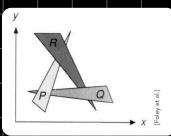


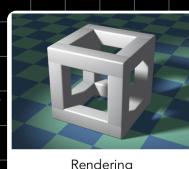
## Visibility/Occlusion <z-buffering>

Painter's Algorithm — paint from back to front, overwrite in the framebuffer

- requires sorting in depth  $< O(n \log n)$  for  $n$  triangles
- could have unresolvable depth order

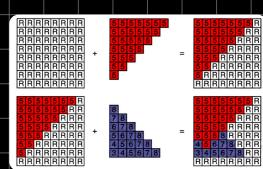


**Z-Buffer**  
This is the algorithm that eventually won.  
Idea:  
• Store current min. z-value for each sample (pixel)  
• Needs an additional buffer for depth values  
- frame buffer stores color values  
- depth buffer (z-buffer) stores depth  
  
IMPORTANT: For simplicity we suppose  
z is always positive  
(smaller z  $\rightarrow$  closer, larger z  $\rightarrow$  further)



```
Initialize depth buffer to ∞
During rasterization:
for (each triangle T)
    for (each (x,y,z) in T)
        if (z < zbuffer(x,y))
            framebuffer(x,y) = rgb; // update color
            zbuffer(x,y) = z; // update depth
        else;
            // do nothing, this sample is occluded
```

在每一个像素内记录最近深度



- $O(n)$  for  $n$  triangles  $< O(n \log n)$  assuming constant coverage> 并未执行排序
- Most important visibility algorithm — implemented in hardware for all GPUs
- 深度完全相等  $< float>$ , 几乎不会出现、在此不作讨论

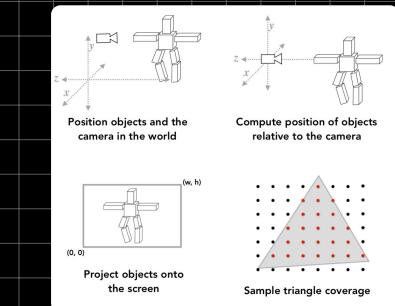
## Illumination, shading & graphic pipeline

In Merriam-Webster Dictionary  
shading, [fə'deɪn], noun  
The darkening or coloring of an illustration or diagram with parallel lines or a block of color.  
In this course  
The process of applying a material to an object.

▲ Shading is local - NO SHADOW is generated

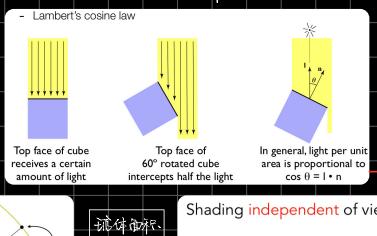


Compute light reflected toward camera at a specific shading point  
Inputs:  
• Viewer direction, v  
• Surface normal, n  
• Light direction, l  
• Surface parameters (color, roughness, etc.)



### Diffuse Reflection

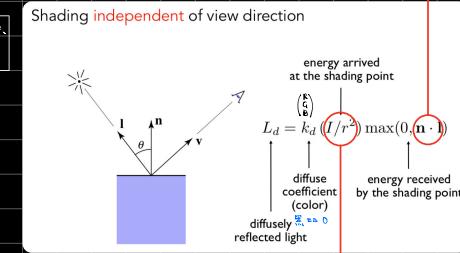
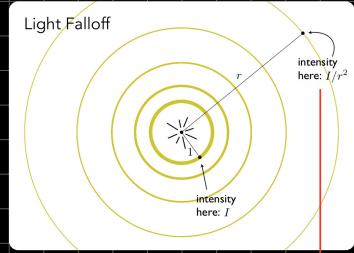
Light is scattered uniformly in all directions  
Surface color is the same for all viewing directions



$$\alpha \cdot C = I \cdot A \cdot \cos(\theta) / \pi \cdot A \cdot \cos(\theta)$$

is a constant

Diffusely Reflected Light Formula



漫反射  
与观看角度无关



### Specular Highlights

**Blinn-Phong**

$V$  close to mirror direction  $\Rightarrow$  half vector near normal

- Measure "near" by dot product of unit vectors

$$\begin{aligned} h &= \text{bisector}(v, l) \\ &= \frac{v + l}{\|v + l\|} \end{aligned}$$

