Homework 1 – Due by 11:59 PM on Sunday Feb 9

(1) Compute $\nabla f(x)$ and $\nabla^2 f(x)$ for f where f is defined as one of the following.

(a) $f(x_1, x_2, x_3) = x_2^2$ (b) $f(x_1, x_2, x_3) = x_1^2 + x_3^2 + 2x_1x_2 + 2x_1x_3 + 2x_2x_3$.

(2) Consider the function

$$f(x) = (a^\top x)(b^\top x),$$

where a, b, and x are $n-$dimensional vectors. Find $\nabla_x f(x)$ *(Hint: use product rule)*

(3) Are the following vectors linearly independent or dependent? Why?

$$a_1 = \begin{bmatrix} 0.2 \\ -7 \\ 8.6 \end{bmatrix}, \quad a_2 = \begin{bmatrix} -0.1 \\ 2 \\ -1 \end{bmatrix}, \quad a_3 = \begin{bmatrix} 0 \\ -1 \\ 2.2 \end{bmatrix}$$

(4) Let $\lambda_1, \ldots, \lambda_n$ be the eigenvalues of $A \in \mathbb{R}^{n\times n}$. Show that the eigenvalues of the matrix $I_n - A$ are $(1 - \lambda_1), \ldots, (1 - \lambda_n)$.

(5) Show that for any $x, y, z \in \mathbb{R}^n$, $\|x - z\| \le \|x - y\| + \|y - z\|$.

(6) Show that $\|\cdot\|_{1/2}$ is not a norm. *(Hint: Show that it may not satisfy the triangle inequality)*

Students enrolled in SIE 549 must solve the following problems.
Students in SIE 449 will get extra credit by solving them.

(7) Show that $\|x\|_\infty \le \|x\|_2 \le \|x\|_1$.

(8) Consider matrix A, from the definition of matrix norms, we know that $\|A\|_2 = \sigma_1$ where σ_1 is the largest singular value of A, and $\|A\|_\infty = \|A^T\|_1$. Show that $\|A\|_2^2 \le \|A\|_1\|A\|_\infty$.

(Hint: Consider first singular vector v of A, we know that $A^TAv = \sigma_1^2 v$)