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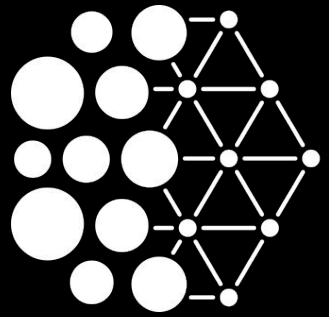

Mila

Introduction to Convolutional Neural Networks, Part I

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Warning to .pdf readers

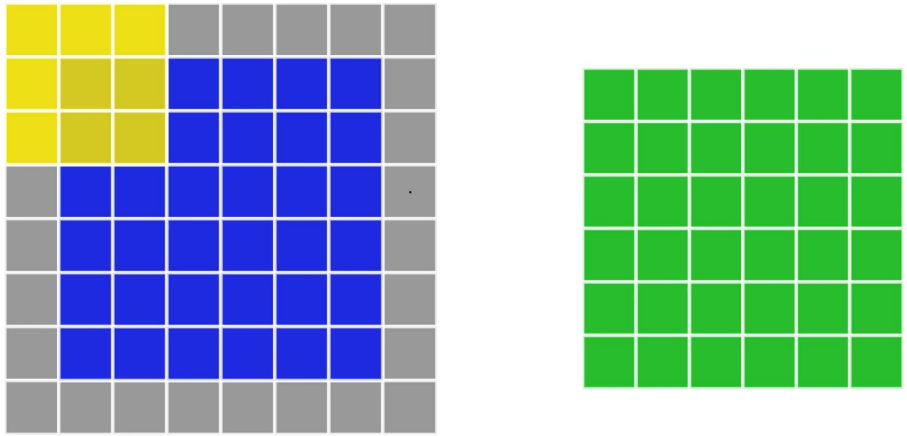
Some slides contained in this presentation contain .gif animations. Pdf files cannot render .gifs so this explains some blank slides in the .pdf format.

All animations in the slide deck can be found here:

https://github.com/jerpint/cnn-cheatsheet

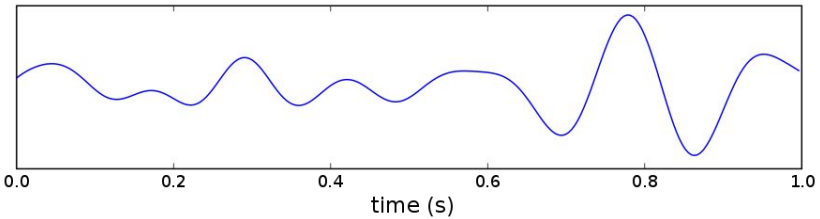
Contents of Part I

- Introduction + intuition to Convolutional Neural Networks (CNNs)
- Mathematical operations in CNNs
- Motivations for using CNNs
- CNN Building Blocks

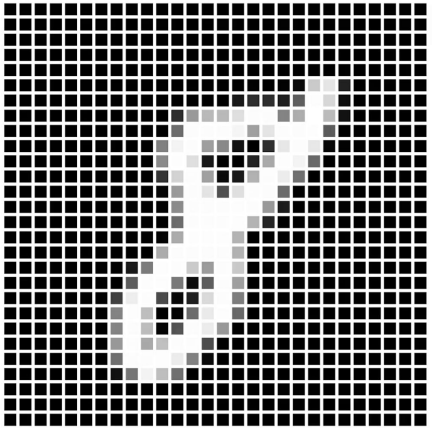


Introduction

Convolutional neural networks, or **CNN**s, are a special type of **neural network** designed for processing data with a **grid-like structure**. These can be, for example:



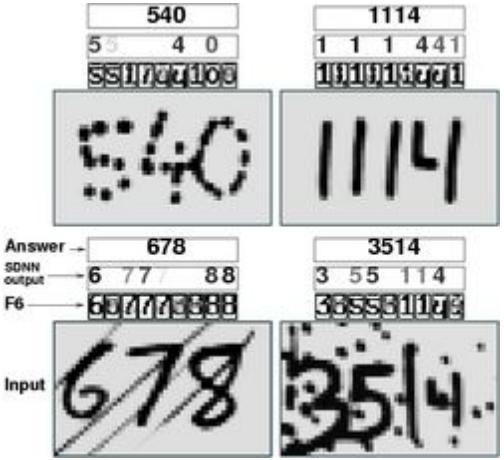
1D grids, e.g. time-series data taken at regular intervals



