(2) Practice Problems

(1) Find all the stationary points of $f(x,y) = \dfrac{x+y}{x^2+y^2+1}$.

(2) Define $f : \mathbb{R}^2 \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}^2$ by $f(x) = x_1^2/6 + x_2^2/4$ and $g(t) = \begin{bmatrix} 3t+5 \\ 2t-6 \end{bmatrix}$. Let $F(t) = f(g(t))$.

(a) Evaluate $\nabla F(t)$ by the chain rule.

(b) Find all the stationary points and classify them as saddle points, minimum or maximum points.

(3) Consider the following function:

$$f(x_1, x_2) = x_1 x_2 + \frac{1}{3}(x_1^3 + x_2^3),$$

find all the stationary points and classify them as saddle points, minimum points, maximum points.