

1. find  $\det(A^T A)$

(a)  $\begin{pmatrix} 1 & 3 & 2 \end{pmatrix}$

(b)  $\begin{pmatrix} 1 & 1 & 1 \\ 1 & 3 & 2 \\ 1 & 3 & 2 \end{pmatrix}$

(c)  $\begin{pmatrix} \cos u & -r \sin u & 0 \\ \sin u & r \cos u & 0 \\ 0 & 0 & 1 \end{pmatrix}$

2.  $A_{m \times n} = Q_{m \times n} R_{n \times n}$

$A$  is  $m \times n$  matrix, with  $\text{rank } A = n$

$Q$  is orthogonal,  $R$  is upper triangle

find  $R$  for  $A = \begin{pmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$

3. Proof for  $f(x, y, z)$ .

Volume element is  $\sqrt{1 + f_x^2 + f_y^2 + f_z^2} \, dx \, dy \, dz$

Hence find the volume of  $\frac{x^2}{4} + \frac{y^2}{9} \leq z \leq a^2$

4. 
$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} (R+r \cos u) \cos v \\ (R+r \cos u) \sin v \\ r \sin u \end{pmatrix} \quad u, v \in [0, 2\pi]$$
 find the volume

5.  $f(z) = z^2 \quad z = x + iy, \quad |z| \leq 1$   
find the surface area of  $f(z)$   
(hint: use 4 coordinates)

6.  $f(z) = \cos(z) \quad z = x + iy$   
find the area  $x \in [-\pi, \pi], \quad y \in [-1, 1]$