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THREE-AXIS SATELLITE ATTITUDE CONTROL BASED ON MAGNETIC TORQUING

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Important equation on page 4

Abstract: Magnetic torquing is attractive as a control principle on small satellites. The actuation principle is to use the interaction between the earth's magnetic field and magnetic field generated by a coil set in the satellite. This control principle is inherently nonlinear, and difficult to use because control torques can only be generated perpendicular to the geomagnetic field vector. This has been a serious obstacle for using magnetorquer based control for three-axis attitude control.
This paper considers the problem from time-varying, nonlinear system point of view and suggests a new controller which is shown to be globally stable. A rigorous stability analysis is presented, and simulation results show convincing performance over the entire envelope of operation.

Keywords: Attitude control, satellite control, time-varying systems, periodic motion, Lyapunov stability, quaternion feedback.

1 INTRODUCTION

Several control methods have been developed over the past years since the first satellite was launched. Generally speaking all those techniques may be classified as active or passive. For missions where pointing accuracy is vital, active techniques are used. The actuators typically used for active techniques are: jet actuators, reaction wheels and finally electromagnetic coils. Magnetic torquing is attractive for small, cheap satellites being in operation in low earth orbits. Magnetic control systems are relatively lightweight, require low power and are inexpensive. This actuation principle was chosen for the Danish Ørsted satellite mission. Due to planned scientific observations it is desired to maintain the Ørsted satellite in a fixed orientation with respect to the earth. There are three mutually perpendicular electromagnetic coils on board. Measurements of the geomagnetic field are possible via a 3-axis magnetometer. Knowledge of attitude information is available through an onboard attitude determination system.

There is broad literature covering satellite attitude control design. Though most of the algorithms presented assume application of jet actuators or reaction wheels. The problem of three-axis magnetic control was addressed in (Musser and Ward, 1989). The local stabilization of the satellite was achieved via implementation of the infinite-time-horizon linear quadratic regulator. Another linear approach was given by Martel *et al.* (1988), where the linearized time-varying satel-

This paper assumes that you want to align t
he BCS (Local) frame with the OCS (Global) frame
for your goal orientation

lite motion model was approximated by a linear time-invariant counterpart. Three-axis stabilization with use of magnetic torquing of a satellite without appendages was treated by Wisniewski (1994), where sliding control law stabilizing a tumbling satellite was proposed.

The paper is organized as follows. Section 2 describes satellite motion. A locally stabilizing controller is proposed in section 4. It is extended to a globally stable, time-varying controller in section 5. Simulation results are shown in section 6.

2 NOTATION

The following notation is used throughout the paper:

BCS=Local Frame

BCS coordinate system built on principal axes,
OCS reference coordinate system fixed in orbit,
WCS inertial right orthogonal system. OCS = Global

WCS-Global but one axis is
always pointing at earth.

$^c\mathbf{v}, {}^o\mathbf{v}, {}^w\mathbf{v}$ — $\mathbf{v}$ resolved in BCS, OCS, WCS,
$\mathbf{\Omega}_{cw}$ — angular velocity of BCS w.r.t. WCS,
$\mathbf{\Omega}_{co}$ — angular velocity of BCS w.r.t. OCS,
$\mathbf{\Omega}_{ow}$ — angular velocity of OCS w.r.t. WCS,
ω_o — orbital rate,
$\mathbf{I}$ — inertia tensor of the satellite,
I_x, I_y, I_z — moments of inertia about x-, y- and z-principal axes,
$\mathbf{N}_{ctrl}$ — control torque,
$\mathbf{N}_{gg}$ — gravity gradient torque,
$\mathbf{N}_{dist}$ — disturbance torques,
$\mathbf{m}$ — magnetic moment,
$\mathbf{B}$ — geomagnetic field vector,
$\tilde{\mathbf{B}}$ — skew symmetric matrix representing a cross product operator $^c\mathbf{B}\times$,
$^c_o\mathbf{q}$ — attitude quaternion representing rotation of BCS relative to OCS,
$\mathbf{q}, q_4$ — vector part and scalar part of $^c_o\mathbf{q}$,
$\mathbf{A}(^c_o\mathbf{q})$ — attitude matrix based on $^c_o\mathbf{q}$,
$\mathbf{i}_o, \mathbf{j}_o, \mathbf{k}_o$ — unit vector along x-, y-, z-axis of OCS,
$\mathbf{E}$ — 3x3 identity matrix.