2D grids, e.g. images composed of pixels with intensity values

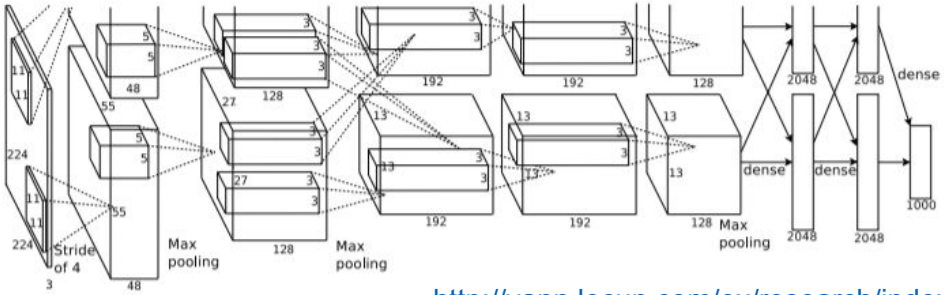
Introduction

CNNs were some of the **first deep learning models** to be shown to **generalize** well to “useful” tasks.



Modern interest in CNNs came about when Alex Krizhevsky et al. won the 2012 **ImageNet** object recognition challenge using **AlexNet**.

IMAGENET



http://yann.lecun.com/ex/research/index.html

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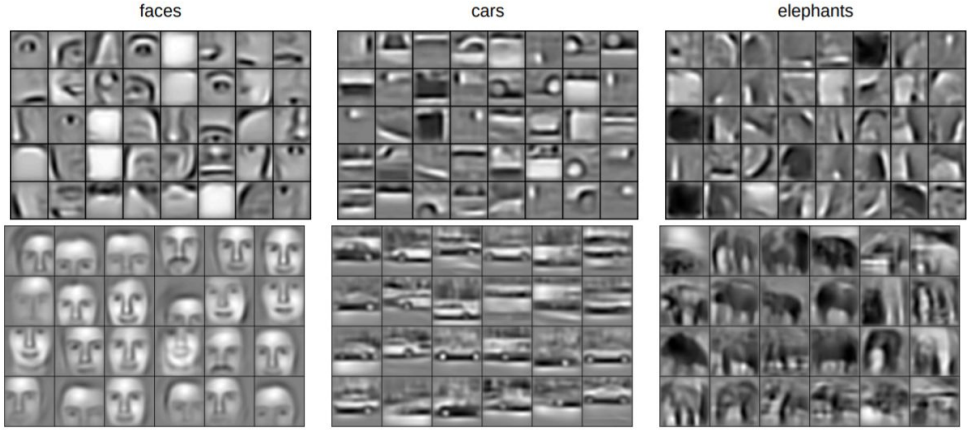
Introduction

CNNs were introduced to encourage neural networks to focus on **local features**, or “patches”, in images.