$$L_s = k_s (I/r^2) \max(0, \cos \alpha)^p$$

$$L_s = k_s (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{h})^p$$

法线方向与半径向量相近

使用  $\cos \theta$  的高次项  
即彼相类达利得值  
化无明里差值

Increasing  $p$  narrows the reflection lobe

**Blinn-Phong**  $L_s = k_s (I/r^2) \max(0, \mathbf{n} \cdot \mathbf{h})^p$

$k_s$

Note: showing  $L_d + L_s$  together

### Ambient lighting 环境光 与光源、观测角度无关

Shading that does not depend on anything

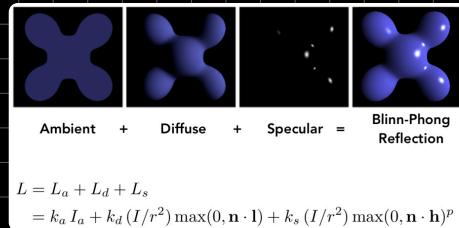
- Add constant color to account for disregarded illumination and fill in black shadows
- This is approximate / fake!

$L_a = k_a I_a$

ambient coefficient

reflected ambient light

### Blinn-Phong Reflection Model



### Shading Frequency 着色频率

Flat Shading | Gouraud Shading | Phong Shading

Flat shading

- Triangle face is flat one normal vector
- Not good for smooth surfaces

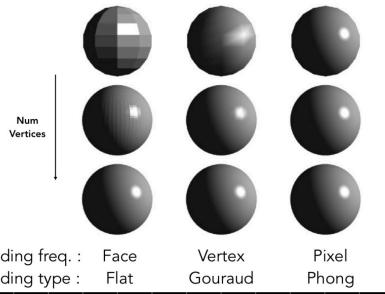
Gouraud shading

- Interpolate colors from vertices across triangle
- Each vertex has a normal vector (how?)

Phong shading

- Interpolate normal vectors across each triangle
- Compute shading using reflection
- Not the Blinn-Phong Reflectance Model

### Shading Frequency: Face, Vertex or Pixel



Defining Per-Vertex Normal Vectors

Best to get vertex normals from the underlying geometry

- e.g. consider a sphere

Otherwise have to infer vertex normals from triangle faces

- Simple when average surrounding face normals

$$N_v = \frac{\sum N_i}{\|\sum N_i\|}$$

Defining Per-Pixel Normal Vectors

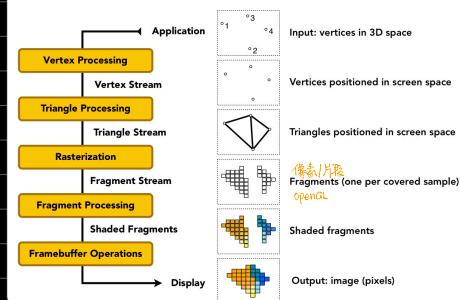
Barycentric interpolation (introducing soon)

of vertex normals



# Graphics < Real-time Rendering > Pipeline

## Graphics Pipeline



## Shader Programs

- Program vertex and fragment processing stages
- Describe operation on a single vertex (or fragment)

Example GLSL fragment shader program

```
uniform sampler2D myTexture; //纹理
uniform vec3 lightDir; //光向量
varying vec2 uv; //纹理坐标
varying vec3 norm; //法线向量

void diffuseShader()
{
    vec3 kd;
    kd = texture2D(myTexture, uv); //取纹理颜色
    kd *= clamp(dot(-lightDir, norm), 0.0, 1.0); //进行阴影
    gl_FragColor = vec4(kd, 1.0); //输出片段颜色
}
```

- Shader function executes once per fragment.
- Outputs color of surface at the current fragment's screen sample position.
- This shader performs a texture lookup to obtain the surface's material color at this point, then performs a diffuse lighting calculation.

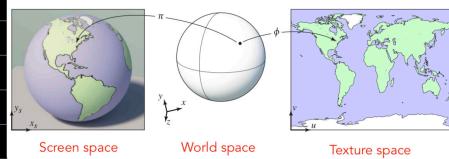
# Texture Mapping

How to define  $k_d$  for every pixel

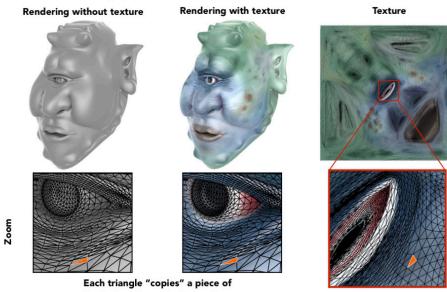


Surface lives in 3D world space

Every 3D surface point also has a place where it goes in the 2D image (**texture**).

