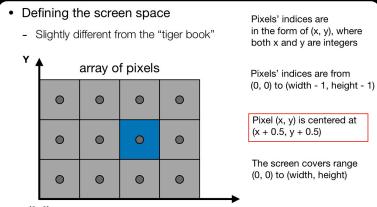


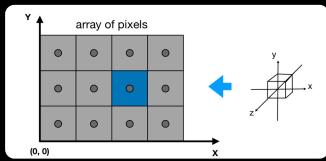
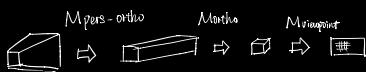
Orthographic Perspective \Rightarrow Canonical cube \Rightarrow ? 多边形转化成屏幕上的像素值。

- What is a screen?
 - An array of pixels
 - Size of the array: resolution
 - A typical kind of raster display
- Raster == screen in German
 - Rasterize == drawing onto the screen
- Pixel (FYI, short for "picture element")
 - For now: A pixel is a little square with uniform color
 - Color is a mixture of (red, green, blue)



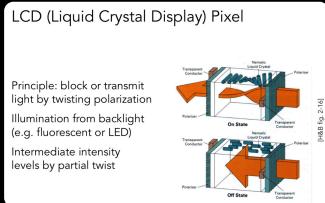
Defining Screen Space

Canonical Cube to Screen



- Irrelevant to z
 - Transform in xy plane: $[x, y, 1]^T \Rightarrow [x, \text{width}, \text{height}]^T$
 - Viewport transform matrix
- $$M_{\text{viewport}} = \begin{pmatrix} \text{width} & 0 & 0 \\ 0 & \text{height} & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad \begin{matrix} \xrightarrow{\text{width}} \\ \xrightarrow{\text{height}} \end{matrix}$$

LCD



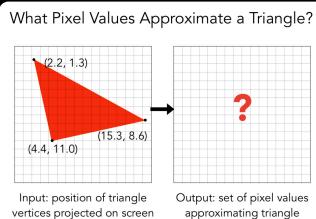
Rasterization

Drawing to raster display

Meshes $\left\{ \begin{array}{l} \text{polygon } \text{[image]} \\ \text{triangle } \text{[image]} \end{array} \right.$ Why triangles?

- Most basic polygon \rightarrow can break up other shapes
- Unique properties \rightarrow guaranteed to be planar
 \rightarrow well-defined interior
 \rightarrow well-defined method for interpolating values of vertices over triangle
barycentric interpolation

Triangle



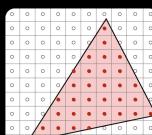
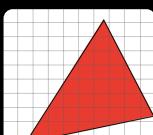
Sampling

a simple approach

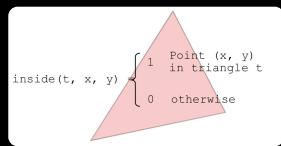
Evaluating a function at a point is sampling.
We can **discretize** a function by sampling.

```
for (int x = 0; x < xmax; ++x)
    output[x] = f(x);
```