2.1 Equations of Dynamics

The dynamics relates torques acting on the satellite to the satellite angular velocity in WCS. The dynamic equation of motion for a satellite in low earth orbit, see (Wertz, 1990), is

$$I\,{}^c\dot{\mathbf{\Omega}}_{cw}(t) = -{}^c\mathbf{\Omega}_{cw}(t) \times \mathbf{I}\,{}^c\mathbf{\Omega}_{cw}(t) + {}^c\mathbf{N}_{ctrl}(t) + {}^c\mathbf{N}_{gg}(t) + \mathbf{N}_{dist}. \quad (1)$$

There are three torques represented in equation (1): the control torque, the gravity gradient torque and disturbance torques (e.g., aerodynamic drag). The magnetic control torque is generated according to

$$^c\mathbf{N}_{ctrl}(t) = {}^c\mathbf{m}(t) \times {}^c\mathbf{B}(t), \quad (2)$$

where $\mathbf{m}(t)$, the magnetic moment generated by the coils, is the control quantity. Gravity gradient torque is given by

$$^c\mathbf{N}_{gg}(t) = 3\omega_o^2({}^c\mathbf{k}_o \times \mathbf{I}\,{}^c\mathbf{k}_o). \quad (3)$$

The disturbance torque is considered to be small comparing with the gravity gradient torque.

2.2 Equation of kinematics

The attitude of the satellite is given through orientation of the BCS with respect to OCS. In this article the attitude is represented by a unit quaternion. The quaternion provides global represented of the attitude without singularities, see (Wertz, 1990), is

$$\dot{\mathbf{q}} = \frac{1}{2}{}^c\mathbf{\Omega}_{co}q_4 + \frac{1}{2}{}^c\mathbf{\Omega}_{co} \times \mathbf{q},$$
$$\dot{q}_4 = -\frac{1}{2}{}^c\mathbf{\Omega}_{co} \cdot \mathbf{q}. \quad (4)$$

Relation between satellite angular velocity relative to WCS and angular velocity with respect to OCS is obtained

$$^c\mathbf{\Omega}_{co} = {}^c\mathbf{\Omega}_{cw} - \omega_o{}^c\mathbf{i}_o. \quad (5)$$

3 ATTITUDE STABILITY AT LARGE

We concentrate on global stability of velocity feedback control in this section. A magnetic generated control torque seen from equation (2) is always perpendicular to the geomagnetic magnetic field vector. This is a serious limitation on the use of magnetic torquing, and the consequence is that a linearized version of equations (1) to (5) is only controllable in two directions at any single point in time.

The control torque is always perpendicular to the geomagnetic field vector, therefore it is desirable that the magnetic moment is also perpendicular to geomagnetic field vector, since only this component has actual influence on the control torque. Consider the following control law

As far as can I can tell, H is just some
constant times the identity matrix

$$^c\mathbf{m}(t) = {}^c\mathbf{B}(t) \times \mathbf{H}{}^c\mathbf{\Omega}_{co}(t), \quad (6)$$

where $\mathbf{H}$ is a 3×3 positive definite constant matrix. Global stability of control law (6) is expressed in the following theorem.

This seems to be essentially a bDot
algorithm. Check Note 1.

Theorem 1 *Consider control law (6) then the satellite (1) to (5) has 4 asymptotically stable local equilibria:*

$$\{({}^c\boldsymbol{\Omega}_{co},\ {}^c\mathbf{k}_o,\ {}^c\mathbf{i}_o) : (\mathbf{0},\ \pm^o\mathbf{k}_o,\ \pm^o\mathbf{i}_o)\}.$$

Proof of Theorem 1 *Consider a scalar function expressing total energy of the satellite. The total energy of the system is sum of kinetic energy of rotary motion, potential energy generated by the gravity gradient and energy originating from rotation of the satellite around the earth.*

$$E_{tot} = E_{kin} + E_{gg} + E_{gyro}.$$

This leads to

$$\begin{aligned} E_{tot} = {} & \tfrac{1}{2}{}^c\boldsymbol{\Omega}_{co}^T\mathbf{I}^c\boldsymbol{\Omega}_{co} + \tfrac{3}{2}\omega_o^2({}^c\mathbf{k}_o^T\mathbf{I}^c\mathbf{k}_o - I_z) \\ & + \tfrac{1}{2}\omega_o^2(I_x - {}^c\mathbf{i}_o^T\mathbf{I}^c\mathbf{i}_o), \end{aligned} \tag{7}$$

where I_x is the satellite maximum moment of inertia, and I_z is the minimum.