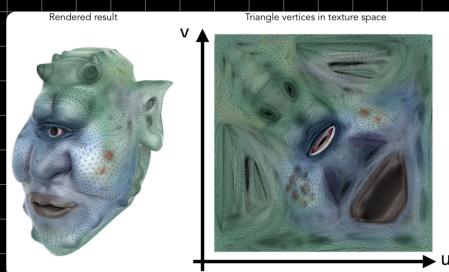
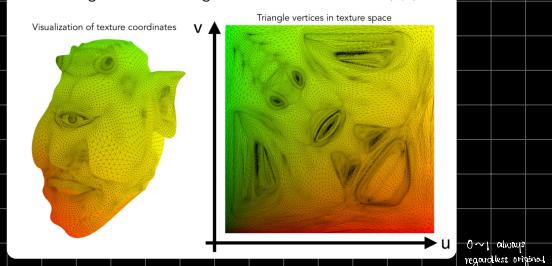


## Texture Applied to Surface



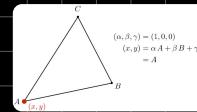
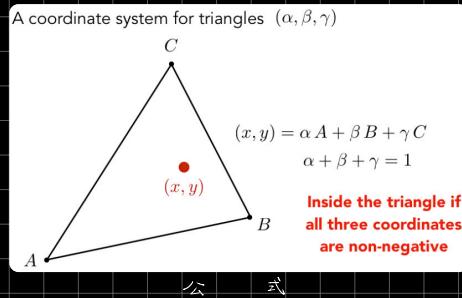
## Visualization of Texture Coordinates

Each triangle vertex is assigned a texture coordinate ( $u, v$ )



## Barycentric Coordinates

重心坐标 - 计算三角形内部的插值 <像素的法线>  
Interpolation across triangle - 一个三角形对应一个重心坐标



例：1点之重心坐标

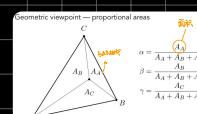
$$(x, y) = \alpha A + \beta B + \gamma C$$

$$\alpha + \beta + \gamma = 1$$

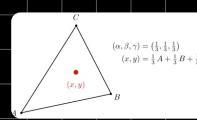
$$\alpha = \frac{-(x - x_B)(y_C - y_B) + (y - y_B)(x_C - x_B)}{-(x_A - x_B)(y_C - y_B) + (y_A - y_B)(x_C - x_B)}$$

$$\beta = \frac{-(x - x_C)(y_A - y_C) + (y - y_C)(x_A - x_C)}{-(x_B - x_C)(y_A - y_C) + (y_B - y_C)(x_A - x_C)}$$

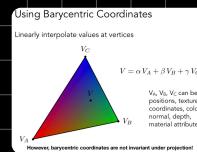
$$\gamma = 1 - \alpha - \beta$$



通过面积比例重心坐标



重心点之重心坐标



使用重心坐标计算属性

法线 色度 纹理 位置

## Simple Texture Mapping: Diffuse Color

Usually a pixel's center

for each rasterized screen sample  $(x, y)$ :

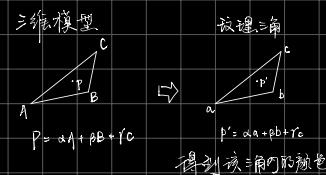
$(u, v) = \text{evaluate texture coordinate at } (x, y)$

`texcolor = texture.sample(u, v);`

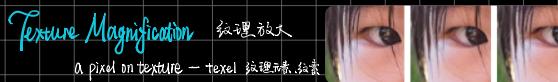
set sample's color to texcolor;

Usually the diffuse albedo  $K_d$   
(recall the Blinn-Phong reflectance model)

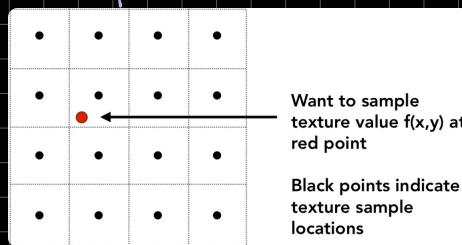
Using barycentric coordinates!



