THE LANGUAGE MODELING PROBLEM

The Language Modeling Problem

- One of the oldest problems studied in statistical NLP because very useful for many applications

The Language Modeling Problem

- We have some finite vacabulary, say V = {the, a , man, telescope, Beckham, two, …} => usually several tens of thousands
- We have an (infinite) set of strings, $V^{\dagger}$ => set of all possible sentences in this language
- A sentence must have 0 or more words and each word must come from V, any sequence is possible.

The Language Modeling Problem

- Possible sentences :
  - The STOP
  - A STOP
  - The fan STOP
  - The fan saw Beckham STOP
  - The fan saw saw STOP
  - The the the STOP
  - STOP

The Language Modeling Problem

- We have a *training* s*ample* of example sentences in English

- Collection of sentences from the New York Times during the last ten years for example,
- or large sample of sentences from the web

- 90s => 20 million words
- 2000s => 1 billion words
- Nowadays => 100s billions words

The Language Modeling Problem

- Our task is to « learn » a probability distribution *p* over the sentences in our language.
- 2 conditions :
  - $p(x) \geq 0 \; \forall \; x$ ∈ $V^{\dagger}$ => For any sentence x, the probability of that sentence must be greater or equal to 0
  - $\sum_{(x \in V^{\dagger})} p\,(x) = 1$ => If we sum over all of the probabilities of the sentences in the language we obtain 1, meaning *p* is a well-formed distribution.

The Language Modeling Problem

- p is essentially a function which returns the probability for a sequence in a given language.

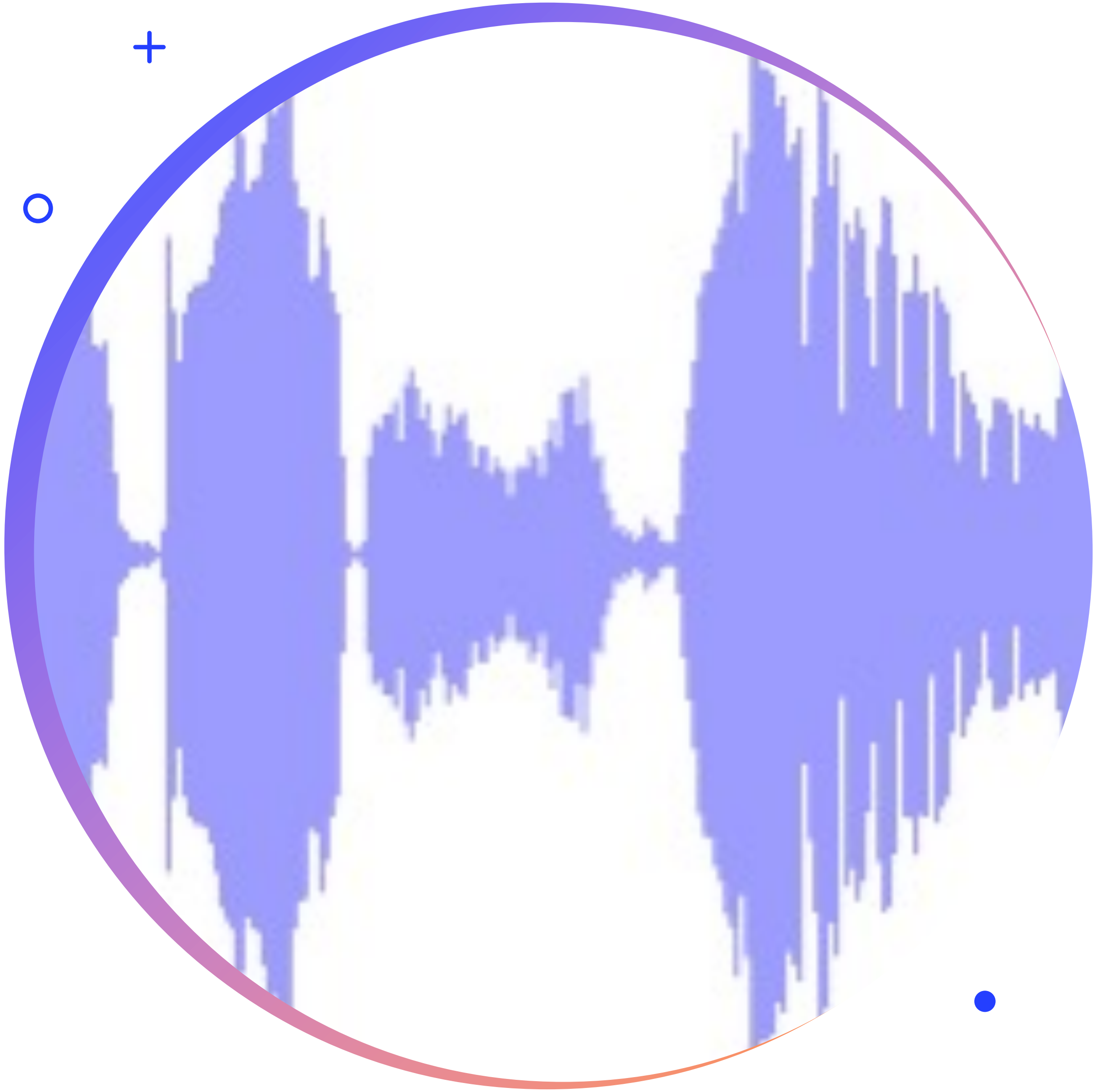
- P(the STOP) = 10^{-12}
- P(the fan STOP) = 10^{-8}
- P(the fan saw Beckham STOP) = 2×10^{-8}
- P(the fan saw saw) = 10^{-15}
- *…* assign a probability to every sequence in the language

