Vector Space Models and Vector Similarity

Some slides and screenshots taken from https://www.coursera.org/learn/classification-vector-spaces-in-nlp

Why use vector space models

- How old are you ?
- What is your age ?
- => different words, same meaning…

- We want to try and capture the meaning of sentences/words, while not being too sensitive to the forms of the words used, but to their meaning !
- For QA, information extraction…

Why use vector space models

- Capture depencies between words :

- I like to eat apples.
- I like to eat pears.
- Using the context of apples and pears, we can deduce that these are both food ! We see they are « surrounded » by the same words and occur in similar positions.

- Going too fast is dangerous, but going slow is not dangerous…
- Given the context, we can deduce fast and slow are antonyms !

Vectors

- Vectors are used as a way to represent the information found in a word or a sentence (/document).

- They are an effective way of transforming words and their relative meaning into mathematical objects to feed to an algoithm.

Fundamental Concept

« You shall know a word by the company it keeps » (Firth, 1957)

Indeed, the vector is built by observing the context around the word and this captures the word’s relative meaning !

How do we construct these vectors ?

- Using a coocurence matrix
- To extract vector representations of
  - A word
  - A Document
  - Depending on the application

- These are called *designs*

Word by Word design

- The co-occurrence of 2 different words is defined by the *# of times they occur together within a certain distance/window k*

I like simple data

I prefer simple raw data

k=2

	simple	raw	like	I
data	2	1	1	0

Practice

- « In general, I love music. But I love pop music more than any other musical genre. To me, music is my greatest love. »

- What is the value for the co-occurrence of “love” and “ music”, if k=2?

Word by Document Design

- Number of times a word *occurs within a specific category*
- Imagine our corpus is divided into three topics :

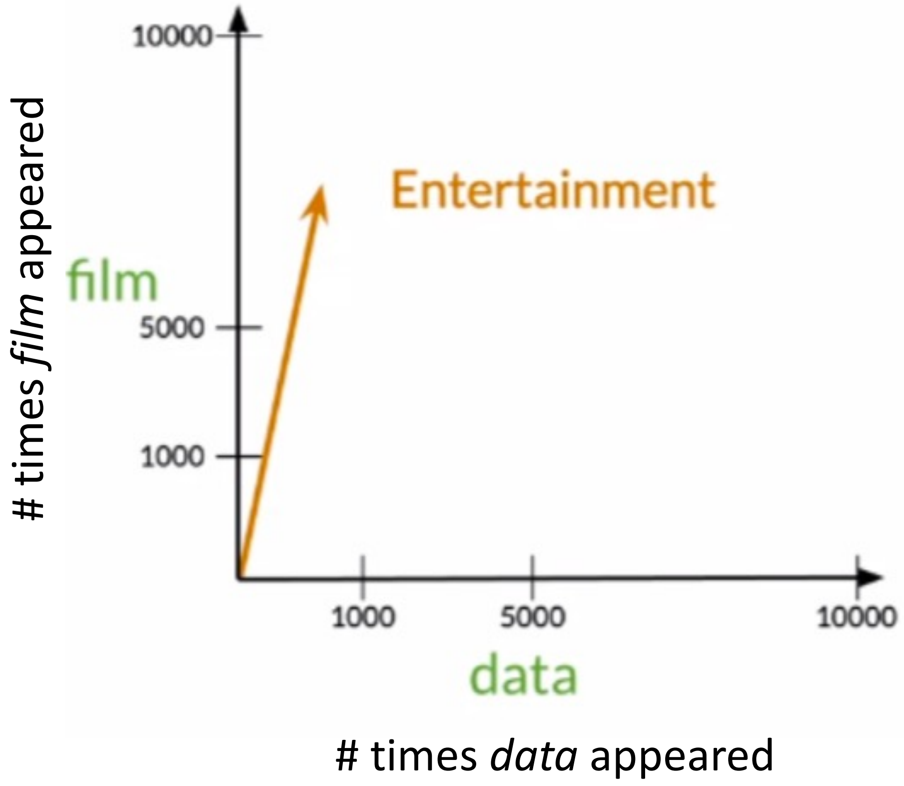
	Entertainment	Economy	Machine Learning
data	500	6620	9320
film	7000	4000	1000

Vector Space

- Given our matrix in the previous slide, we could represent the words *data* and *film* using the rows of our matrix, which would give us two 3-D vectors.

- To make things more visual let's take the vectors of the topics (2-D vectors), using the columns :

Vector space

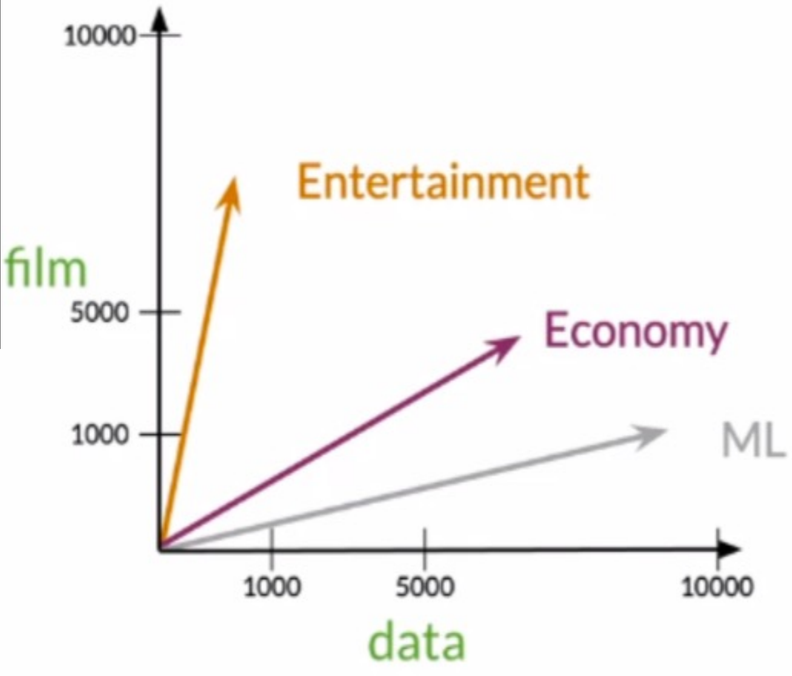


	Entertainment	Economy	ML
data	500	6620	9320
film	7000	4000	1000

Vector Spaces

- We can determine relationships between types of documents
- We can see that the documents about economy and ML are more similar than those about entertainement…