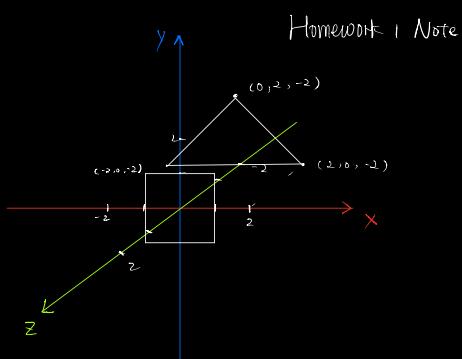
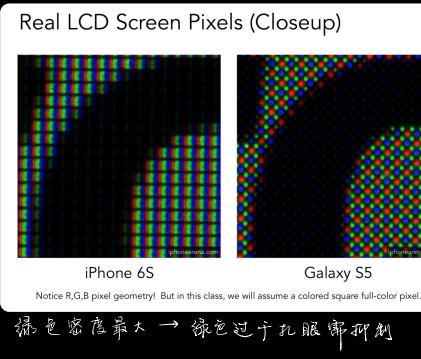
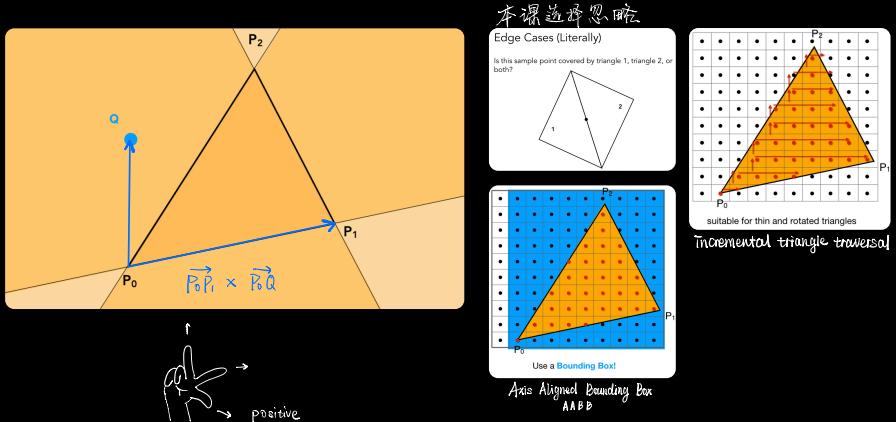
Sampling is a core idea in graphics.
We sample time (1D), area (2D), direction (2D), volume (3D) ...



采样函数
判断像素中心是否在图形内



```
for (int x = 0; x < xmax; ++x)
    for (int y = 0; y < ymax; ++y)
        image[x][y] = inside(tri,
            x + 0.5,
            y + 0.5);
```



Anti-aliasing

Sampling is ubiquitous in computer graphics
Sampling artifact < error / mistakes / inaccuracies >



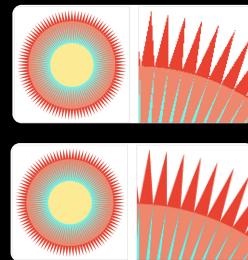
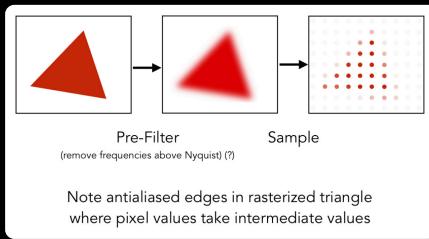
Artifacts due to sampling - "Aliasing"

- Jagged – sampling in space
- Moire – undersampling images
- Wagon wheel effect – sampling in time
- [Many more] ...

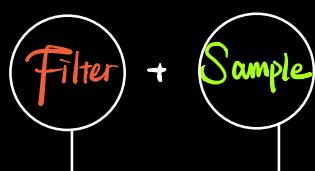
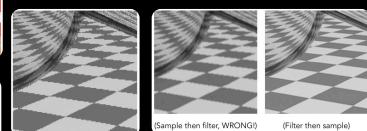
Behind the Aliasing Artifacts

- Signals are **changing too fast** (high frequency), but sampled **too slowly**

Anti-aliasing idea: **blurring** < pre-filtering > before sampling

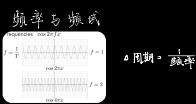


BLURRED ALIASING



Fourier Transform

傅里叶级数展开与傅里叶变换



Fourier Transform

Represent a function as a weighted sum of sines and cosines

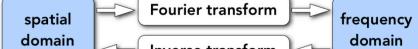


$$f(x) = \frac{A}{2} + \frac{2A \cos(\omega_1 x)}{\pi} + \frac{2A \cos(3\omega_1 x)}{3\pi} + \frac{2A \cos(5\omega_1 x)}{5\pi} + \frac{2A \cos(7\omega_1 x)}{7\pi} + \dots$$