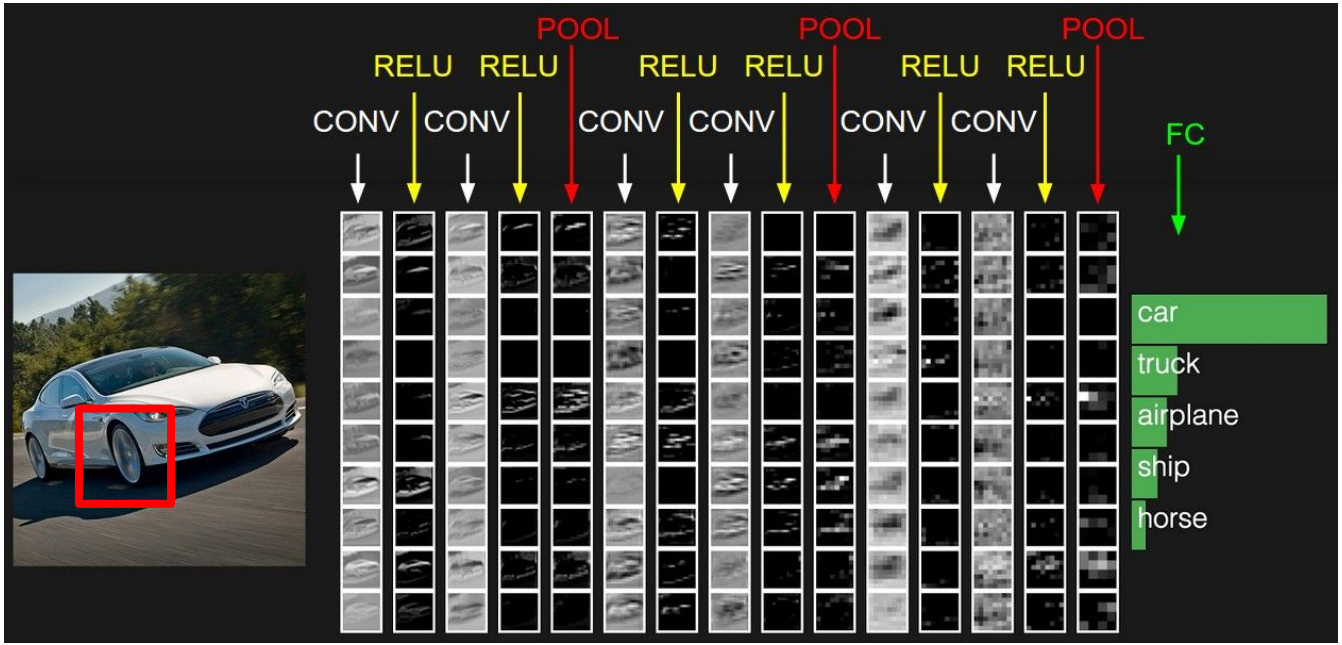


Think of a detector that scans images looking for “eye-like” and “nose-like” features to detect a human.

http://robotics.stanford.edu/~ang/papers/icml09-ConvolutionalDeepBeliefNetworks.pdf

Introduction

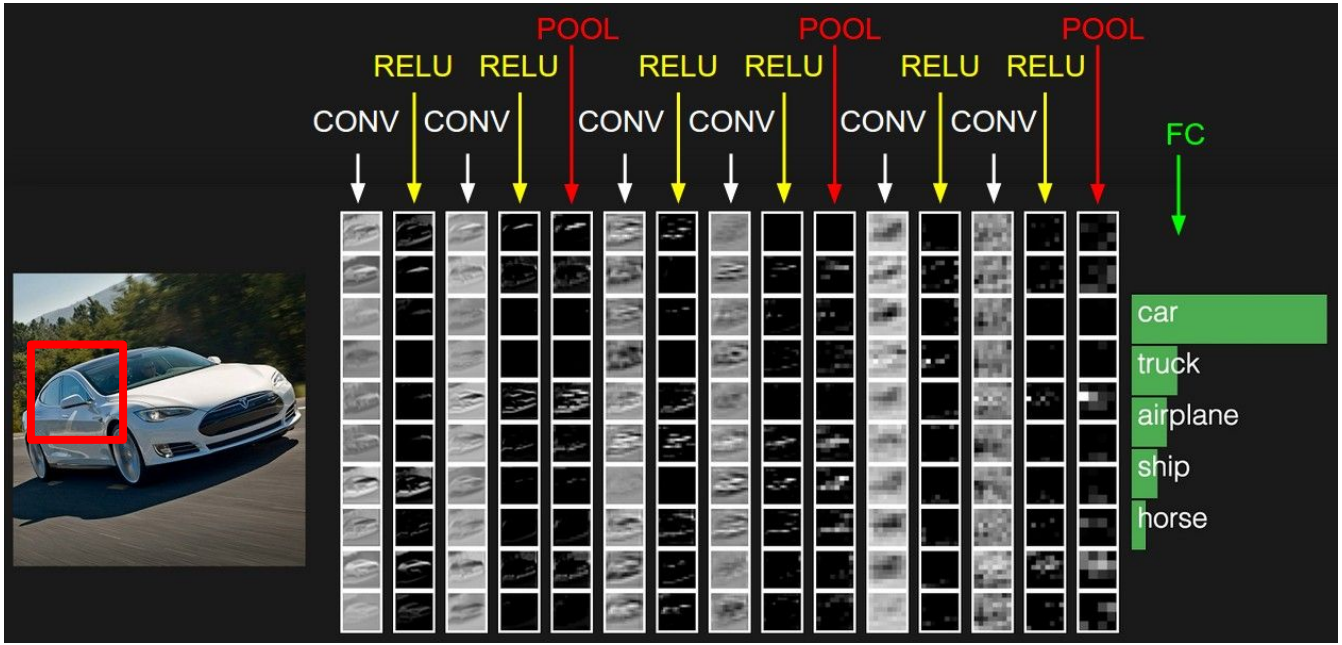
For example, features can be extracted at different local regions to predict whether a car is present in an image or not.