Now, the time derivative of E_{tot} is shown to be positive semidefinite.

$$\dot{E}_{tot} = {}^c\boldsymbol{\Omega}_{co}^T\mathbf{I}^c\dot{\boldsymbol{\Omega}}_{co} + 3\omega_o^2{}^c\mathbf{k}_o^T\mathbf{I}^c\dot{\mathbf{k}}_o - \omega_o^2{}^c\mathbf{i}_o^T\mathbf{I}^c\mathbf{i}_o \tag{8}$$

The equations (1) to (5) are substituted into (8) yielding

$$\begin{aligned} \dot{E}_{tot} = {} & {}^c\boldsymbol{\Omega}_{co}^T(-{}^c\boldsymbol{\Omega}_{cw} \times \mathbf{I}^c\boldsymbol{\Omega}_{cw} + 3\omega_o^2\ {}^c\mathbf{k}_o^T \times \\ & \mathbf{I}^c\mathbf{k}_o + {}^c\mathbf{N}_{ctrl}) - \omega_o{}^c\boldsymbol{\Omega}_{co}^T\mathbf{I}({}^c\mathbf{i}_o \times {}^c\boldsymbol{\Omega}_{co}) \\ & + 3\omega_o^2{}^c\mathbf{k}_o^T\mathbf{I}({}^c\mathbf{k}_o \times {}^c\boldsymbol{\Omega}_{co}) - \omega_o^2{}^c\mathbf{i}_o^T\mathbf{I}({}^c\mathbf{i}_o \times {}^c\boldsymbol{\Omega}_{co}). \end{aligned} \tag{9}$$

Since

$$\begin{aligned} {}^c\boldsymbol{\Omega}_{co}^T({}^c\boldsymbol{\Omega}_{cw} \times \mathbf{I}^c\boldsymbol{\Omega}_{cw}) = {} & \omega_o{}^c\boldsymbol{\Omega}_{co}^T({}^c\mathbf{i}_o \times \mathbf{I}^c\boldsymbol{\Omega}_{co}) \\ & + \omega_o^2\ {}^c\boldsymbol{\Omega}_{co}^T({}^c\mathbf{i}_o \times \mathbf{I}^c\mathbf{i}_o), \end{aligned} \tag{10}$$

equation (10) is reduced to

$$\dot{E}_{tot} = {}^c\boldsymbol{\Omega}_{co}^T\mathbf{N}_{ctrl}. \tag{11}$$

If proposed control law (6) is applied then equation (11) becomes

$$\dot{E}_{tot} = -{}^c\boldsymbol{\Omega}_{co}^T\tilde{\mathbf{B}}\tilde{\mathbf{B}}\mathbf{H}^c\boldsymbol{\Omega}_{co}. \tag{12}$$

Here $\tilde{\mathbf{B}}$ is the skew symmetric matrix representing a cross product operator: ${}^c\mathbf{B}\times$. $\tilde{\mathbf{B}}\tilde{\mathbf{B}}$ is semipositive definite and $\mathbf{H}$ is positive definite. The derivative of the total energy is thus positive semidefinite. Moreover $E_{tot}(t)$ is limited by the initial value $E_{tot}(t_0)$. $\ddot{E}_{tot}(t)$ is a continuous function of ${}^c\mathbf{B}(t)$ and ${}^c\dot{\boldsymbol{\Omega}}_{co}(t)$ which are bounded (${}^c\boldsymbol{\Omega}_{co}(t) < {}^c\boldsymbol{\Omega}_{co}(t_0)$). Thus $\dot{E}_{tot}(t)$ is uniformly continuous in time. According to the Barbalat's lemma, see (Slotine and Li, 1991), lemma 4.2, $\lim_{t\to\infty}\dot{E}_{tot}(t) = 0$.