Vector Space



	Entertainment	Economy	ML
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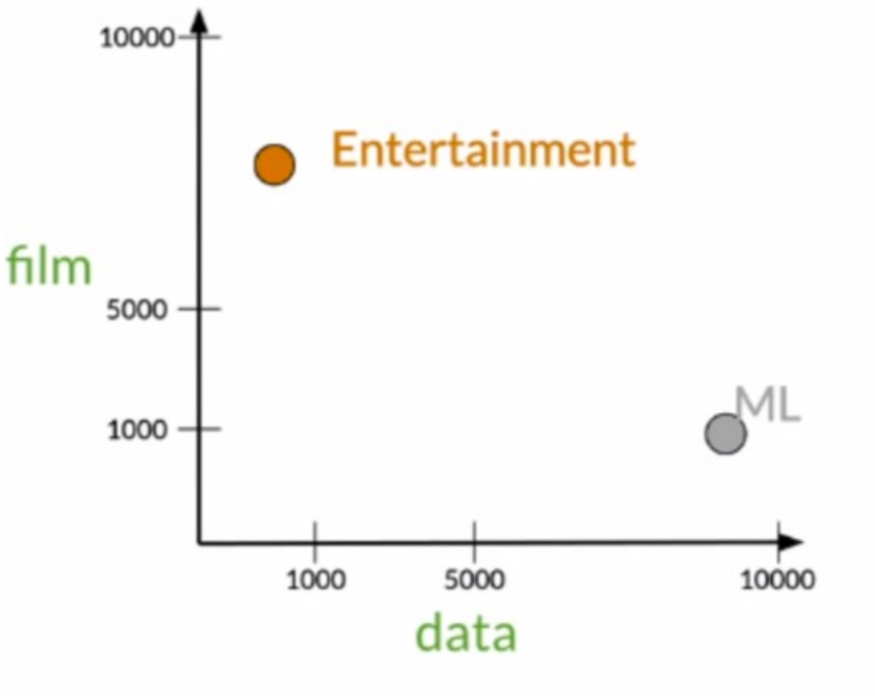
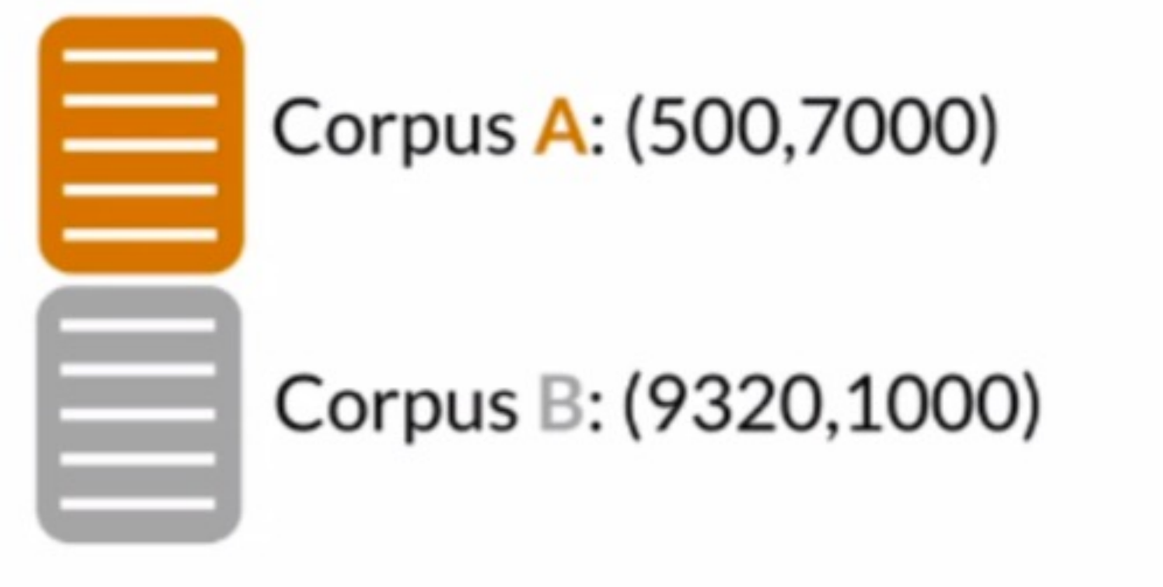
How do we measure the degree of similarity ?

