Student Name: .... SIE 449/549

Practice Problems for Midterm Exam

(1) Indicate whether the following statements are true or false. Justify your answer if the statement is false. (4 points)

(a) If f is a convex function, then its minimizer must be unique.

☐ True

☐ False

(b) If a function $f : \mathbb{R}^n \to \mathbb{R}$ is twice continuously differentiable and its Hessian satisfies $\|\nabla^2 f(x)\| \leq L$ for all x, then its gradient ∇f is L-Lipschitz continuous.

☐ True

☐ False

(2) For the following function, find all the stationary points and classify them as saddle points, minimum points, maximum points:

$$f(x, y) = x^2 + y^2 + 4x$$

(4) Suppose f and g are Lipschitz continuous functions with constants L_f and L_g and they are bounded, i.e., $|f| \leq M_f$ and $|g| \leq M_g$. Show that:

(a) $\alpha f(x) + \beta$ is a Lipschitz continuous function with constant $|\alpha| L_f$.

(b) $f(x) + g(x)$ s a Lipschitz continuous function with constant $L_f + L_g$.

(c) $f(x)g(x)$ is a Lipschitz continuous function with constant $M_f L_g + M_g L_f$.

(5) Let $C \subseteq \mathbb{R}^n$ is a convex set. Let f be a convex function over C and let g be a strictly convex function over C. Suppose x^* and y^* are the optimal solutions of $\min h(x) \triangleq f(x) + g(x)$. Show that $x^* = y^*$.

(6) If $f : \mathbb{R}^n \to \mathbb{R}$ is convex and continuously differentiable , then show that

$$(\nabla f(x) - \nabla f(y))^T (x - y) \geq 0.$$

(7) Suppose function f is twice continuously differentiable and has a Lipschitz continuous gradient with constant L. Consider function $g(x) = f(Ax + b)$ and show that it has a Lipschitz continuous gradient with constant $\|A\|^2 L$.

(8) (a) Let $C_i \subseteq \mathbb{R}^{k_i}$ be a convex set for any $i = 1, 2, \ldots, m$. Then the Cartesian product is convex.

$$C \triangleq C_1 \times C_2 \times \ldots \times C_m = \{(x_1, x_2, \ldots, x_m) \mid x_i \in C_i, i = 1, 2, \ldots, m\}$$

(b) Show that $C = \{x : \max_{i=1,2,\ldots,n} x_i \leq 1\}$ is a convex set.

(9) Consider the following optimization problem:

$$\min_{x \in \mathbb{R}^n} \|Ax - b\|^2$$

(a) What is the gradient of the objective function?

(b) Find the Lipschitz constant of the gradient.

(c) Complete the following code to solve the problem via **gradient**. Choose the step size as $1/L$ where L is the Lipschitz constant of the gradient and set the accuracy to $\epsilon = 1e - 3$.

```
n = 50;
x = zeros(n,1);
A = randn(n,n);
b = randn(n,1);
epsilon = 1e-3;

while



end
display(x);
```