NOTIONS DE PROBABILITÉ (RAPPELS)

The Basics

- Probability is simply how likely something is to happen.
- Whenever we're unsure about the outcome of an event, we can talk about the probabilities of certain outcomes—how likely they are.
- The analysis of events governed by probability is called statistics.

Flipping a Coin

- **The best example for understanding probability is flipping a coin:**
- There are two possible outcomes – heads or tails.
- What's the probability of the coin landing on Heads?
  - You might intuitively know that the likelihood is half/half, or 50%. But how do we work that out?
- Probability = $\frac{\#\ of\ possibilities\ that\ meet\ our\ condition}{\#\ of\ equally\ likely\ possibilities}$
- In this case :
  - P (h) = ½ = 50%

Example

- More generally :
  - **Probability of an event = (# of ways it can happen) / (total number of outcomes)**
- Rolling a die:
  - What is the probability of rolling a 1 ?
  - What is the probability of rolling a 2 or a 6 ?
  - What is the probability of rolling an even number ?

Sample Space

- Sample space => all of the possible outcomes of an « experiment »
- Example :
  - What is the sample space in the case where our experiment consists in flipping 3 coins ?
  - HHH
  - HHT
  - HTH
  - HTT
  - THH
  - THT
  - TTH
  - TTT

Sample Space

- What is the probability of getting exactly 2 heads when flipping 3 coins :

- P(« exactly 2 heads ») = $\frac{how\ many\ possible\ outcomes\ are\ associated\ with\ the\ event\ ?}{number\ of\ possible\ outcomes}$

  = 3 / 8

- And the probability of at least 1 head ?

Union, Intersection, and Complement

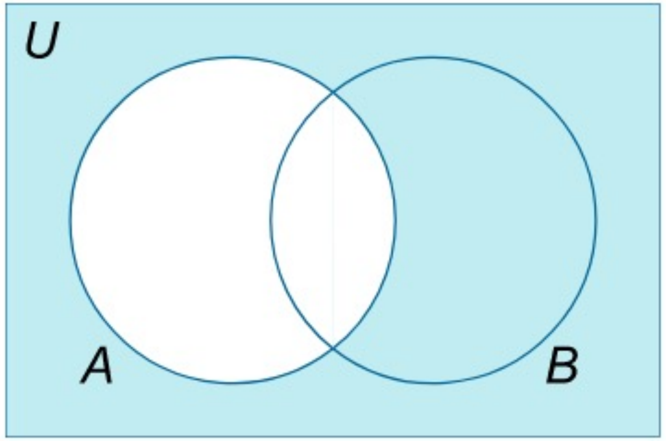
- What is a set ?
  - Fundamentally, just a collection of distinct objects, which can be anything :
    - A = {56, 😀, blue, orange, 99, Jim, Claire}
    - B = {65, 🙁, blue, red, 100, James, Clara}

- The **union** of two sets contains all the elements contained in either set (or both sets).
- The union is notated $A \cup B$.
- More formally, $x \in A \cup B$ if $x \in A$ or $x \in B$ (or both)

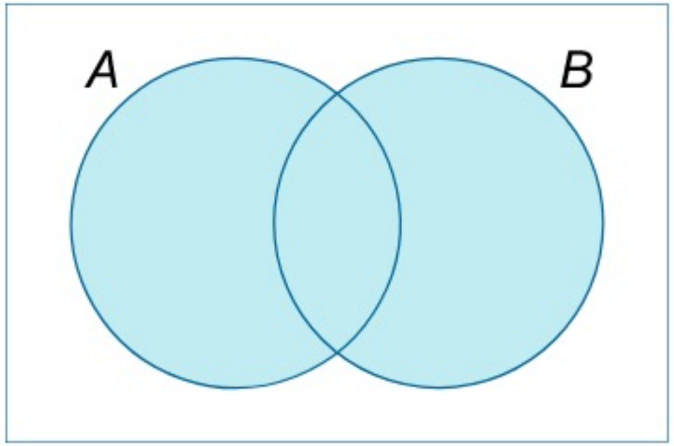
Union, Intersection, and Complement

- The **intersection** of two sets contains only the elements that are in both sets.
- The intersection is notated $A \cap B$.
- More formally, $x \in A \cap B$ if $x \in A$ and $x \in B$

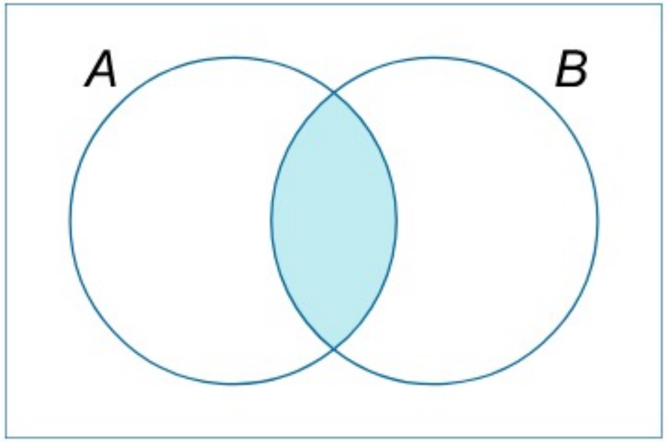
- The **complement** of a set A contains everything in a given universe that is *not* in the set A.
- The complement is notated A'.