http://cs231n.github.io/convolutional-networks/

Introduction

For example, features can be extracted at different local regions to predict whether a car is present in an image or not.

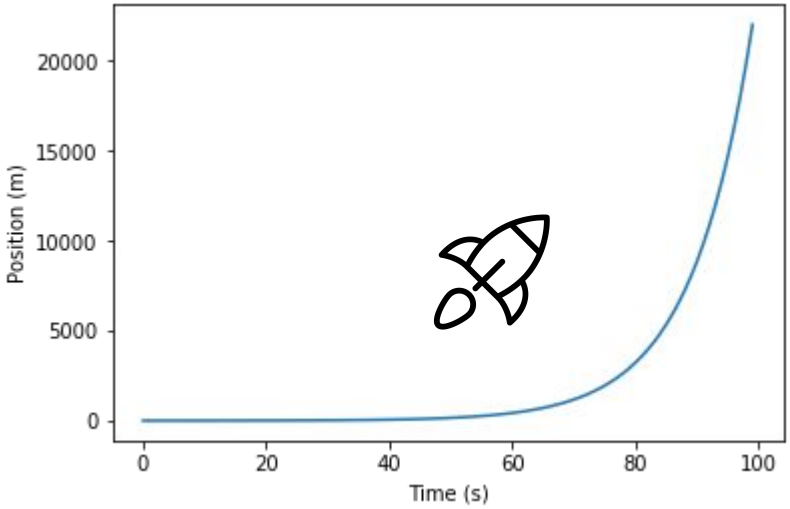


http://cs231n.github.io/convolutional-networks/

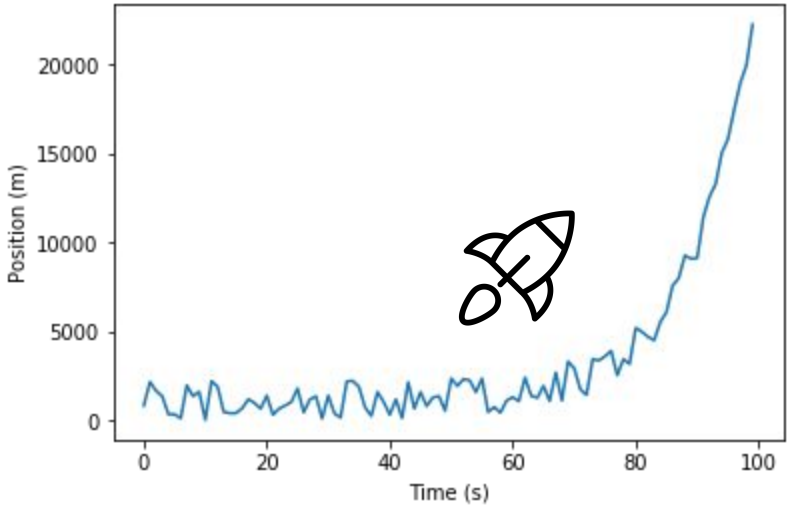
Convolution - Intuition

Suppose you are recording the **position** of a spaceship with a laser. However, you notice inherent **noise** in your **measurements** (radiation, temperature, etc.).