频率逆推

Fourier Transform Decomposes A Signal Into Frequencies

$$f(x) \quad F(\omega) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i \omega x} dx \quad F(\omega)$$

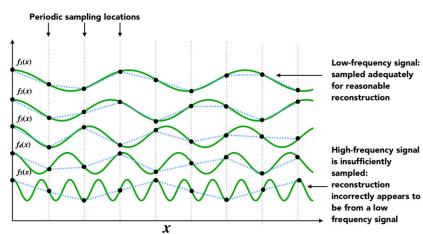


$$f(x) = \int_{-\infty}^{\infty} F(\omega) e^{2\pi i \omega x} d\omega$$

Recall $e^{ix} = \cos x + i \sin x$

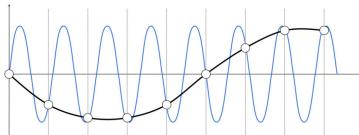
通过存储频率的后援原

Higher Frequencies Need Faster Sampling



高频率函数难以精确采样 → 类似于丢样

Undersampling Creates Frequency Aliases



High-frequency signal is insufficiently sampled: samples erroneously appear to be from a low-frequency signal
Two frequencies that are indistinguishable at a given sampling rate are called "aliases"

Filtering = Getting rid of certain frequency contents

高倍数放大量



时 域

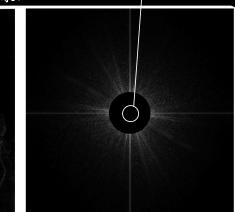


频 率 图 像 边 界 之 间 有 互 相 补 补 频 率

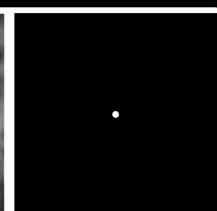
滤波低频源

突出面部变化的高频边缘

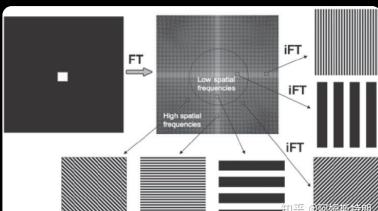
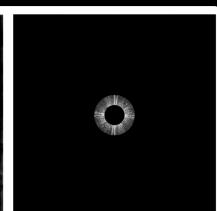
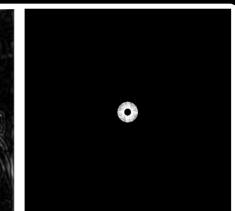
High Pass Filter