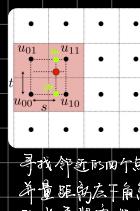
该三角形的颜色



## Bilinear Interpolation 双线性插值



采样点非整数



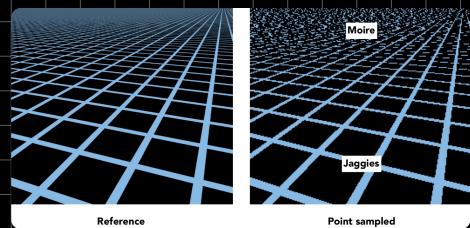
Linear interpolation (1D)  
 $\text{lerp}(x, v_0, v_1) = v_0 + x(v_1 - v_0)$

Two helper lerps  
 $u_0 = \text{lerp}(s, u_{00}, u_{10})$   
 $u_1 = \text{lerp}(s, u_{01}, u_{11})$

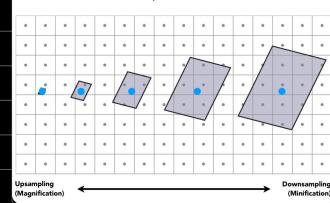
Final vertical lerp, to get result:  
 $f(x, y) = \text{lerp}(t, u_0, u_1)$

## Bicubic Interpolation 双三次插值 - 取周围16块计算

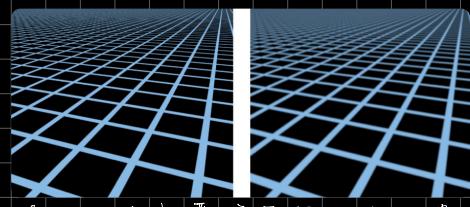
纹理过大



Screen Pixel "Footprint" in Texture



单个像素覆盖的纹理区域不连续变大



使用双倍超采样可解决但太贵

Mipmap allowing <fast, approx, square> range queries 只能做近似及方形

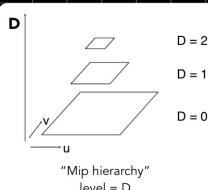
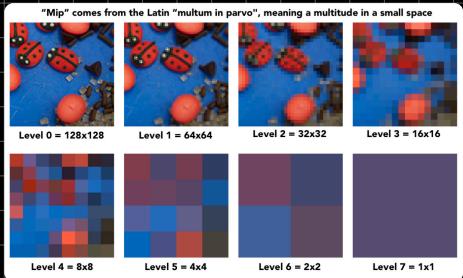
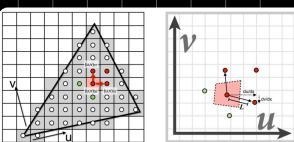
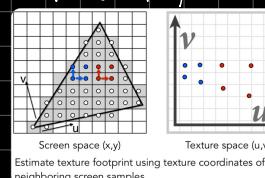


image pyramid

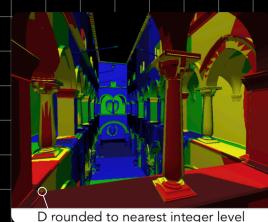


Computing Mipmap Level D

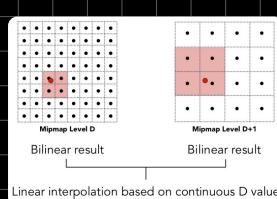


通过映射邻近点到 u-v 因此进行插值计算主要的区域大小

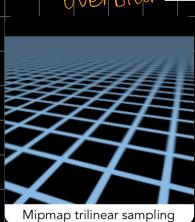
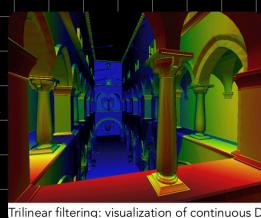
Mipmap limitation  
overblur



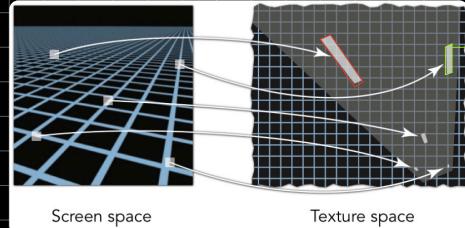
但是不连续  
会有断裂感



Bilinear interpolation



# Anisotropic Filtering



Irregular footprint in Texture

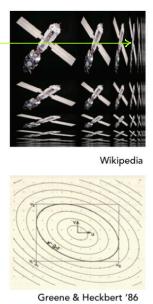
## Anisotropic Filtering

Ripmaps and summed area tables

- Can look up axis-aligned rectangular zones
- Diagonal footprints still a problem

