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Digital Foundations: Mastering Number Systems Fundemental To Computing

1ST ADDITION

TIKUM

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CHAPTER 1: NUMBERING SYSTEMS & MACHINE LANGUAGE

INTRODUCTION TO NUMBERING SYSTEMS

Number Systems

Binary: The foundation of all modern computing. A base 2 system Uses 0's and 1's

Octal: A base 8 system that uses 8 digits from 0-7

Hexadecimal: A base 16 numbering system that uses 16 symbols 0-9 and then A-F

Decimal: A base 10 numbering system that uses the numbers 0-9

Corporate newsletter

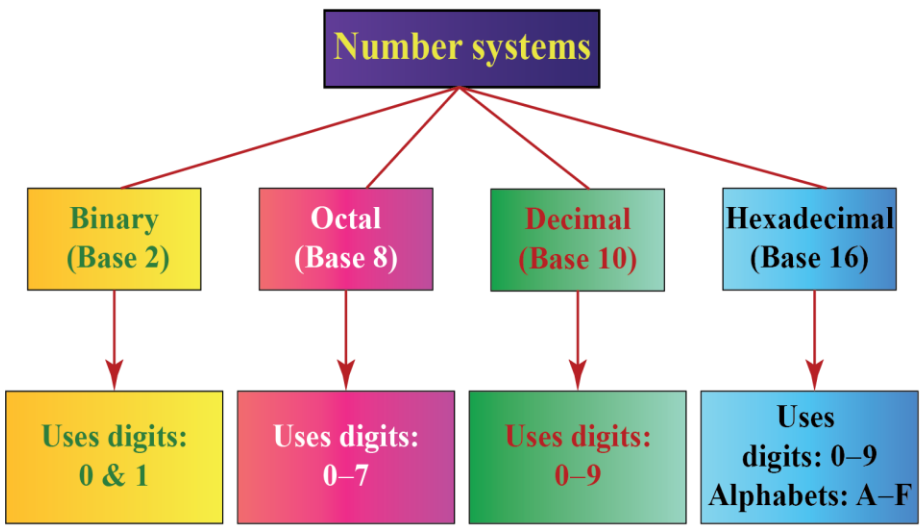

Figure 1: 4 major numbering systems including their bases and alphabets

Introduction

In computer science, numbers are the foundation of everything. Numbers are used in every aspect including from the algorithms of each program, to the lower level compiling of those algorithms. Computers use different number systems to perform calculations, store data, and represent information (Jones, 2024). While humans commonly use the decimal (base-10) system, computers operate using the binary (base-2) system. Additionally, hexadecimal (base-16) and octal (base-8) systems are often used to represent large binary numbers more easily (Jones, 2024).

In this section, we will explore the four main number systems—**binary**, **decimal**, **octal**, and **hexadecimal**—and learn how they are used in computing.

Section 1

DECIMAL SYSTEM (BASE-10)

The Decimal System is the number system we use in everyday life, and is how we typically count. It is a **base-10** system, meaning it uses ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Each place value represents a power of 10 (Jones, 2024). For example, the number 348 in decimal can be broken down as:

$$348_{10} = (3\ x\ 10^2) + (4\ x\ 10^1) + (8\ x\ 10^0)$$

This system is intuitive for humans because we commonly use it every day in our daily functionalities such as math, money, and counting.

Knowledge Check

What number system do Humans most commonly use?

