Solution to Homework 2

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1 Gradient and Hessian of $NLL(\theta)$ for logistic regression

1.1

Given

$$g(z) = \frac{1}{1+e^{-z}} \tag{1}$$

Proof.

$$\frac{\partial g(z)}{\partial z} = \frac{(1+e^{-z}).0 + e^{-z}}{(1+e^{-z})^2} = \frac{e^{-z}}{(1+e^{-z})^2} = \frac{g(z)}{(1-g(z))} \tag{2}$$

□

1.2

For logistic regression, negative log likelihood is

$$NLL(\theta) = -\sum_{i=1}^{m} \left(y_i log(h_\theta(x_i)) + (1-y_i) log(1-h_\theta(x_i))\right) \text{ where } h_\theta(x_i) = g(\theta^T x^i) \tag{3}$$

Proof. Using equation **??** and chain rule for differentiation we have

$$\begin{aligned}
NLL(\theta) &= -\sum_{i=1}^{m} \left(\frac{y_i}{h_\theta(x_i)} h_\theta(x_i)(1-h_\theta(x_i)) \frac{\partial \theta^T x^i}{\partial \theta} - \frac{(1-y_i)}{(1-h_\theta(x_i))} h_\theta(x_i)(1-h_\theta(x_i)) \frac{\partial \theta^T x^i}{\partial \theta} \right) \\
&= -\sum_{i=1}^{m} \left(y_i(1-h_\theta(x_i)) - (1-y_i)h_\theta(x_i)\right) x_i \text{ because } \frac{\partial}{\partial \theta}\theta^T x^i = x^i \qquad (4) \\
&= \sum_{i=1}^{m} \left(h_\theta(x_i) - y_i\right)
\end{aligned}$$

□

1.3

Given

$$
\begin{aligned}
H = X^T S X \text{ where } S = diag(\mu_1 \dots \mu_m) \\
\mu_i = h_\theta(x_i)(1 - h_\theta(x_i)) \text{ for } i = 1 \dots m \\
\text{and } 0 < \mu_i < 1 \text{ for } i = 1 \dots m
\end{aligned} \tag{5}
$$

Proof. For any vector $u \neq 0$ we have,

$$
\begin{aligned}
u^T H u &= u^T (X^T S X) u \\
&= (Xu)^T S (Xu) \\
&= v^T S v \text{ where } v = [v_1 \dots v_m]^T = Xu \neq 0 \text{ since } X \text{ is full rank} \\
&= \sum_{i=1}^{m} v_i^2 \mu_i \\
&> 0 \text{ since } \mu_i \text{ is positive and } v_i \neq 0
\end{aligned} \tag{6}
$$

Thus, H is positive definite. □

2 Regularizing logistic regression

Proof. The maximal likehood and MAP estimates for θ are

$$
\begin{aligned}
\theta_{MLE} &= argmax_\theta \prod_{i=1}^{m} P(y^{(i)} | x^{(i)}; \theta) \\
\theta_{MAP} &= argmax_\theta P(\theta) \prod_{i=1}^{m} P(y^{(i)} | x^{(i)}; \theta) \text{ where } P(\theta) \sim N(0, \alpha^2 I)
\end{aligned} \tag{7}
$$

Equation **??** can be rewritten using log likelihood $LL(\theta)$:

$$
\begin{aligned}
\theta_{MLE} &= argmax_\theta LL(\theta) \text{ where } LL(\theta) = \sum_{i=1}^{m} log(P(y^{(i)} | x^{(i)}; \theta)) \\
\theta_{MAP} &= argmax_\theta log(P(\theta)) + LL(\theta) \\
&= argmax_\theta K - \frac{d}{2\alpha^2} \theta^T \theta + LL(\theta) \text{ where } K \text{ is constant. This follows from } P(\theta) \sim N(0, \alpha^2 I) \\
&= argmax_\theta LL(\theta) - \frac{d}{2\alpha^2} \|\theta\|_2^2
\end{aligned} \tag{8}
$$