We will prove that E_{tot} converges to 0 by contradiction. Let E_{tot} not converge to 0. Then equation (12) implies that $\dot{E}_{tot} = 0$ and E_{tot} is constant. Hence the vector ${}^c\mathbf{B}(t)$ is parallel to ${}^c\boldsymbol{\Omega}_{co}(t)$ or ${}^c\mathbf{B}(t)$ is parallel to $\mathbf{H}^c\boldsymbol{\Omega}_{co}(t)$ for each t. It can be expressed by

Upside down A means "for all"

$$\forall_{t\geq 0}\ {}^c\mathbf{B}(t) = \alpha^c\boldsymbol{\Omega}_{co}(t)\ \vee\ {}^c\mathbf{B}(t) = \beta\mathbf{H}^c\boldsymbol{\Omega}_{co}(t), \tag{13}$$

V means or

c=Local
o=Global

where α and β are some non zero constants. Differentiating ${}^c\boldsymbol{\Omega}_{co}(t)$ and ${}^c\mathbf{B}(t)$ in equation (13) with respect to time gives

I think A converts a quaternion to a rot matrix

$$\begin{aligned} & \forall_{t\geq 0}\ \mathbf{A}({}^c_o\mathbf{q})^o\dot{\mathbf{B}}(t) = \alpha^c\dot{\boldsymbol{\Omega}}_{co}(t) \\ & \vee\ \mathbf{A}({}^c_o\mathbf{q})^o\dot{\mathbf{B}}(t) = \beta\mathbf{H}^c\dot{\boldsymbol{\Omega}}_{co}(t), \end{aligned} \tag{14}$$

since

$${}^c\dot{\mathbf{B}}(t) = {}^c\mathbf{B}(t) \times {}^c\boldsymbol{\Omega}_{co}(t) + \mathbf{A}({}^c_o\mathbf{q})^o\dot{\mathbf{B}}(t). \tag{15}$$

Note 1.

${}^o\mathbf{B}$ and ${}^o\dot{\mathbf{B}}$ are given by the geomagnetic field model in Wertz (1990), the development of ${}^c\dot{\boldsymbol{\Omega}}_{co}(t)$ with no control torque (because of (13)) is shown in equations (1) and (3). It follows that equation (14) is not valid. This shows the contradiction.

Remark 3.1 *The control law (6) can be used for three-axis magnetic stabilization of the satellite in a neighbourhood of one of 4 equilibria stated in theorem 1 if $I_x > I_y > I_z$.*

4 LOCAL ATTITUDE STABILITY

Three-axis attitude control at equilibrium $(\mathbf{0},\ {}^o\mathbf{k}_o,\ {}^o\mathbf{i}_o)$ is addressed in this section. Control law (6) can give any of four attitude equilibria in theorem 1. This is not operationally acceptable. Therefore a new control law is introduced

$${}^c\mathbf{m}(t) = {}^c\mathbf{B}(t) \times \mathbf{H}^c\boldsymbol{\Omega}_{co}(t) - \epsilon^c\mathbf{B}(t) \times {}^c_o\mathbf{q}(t), \tag{16}$$

where $\mathbf{H}$ is a positive definite constant matrix and ϵ is a constant scalar. In equation (16) a small perturbation in attitude was added comparing with the control law (6). For small ϵ the satellite is stable in the neighbourhood of reference $(\mathbf{0},\ {}^o\mathbf{k}_o,\ {}^o\mathbf{i}_o)$, since the differential equations describing motion of the satellite are well posed. The question is, how large ϵ shall be that the system is still locally stable.

The system is first linearized. The attitude quaternion is linearized in a special way as a multiplication of a small perturbation by the quaternion in reference $(\mathbf{0},\ {}^o\mathbf{k}_o,\ {}^o\mathbf{i}_o)$.

$${}^c_o\mathbf{q}_{ref} = \mathbf{q}_{ref} \cdot [\delta q^T\ 1]^T, \tag{17}$$

where $\mathbf{q}_{ref} = [0\ 0\ 0\ 1]^T$ and $\delta q = [\delta q_1\ \delta q_2\ \delta q_3]^T$.

The linearized equation of motion (1) to (5) is

$$\frac{d}{dt}\begin{bmatrix}\delta\Omega\\ \delta q\end{bmatrix} = \tilde{\mathbf{A}}\begin{bmatrix}\delta\Omega\\ \delta q\end{bmatrix} + \tilde{\mathbf{B}}(t)(\mathbf{H}\delta\Omega - \epsilon\delta q), \qquad (18)$$