Complement



Union



Intersection

VENN DIAGRAMS

Set Operations Example

- If :
- A = {3, 7, -5, 0, 13}
- B = {0, 17, 13, ⭐, Blue}
- C = {Pink, ⭐, 3, 17}
- **Then what would this set be : $(A \cap C') \cup (B \cap C)$?**

General Addition Rule for Probability

- Imagine we have a bag in which there are:
  - *2 green cubes*
  - *3 green spheres*
  - *4 yellow cubes*
  - *5 yellow spheres*
- We consider the event where we pull out an object, each object having the same probability of being pulled out from the bag :
  - **What is P (cube) ?**
  - **What is P (yellow) ?**
  - **What is P(yellow cube) ?**
  - **What is P (yellow or cube) ?**

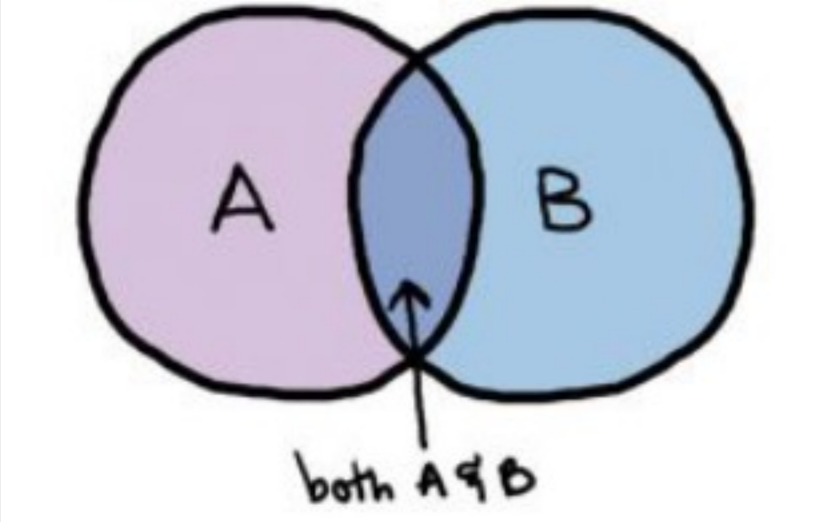
General Addition Rule for Probability

- P (yellow or cube) $= \frac{\#\, of\ yellow\ objects + \#\, of\ cubes - \#of\ elmts\ counted\ twice}{total\ number\ of\ objects}$

  = (9 + 6 – 4) / 14 = 11/14

General rule :

- P (A or B) = P (A U B) = P (A) + P (B) – P (A ∩ B)



Monty Hall Problem/Brain-teaser

- You are a contestant at a game show:
- There are three doors behind which are hidden
  - A goat
  - A car
  - And another goat
- The rules are that you must first pick a door.
- After that, the game show host opens one of the remaining doors (behind wihich there is a goat) and asks you if you wish to switch and choose the other remaining door…
- So the question is:
  - Should you switch or should you stick to your guns and keep your initial choice ?

Solution

Door A: Goat   Door B: Car   Door C: Goat

- We need to compare the probability of winning if we switch our answer with the probability of winning if we don't.
- Let's enumerate the possible outcomes for the 2 scenarios:
  - Scenario 1: we don't switch:
    - Choose door A => ends in a loss
    - Choose door B => ends in a win
    - Choose door C => ends in a loss
    - Therefore P (winning by not switching) = $\frac{how\ many\ possible\ outcomes\ are\ associated\ with\ the\ event\ of\ winning\ ?}{number\ of\ possible\ outcomes}$ = 1/3

Solution

- Scenario 2, we do decide to switch:
  - Choose door A => ends in a win (since the host has to open one of the remaining doors behind which there is a goat, it means the only door we can switch to is the one with the car, so we necessarily end up winning)
  - Choose door B => ends in a loss (only remaining door will be a goat so we necessarily lose since we'll be switching to that one)
  - Choose door C => ends in a win (same situation as with door A)

  - Therefore P (winning by switching) = $\frac{\textit{how many possible outcomes are associated with the event ?}}{\textit{number of possible outcomes}}$ = 2/3

- Careful => When the host eliminates one of the possible doors, the probaility of winning does not become $\frac{1}{2}$....
- As we saw, there are 3 doors to begin with and in one scenario we have 1 opportunity out of 3 to choose the correct door, while in the other scenario we have 2 opportunities out of 3 to end up on the correct door.
- Winning with a probability of $\frac{1}{2}$ can only be true if the problem was choosing between 2 equiprobable options, in situations such as :
  - There is no third door to begin with
  - Or you are a spectator arriving mid-game and the third door has already been opened, meaning you don't know which door the player chose initially

Solution

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Solution

- For more detailed explanations :
- https://www.youtube.com/watch?v=Xp6V_IO1ZKA
- https://www.youtube.com/watch?v=XiKMMt3Mm4k
- https://www.youtube.com/watch?v=4Lb-6rxZxx0