Now,

$$
\begin{aligned}
LL(\theta_{MAP}) - \frac{d}{2\alpha^2}\|\theta_{MAP}\|_2^2 &\geq LL(\theta_{MLE}) - \frac{d}{2\alpha^2}\|\theta_{MLE}\|_2^2 && \text{from definition for } \theta_{MAP} \\
&\geq LL(\theta_{MAP}) - \frac{d}{2\alpha^2}\|\theta_{MLE}\|_2^2 && \text{from definition for } \theta_{MLE} \\
\implies \frac{d}{2\alpha^2}\|\theta_{MAP}\|_2^2 &\leq \frac{d}{2\alpha^2}\|\theta_{MLE}\|_2^2 \\
\implies \|\theta_{MAP}\|_2^2 &\leq \|\theta_{MLE}\|_2^2 \\
\implies \|\theta_{MAP}\|_2 &\leq \|\theta_{MLE}\|_2
\end{aligned} \tag{9}
$$

$\square$

3 Implementing logistic regression

3.1 Part A

Implementing logistic regression : the sigmoid function

Implemented `sigmoid` method in `utils.py`.

```
def sigmoid (z):
    sig = np.zeros(z.shape)
    # Your code here
    sig = 1/(1+np.exp(-1*z))
    # End your ode
    return sig
```

Cost function and gradient of logistic regression

Implemented `loss` and `grad_loss` methods in the `LogisticRegressor` class in `logistic_regressor.py`.

```
J=-1*np.sum(np.log(utils.sigmoid(np.dot(theta,X.T)))*y+np.log(1-utils.sigmoid(np.dot(theta,X.T)))*(1-y))/m
```

```
grad = np.dot((utils.sigmoid(np.dot(theta,X.T))-y),X)/m
```

Prediction using a logistic regression model

Implemented `predict` method in the `LogisticRegressor` class in `logistic_regressor.py`.

```
y_pred = utils.bin_features(utils.sigmoid(np.dot(self.theta,X.T))-0.5)
```

Add code in `ex1.py`.

```
pred_prob = utils.sigmoid(np.dot(log_reg1.theta,np.array([1,45,85]).T))
```

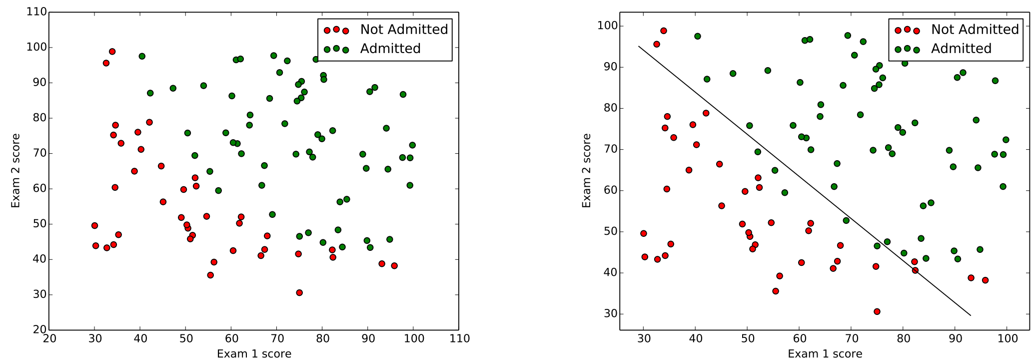
```
accuracy  = 1-float(np.sum(np.abs(predy-y)))/y.shape[0]
```