where

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0 & 0 & 0 & -2k\sigma_x & 0 & 0\\ 0 & 0 & \omega_o\sigma_y & 0 & 2k\sigma_y & 0\\ 0 & \omega_o\sigma_z & 0 & 0 & 0 & 0\\ \frac{1}{2} & 0 & 0 & 0 & 0 & 0\\ 0 & \frac{1}{2} & 0 & 0 & 0 & \omega_o\\ 0 & 0 & \frac{1}{2} & 0 & -\omega_o & 0 \end{bmatrix},$$

$$\sigma_x = \frac{I_y - I_z}{I_x},\ \sigma_y = \frac{I_z - I_x}{I_y},\ \sigma_z = \frac{I_x - I_y}{I_z},$$

$$\tilde{\mathbf{B}} = \mathbf{I}^{-1}\begin{bmatrix} {}^oB_y^2 + {}^oB_z^2 & {}^oB_x{}^oB_y & {}^oB_x{}^oB_z\\ {}^oB_x{}^oB_y & {}^oB_x^2 + {}^oB_z^2 & {}^oB_y{}^oB_z\\ {}^oB_x{}^oB_z & {}^oB_y{}^oB_z & {}^oB_x^2 + {}^oB_y^2\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0 \end{bmatrix}$$

Observing that the geomagnetic field in OCS is approximately periodic with one orbit period, local stability of the satellite is analyzed with the use of Floquet theory in (Mohler, 1991).

It was proven in section 3 that the satellite motion is locally stable at the reference $(\mathbf{0},\ {}^o\mathbf{k}_o,\ {}^o\mathbf{i}_o)$ for $\epsilon = 0$. Now we find all ϵ for which the linearized satellite system (18), with a certain fixed value of matrix $\mathbf{H}$, is stable by plotting a locus of characteristic multipliers. An example of characteristic multipliers locus is depicted in figure (1), where $\epsilon = 0.6\ 10^{-3}$ is the limit of stability.

5 GLOBALLY STABILIZING CONTROLLER

The control law (16) was shown to be locally stable. A globally stable controller is the ultimate goal. The main obstacle is again the cross product with the geomagnetic field vector.

5.1 *Idealized quaternion feedback*

If it were possible to produce a control torque proportional to the quaternion error, a globally stable controller would result. This is shown in the following theorem.

Theorem 2 *The control law*

$${}^c\mathbf{m}(t) = {}^c\mathbf{B}(t) \times \mathbf{H}{}^c\mathbf{\Omega}_{co}(t) - \epsilon{}^c_o\mathbf{q}(t), \qquad (19)$$

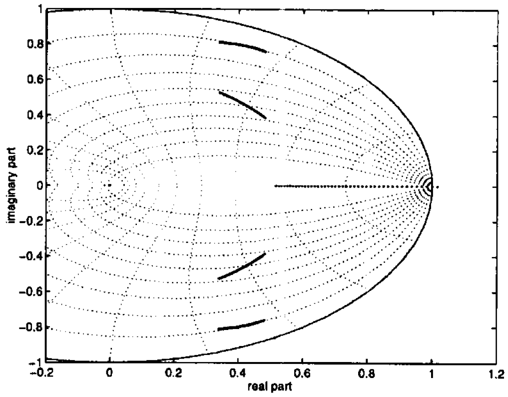

Fig. 1. Locus of characteristic multiplier $\lambda(\epsilon)$ for ϵ in $[0, 0.6\ 10^{-3}]$, $\mathbf{H} = 0.3\mathbf{E}$.

where $\mathbf{H}$ is a positive definite constant matrix, ϵ is a positive scalar, makes the system globally asymptotically stable at the reference $(\mathbf{0},\ {}^o\mathbf{k}_o,\ {}^o\mathbf{i}_o)$.

Proof of Theorem 2 *Total energy in the system is slightly modified from (7), proof of theorem 1 and equal*

$$\begin{aligned} E_{tot} = {} & \tfrac{1}{2}{}^c\mathbf{\Omega}_{co}^T\mathbf{I}{}^c\mathbf{\Omega}_{co} + \tfrac{3}{2}\omega_o^2({}^c\mathbf{k}_o^T\mathbf{I}{}^c\mathbf{k}_o - I_z)\\ & + \tfrac{1}{2}\omega_o^2(I_x - {}^c\mathbf{i}_o^T\mathbf{I}{}^c\mathbf{i}_o) + \epsilon(q_1^2 + q_2^2\\ & + q_3^2 + (1 - q_4)^2). \end{aligned} \qquad (20)$$

