Homework 2 – Due by 11:59 PM on Sunday Feb 16

(1) For each of the following functions, find all the stationary points and classify them as saddle points, minimum points, maximum points:

(a) $f(x_1, x_2, x_3) = x_1^4 - 2x_1^2 + x_2^2 + 2x_2x_3 + 2x_3^2$

(b) $f(x_1, x_2) = x_1^4 + 2x_1^2x_2 + x_2^2 - 4x_1^2 - 8x_1 - 8x_2$

(2) (a) Let $C_1, \dots, C_k \subseteq \mathbb{R}^n$ be convex sets and $\mu_1, \dots, \mu_k \in \mathbb{R}$. Then the set $C \triangleq \sum_{i=1}^k \mu_i C_i$ is convex.

(b) Let $M \subseteq \mathbb{R}^n$ be a convex set and $A \in \mathbb{R}^{m\times n}$. Then the set $\mathbf{A}(M) = \{Ax : x \in M\}$ is convex.

(c) Let $D \subseteq \mathbb{R}^m$ be a convex set and let $A \in \mathbb{R}^{m\times n}$. Then the set

$$A^{-1}(D) = \{x \in \mathbb{R}^n \mid Ax \in D\}$$

is convex.

(3) Show that the set of all positive semidefinite matrices is a convex set.

(4) Is $f(x) = -x_1x_2$ a convex function or not? Why?

(5) (a) Let A be a symmetric 2×2 matrix. Then A is positive semidefinite if and only if $\mathrm{tr}(A) \geq 0$ and $\det(A) \geq 0$.

(b) Use part (a) and prove that $f(x) = \dfrac{x_1^2}{x_2}$ is a convex function for $x_2 > 0$.

Students enrolled in SIE 549 must solve the following problems.
Students in SIE 449 will get extra credit by solving them.

(6) Function $f : \mathbb{R}^n \to \mathbb{R}$ is called a *strongly convex function*, if there exists $\alpha > 0$ such that $f(x) - \alpha\|x\|^2$ is convex. Use this definition and answer the following questions.

(a) Show that if f is strongly convex then f is convex.

(b) Show that x^4 is a convex function but it is not strongly convex.