Result

Run `ex1.py` in the shell:

```
Plotting data with green circle indicating (y=1) examples and red circle indicating (y=0) examples ...
Loss on all-zeros theta vector (should be around 0.693) =  0.69314718056
Gradient of loss wrt all-zeros theta vector (should be around [-0.1, -12.01, -11.26]) =  [ -0.1         -12.00921659  -11.26284221
Optimization terminated successfully.
         Current function value: 0.203498
         Iterations: 19
         Function evaluations: 20
         Gradient evaluations: 20
```

```
Theta found by fmin_bfgs:  [-25.16056945    0.20622963    0.20146073]
Final loss =  0.203497702351
For a student with 45 on exam 1 and 85 on exam 2, the probability of admission =  0.776246678481
Accuracy on the training set =  0.89
Theta found by sklearn:  [[-25.15293066    0.20616459    0.20140349]]
```



(a) The training data

(b) The decision boundary

Figure 1

All as expected.

3.2 Part B

Cost function and gradient for regularized logistic regression

Implemented `loss` and `grad_loss` methods in the `RegLogisticRegressor` class in `logistic_regressor.py`.

```
J=-1*np.sum(np.log(utils.sigmoid(np.dot(theta,X.T)))*y+np.log(1-utils.sigmoid(np.dot(theta,X.T)))*(1-y))/m
+reg*np.sum(theta[1:]**2)/2/m
```

```
grad = np.dot((utils.sigmoid(np.dot(theta,X.T))-y),X)/m
```

Prediction using the model

Implemented `predict` method in the `LogisticRegressor` class in `logistic_regressor.py`.

```
y_pred = utils.bin_features(utils.sigmoid(np.dot(self.theta,X.T))-0.5)
```

Add code in `ex1_reg.py`.

```
accuracy  = 1-float(np.sum(np.abs(predy-y)))/y.shape[0]
```

Varying

Add code in `ex1_reg.py`.

```
for reg in [0,1.0,100.0]:
...
```

Exploring L1 and L2 penalized logistic regression

Add code in `ex1_reg.py`.

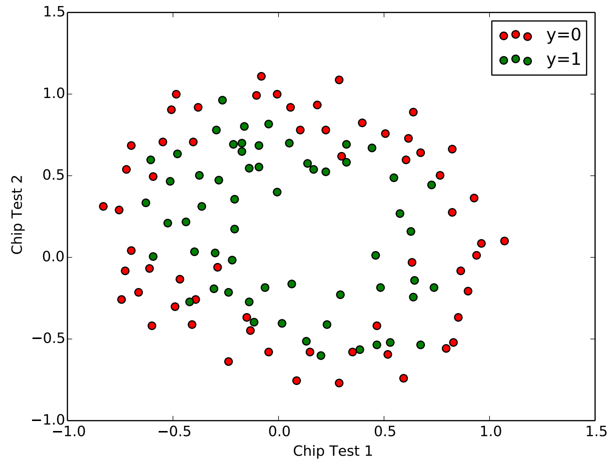
```
for reg in [0.1,0.5,1.0,5.0,10.0]:
...
```