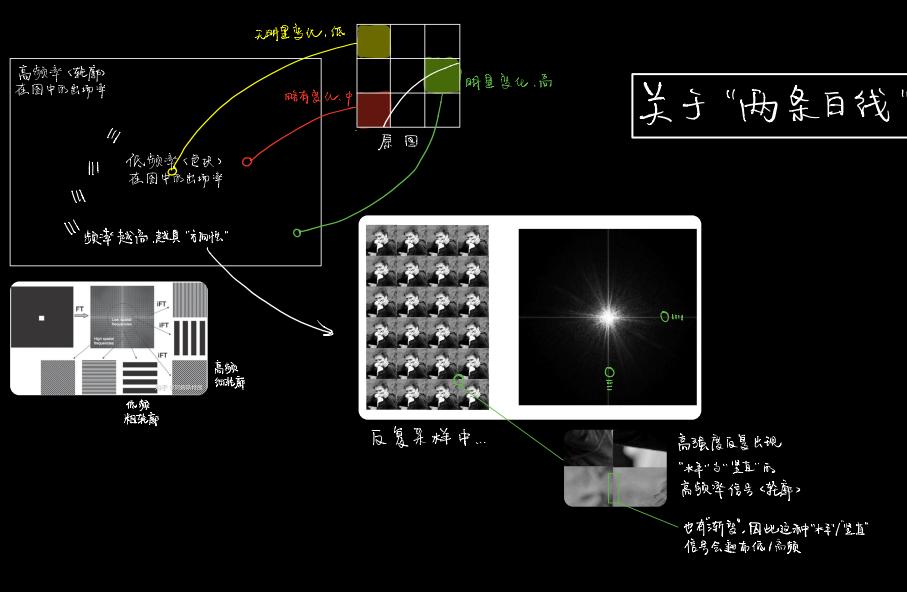


Low Pass Filter < blur >

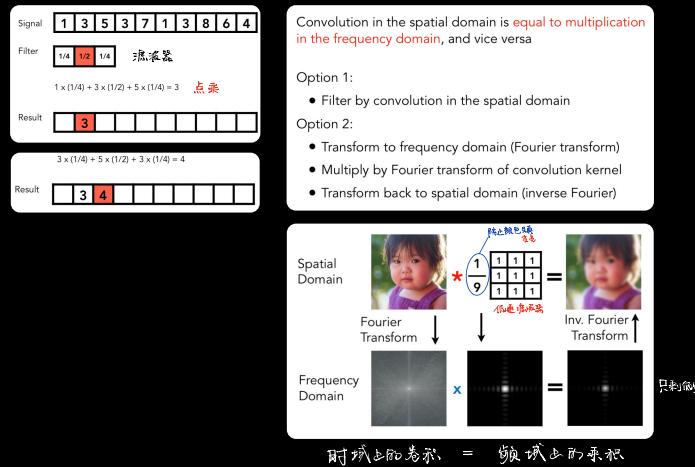


High Pass Filter

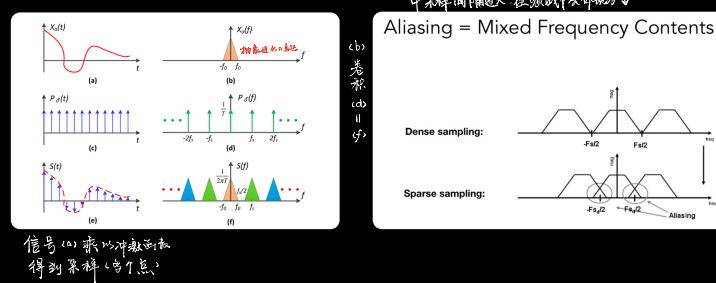
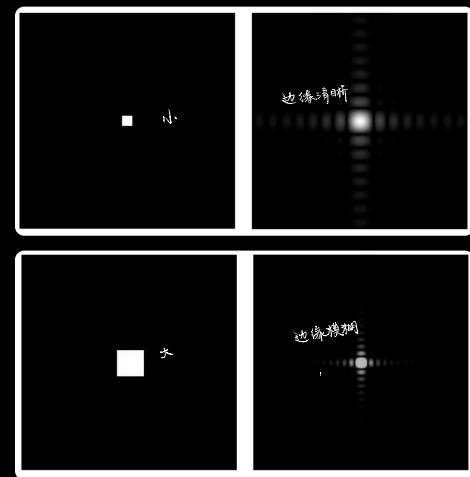




Convolution 卷积



filtering = Convolution 卷积
= Averaging 模糊 平均



Sampling = Repeating Frequency Contents

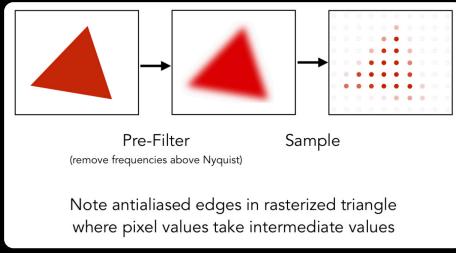
How can we reduce aliasing error

Option 1: Increase sampling rate

- Essentially increasing the distance between replicas in Fourier domain
- Higher resolution displays, sensors, framebuffers...
- But: costly & may need very high resolution

