

Calculating the Dipole moment.

$$D = \text{Drag} \left(\frac{kg \cdot m}{s^2} \right)$$

C_d = Coefficient of Drag (*unitless*)

V = Velocity (m/s)

A = Area normal to Velocity vector (m^2)

ρ = Atmosphere Density (kg/m^3)

B = Earth's Magnetic Strength (T)

μ = Magnetorquer dipole moment

τ = Atmospheric Torque

Unit Conversions

$$A * m^2 = \frac{N \cdot m}{T} = \frac{kg \cdot m^2}{s^2 \cdot T}$$

Finding the density of atmosphere at orbital altitude

{ref: <http://www.braeunig.us/space/atmos.htm> }

To make a conservative estimate, I used the 360 km data for atmospheric density. This will give me a higher density than the estimated 450 km orbital path density, thus a higher drag force.

$$\rho = 7.99 * 10^{-12} \frac{kg}{m^3}$$

Velocity for this altitude was attained from

{ref: http://www.calctool.org/CALC/phys/astronomy/earth_orbit } but a formula could be used to achieve the same values.

$$V = 7695.3 \frac{m}{s}$$

The Coefficient of Drag value was taken from page 5 of

{ref: <https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19710016459.pdf> }

$$C_d = 2.6$$

The magnetic field strength of the Earth was attained from

{ref: <https://academic.oup.com/gji/article/183/3/1216/637157#ss8> } and I used the minimum value of

$$B = 2.5 * 10^{-5} T$$

The Area of the cube was generalized at 10 cm per side resulting in

$$A = 0.01 m^2$$

Putting all this into the drag force equation

$$D_{360\text{ km}} = \frac{1}{2} C_d \rho_{360\text{ km}} A V_{360\text{ km}}^2$$

$$D_{360\text{ km}} = \frac{1}{2} (2.6) \left(7.99 * 10^{-12} \frac{\text{kg}}{\text{m}^3} \right) (0.01\text{ m}^2) \left(7695.3 \frac{\text{m}}{\text{s}} \right)^2 = 6.15 * 10^{-6} \frac{\text{kg} * \text{m}}{\text{s}^2}$$

Then using the moment arm estimation from page 8 of

{ref: <https://ntrs.nasa.gov/archive/nasa/casi.ntrs.nasa.gov/19710016459.pdf> }

$$l = \frac{(\text{maximum Dimension of spacecraft})}{3}$$

The maximum dimension was calculated from the cube shape and using the distance between opposite corners

$$\text{distance} = \sqrt{3} (0.1\text{ m}) = 0.1732\text{ m}$$

Therefor the Atmospheric torque is calculated by

$$\tau = \frac{\text{distance}}{3} (D_{360}) = \left(\frac{0.1732}{3} \text{ m} \right) \left(6.15 * 10^{-6} \frac{\text{kg} * \text{m}}{\text{s}^2} \right) = 3.551 * 10^{-7} \frac{\text{kg} * \text{m}^2}{\text{s}^2}$$

The dipole moment would then just be a small manipulation to the torque equation here

$$\tau = \mu \times B$$

$$\mu = \frac{\tau}{B} = \frac{3.551 * 10^{-7} \frac{\text{kg} * \text{m}^2}{\text{s}^2}}{2.5 * 10^{-5} \text{ T}} = 0.0142 \text{ A} * \text{m}^2$$