Result

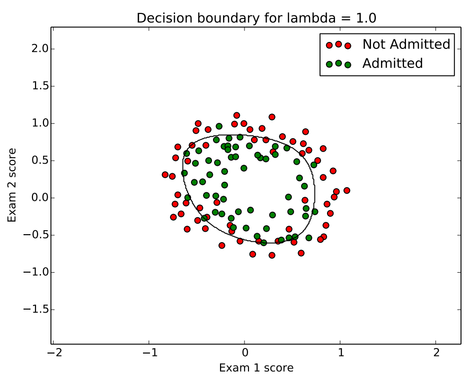
Run `ex1_reg.py` in the shell:

```
Plotting data with green circle indicating (y=1) examples and red circle indicating (y=0) examples ...
Optimization terminated successfully.
         Current function value: 0.224569
         Iterations: 546
         Function evaluations: 547
         Gradient evaluations: 547
Theta found by fmin_bfgs:with reg = 0.0 [   35.10191556    44.11916104    69.27187135  -344.27909285  -198.23463329
  -184.2284154   -295.82041756  -621.7326024   -510.84919909  -328.31173228
  1094.70040538  1269.58583356  1757.74907248   900.93789211   436.58879589
   471.12031352  1236.23835236  1822.81976807  1929.66695582  1131.05273288
   463.79908073 -1142.11739081 -2020.95888645 -3463.39935641 -3484.5099411
 -3252.26696072 -1546.00910831  -510.41253513]
Final loss =  0.224568734073
Accuracy on the training set =  0.915254237288
Optimization terminated successfully.
         Current function value: 0.529003
         Iterations: 47
         Function evaluations: 48
         Gradient evaluations: 48
Theta found by fmin_bfgs:with reg = 1.0 [ 1.27268739  0.62557016  1.1809665  -2.01919822 -0.91761468 -1.43194199
  0.12375921 -0.36513086 -0.35703388 -0.17485805 -1.45843772 -0.05129676
 -0.61603963 -0.2746414  -1.19282569 -0.24270336 -0.20570022 -0.04499768
 -0.27782709 -0.29525851 -0.45613294 -1.04377851  0.02762813 -0.29265642
  0.01543393 -0.32759318 -0.14389199 -0.92460119]
Final loss =  0.4624583499
Accuracy on the training set =  0.830508474576
Optimization terminated successfully.
         Current function value: 0.621828
         Iterations: 27
         Function evaluations: 28
         Gradient evaluations: 28
Theta found by fmin_bfgs:with reg = 5.0 [  5.26750986e-01   8.29000343e-02   3.51747540e-01  -7.63457302e-01
  -2.16894856e-01  -4.73447564e-01  -6.09029000e-02  -1.03822986e-01
  -1.12858258e-01  -1.35090185e-01  -5.64095465e-01  -2.15435150e-02
  -2.05546999e-01  -5.63112422e-02  -4.64839657e-01  -1.56171750e-01
  -6.57923576e-02  -3.37041246e-02  -8.58443620e-02  -7.72930748e-02
  -2.70622817e-01  -4.14876764e-01  -1.61078926e-03  -1.01471865e-01
   2.46885978e-05  -1.10366999e-01  -2.49549621e-02  -4.24374938e-01]
Final loss =  0.577292805563
Accuracy on the training set =  0.813559322034
Theta found by sklearn with L2 reg: with reg = 0.1 [ 2.65855183  1.76427994  2.91364412 -4.03385629 -3.34849756 -4.0181188
  0.76777199 -1.08648166 -0.47195071 -0.4774888  -3.27598952  0.54686285
 -1.80180787 -1.17932445 -2.79104067 -0.62127841 -0.4711418   0.61454641
 -1.14697992 -1.20796935 -0.10569617 -2.66246949  0.45857402 -0.76144039
  0.43744164 -1.17502213 -0.93753591 -1.20049576]
Loss with sklearn theta:  0.353830932899
Theta found by sklearn with L1 reg:  [ 4.00212755  2.56718307  3.56329211 -7.68389893 -6.8113113  -8.66237057
  0.59188338 -0.20095689  0.          0.          0.          2.445711    0.
  0.         -1.7055534   0.          0.          0.36343109 -0.67166876
  0.          0.         -6.72105236  0.          0.          0.          0.
 -0.06140466  0.        ]
Loss with sklearn theta:  0.33643326718
Theta found by sklearn with L2 reg: with reg = 0.5 [  1.57595698e+00   9.41010433e-01   1.64795490e+00  -2.54342145e+00
  -1.48161546e+00  -1.93049559e+00   2.76332771e-01  -5.72906659e-01
  -4.89795242e-01  -1.92532820e-01  -1.94870434e+00  -1.23689578e-02
  -8.94257808e-01  -4.62996758e-01  -1.60813528e+00  -3.02851338e-01
  -2.97201842e-01   1.63127043e-03  -4.40179153e-01  -4.71492765e-01
  -4.92042624e-01  -1.43161435e+00   8.76115308e-02  -4.19765000e-01
   5.79982479e-02  -4.93712970e-01  -2.66131793e-01  -1.15377891e+00]
Loss with sklearn theta:  0.422346235529
Theta found by sklearn with L1 reg:  [ 2.74906778  1.55163984  2.19592256 -5.59295394 -3.43490014 -4.93205464
  0.          0.          0.          0.         -2.27853613  0.          0.
  0.         -2.11210668  0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.          0.
  0.        ]
Loss with sklearn theta:  0.37758866439
Theta found by sklearn with L2 reg: with reg = 1.0 [ 1.1421394   0.60141117  1.16712554 -1.87160974 -0.91574144 -1.26966693
  0.12658629 -0.3686536  -0.34511687 -0.17368655 -1.42387465 -0.04870064
 -0.60646669 -0.26935562 -1.16303832 -0.24327026 -0.20702143 -0.04326335
 -0.28028058 -0.286921   -0.46908732 -1.03633961  0.02914775 -0.29263743
  0.01728096 -0.32898422 -0.13801971 -0.93196832]
Loss with sklearn theta:  0.46843403006
Theta found by sklearn with L1 reg:  [ 1.86960916  0.68659701  1.2804614  -4.86238046 -1.62173321 -2.34246341
  0.          0.          0.          0.          0.          0.          0.
  0.         -2.36718825  0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.          0.
  0.        ]
```