- Distance between vectors

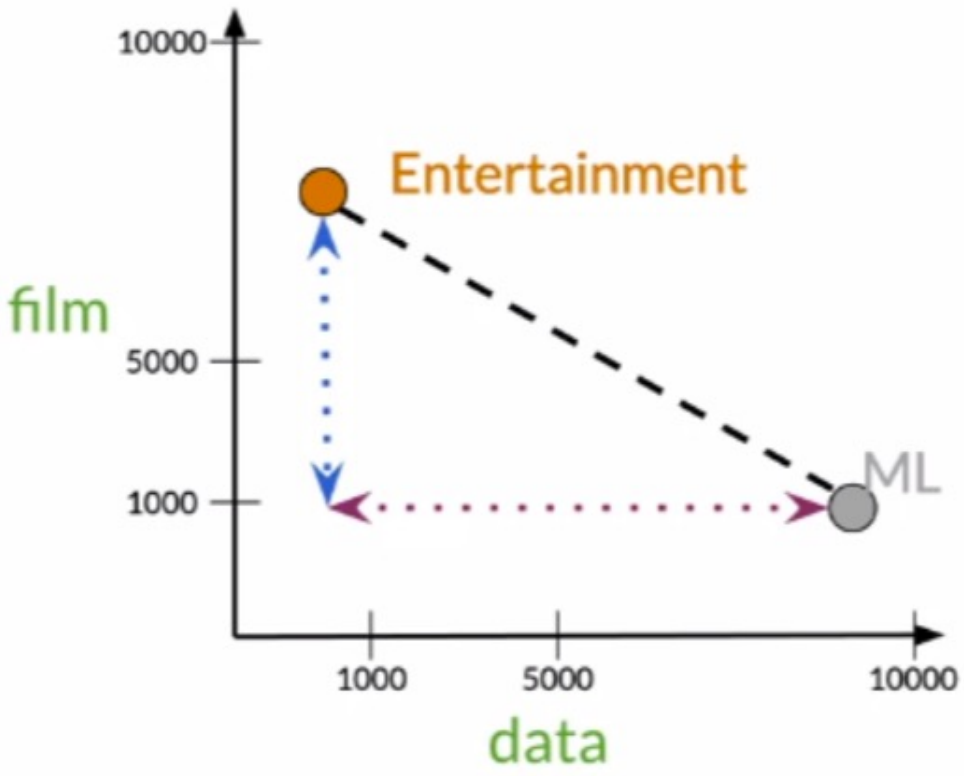
- Angle between vectors

Euclidian distance between 2 points/vectors





Euclidian distance



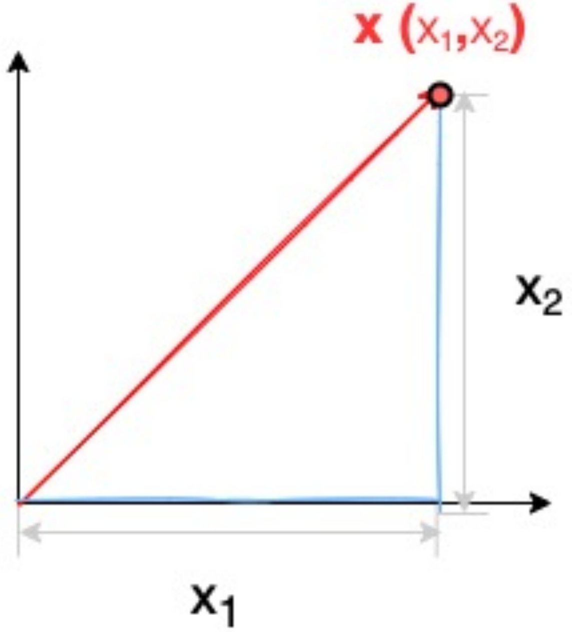
$$d(B,A) = \sqrt{\underline{(B_1 - A_1)^2} + \underline{(B_2 - A_2)^2}}$$

- 1st term : distance between their x coordinates
- 2nd term : distance between the y coordinates

$$c^2 = a^2 + b^2$$

Vector Norm – Euclidian Norm

- How can we calculate the length of a vector ?
- $\| \boldsymbol{a} \| = \sqrt{a_1^2 + a_2^2 + \cdots + a_n^2} = \sqrt{\sum_{i=1}^{n} a_i^2}$
- Euclidian norm or L2 norm
- Measures the shortest distance from the origin



Distance / Vector norm

- Finding the distance between 2 vectors comes down to
- *calculating the length of the vector that allows you to reach the 2nd vector from the tip of the 1st*
- This is the vector $\mathbf{c} = \boldsymbol{b} - \boldsymbol{a}$

