(8) Practice Problems

(1) Show that for a convex function, the subdifferential is convex.

(2) For a convex function h, show that $\|\text{prox}_h(x) - \text{prox}_h(y)\| \leq \|x - y\|$.

(3) Let $f_i : \mathbb{R}^n \to \mathbb{R}$, $i = 1, \ldots, m$ be convex functions and let $f = f_1 + \ldots + f_m$. Show that

$$\partial f_1 + \ldots + \partial f_m \subseteq \partial f.$$

(4) Define $G_\eta(x) = \frac{1}{\eta}\left[x - \text{prox}_{h,\eta}(x - \eta \nabla g(x))\right]$ and prove the following holds:

$$G_\eta(x^*) = 0 \iff 0 \in \nabla g(x^*) + \partial h(x^*).$$