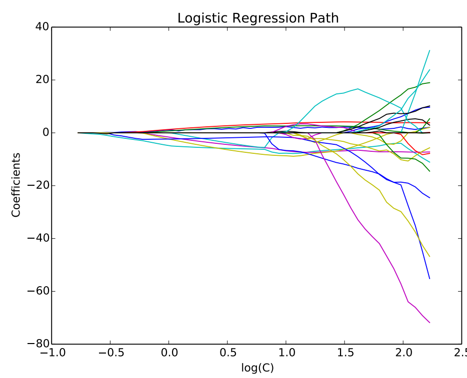
```
Loss with sklearn theta:  0.438149813414
Theta found by sklearn with L2 reg: with reg = 5.0 [  4.01129749e-01   7.86245631e-02   3.59319093e-01  -6.87459023e-01
  -2.25095774e-01  -3.94964659e-01  -5.66501493e-02  -1.02117116e-01
  -1.07302805e-01  -1.26464125e-01  -5.35417213e-01  -2.40646947e-02
  -1.94101075e-01  -5.88107997e-02  -4.39124052e-01  -1.52471333e-01
  -6.51808095e-02  -3.18085564e-02  -8.46692847e-02  -7.44533648e-02
  -2.68017771e-01  -4.01221135e-01  -2.79125965e-03  -9.76575762e-02
  -6.04215794e-04  -1.07053968e-01  -2.56309837e-02  -4.16212768e-01]
Loss with sklearn theta:  0.58378743523
Theta found by sklearn with L1 reg:  [ 0.          0.          0.         -0.26092308  0.          0.
0.
  0.          0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.          0.
  0.          0.          0.          0.          0.          0.          0.        ]
Loss with sklearn theta:  0.681052511162
Theta found by sklearn with L2 reg: with reg = 10.0 [ 0.21469236 -0.00761966  0.17611687 -0.4012903  -0.11745553 -0.23188083
 -0.06668596 -0.05584267 -0.06215384 -0.09710193 -0.31766892 -0.01468057
 -0.10913398 -0.03014551 -0.26764027 -0.11186999 -0.03627398 -0.02114738
 -0.04753651 -0.04038118 -0.18117647 -0.24308692 -0.00364108 -0.05525352
 -0.00101451 -0.06094026 -0.01293964 -0.26287463]
Loss with sklearn theta:  0.621592068026
Theta found by sklearn with L1 reg:  [ 0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.
0.
  0.  0.  0.  0.  0.  0.  0.  0.  0.  0.]
Loss with sklearn theta:  0.69314718056
```