## EWA filtering

- Use multiple lookups
- Weighted average
- Mipmap hierarchy still helps
- Can handle irregular footprints



Ripmap  
空间开销为恒定的<黑>

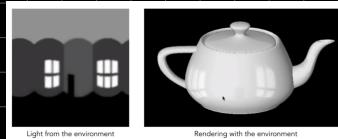


消耗计算

Texture = Memory + Range Query < filtering >

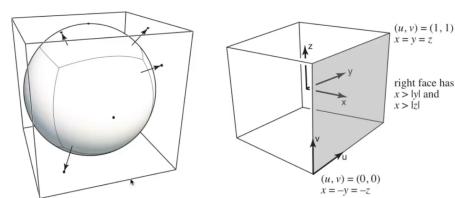
Environment Map 记录各方向的光照  
并形成纹理

环境光照/贴图



## Spherical Environment Map

Cube Map 用六张图片表示环境光



A vector maps to cube point along that direction.  
The cube is textured with 6 square texture maps.

减少扭曲



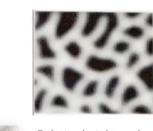
减少计算在哪个面上

## 凹凸贴图 法线贴图

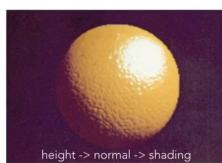
### Bump / Normal mapping

Textures doesn't have to only represent colors

- What if it stores the height / normal?
- Bump / normal mapping
- Fake the detailed geometry



Relative height to the underlying surface

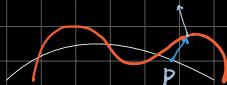


height -> normal -> shading

假的凹凸贴图

高低 → 法线 → 着色

增加纹理  
但不改变模型三角形



## How to perturb the normal <flatland>



Original surface normal  $n(p) = (0, 0, 1)$

Derivative at  $p$  is  $dp = c * [h(p+1) - h(p)]$

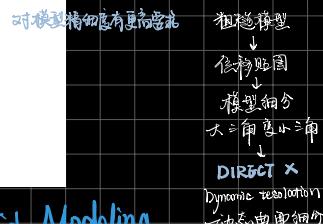
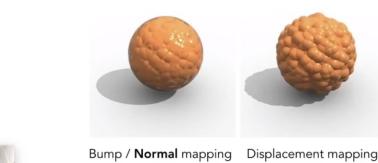
Perturbed normal is then  $n(p) = (1 - dp \cdot I) \cdot \text{normalized}$

### How to perturb the normal (in 3D)

- Original surface normal  $n(p) = (0, 0, 1)$
- Derivatives at  $p$  are
  - $dp/du = c1 * [h(u+1) - h(u)]$
  - $dp/dv = c2 * [h(v+1) - h(v)]$
- Perturbed normal is  $n = (-dp/du, -dp/dv, 1). \text{normalized}$
- Note that this is in **local coordinate!** 寄生概念起搏

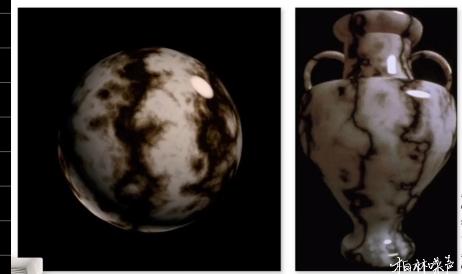
## 位移贴图 Displacement mapping 位移贴图顶点

- Displacement mapping** — a more advanced approach
  - Uses the same texture as in bumping mapping
  - Actually **moves the vertices**



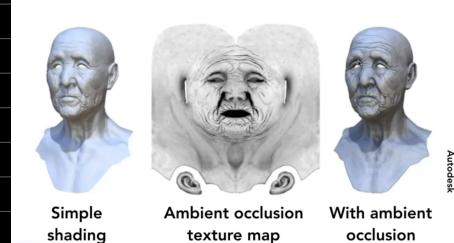
## 3D Procedural Noise + Solid Modeling

过程纹理



## 环境光遮蔽 Ambient Occlusion

### Provide Precomputed Shading



## 3D Textures and Volume Rendering

### 3D Textures and Volume Rendering

