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CSC 355. Discrete Structures and Basic Algorithms
Homework Assignment 1

Instructions: Solve the following questions.

Question 1.
Simplify each of the following without using a calculator. Type your answer into the blank as an integer or in the form “a/b” for a fraction. (e.g. 3/4 for ¾)

(a) $3^{log_3 9+1} = 9+1 = \boxed{10}$

(b) $log_4\left(\frac{16}{64}\right) = \log_4 16 - \log_4 64 = 2-3 = \boxed{-1}$

(c) $log_{32}128 = \log_{32}(32\cdot 4) = 1 + \log_{32} 4 = 1 + 2\log_{32} 2 = 1 + 2\left(\frac{1}{\log_2 32}\right) = 1 + 2\left(\frac{1}{5}\right) = \boxed{7/5}$

Question 2.
Consider the following proof that $log_2(4^n) = 2^n$ for all $n \geq 0$.
(a) Is this proof valid? Type “Y” for yes or “N” for no. N
(b) If your answer to (a) was “N”, type the number of the line where the error occurs. If your answer was “Y”, just type “Y” again. 3

1 **Conjecture.** $log_2(4^n) = 2^n$ for all $n \geq 0$.
2 $log_2(4^n)$
3 $= log_2(2^2)^n$
4 $= 2^n$

Question 3. Represent the sigma notation for the following series:

$$1^3 + 2^3 + 3^3 + 4^3 + 5^3 \qquad \sum_{i=1}^{5} i^3$$

Question 4. Represent the sigma notation for the following series:

$$\frac{1}{3} - \frac{1}{5} + \frac{1}{7} - \frac{1}{9} \qquad \sum_{i=1}^{4} -(-1^{i})(2i+1)^{-1}$$

Question 5. Represent the sigma notation for the following series:

$$f_1x_1 + f_2x_2 + f_3x_3 + \ldots + f_Nx_N$$

$$\sum_{i=1}^{N} f_i x_i$$

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Question 6. Write an algorithm that is correct, composed with concrete steps, has no ambiguity and terminates when it meets the solution. The algorithm must solve one of the following math problems.

a) Tower of Hanoi
b) Factorial
c) Fibonacci series

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START
Fib(n)
 a[0] =                 0
 a[1] = 1

 FOR(i                 = 2; i < n; i++)
 a[i] =                 a[i-2] + a[i-1]
 ENDFOR

 return                 a[]
END Fib
END
```

Question 7.
Match each function with its appropriate big-Theta approximation.

(a) $5nlog(16^n)$ theta(n^2)
(b) $8n^2 + 10n - 4$ theta(n^2)
(c) $12log15$ theta(1)
(d) $8^{log_2 n+1} - 1$ theta(n^3)
(e) $log\left(\frac{n^2}{4}\right) + 3logn$ theta(logn)
(f) $\frac{3^{log_3 n+1}-1}{2}$ theta(n)
(g) $log(n!)$ theta(logn)
(h) $5(2^{n+1})$ theta(2^n)

Question 8. Simplify each of the following without using a calculator.

(a) $4 \times (4^2)^6 =$ 4^13

(b) $5^{(2^3)} =$ 5^8

(c) $log_2 16^2 =$ 2log(2, 2^4) = 8log(2, 2) = 8

(d) $log_3\left(\frac{1}{9}\right) =$ log(3, 1) - log(3, 9) = -log(3, 9)

(e) $log_{16} 32 =$ log(16, 2*16) = log(16, 16) + log(16, 2) = 1 + 1 / [log(2,16)]

Question 9.
Determine if each of these statements is true or false. Type "T" for true or "F" for false.
(a) $7n + 3n^2 + nlog(2^{n^2})$ is $O(n^2)$ F
(b) $4nlog^2n + 2^n + 3n^3$ is $\Omega(n^3)$ T
(c) $16nlog(16) + 16^{log_2 n}$ is $\theta(n)$ F

(d) $nlog(n^2) + n^2logn$ is $O(nlogn)$ F
(e) $4nlog(\frac{n}{4}) + 5n$ is $\Omega(nlogn)$ T

Question 10.
Given that $f(n)\ is\ \Omega(g(n))$, $g(n)\ is\ O(h(n))$, and $h(n)\ is\ \theta(i(n))$, determine if each of the statements below is definitely true (DT), definitely false (DF), or possibly true/possibly false (PT). Type "DT", "DF", or "PT" into the space provided.

(a) $f(n)\ is\ O(h(n))$ PT
(b) $g(n)\ is\ O(f(n))$ DF
(c) $i(n)\ is\ \Omega(g(n))$ PT

Submission Instructions

You must upload your homework in a **pdf** file in the designated area in D2L.

Grading Points

Total Score: 25 points

**Each question has a value of 2.5 points*