(a) The training data

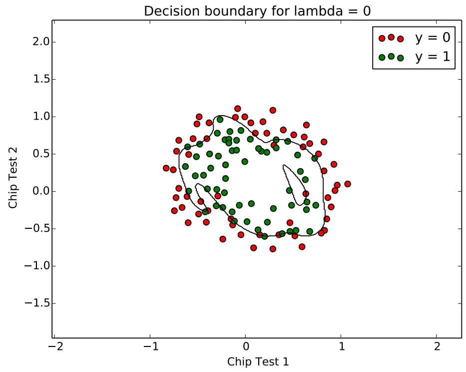


(b) Training data with decision boundary of reg = 1 with sk(l2)

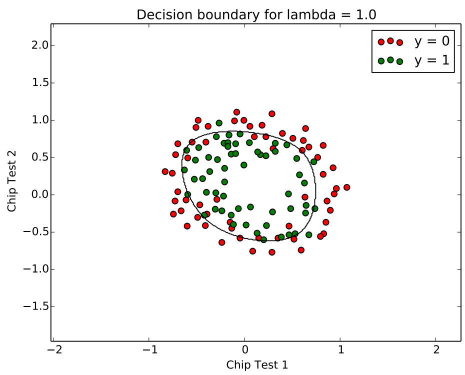


(c) The regression path of reg = 1

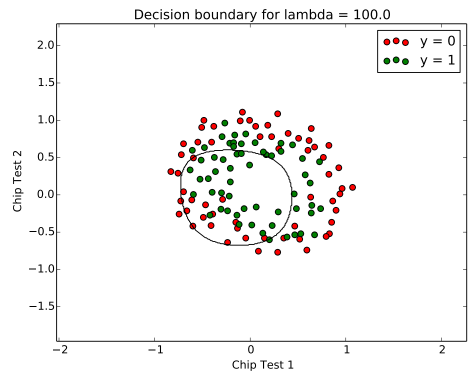
Figure 2



(a) Overfitted boundary



(b) Properly fitted boundary



(c) Underfitted boundary

Figure 3: Boundary with different reg

All as expected. We can see overfitted and underfitted boundary when =0 and 100 respectively. When increases from 0.1 to 10, the output of l1 reg decreases to 0 more quickly than l2 reg, and the cost of l1 is higher than l2.

3.3 Part C

Feature transformation

———

Implemented `stdFeatures`, `logTransformFeatures` and `binarizeFeatures` method in `utils.py`.

```
def log_features(X):
    logf = np.zeros(X.shape)
    # Your code here
    logf = np.log(X+0.1)
    # End your ode
    return logf
def bin_features(X):
    tX = np.zeros(X.shape)
    # your code here
    tX = np.array(X>0,dtype=int)
```

```
    # end your code
    return tX
```

Feature transformation

Implemented `select_lambda_crossval` method in `utils.py`.

```
def select_lambda_crossval(X,y,lambda_low,lambda_high,lambda_step,penalty):

    best_lambda = lambda_low

    # Your code here
    # Implement the algorithm above.
    best_accu=0
    kf=cross_validation.KFold(y.shape[0],10)
    l=lambda_low
    while l<=lambda_high:
        accu=0
        for train_index, test_index in kf:
            xx=X[train_index]
            yy=y[train_index]
            xt=X[test_index]
            yt=y[test_index]
            if penalty == "l2":
                lreg = linear_model.LogisticRegression(penalty=penalty,C=1.0/best_lambda, solver='lbfgs',fit_intercept=True)
            else:
                lreg = linear_model.LogisticRegression(penalty=penalty,C=1.0/best_lambda, solver='liblinear',fit_intercept=True)
            lreg.fit(xx,yy)
            predy = lreg.predict(xt)
            accu=accu+np.mean(predy==yt)
        accu=accu/10
        if accu>best_accu:
            best_accu=accu
            best_lambda=l
        l=l+lambda_step
    # end your code

    return best_lambda
```