Real Position



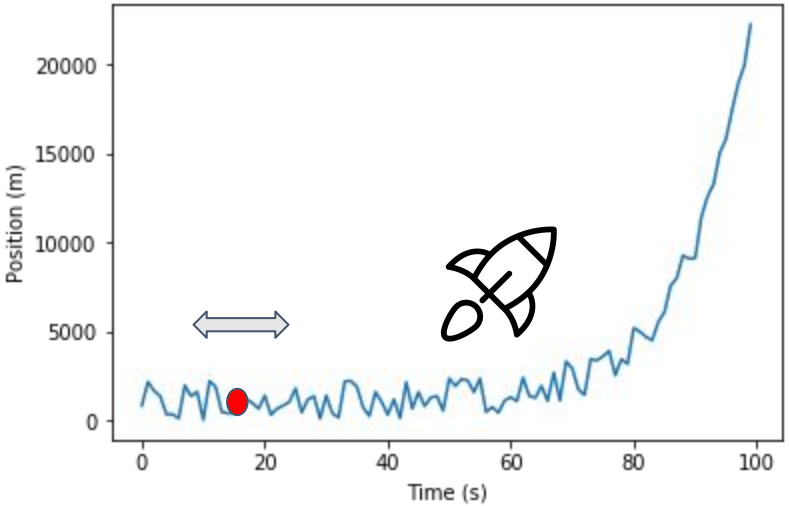
Measured Position



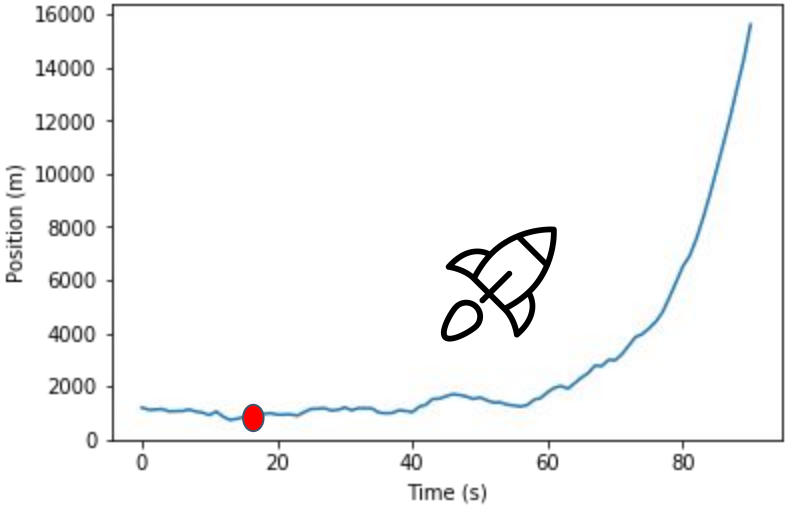
Convolution - Intuition

A “simple” way to clean the obtained signal is to use a moving average. This is an example of **smoothing**, and can easily be defined using a **convolution**.

Measured Position



Smoothed Signal



Convolution

Convolutions are a special type of mathematical **operation**. They are the foundation behind CNNs. Mathematically, a convolution is defined using the $*$ operator:

$$s(t) = (x * w)(t)$$

where **s(t)** is said to be the convolution of **x(t)** with **w(t)** and **t** is typically time. In the context of CNNs, ***x*** is typically referred to as the *input*, ***w*** as the *kernel* and **s** as the *feature map*.