$$\| \boldsymbol{c} \| = \sqrt{c_1^2 + c_2^2 + \cdots + c_n^2}$$

$$= \sqrt{(b_1 - a_1)^2 + (b_2 - a_2)^2 + \cdots + (b_n - a_n)^2}$$

Euclidian distance for an n-dimensional matrix

- We can now generalize to any number of dimensions

	data	boba ($\vec{w}$)	ice-cream ($\vec{v}$)
AI	6	0	1
drinks	0	4	6
food	0	6	8

$$= \sqrt{(1-0)^2 + (6-4)^2 + (8-6)^2}$$

$$= \sqrt{1+4+4} = \sqrt{9} = 3$$

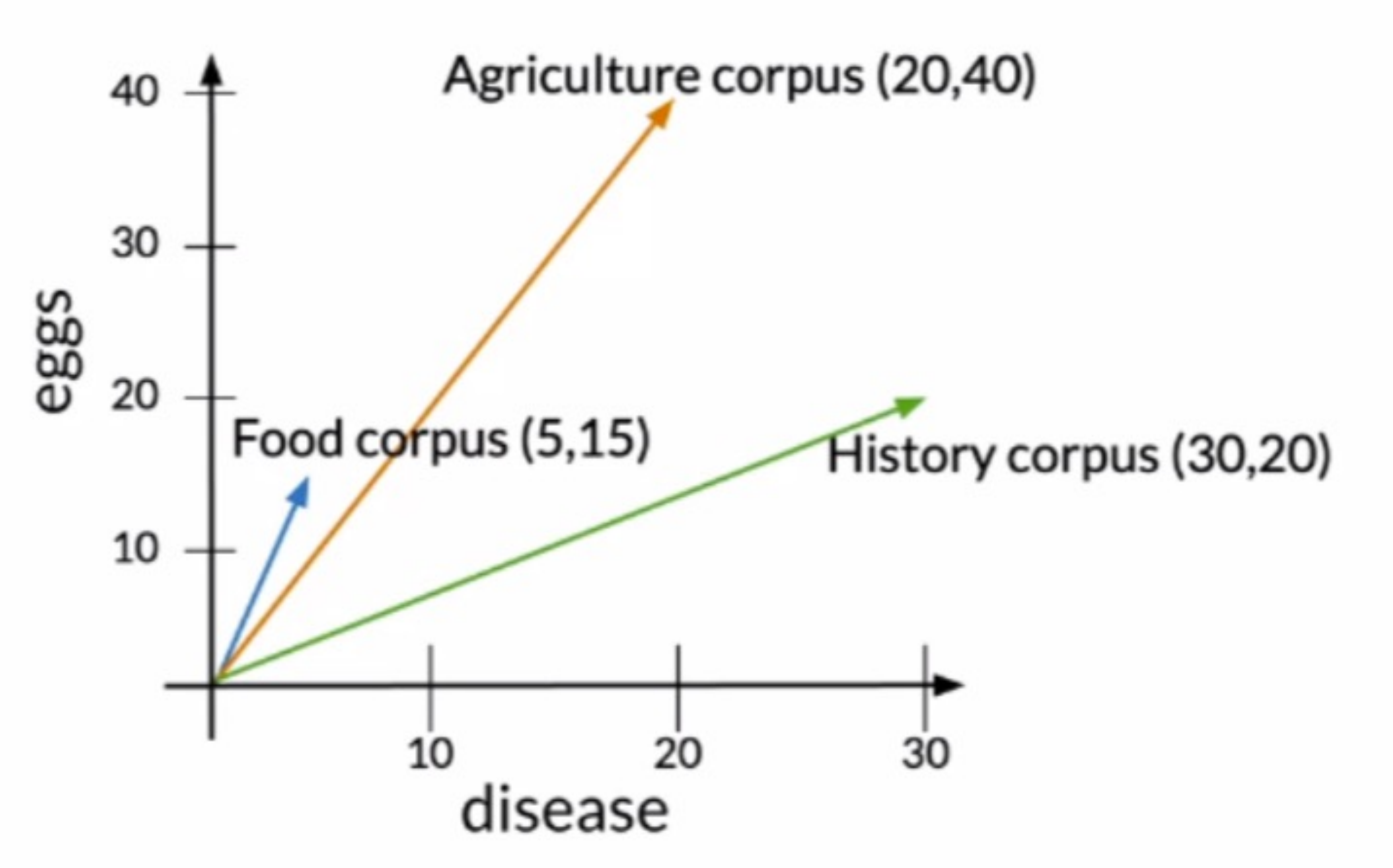
$$d(\vec{v}, \vec{w}) = \sqrt{\sum_{i=1}^{n} (v_i - w_i)^2}$$