The attitude quaternion satisfies the constraint equation $q_1^2 + q_2^2 + q_3^2 + q_4^2 = 1$, thus

$$\begin{aligned} E_{tot} = {} & \tfrac{1}{2}{}^c\mathbf{\Omega}_{co}^T\mathbf{I}{}^c\mathbf{\Omega}_{co} + \tfrac{3}{2}\omega_o^2({}^c\mathbf{k}_o^T\mathbf{I}{}^c\mathbf{k}_o - I_z)\\ & + \tfrac{1}{2}\omega_o^2(I_x - {}^c\mathbf{i}_o^T\mathbf{I}{}^c\mathbf{i}_o) + 2\epsilon(1 - q_4). \end{aligned} \qquad (21)$$

The time derivative of (21) gives

$$\dot{E}_{tot} = {}^c\mathbf{\Omega}_{co}^T\mathbf{N}_{ctrl} + \epsilon{}^c\mathbf{\Omega}_{co}^T\,{}^c_o\mathbf{q}. \qquad (22)$$

Applying the control law defined in (19), $\dot{E}_{tot}$ is

$$\dot{E}_{tot} = -{}^c\mathbf{\Omega}_{co}^T\tilde{\mathbf{B}}\tilde{\mathbf{B}}\mathbf{H}{}^c\mathbf{\Omega}_{co}. \qquad (23)$$

This complies with equation (12). Thus the satellite with control law (19) is globally asymptotically stable at the reference $(\mathbf{0},\ {}^o\mathbf{k}_o,\ {}^o\mathbf{i}_o)$.

5.2 *Quaternion feedback with magnetic torquing*

Achievable control with magnetorquers involves the cross product with the geomagnetic field. The purpose of this subsection is to derive a global stabilizing controller under this limitation.

The proposed feedback with magnetic torquing is

$${}^c\mathbf{m}(t) = {}^c\mathbf{B}(t) \times \mathbf{H}{}^c\mathbf{\Omega}_{co}(t) + \epsilon(t){}^c\mathbf{B}(t) \times {}^c_o\mathbf{q}(t), \qquad (24)$$

This is the equation that is used to find the dipole moment. It can be seen as "bDot vector plus quaternion axis crossed with earthMF". Our model doesn't use this exactly because we only want to point one vector, not control our full rotation.

where $\mathbf{H}$ is a positive definite constant matrix and $\epsilon(t)$ is a piecewise continuous positive scalar function satisfying

$$\epsilon(t) = const > 0, \; t \in (kT, (k+1)T), \; k = 1, 2, ... \\ \epsilon(kT) > \epsilon((k+1)T) > 0 \tag{25}$$

is addressed in the sequel.

Theorem 3 *Consider the control law (24) then the satellite, equations (1) to (5) has 4 asymptotically stable local equilibria:*
$\{({}^c\mathbf{\Omega}_{co}, \; {}^c\mathbf{k}_o, \; {}^c\mathbf{i}_o) : (\mathbf{0}, \; \pm^o \mathbf{k}_o, \; \pm^o \mathbf{i}_o)\}$.

Proof of Theorem 3 *For simplicity of notation the equations of satellite (1) to (5) actuated by (6) are represented by*

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), t), \tag{26}$$

and the equations of satellite motion actuated by (24), for constant $\epsilon(t) = \epsilon(kT)$, *are*

$$\dot{\mathbf{x}}(t) = \mathbf{f}_k(\mathbf{x}(t), t). \tag{27}$$

Moreover, the differential equation (26) has solution $\mathbf{x}(t, t_0, \mathbf{x}_0)$ *for the initial condition* $\mathbf{x}(t_0) = \mathbf{x}_0$, *and (27) has solution* $\mathbf{x}_k(t, t_0, \mathbf{x}_0)$ *for the initial condition* $\mathbf{x}_k(t_0) = \mathbf{x}_0$.

Since the kinematic and dynamic differential equations are Lipschitz, the following is true

$$if \; lim_{k \to \infty} \mathbf{f}_k(\mathbf{x}(t), t) = \mathbf{f}(\mathbf{x}(t), t) \; then$$

$$lim_{k \to \infty} \mathbf{x}_k(t, t_0, \mathbf{x}_0) = \mathbf{x}(t, t_0, \mathbf{x}_0),$$

thus

$$if \; lim_{t \to \infty} \mathbf{x}(t, t_0, \mathbf{x}_0) = \mathbf{y}_f \; then$$

$$lim_{t \to \infty} \mathbf{x}_k(t, t_0, \mathbf{x}_0) = \mathbf{y}_f.$$

This means, if $lim_{t \to \infty} \epsilon(t) = 0$, *each trajectory of the satellite actuated by the control law (24) converges to one of the equilibria:*
$\{({}^c\mathbf{\Omega}_{co}, \; {}^c\mathbf{k}_o, \; {}^c\mathbf{i}_o) : (\mathbf{0}, \; \pm^o \mathbf{k}_o, \; \pm^o \mathbf{i}_o)\}$.