The Language Modeling Problem

- We want to try and assign a high probability to likely sentences in English and low probability to unlikely sentences in English

Why would we want to do this ?!

- Language models are useful in many applications:
  - Speech recognition: language models are critical for modern speech recognizers (handwriting recognition also)
  - The estimation techniques used for this problem are useful for other NLP problems such as POS tagging or automatic translation.



Language modeling for Speech Recognition

- Quick sketch :
  - Input => an acoustic recording
  - Then map this input to the words which are actually spoken

Language modeling for Speech Recognition

- Imagine the person says « recognize speech »

- In practice, there are actually many alternative sentences which could have been spoken :
  - « wreck a nice beach »
  - ....
  - ....

- Similar sentences from an acoustic point of view

Language modeling for Speech Recognition

- A language model allows us to produce a probability for each sentence and estimate that « recognize speech » is more probable than another option.

- => Adds some very useful info to get rid of these kinds of confusions

A naive method for Language Modeling

- We have N sentences
- For any sentence or sequence $x_1 \dots x_n$, $\mathrm{c}(x_1 \ \dots \ x_n)$ is the number of times the sentence was seen in our training data.

- A naive estimate :

- $p(x_1 \dots x_n) = \frac{c(x_1 \dots x_n)}{N}$

A naive method for Language Modeling

- Has some deficiencies, although it’s a well-formed language model:

- Mainly it assigns proba 0 to any sentence not seen in our training sample…

- Cannot generalize to new sentences

Trigram Models

- Widely used language model

- Build heavily on the idea of Markov processes…

Markov Processes

- Consider a sequence of random variables $X_1, \ldots, X_n$.
- Each random variable can take any value in a finite set V (vocab).
- We can assume the length n is fixed for now. (n=100 for ex.)
- We want to model the joint probability

$$P\ (X_1 = x_1, \ldots, X_n = x_n)$$

Markov Processes

- Huge number of possible values.

- $|V|^n$ possible sequences in our example

First-Order Markov Process

- Going to use chain rule to decompose this joint proba

- Remember: P(A,B) = P(A) x P(B|A)
- And therefore P(A,B,C) = P(A) x P(B|A) x P(C|A,B)
- So :

$$P(X_1 = x_1, \dots, X_n = x_n)$$
$$=$$
$$P(X_1 = x_1) \prod_{i=2}^{n} P(X_i = x_i | X_1 = x_1, \dots, X_{i-1} = x_{i-1})$$

First-Order Markov Process

- The 1st order Markov asumption states that

$$\prod_{i=2}^{n} P(X_i = x_i | X_1 = x_1, \ldots, X_{i-1} = x_{i-1})$$

Is equal to

$$\prod_{i=2}^{n} P(X_i = x_i | X_{i-1} = x_{i-1})$$

First-Order Markov Process

$$P(X_1 = x_1, \ldots, X_n = x_n)$$

(exact equality)

$$P(X_1 = x_1) \prod_{i=2}^{n} P(X_i = x_i | X_1 = x_1, \ldots, X_{i-1} = x_{i-1})$$