Numpy exercise

- Calculate the euclidian distance

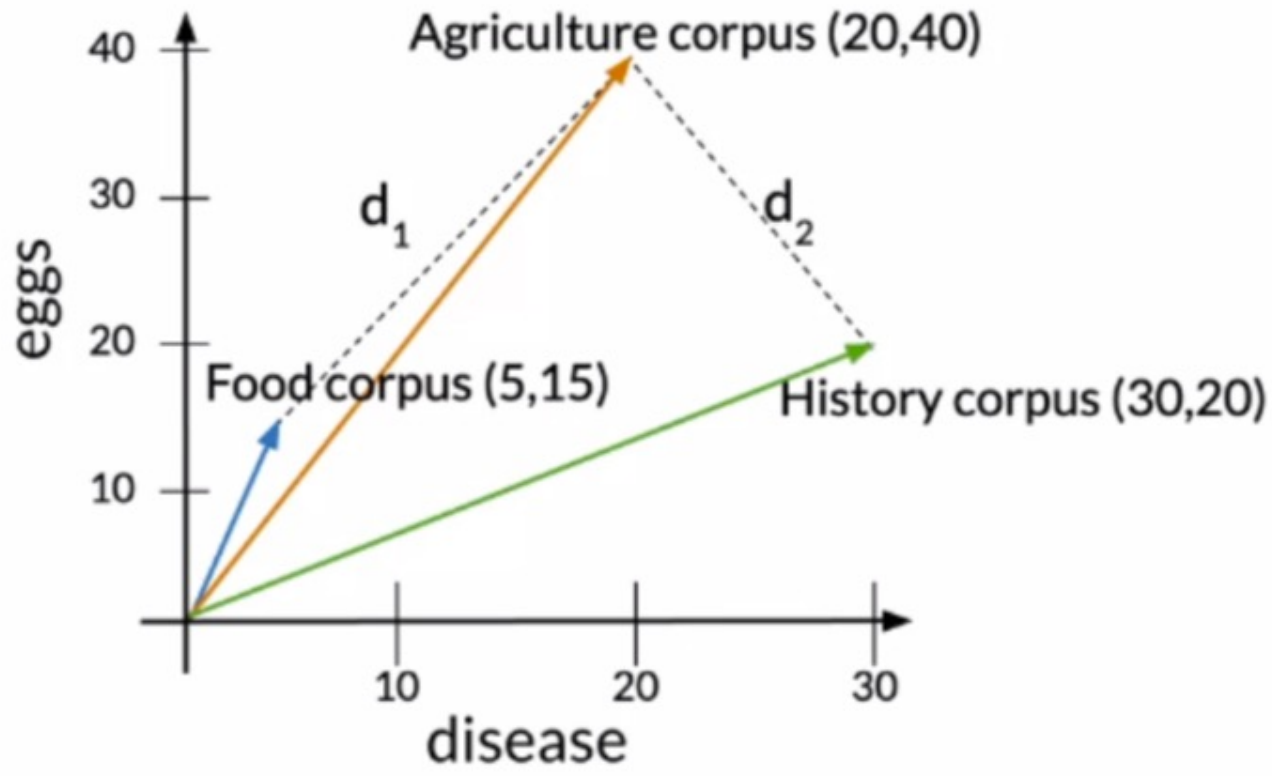
Cosine similarity

- Euclidian distance vs Cosine Similarity



Euclidian distance limitations

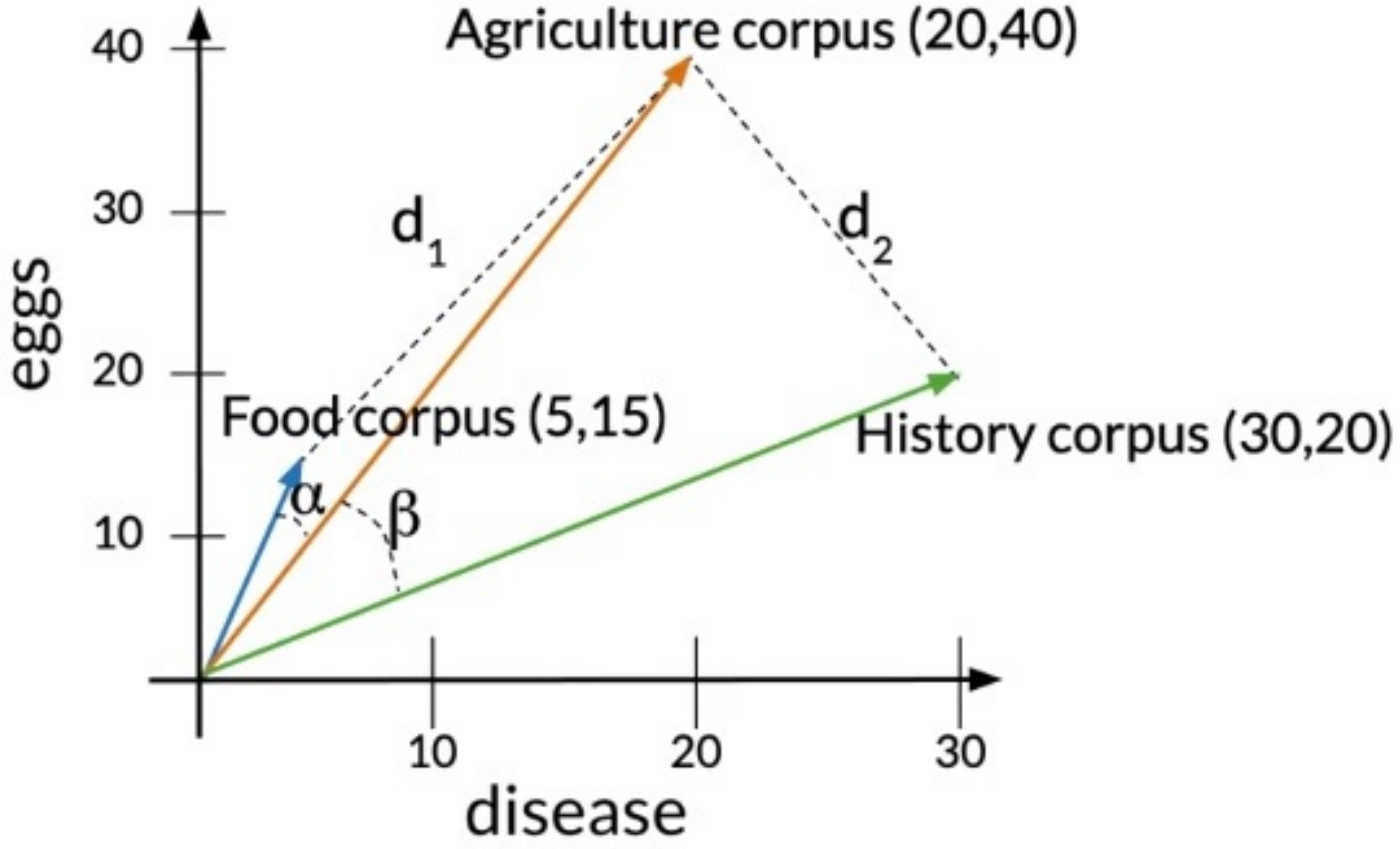
- IN this case, measuring the eulidian distance suggests the agriculter and food corpora have less in common than the agriculture and history corpora…



Euclidean distance: $d_2 < d_1$

Cosine Similarity

- Another method for computing similarity is to compute ths cosine of the inner angle between 2 vectors
- See if 2 vectors are pointing in the same direction
- $\beta > \alpha$
- This metric is not biased by the magnitude of the vector representations
- So this is a more adapted metric when the corpora are of different sizes



Law of Cosines and the Dot Product

- Now you know about vector norms, another way of expressing the dot product that we haven't seen is :

$$\mathbf{a} \cdot \boldsymbol{b} = ||\boldsymbol{a}||\,||\boldsymbol{b}||cos\theta$$

- This comes from the law of cosines

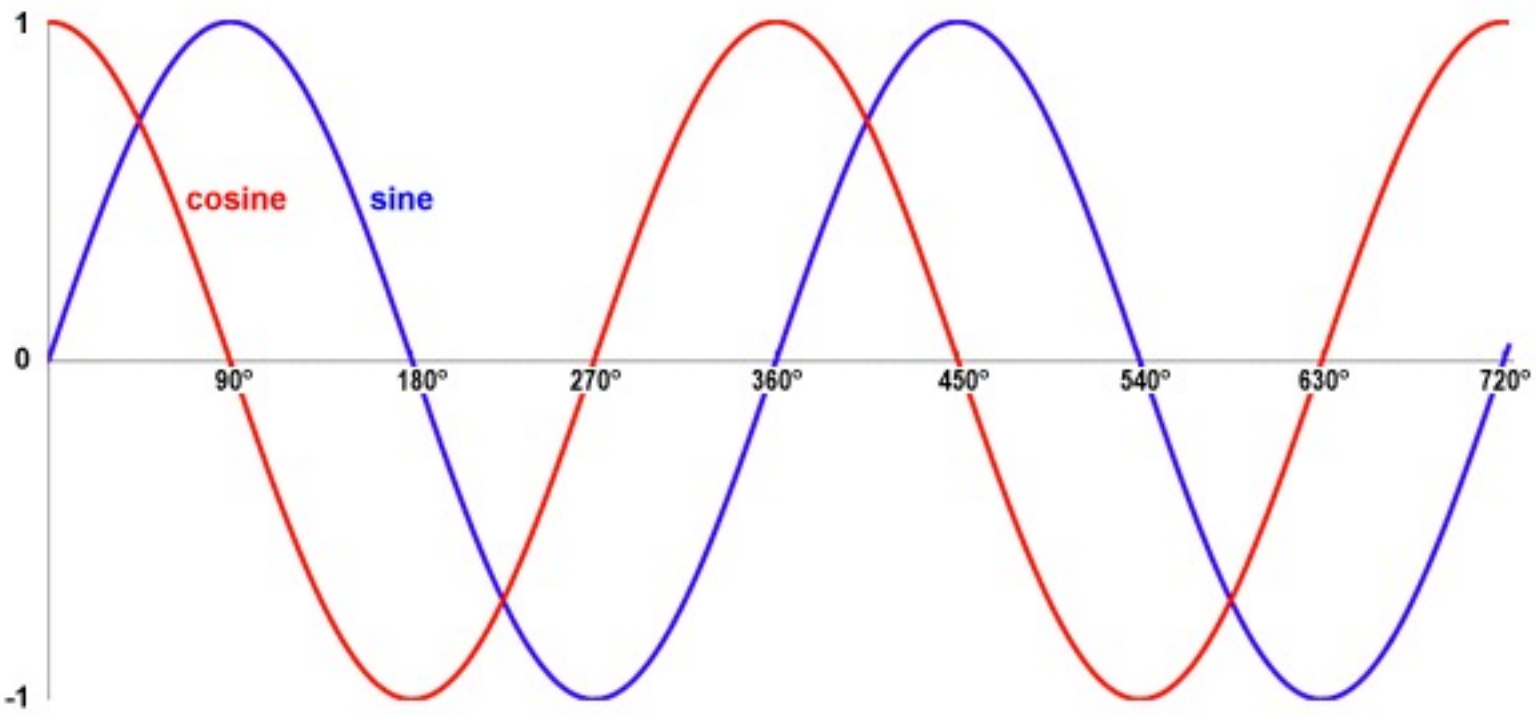
- (See here for proof)

