

CSC 355. Discrete Structures and Basic Algorithms
Homework Assignment 2

Instructions: Solve the following questions.

1. Using induction to prove that for any natural number n , $0+1+\dots+n = n(n+1)/2$
2. Using the literature provided in Module 2, explain the following:
 - a. Describe the upper bound.
 - b. Describe the lower bound.
 - c. Describe the grow rate.
3. State the recursive algorithm or pseudocode to solve the following exercises. For each algorithm analyze the best, worst and average cases.
 - a. Fibonacci Series
 - b. Factorial
 - c. Hanoi Tower
4. Does the Linear Search is $\Omega(1)$ in its best case? Why?
5. The sequential search algorithm is $\Theta(n^2)$ or not? Why?

Submission Instructions

You must upload your homework in a **pdf** file in the designated area in D2L.

Grading Points

Total Score: 25 points

**Each question has a value of 5 points*

(1) Assume $0+1+2+\dots+n = n(n+1)/2$ is true for $n=k$ where $k=5$

$$0+1+2+3+4+5 = 5(6)/2 = 15$$

if $0+1+2+\dots+(k+1) = (k+1)(k+2)/2$ then $0+1+2+\dots+n = n(n+1)/2$ is true for all n

proof: $k+1=6$

$$0+1+2+3+4+5+6 = 21$$

$$6(7)/2 = 42/2 = 21$$



(2)

a. Worst case scenario for an algorithm in runtime

b. Best case scenario for an algorithm in runtime

c. The average scenario for an algorithm in runtime

(3)

a.

```
Fibonacci(n){
    if (n == 0)
        return 0;
    else if (n == 1)
        return 1;
    else
        return Fibonacci(n-1) + Fibonacci(n-2)
}
```

// Best case is $n = 0$; $T = 1$
// Worst case is n is very large; $O(2^n)$
// Average case is any number n ; $O(2^n)$

b.

```
Factorial(n) {
    if (n == 0)
        return 1;
    return n * Factorial(n - 1)
}
```

// Best case is $n = 0$; $T = 1$
// Worst case is N is very large; $O(N)$
// Average case is N is a number; $O(N)$

c.

```
Hanoi(n, src, aux, tgt){
    if (n == 1)
        move disk 1: src -> tgt
    else
        Hanoi(n - 1, src, tgt, aux)
        move disk n: src -> tgt
        Hanoi(n - 1, aux, src, tgt)
}
```

// Best case is $n = 1$; $T = 1$
// Worst case case is N is very large; $O(2^N)$
// Average case is N is a number; $O(2^N)$

(4.) Yes, because in the best case scenario it would only take one iteration to find the matching item.

(5.) No, because the worst case scenario (big O) of a sequential search is n . This means that the theta value can't be larger than big O.