It was stated in theorem (3) that the satellite with control law (24) has 4 asymptotically stable equilibria. Local analysis demonstrated that the equilibrium $(\mathbf{0}, \; {}^o\mathbf{k}_o, \; {}^o\mathbf{i}_o)$ is locally stable if $\epsilon(t) < \tilde{\epsilon}$, where $\tilde{\epsilon}$ is the maximum value of ϵ in (16) providing local stability. On the other hand if $\epsilon(t)$ is large enough in Euclidean norm sense so that the quaternion feedback is the most significant component on the r.h.s. of equation (1), then vectors $\mathbf{I}^c\mathbf{\Omega}_{co}(t)$ and $\mathbf{q}(t)$ become parallel

$$\mathbf{I}^c\dot{\mathbf{\Omega}}_{co}(t) \approx -\epsilon(t)^c\mathbf{B} \times ({}^c\mathbf{B} \times \mathbf{q}(t)). \tag{28}$$

Equation (28) implies that for large $\epsilon(t) > 0$

$${}^c\mathbf{\Omega}_{co}^T({}^c\mathbf{B} \times ({}^c\mathbf{B} \times \mathbf{q}, (t))) \leq 0, \tag{29}$$

and

$${}^c\mathbf{\Omega}_{co}^T\mathbf{q} \leq 0, \tag{30}$$

since I is positive definite, and $\tilde{\mathbf{B}}\tilde{\mathbf{B}}$ is positive semidefinite.

If equation (29) is always satisfied, then the satellite is asymptotically stable at the reference $(\mathbf{0}, \; {}^o\mathbf{k}_o, \; {}^o\mathbf{i}_o)$. Proof of this statement is similar to the proof of asymptotic stability of the control law (19). The total energy in the system is defined by (21), where $2\epsilon(1 - q_4)$ is substituted for

$$2\epsilon(t_0)(1 - q_4) \sup_{t \in [0,T)} |eig(\tilde{\mathbf{B}}^T\tilde{\mathbf{B}})|,$$

thus time derivative of E_{tot} is negative definite for each nonzero ${}^c\mathbf{\Omega}_{co}$ and $\mathbf{q}$.

From the analysis carried out so far it follows that for $\epsilon(t_0)$ large enough and $\Delta = |\epsilon((k+1)T) - \epsilon(kt)|$ small enough, the satellite trajectory diverges from equilibria:

$$(\mathbf{0}, \; -^o \mathbf{k}_o, \; {}^o\mathbf{i}_o), \; (\mathbf{0}, \; -^o \mathbf{k}_o, \; -^o \mathbf{i}_o), \; (\mathbf{0}, \; {}^o\mathbf{k}_o, \; -^o \mathbf{i}_o)$$

toward equilibrium $(\mathbf{0}, \; {}^o\mathbf{k}_o, \; {}^o\mathbf{i}_o)$, which is in turn locally stable.

Simulations results have confirmed our hypothesis that the satellite actuated according to (24) is globally asymptotically stable. This control algorithm is especially useful when a boom is in the upside-down position. In practical implementation for the Ørsted satellite $\epsilon(t)$ was chosen continuous and convergent to some constant $\epsilon(t), \; \hat{\epsilon} \leq \tilde{\epsilon}$.

The similarity between the locally and globally stable controllers is striking. The only difference is the gain factor $\epsilon(t)$. This makes implementation exceptionally simple: 1) if the boom is in upright position, apply control law (24) with constant $\epsilon(t), \; \hat{\epsilon} \leq \tilde{\epsilon}$, 2) if the boom is detected to be upside-down, apply (24) with $\epsilon(t)$ gradually decreasing to $\hat{\epsilon}$.