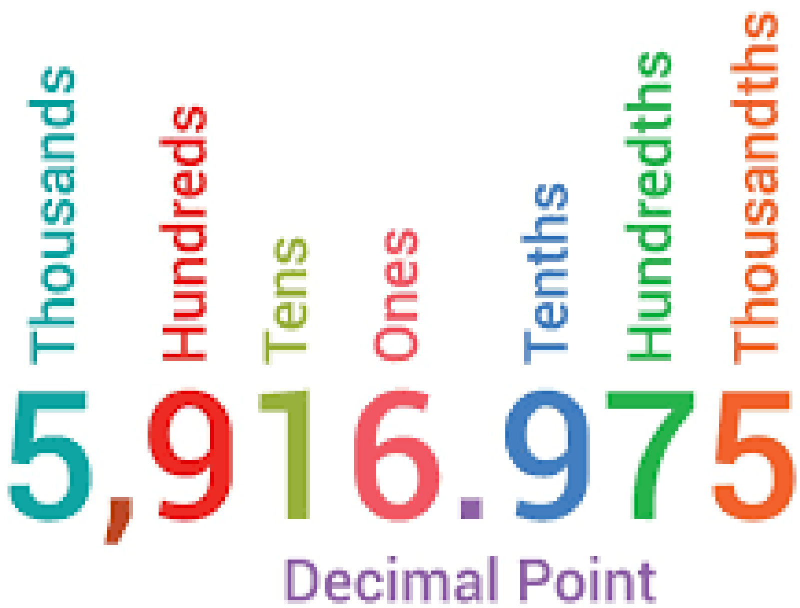

Figure 2: A decimal number broken down

The decimal number system is mainly used on the face of programs, and is then interpreted by the computer through the use of another numbering system. This being the Binary numbering system.

Section 2

UNDERSTANDING THE BIANRY SYSTEM (BASE-2)

What is Binary?

In binary, each digit is referred to as a **bit**, which is short for **binary digit**. A binary number is simply a series of bits. The rightmost bit is the **least significant bit (LSB)**, and the leftmost bit is the **most significant bit (MSB)** (Lenovo, 2024).

- **Example**:
  The binary number 1011_2 is composed of four bits, with the LSB being **1** and the MSB being **1**.

Binary numbers work on a **base-2** system, meaning each position represents a power of 2, unlike the decimal system (base-10) where each position represents a power of 10 (Nicholls, 2022).

- In the number 1011_2, the positions represent: $2^3 2^2 2^1 2^0$
- Using the powers of 2, we can convert 1011_2 to its decimal (base-10) equivalent: $(1x2^3) + (0x2^3) + (1x2^3) + (1x2^3) = 8 + 0 + 2 + 1 = 11_{10}$

Thus, $1011_2 = 11_{10}$

Binary is a big part of computer logic and operations. Computers use binary because it simplifies the design of circuits. With only two states (0 and 1), electrical switches can easily represent data. This simplicity allows computers to process complex operations with great speed and efficiency (siddhu86t9, 2024).

Denary number	Binary number
1	1
2	10
3	11
4	100
5	101
6	110
7	111
8	1000
9	1001
10	1010

Figure 3: The binary equivalent of denary numbers

Conversion Example: Decimal to Binary

To convert a decimal number to binary, repeatedly divide the number by 2, noting the remainders. The binary number is formed by reading the remainders from bottom to top.

Example: Convert 19_{10} to binary.

Divide 19 by 2: 19 ÷ 2 = 9 with a remainder 1.

Divide 9 by 2: : 9 ÷ 2 = 4 with a remainder 1.

Divide 4 by 2: : 4 ÷ 2 = 2 with a remainder 0

Divide 2 by 2: : 2 ÷ 2 = 1 with a remainder 0

Divide 1 by 2: : 1 ÷ 2 = 0 with a remainder 1.

Read the remainders from bottom to top: $19_{10} = 10011_2$.

Knowledge Check!

Example Problems: Conversion Between Binary and Decimal

Convert 10011_2 to decimal.

Convert 29_{10} to binary.

Convert 47_{10} to binary.

Convert 1110_2 to decimal.

Convert 1100_2 to decimal.

Convert 25_{10} to binary

- BINARY NUMBERS ARE THE FOUNDATION FOR ALL COMPUTER SYSTEMS
- BINARY, DECIMALS AND OTHER NUMBERS CAN BE CONVERTED AMONGST EACH OTHER

Section 3

EXPLORING THE HEXADECIMAL (BASE-16) & OCTAL (BASE-8) SYSTEM

While binary is fundamental to computer systems, other number systems like **hexadecimal** (base 16) and **octal** (base 8) are also commonly used in computing. These systems provide more compact ways to represent large binary numbers and are frequently used in programming, memory addressing, and color codes in web design

What is Hexadecimal?

The hexadecimal system (or hex for short) is a base-16 numbering system. It uses the digits 0-9 and the letters A-F, where:

- A = 10
- B = 11
- C = 12
- D = 13
- E = 14
- F = 15

Why Use Hexadecimal?

