Appendix

A. Proof of Lemma 1

In the proof, we first consider the case over a total of T iterations, and then divide the DP constraint equally among each iteration. The proof is similar to that of [1]. We denote $\mathbf{y}_i = [\mathbf{y}_i^1, \cdots, \mathbf{y}_i^T]$ as the T successive received signals from IoMT device i, and $m_i^t = \sqrt{(h_i\sqrt{\tau_i p_i^t})^2 + N_0}$ is the standard deviation of the effective noise in $\mathbf{y}_i^t$. According to the definition of DP loss, for the i-th device, the privacy loss after T iterations can be represented as

$$
\begin{aligned}
\mathcal{L}_{\mathcal{D},\mathcal{D}'}(\mathbf{y}_i) &= \ln\left(\prod_{t=1}^{T} \frac{P\Big[\mathbf{y}_i^t \big| \mathbf{y}_i^{t-1}, \cdots, \mathbf{y}_i^1, \mathcal{D}_i\Big]}{P\Big[\mathbf{y}_i^t \big| \mathbf{y}_i^{t-1}, \cdots, \mathbf{y}_i^1, \mathcal{D}_i'\Big]}\right) \\
&= \sum_{t=1}^{T} \ln\left(\frac{P\Big[\mathbf{y}_i^t \big| \mathbf{y}_i^{t-1}, \cdots, \mathbf{y}_i^1, \mathcal{D}_i\Big]}{P\Big[\mathbf{y}_i^t \big| \mathbf{y}_i^{t-1}, \cdots, \mathbf{y}_i^1, \mathcal{D}_i'\Big]}\right) \\
&= \sum_{t=1}^{T} \ln\left(\frac{\exp\left(-\frac{\|\mathbf{y}_i^t - h_i^t \frac{\sqrt{\theta_i p_i^t}}{\Lambda} \nabla F_k(\mathbf{w}^t;\mathcal{D}_i)\|^2}{2(m_i^t)^2}\right)}{\exp\left(-\frac{\|\mathbf{y}_i^t - h_i^t \frac{\sqrt{\theta_i p_i^t}}{\Lambda} \nabla F_k(\mathbf{w}^t;\mathcal{D}_i')\|^2}{2(m_i^t)^2}\right)}\right) \\
&= \sum_{t=1}^{T} \ln\left(\frac{\exp\left(-\frac{\|\mathbf{r}_i^t\|^2}{2(m_i^t)^2}\right)}{\exp\left(-\frac{\|\mathbf{r}_i^t + \mathbf{v}_i^t\|^2}{2(m_i^t)^2}\right)}\right),
\end{aligned}
$$

where $\mathbf{r}_i^t \sim \mathcal{N}(0, (m_i^t)^2\mathbf{I})$ represents the effective noise, and we set

$$
\begin{aligned}
\mathbf{v}_i^t = h_i^t \frac{\sqrt{\theta_i p_i^t}}{\Lambda}\Big[&\sum_{(\mathbf{u},v)\in\mathcal{D}_i} \nabla f\left(\mathbf{w}^t; \mathbf{u}, v\right) \\
&- \sum_{(\mathbf{u},v)\in\mathcal{D}_i'} \nabla f\left(\mathbf{w}^t; \mathbf{u}, v\right)\Big],
\end{aligned}
$$

where $\|\mathbf{v}_i^t\| = \Delta_i^t$. Following the conclusion in [2], we can then bound privacy violation probability