Result

Run `ex1_spam.py` in the shell:

```
L2 Penalty experiments -----------
best_lambda =  0.1
Coefficients =  [-4.86311314] [[ -2.74144964e-02  -2.25297922e-01   1.21840741e-01   2.29363175e+00
    2.70425757e-01   2.32851060e-01   9.28595406e-01   2.95200115e-01
    1.62205894e-01   6.78255634e-02  -8.32602218e-02  -1.60373332e-01
   -4.72248839e-02   1.07676572e-02   1.87904870e-01   8.19771733e-01
    5.09529185e-01   3.98709124e-02   2.67729599e-01   3.47046562e-01
    2.60498968e-01   3.64607069e-01   7.25020572e-01   1.96728174e-01
   -3.15395736e+00  -4.03134022e-01  -1.25451015e+01  -6.16564148e-02
   -1.56114501e+00  -5.51433441e-02  -3.00843039e-02   4.07264561e-01
   -3.68156729e-01  -1.43612275e+00  -5.87186446e-01   4.44293905e-01
    4.23160725e-02  -1.56897114e-01  -4.55330242e-01  -1.02250011e-01
   -3.54273381e+00  -1.72944271e+00  -4.37530043e-01  -1.05999937e+00
   -9.18599054e-01  -1.75490185e+00  -1.67475688e-01  -9.56877080e-01
   -3.65654247e-01  -1.36535823e-01  -6.58693028e-02   2.06714195e-01
    1.70694494e+00   1.21460221e+00  -3.35271575e-01   1.56142019e+00
    3.68774998e-01]]
Accuracy on set aside test set for  std  =  0.9296875
best_lambda =  0.1
Coefficients =  [-1.40695708] [[-0.1748124  -0.05135689 -0.12339923  1.2828822   0.46456529  0.26013964
   0.96479518  0.48101469  0.11530886  0.15833164 -0.10051294 -0.1852999
  -0.33761946  0.24045718  0.42765064  0.52228884  0.60140119 -0.10265903
   0.12174456  0.29864954  0.20432531  0.18931018  0.54668794  0.56311458
  -1.37213787  0.09174394 -3.52244763  0.25886851 -0.20749972  0.07686118
   0.40469082  1.04149701 -0.15729015  1.29670296 -0.45104347  0.47004354
  -0.35705768  0.36273978 -0.33026736  0.03533685 -0.31829402 -0.93876559
  -0.68715245 -0.80497352 -0.44309598 -0.93099029  0.18540049 -0.84891725
  -0.43476012 -0.12763685  0.19196783  0.7830564   1.47892381  0.02683296
   0.69607044  0.04929847  0.36289482]]
Accuracy on set aside test set for  logt  =  0.943359375
best_lambda =  0.1
Coefficients =  [-1.77183673] [[-0.27016277 -0.13652572 -0.44105224  0.19593277  1.09940287  0.28381492
   2.39371265  0.89847243  0.26535465  0.435632   -0.39728324 -0.42882819
  -1.07464275  0.32304905  0.66670341  1.58604221  1.10348806 -0.17358757
   0.2491685   0.767151    0.78417213  1.4247451   1.03887036  1.61862715
  -2.93738195 -0.23315908 -5.85002477  1.26265761 -0.83080782 -0.06708161
  -1.65512023 -1.71956092 -0.92718438  0.44756264 -0.73938996  0.51818083
  -1.06394957  1.07675048 -0.98242564 -0.32995306 -2.96965659 -2.5398783
  -1.21092653 -2.16550138 -0.85215133 -2.56736788  0.03483552 -2.02741466
```