(Markov assumption)

$$P(X_1 = x_1) \prod_{i=2}^{n} P(X_i = x_i | X_{i-1} = x_{i-1})$$

First-Order Markov Process

- Huge assumption to state that the probabilty of a word here is only conditioned on the previous word…

Second-Order Markov Processes

- Very similar model :

$$P(X_1 = x_1, \dots, X_n = x_n)$$

$$= P(X_1 = x_1)\ P(X_2 = x_2 | X_1 = x_1) \prod_{i=3}^{n} P(X_i = x_i | X_{i-2} = x_{i-2}, X_{i-1} = x_{i-1})$$

- Condition on previous 2 elements vs only the previous element

Second-Order Markov Processes

- We would also like to make the length of a sentence be a random variable: not all sentences will have 100 words…

- So we can define X_n to always be equal to STOP where STOP is a special symbol.

- Basically, if STOP is at position i, then this marks the end of the sentence and $i = n$

Trigram Language Model

- Given these concepts we can define a trigram language model, which consists of :

- A finite set V

- A parameter $q(w|u, v)$ for each trigram u, v, w such that $w \epsilon V \cup \{STOP\}$ and $u, v \epsilon V \cup \{*\}$ (special start symbols)

Trigram Language Model Formal Definition

- For any sentence $x_1, \ldots, x_n$
  - where $x_i \in V$ for $i = 1, \ldots, n - 1$
  - and $x_n =$ STOP
- The probability of the sentence under the trigram language model is

$$p(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} q(x_i | x_{i-2}, x_{i-1})$$

- Where we define $x_{-1} = x_0 = *$

An example to make things clearer

- Sentence : « * * The dog barks STOP »
  - $p(** \; the \; dog \; barks \; STOP) = q(the| *,*) \times q(dog| *, the) \times q(barks|the, dog) \times q(STOP|dog, barks)$
- Product of terms to get the proba of the sentence under this type of language model
- We’re treating sentences as being generated by a second order Markov process, where each word generated is dependent purely on the 2 previous words.

Trigram Language Model

- Advantages :
  - Simple, easy and cheap
  - useful for many applications
  - availablity of statistics over the internet
  - well understood math
- Disadvantages:
  - Language: they do not capture non-local dependencies

Estimating the parameters

- So we need to estimate $q(w_i|w_{i-2}, w_{i-1})$
- Remember, if we have two dependent events :

$$p(A, B) = p(A) \times p(B|A)$$

- Which is equivalent to

$$p(B \mid A) = \frac{p(A,B)}{p(A)}$$

- Which can be generalized to 3 events

$$p(C|A, B) = \frac{p(A, B, C)}{p(A, B)}$$

Estimating the parameters

- A natural estimate is therefore :

$$q(w_i|w_{i-2}, w_{i-1}) = \frac{Count(w_{i-2}, w_{i-1}, w_i)}{Count(w_{i-2}, w_{i-1})}$$

- So for example :

$$q(laughs|the, dog) = \frac{Count(the, dog, laughs)}{Count(the, dog)}$$

Estimating the parameters from a toy corpus

- An example corpus:

1. the cat saw the mouse.
2. the cat heard a mouse.
3. the mouse heard.
4. a mouse saw.
5. a cat saw.
6. a cat heard the mouse.

=> Using the corpus, give the parameter estimates for :

- a bigram language model
- a trigram language model

(the parameters for the first 2 sentences are enough)

Estimating the parameters

Bigram	Count	Unigram	Count	Relative frequency
* the	3	*	6	3/6
the cat	2	the	5	2/5
cat saw	2	cat	4	2/4
saw the	1	saw	3	1/3
the mouse	2	the	5	2/5
mouse STOP	3	mouse	5	3/5
cat heard	2	cat	4	2/4
heard a	1	heard	3	1/3
a mouse	2	a	4	2/4
...	...	...	...	...

Estimating the parameters

Trigrams	Count	Bigram	Count	Relative frequency
* * the	3	**	6	3/6
* the cat	2	* the	3	2/3
the cat saw	1	the cat	2	1/2
cat saw the	1	cat saw	2	1/2
saw the mouse	1	saw the	1	1
the mouse STOP	2	the mouse	3	2/3
the cat heard	1	the cat	2	1/2
cat heard a	1	cat heard	2	1/2
heard a mouse	1	heard a	1	1
a mouse STOP	1	a mouse	2	1/2
…	…	…	…	…