix} = \cos x + i \sin x$$

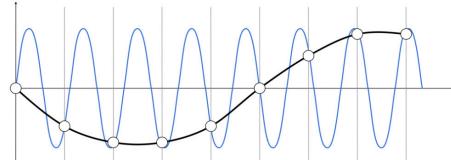
通过存储频率之后复原

## Higher Frequencies Need Faster Sampling



高频率函数难以精确采样 → 类似于走样

## Undersampling Creates Frequency Aliases



High-frequency signal is insufficiently sampled: samples erroneously appear to be from a low-frequency signal

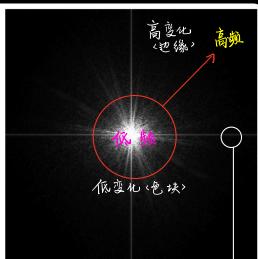
Two frequencies that are indistinguishable at a given sampling rate are called "aliases"

Filtering = Getting rid of certain frequency contents

高保真信息量



时 域



⇒

频 率  
图像边界之间完全无相移  
因而产生副边瓣

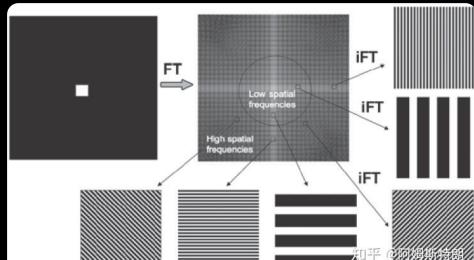
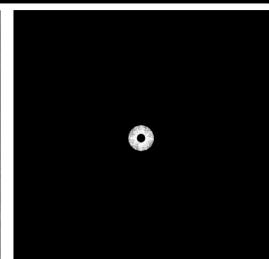


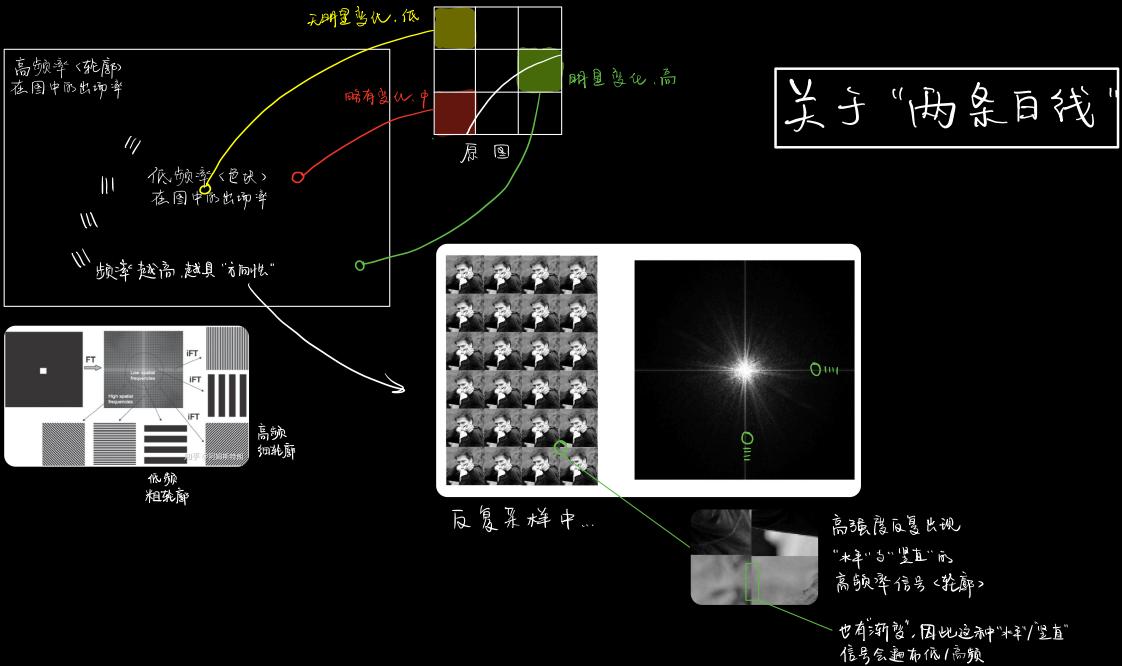
突出附近变化的  
高频 → 边缘  
滤除低频背景

High Pass Filter

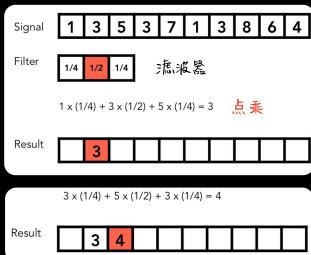


Low Pass Filter < blur >





## Convolution 卷积



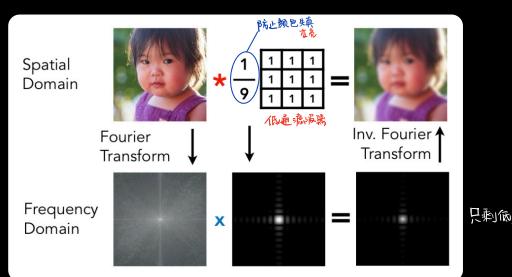
Convolution in the spatial domain is equal to multiplication in the frequency domain, and vice versa

Option 1:

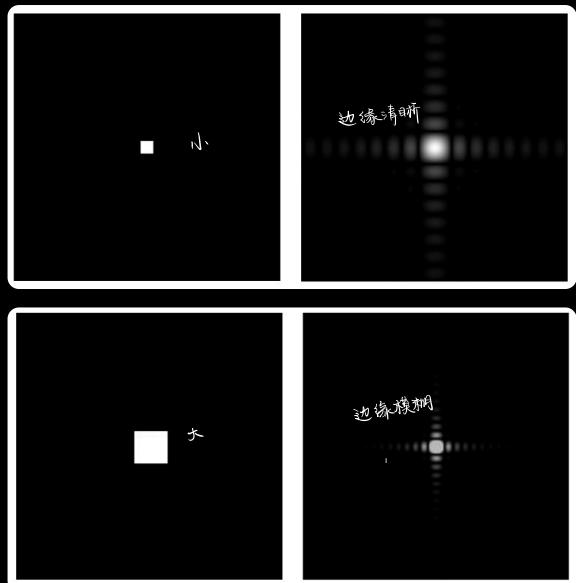
- Filter by convolution in the spatial domain

Option 2:

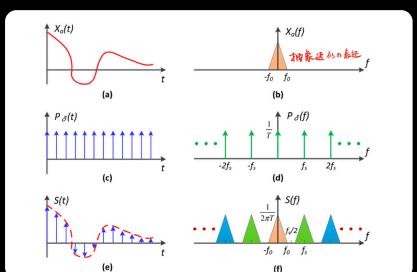
- Transform to frequency domain (Fourier transform)
- Multiply by Fourier transform of convolution kernel
- Transform back to spatial domain (inverse Fourier)



filtering = Convolution 卷积  
= Averaging <模糊> 平均

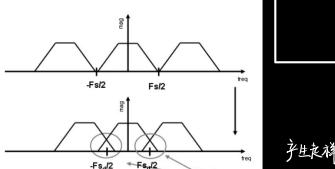


频域中的频率与时域中的间隔为倒数关系，因此时域中采样间隔过大，在频域中会体现为↓



Aliasing = Mixed Frequency Contents

Dense sampling:



信号 (a) 表示冲激函数  
得到采样 (每个点)

Sampling = Repeating Frequency Contents