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  -0.37386944  0.22126778 -0.2152468   1.27339759  1.5956514  -0.03237894
  -0.04463836 -0.04463836 -0.04463836]]
Accuracy on set aside test set for  bin  =  0.92578125
L1 Penalty experiments -----------
best_lambda =  2.1
Coefficients =  [-2.72840332] [[ -1.93809370e-02  -1.90165421e-01   1.26218367e-01   4.56664238e-01
    2.58423508e-01   2.02669188e-01   9.10326804e-01   2.92930859e-01
    1.58447578e-01   5.59232984e-02  -5.60502424e-02  -1.51431784e-01
   -2.86214584e-02   1.08977734e-02   1.74617696e-01   7.93158066e-01
    4.83667987e-01   5.45908763e-02   2.63677888e-01   2.67168004e-01
    2.50977911e-01   3.57024814e-01   7.25613219e-01   2.17779610e-01
   -2.75446443e+00  -3.78683120e-01  -6.94538791e+00  -3.94769408e-02
   -6.47743850e-01  -9.90081163e-03   0.00000000e+00   0.00000000e+00
   -3.58637375e-01   0.00000000e+00  -1.57522300e-01   3.32751667e-01
    4.09903100e-03  -1.34993641e-01  -3.79065669e-01  -6.46707542e-02
   -6.35617521e-01  -1.10564315e+00  -2.72601780e-01  -7.83678731e-01
   -8.24016676e-01  -1.45018868e+00  -1.23169859e-01  -7.13330756e-01
   -3.01341655e-01  -1.31923358e-01  -5.95346647e-02   2.09510366e-01
    1.67669131e+00   5.59770230e-01  -7.64465943e-02   9.25879899e-01
    3.43024768e-01]]
Accuracy on set aside test set for  std  =  0.92578125
best_lambda =  5.1
Coefficients =  [ 0.] [[-0.02237023  0.          -0.05154852  0.16878365  0.42402059  0.15412008
   0.93326218  0.43544729  0.02545247  0.08676809  0.          -0.18887613
  -0.14290738  0.14542169  0.03694076  0.5082957   0.47578605  0.
   0.0795449   0.12335347  0.20266011  0.12228524  0.53917235  0.52854023
  -1.08571406  0.          -1.54138485  0.02033182  0.          0.          0.
   0.          -0.04969466  0.          0.          0.31653286 -0.36289535
   0.          -0.11699187  0.          0.          -0.55663169  0.
  -0.35628821 -0.33859048 -0.77802727  0.          -0.23452883 -0.02391159
   0.          0.          0.75476966  1.30466053  0.          0.55834284
   0.15493247  0.16648669]]
Accuracy on set aside test set for  logt  =  0.940755208333
best_lambda =  0.1
Coefficients =  [-0.23362403] [[-0.27341794 -0.1210642  -0.43824938  0.13312448  1.10360255  0.29869675
   2.42028679  0.90527665  0.26731351  0.44562181 -0.41187711 -0.42707819
  -1.09389899  0.3194352   0.66224276  1.59519397  1.13534114 -0.1819026
   0.24500927  0.80099193  0.78677071  1.41392778  1.03768696  1.65676144
  -3.01060339 -0.19787025 -6.21759202  1.30722175 -0.83095457 -0.04058604
  -1.74016234 -1.90105145 -0.9445934   0.20424823 -0.72605373  0.52532324
  -1.0517627   1.10852783 -0.99529849 -0.30705931 -4.06519585 -2.61231263
  -1.25451571 -2.21821251 -0.8609704  -2.58705802  0.          -2.10652216
  -0.3809371   0.22010838 -0.20768955  1.28165763  1.61412763 -0.0199922
  -0.23270361 -1.0056229  -0.43930022]]
Accuracy on set aside test set for  bin  =  0.92578125
```

All as expected.

We can see that the l1 regularization could exclude unrelated features better. For 3 normalization mathods, the logt method works the best.