New intuition about the dot product

- The result of the dot product is therefore impacted by
  - The magnitude of the vectors
  - their direction / the angle between them

- When $\theta < 90°$ dot product is positive
- When $\theta = 90°$ dot product = 0
- When $90° < \theta < 180°$ dot product is negative

Sine and Cosine functions



Cosine Similarity

- Remember

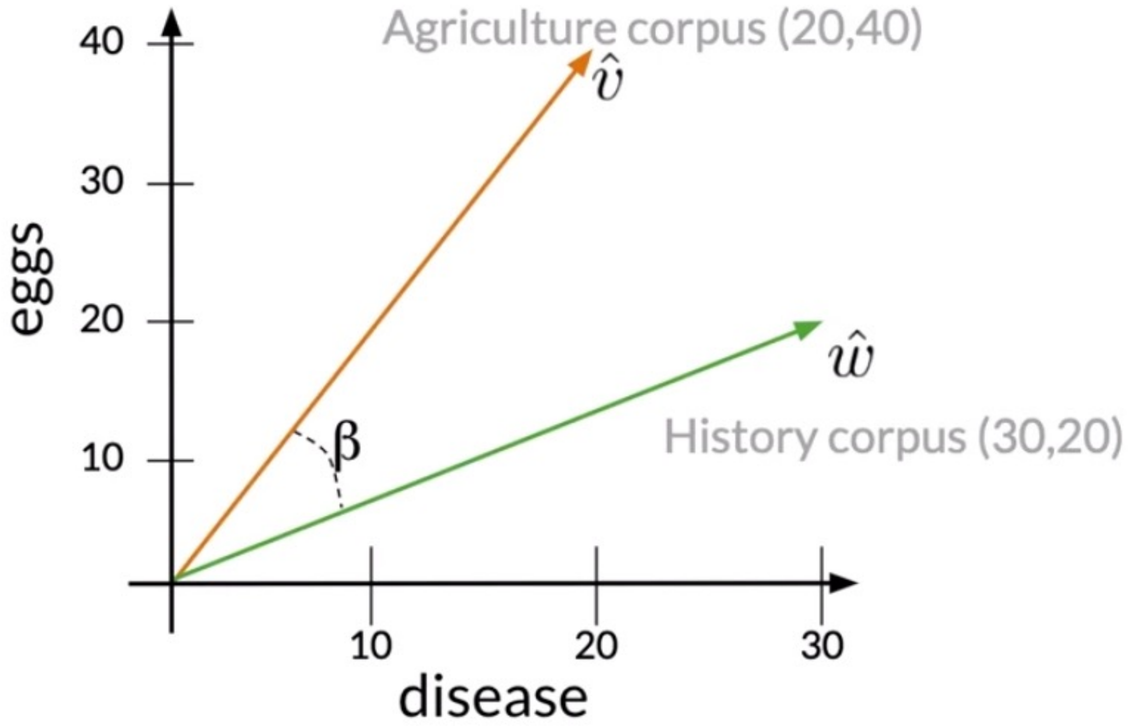
$$\mathbf{a} \cdot \boldsymbol{b} = \|\boldsymbol{a}\|\|\boldsymbol{b}\| cos\theta$$

- So

$$Similarity(\boldsymbol{a}, \boldsymbol{b}) = \cos\theta = \frac{\boldsymbol{a} \cdot \boldsymbol{b}}{\|\boldsymbol{a}\|\|\boldsymbol{b}\|}$$

Cosine similarity

In our example, because word counts are positive, the cosine similarity cannot be negative