6 SIMULATION RESULTS

The global stabilizing controller developed in this article has been implemented for the Ørsted satellite in the operational phase. A circular orbit with inclination of 96 degrees has been simulated. The geomagnetic field has been computed using 8th order spherical harmonic model. The satellite has the following moments of inertia: $I_{xx} = 178.3 \; kgm^2$, $I_{yy} = 177.8 \; kgm^2$, $I_{zz} = 1.3 \; kgm^2$.

Simulation results are shown in figures (2) to (4). The initial conditions are such that the satellite is in upside-down position corresponding to the equilibrium $(\mathbf{0},\ -{}^{o}\mathbf{k}_o,\ {}^{o}\mathbf{i}_o)$. The controller is quite convincing. It takes less than half an orbit to turn the satellite up, and it is stabilized to the operational region within 6 orbits. This is rather satisfactory considering that the available mechanical torque is less than $10^{-3}\ Nm$, which is only three times as much as the magnitude of the maximum gravity gradient torque for this satellite.

7 CONCLUSIONS

This work is believed to contribute to the development of proportional-derivative feedback control based only on magnetic torquing for low earth orbit satellites. Both locally and globally stabilizing controllers were proposed, and a rigorous stability analysis was carried out. Simulation results showed the proficiency of the new controller in the upside-down configuration, a worst case situation for the satellite.

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Epsilon must go to 0 so that the global control law looks like the local control law as t->inf and the satellite stays where it's supposed to? (Not what this graph shows for some reason)

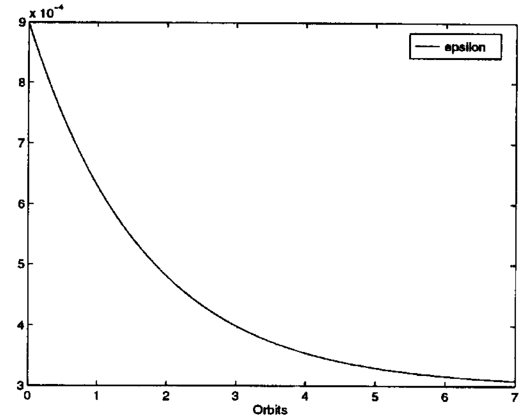

Fig. 2. ϵ converges to $\hat{\epsilon} = 0.0003$, $\mathbf{H} = 0.3\mathbf{E}$.

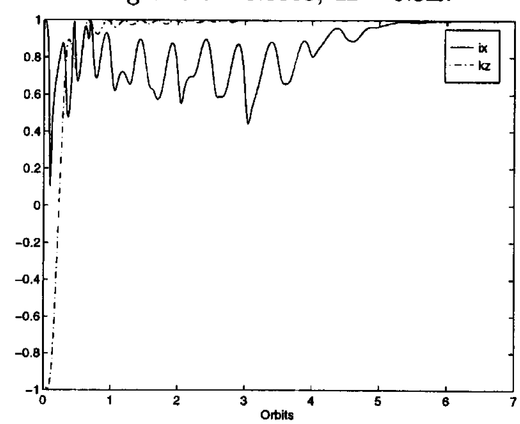

Fig. 3. ${}^{c}\mathbf{k}_{oz}$ characterizes convergence of ${}^{c}\mathbf{k}_o$ toward ${}^{o}\mathbf{k}_o$ (if ${}^{c}\mathbf{k}_{oz} < 0$ satellite is in upside-down position), while ${}^{c}\mathbf{i}_{ox}$ characterizes convergence of ${}^{c}\mathbf{i}_o$ toward ${}^{o}\mathbf{i}_o$

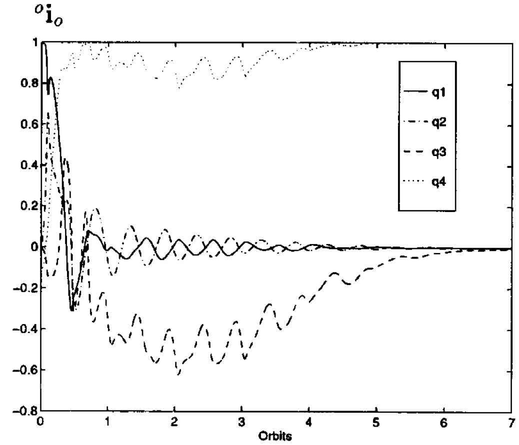

Fig. 4. The attitude quaternion, ${}^{c}_{o}\mathbf{q}$ converges to $[0\ 0\ 0\ 1]^T$ from an upside-down condition.