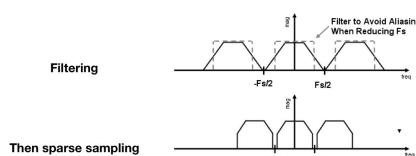
Option 2: Antialiasing

- Making Fourier contents "narrower" before repeating
- i.e. **Filtering out high frequencies before sampling**

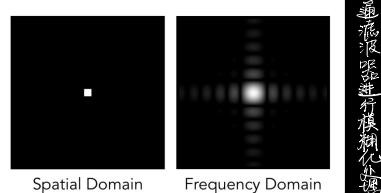


反走样 先模糊 后采样

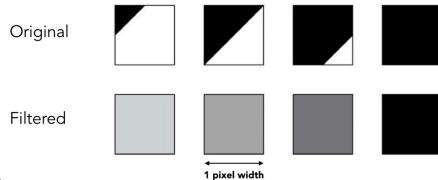
Antialiasing = Limiting, then repeating



A 1 pixel-width box filter (low pass, blurring)



In rasterizing one triangle, the average value inside a pixel area of $f(x,y) = \text{inside}(\text{triangle},x,y)$ is equal to the area of the pixel covered by the triangle.



Averaging values in pixel area

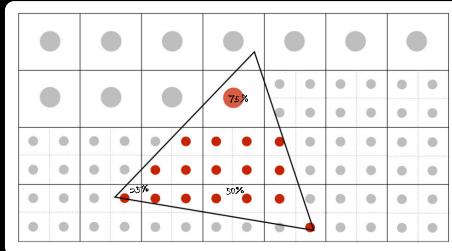
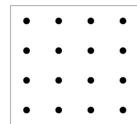
Solution:

- **Convolve** $f(x,y)$ by a 1-pixel box-blur
 - Recall: convolving = filtering = averaging
- **Then sample** at every pixel's center

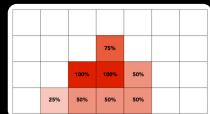
Multisample Antialiasing MSAA

Approximate the effect of the 1-pixel box filter by sampling multiple locations within a pixel and averaging their values:

消除单个像素锯齿
分别采样
并计算此像素值。



- ① 分割
- ② 计算
- ③ 输出<百分比>



Fast Approximate AA FXAA

图像后期处理 → 处理已有锯齿的图

△ 寻找有锯齿的边界并计算

Temporal AA TAA

与时间相关 / temporal /

处理静态时，观测同一像素不同时间的值
并将稳定性延续至下一帧

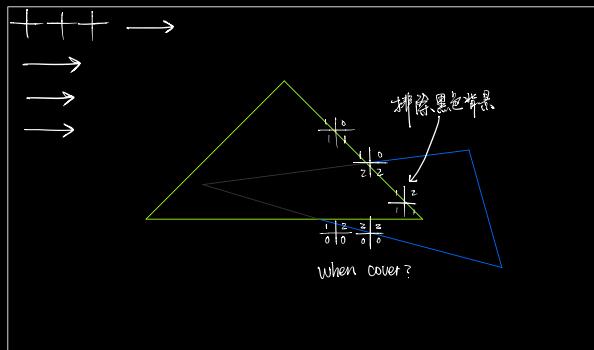
Super resolution / sampling 超分辨率

from low resolution to high resolution 小图放大

Still "not enough sampling" problem

DLSS Deep learning super-sampling 猜测 → 补细节

HW2 : Super-sampling

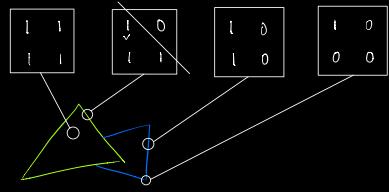


Z-buffering

```

if in-side triangle
if (buffering = -1 (background))
    update & discard blackbk
else
    mix two color & update
end.

```



if all-in triangle
cover

if mixed with black || mixed with
other color
 Z-buffer higher
 replace former color
 else
 mix with other color