Hexadecimal is useful because it can represent large binary numbers in a more readable form for humans. Each hex digit corresponds directly to four binary digits knowm as a **nibble**, making conversion between hex and binary straightforward.

- For example, the hex number $3A7F_{16}$ can be converted to binary by converting each hex digit individually:

  3 = 0011_2, A = 1010_2, 7 = 0111_2, F = 1111_2

  Therefore, $3A7F_{16} = 0011\ 1010\ 0111\ 1111_2$

We can separate each digit and find its corresponding binaryt value, then combine the results to find the conversion. In our example, we knew A was equal to 10, so we found the binary equivalent of 10, and so on and so forth.

Lets convert Hexadecimal to Decimal Next.

Conversion Example: Hexadecimal to Decimal

Example: Convert $2B_{16}$ to decimal.

1. Write the powers of 16:

   $2B_{16}$ = $(2\ x\ 16^1)$ + $(B\ x\ 16^0)$

2. Substitute values for the digits: = $(2\ x\ 16) + (11\ x\ 1) = 32 + 11 =\ 43_{10}$

3. Therefore, $2B_{16} =\ 43_{10}$

What is Octal?

The **octal** system is a **base-8** numbering system, using the digits **0-7**. Similar to hexadecimal, octal is useful for simplifying large binary numbers, as each octal digit represents three binary digits.

- For example, the octal number 17_8 corresponds to the binary number: 1 = $001_2, 7 = 111_2$ Therefore, $17_8 =\ 001\ 111_2$.

Since octal uses the numbers 0 – 7, we can represent each of its digits using just 3 binary bits. This means, similarly to hex, we can seperate each individual digit, and convert it to binary, before combing to find the final result. Now lets Convert Ocatl to decimal:

Conversion Example: Octal to Decimal

Example: Convert 34_8 to decimal.

1. Write the powers of 8:

   34_8 = $(3\ x\ 8^1)$ + $(4\ x\ 8^0)$

2. Calculate the decimal equivalent:

$$(3\ x\ 8) + (4\ x\ 1) = 24 + 4 =\ 28_{10}$$

3. Therefore, $34_8 =\ 28_{10}$

Why Use Octal and Hexadecimal?

- **Hexadecimal**: Widely used in **programming** and **memory addressing** because it's compact and easy to convert to binary.

- **Octal**: Historically used in older computing systems and still useful in some embedded systems and file permissions

Visual Representation: Hexadecimal, Octal, Binary, and Decimal Conversions

Decimal	**Binary**	**Hexadecimal**	**Octal**
0	0000	0	0
1	0001	1	1
2	0010	2	2
3	0011	3	3
4	0100	4	4
5	0101	5	5
6	0110	6	6
7	0111	7	7
8	1000	8	10
9	1001	9	11
10	1010	A	12
11	1011	B	13
12	1100	C	14
13	1101	D	15
14	1110	E	16
15	1111	F	17

Knowledge Check!

Example Problems: Conversion Between Binary, Hexadecimal, Octal, and Decimal

Convert $7D_{16}$ to decimal.

Convert 145_8 to decimal.

Convert 205_{10} to hexadecimal.

Convert 1001_2 to octal.

Convert $4F_{16}$ to binary.

Convert 72_8 to decimal.

Convert 195_{10} to hexadecimal.

Convert 11001010_2 to octal.

SUMMARY:

- Binary numbers are the building blocks of computer systems.
- Learning to convert between binary and decimal is essential for understanding how computers process data.
- Hexadecimal and octal systems provide compact representations of binary numbers.
- Both are essential for simplifying complex binary numbers in programming, memory addressing, and color coding.
- Learning to convert between number systems is key to understanding how computers interpret data.