$$\hat{v} \cdot \hat{w} = \|\hat{v}\| \|\hat{w}\| \cos(\beta)$$

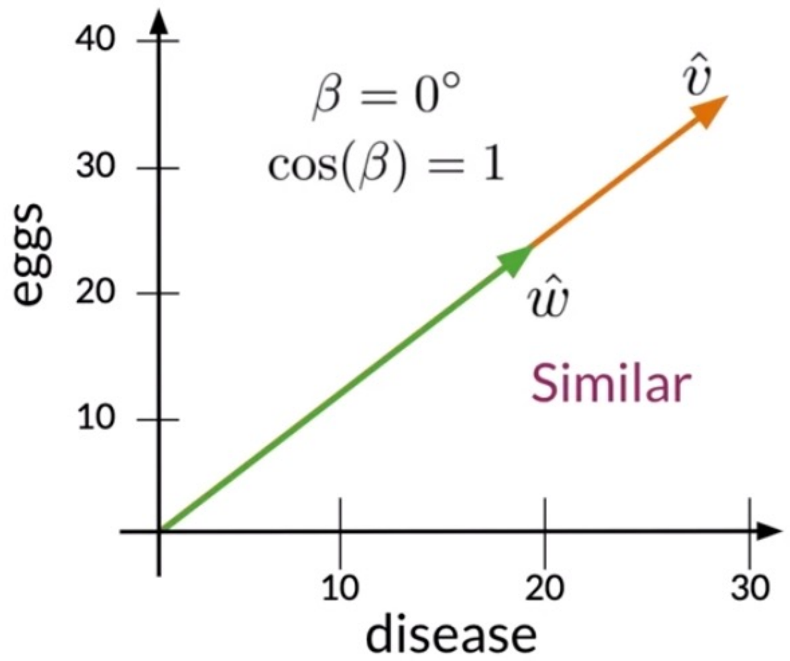
$$\boxed{\cos(\beta) = \frac{\hat{v} \cdot \hat{w}}{\|\hat{v}\| \|\hat{w}\|}}$$

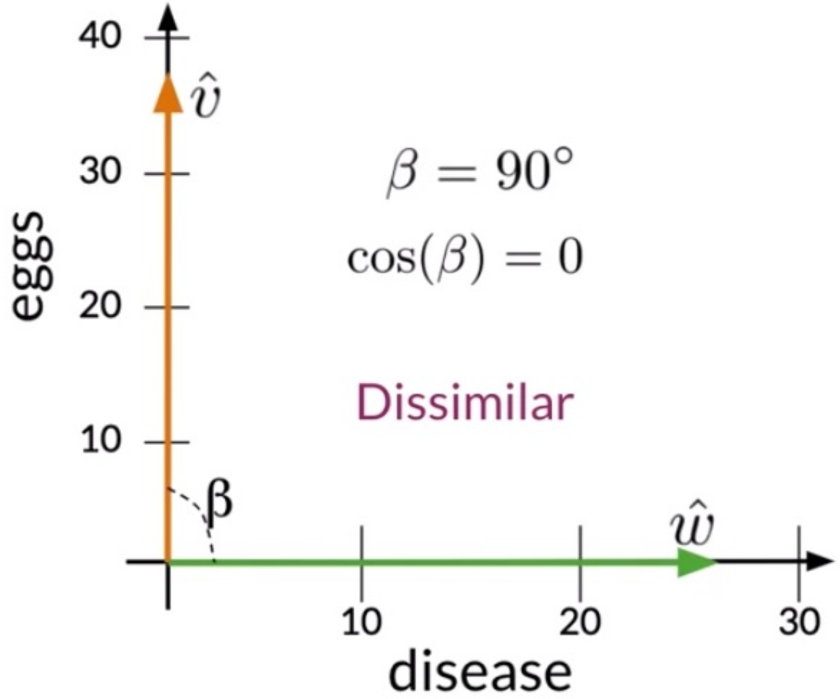
$$= \frac{(20 \times 30) + (40 \times 20)}{\sqrt{20^2 + 40^2} \times \sqrt{30^2 + 20^2}}$$

$$= 0.87$$

What does cosine similarity tell us about the similarity between 2 vectors ?

- Max angle is 90° for reasons explained previously





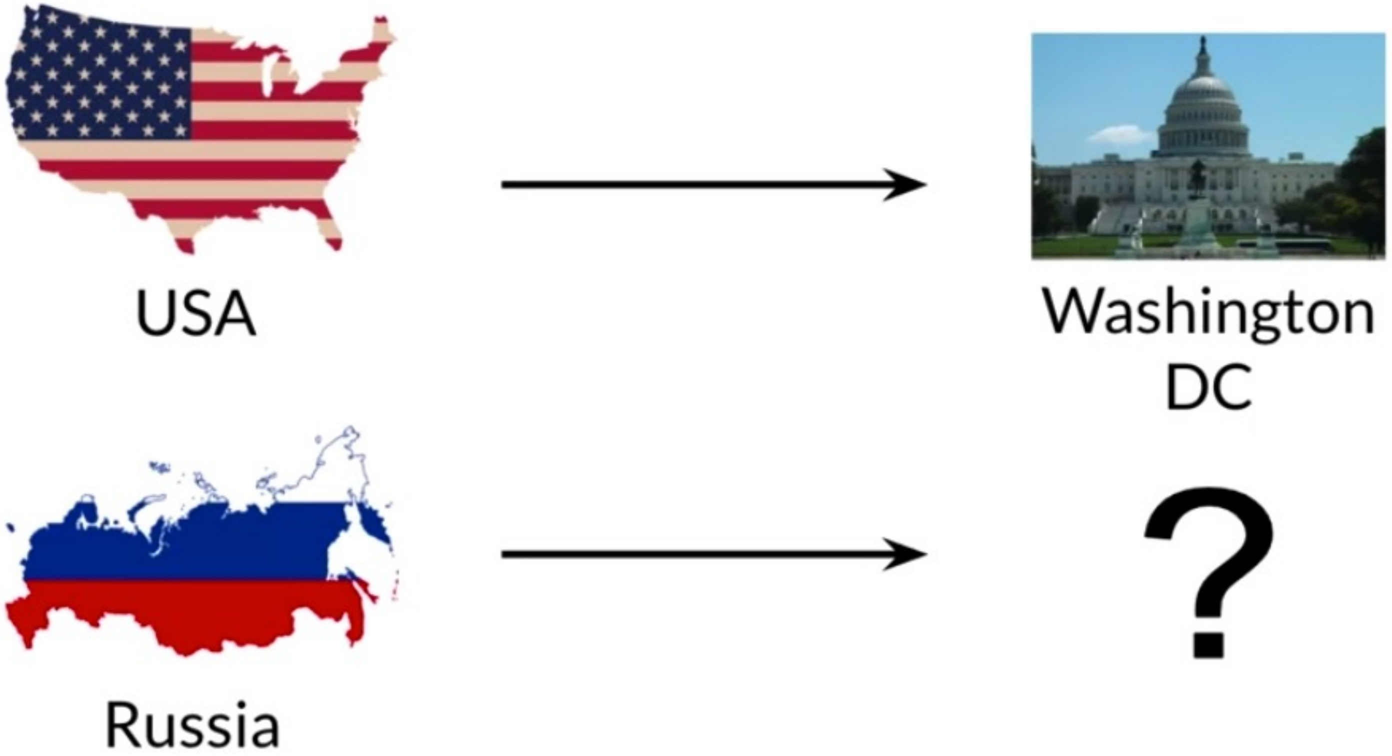
Cosine Similarity

- So cosine similarity is proportional ($\propto$) to the similarity between the directions of the vectors

- 0 < simil < 1 for the vector space we've seen so far.