$$
\begin{aligned}
&\Pr\left(\left|\sum_{t=1}^{T} \frac{2(\mathbf{r}_i^t)^\top \mathbf{v}_i^t + \|\mathbf{v}_i^t\|^2}{2(m_i^t)^2}\right| > \epsilon\right) \\
&\overset{(a)}{\leq} \Pr\left(\left|\sum_{t=1}^{T} \frac{(\mathbf{r}_i^t)^\top \mathbf{v}_i^t}{(m_i^t)^2}\right| > \epsilon - \sum_{t=1}^{T} \frac{\|\mathbf{v}_i^t\|^2}{2(m_i^t)^2}\right) \\
&= 2\Pr\left(\sum_{t=1}^{T} \frac{(\mathbf{r}_i^t)^\top \mathbf{v}_i^t}{(m_i^t)^2} > \epsilon - \sum_{t=1}^{T} \frac{\|\mathbf{v}_i^t\|^2}{2(m_i^t)^2}\right) \\
&\overset{(b)}{\leq} 2\frac{\sqrt{\sum_{t=1}^{T}\left(\frac{\Delta_i^t}{m_i^t}\right)^2}}{\sqrt{2\pi}\left[\epsilon - \sum_{t=1}^{T}\frac{1}{2}\left(\frac{\Delta_i^t}{m_i^t}\right)^2\right]} \\
&\quad \times \exp\left(-\frac{\left[\epsilon - \sum_{t=1}^{T}\frac{1}{2}\left(\frac{\Delta_i^t}{m_i^t}\right)^2\right]^2}{2\sum_{t=1}^{T}\left(\frac{\Delta_i^t}{m_i^t}\right)^2}\right),
\end{aligned}
$$

where (a) is obtained by using $\Pr(X < -\epsilon - b) \leq \Pr(X < -\epsilon + b)$ for an arbitrary $b \geq 0$, and (b) comes from the distribution

$$
\begin{aligned}
X \sim \mathcal{N}(0, \sigma^2) : \Pr(X > s) &= \frac{1}{\sigma\sqrt{2\pi}} \int_s^\infty \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \\
&\leq \frac{1}{\sigma\sqrt{2\pi}} \int_s^\infty \frac{x}{s} \exp\left(-\frac{x^2}{2\sigma^2}\right) dx \\
&= \frac{\sigma}{s\sqrt{2\pi}} \exp\left(-\frac{s^2}{2\sigma^2}\right).
\end{aligned}
$$

Letting $q = \frac{\epsilon - \sum_{t=1}^{T} \frac{1}{2}(\Delta_i^t/m_i^t)^2}{\sqrt{2\sum_{t=1}^{T}(\Delta_i^t/m_i^t)^2}}$, the DP condition is implied by the inequality

$$
\Pr(|\mathcal{L}_{\mathcal{D},\mathcal{D}'}(\mathbf{y}_i)| > \epsilon) \leq \frac{1}{q\sqrt{\pi}} e^{-q^2} < \delta.
$$

Finally, defining the function $C(x) = \sqrt{\pi} x e^{x^2}$ and utilizing its monotonicity yields the result

$$
\begin{aligned}
\sum_{t=1}^{T}\left(\frac{\sqrt{2} h_i^t \sqrt{\tau_i}\Lambda}{m_k^{(t)}}\right)^2 &\leq \left(\sqrt{\epsilon + \left[\mathcal{C}^{-1}\left(1/\delta\right)\right]^2} - \mathcal{C}^{-1}\left(1/\delta\right)\right)^2 \\
&\triangleq \mathcal{R}_{\text{dp}}(\epsilon, \delta).
\end{aligned}
$$

To simplify the subsequent analyses, we divide the DP constraints equally into each iteration, thus obtaining the desired result

$$
\left(\frac{\sqrt{2} h_i \sqrt{\tau_i}\Lambda}{\sqrt{(h_i\sqrt{\tau_i p_i})^2 + N_0}}\right)^2 \leq \frac{\mathcal{R}_{\text{dp}}(\epsilon, \delta)}{T}.
$$

References

[1] D. Liu and O. Simeone, "Privacy for free: Wireless federated learning via uncoded transmission with adaptive power control," *IEEE Journal on Selected Areas in Communications*, vol. 39, no. 1, pp. 170–185, 2020.

[2] C. Dwork, A. Roth *et al.*, "The algorithmic foundations of differential privacy," *Foundations and Trends® in Theoretical Computer Science*, vol. 9, no. 3–4, pp. 211–407, 2014.