Number Systems		
System	Base	Digits
Binary	2	0 1
Octal	8	0 1 2 3 4 5 6 7
Decimal	10	0 1 2 3 4 5 6 7 8 9
Hexadecimal	16	0 1 2 3 4 5 6 7 8 9 A B C D E F

ANSWER SHEET

Section 2:

1. **Convert $\mathbf{10011_2}$ to decimal.**

   $10011_2 = (1\ x\ 2^4) + (0\ x\ 2^3) + (0\ x\ 2^2) + (1\ x\ 2^1) + (1\ x\ 2^0)$

   $16 + 0 + 0 + 2 + 1 = 19_{10}$

   Answer: 19_{10}

2. **Convert $\mathbf{29_{10}}$ to binary.**

   Divide by 2 repeatedly:

   - 29 ÷ 2 = 14 remainder 1
   - 14 ÷ 2 = 7 remainder 0
   - 7 ÷ 2 = 3 remainder 1
   - 3 ÷ 2 = 1 remainder 1
   - 1 ÷ 2 = 0 remainder 1

   The remainders from bottom to top: 11101_2

   Answer: 11101_2

3. **Convert 47_{10} to binary.**

   Divide by 2 repeatedly:

   - 47 ÷ 2 = 23 remainder 1
   - 23 ÷ 2 = 11 remainder 1
   - 11 ÷ 2 = 5 remainder 1
   - 5 ÷ 2 = 2 remainder 1
   - 2 ÷ 2 = 1 remainder 0
   - 1 ÷ 2 = 0 remainder 1

   The remainders from bottom to top: 101111_2

   Answer: 101111_2

4. **Convert 1110_2 to decimal.**

   1110_2=$(1\ x\ 2^3)$ + $(1\ x\ 2^2)$ + $(1\ x\ 2^2)$ + $(0\ x\ 2^0)$

   $8 + 4 + 2 + 0 = 14_{10}$

   Answer: 14_{10}

5. **Convert 1100_2 to decimal.**

   1100_2=$(1\ x\ 2^3)$ + $(1\ x\ 2^2)$ + $(0\ x\ 2^2)$ + $(0\ x\ 2^0)$

   $8 + 4 + 0 + 0 = 12_{10}$

   Answer: 12_{10}

6. **Convert 25_{10} to binary**

   Divide by 2 repeatedly:

   - 25 ÷ 2 = 12 remainder 1
   - 12 ÷ 2 = 6 remainder 0
   - 6 ÷ 2 = 3 remainder 0

- $3 \div 2 = 1$ remainder 1
- $1 \div 2 = 0$ remainder 1

The remainders from bottom to top: 11001_2

Answer: 11001_2

Section 3:

7. **Convert $7D_{16}$ to decimal:**

$7D_{16}$=$(7\ x\ 16^1)$ + $(13\ x\ 16^0)$

= $(7\ x\ 16) + (13\ x\ 1) = 112 + 13 = 125_{10}$

Answer: 125_{10}

8. **Convert 145_8 to decimal:**

145_8=$(1\ x\ 8^2)$ + $(4\ x\ 8^1)$ + $(5\ x\ 8^0)$

= $(1\ x\ 64) + (4\ x\ 8) + (5\ x\ 1) = 64 + 32 + 5 = 101_{10}$

Answer: 101_{10}

9. **Convert 205_{10} to hexadecimal.**

Divide by 16 repeatedly:

- 205 / 16 = 12 remainder 13 (D in hex)
- 12 / 16 = 0 remiander 12 (C in hex)

Answer: CD_{16}

10. **Convert 1001_2 to octal.**
- Group into sets of three from the right: 001 001
- Convert each group to octal:
  001 = 1
  001 = 1

Answer: 11_8

11. **Convert $4F_{16}$ to binary.**

    Convert each hex digit to binary:

    - 4 = 0100
    - F = 1111

    Answer: 01001111_2

12. **Convert 72_8 to decimal.**

    72_8= $(7\ x\ 8^1)$ + $(2\ x\ 8^0)$

    $= 56 + 2 = \ 58_{10}$

    Answer: 58_{10}

13. **Convert 195_{10} to hexadecimal.**

    Divide by 16 repeatedly:

    - 195 ÷ 16 = 12 remainder 3
    - 12 ÷ 16 = 0 remainder 12 (C in hex)

    Answer: $C3_{16}$

14. **Convert 11001010_2 to octal.**

    Group into sets of three from the right: 001 100 101 010

    Convert each group to octal:

    - 001 = 1
    - 100 = 4
    - 101 = 5
    - 010 = 2

    Answer: 1452_8

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