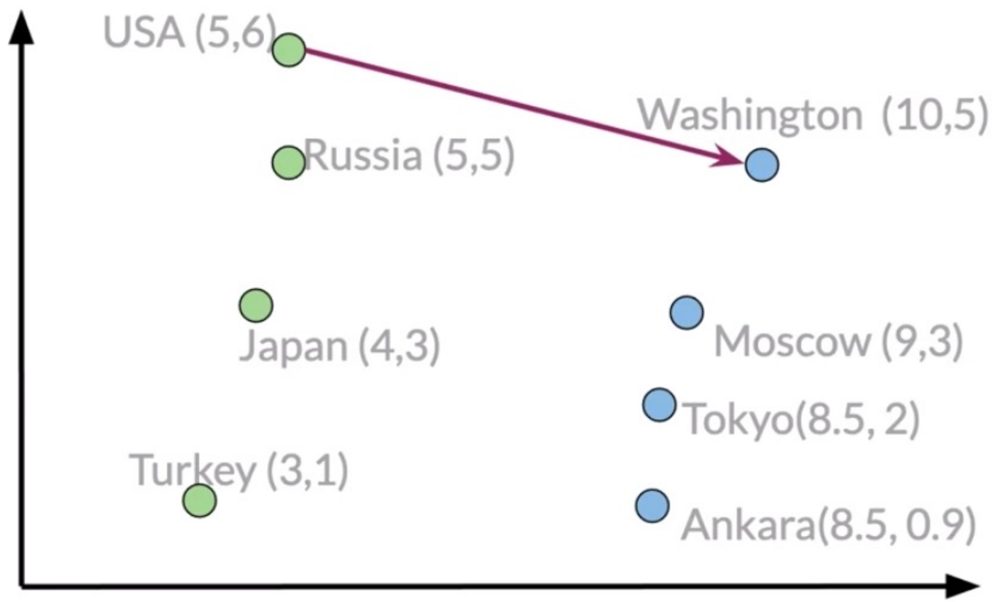
How to manipulate vector representations

- We can us e them to infer unknown relations between words.
- For example you can use the relation between the USA and its capital to infer the capital of Russia !



Manipulating word vectors

- To find the relation between a country and its capital, you can use linear algebra.
  - Find the vector that leads you from a country to its capital (subtract one from the other)
  - This vector encodes the relationship « has capital »

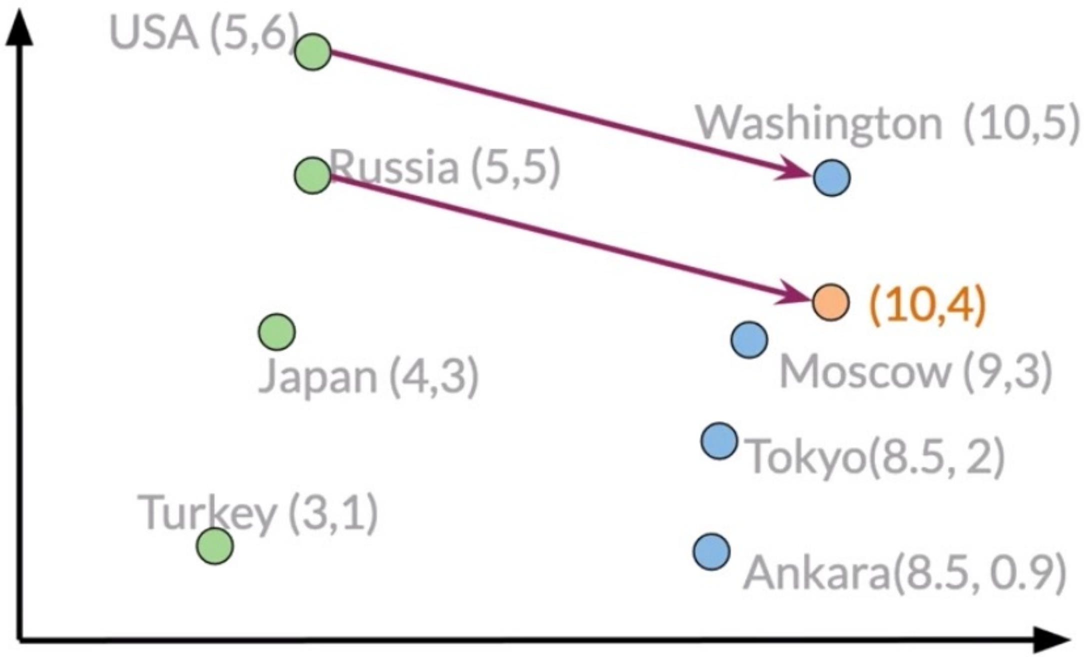


Washington - USA = $\begin{bmatrix} 5 & -1 \end{bmatrix}$

Remember a vector gives you directions, or how to move in space using its coordinates.

Making a prediction

- However the result of your computation may not land perfectly on the desired vector…
- So you need to use euclidian distance or cosine similarity to find the vector closest to your prediction



Washington - USA = $\begin{bmatrix} 5 & -1 \end{bmatrix}$

Russia + $\begin{bmatrix} 5 & -1 \end{bmatrix}$ = $\begin{bmatrix} 10 & 4 \end{bmatrix}$