$$s(t) = \int x(a)w(t-a)da.$$

Convolution for a continuous signal

$$s(t) = \sum_{a=-\infty}^{\infty} x(a)w(t-a).$$

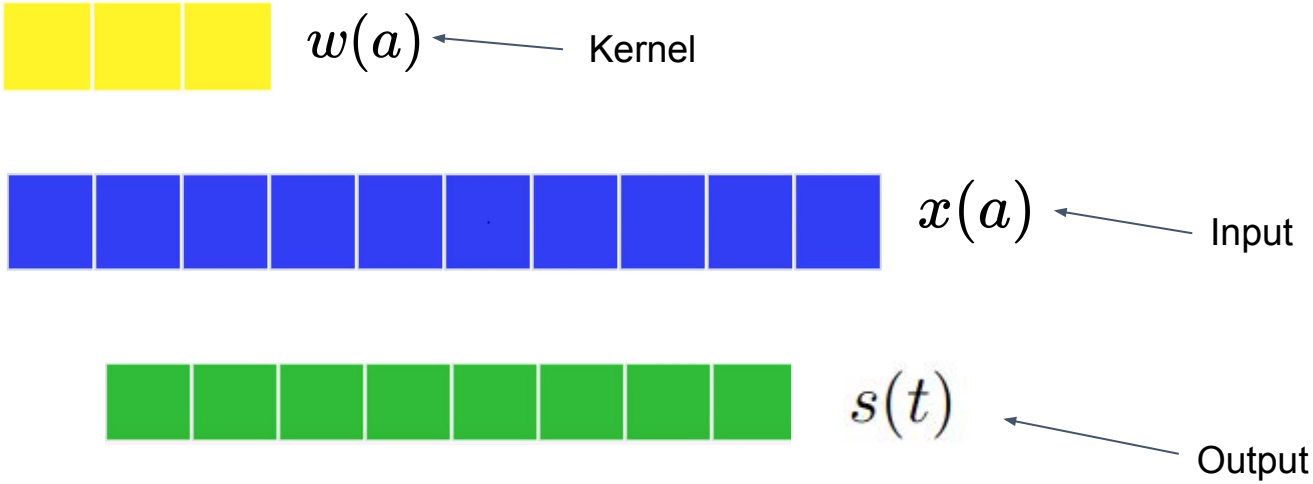
Convolution for a discrete signal

1D Convolution

Let’s begin with an animation of a **1D** convolution:

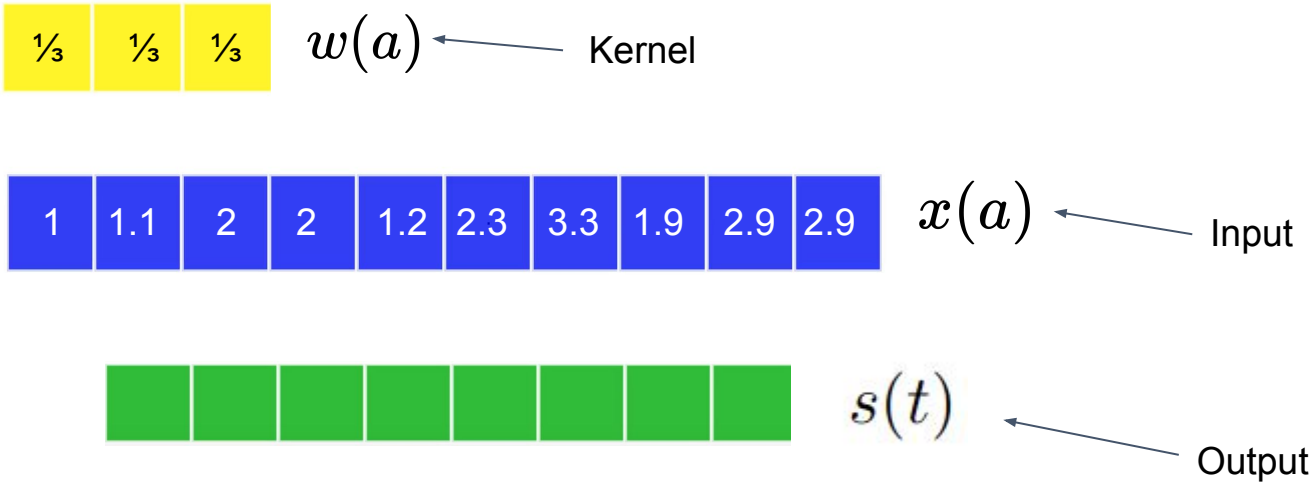
1D Convolution

$$s(t) = \sum_{a=-\infty}^{\infty} x(a)w(t-a).$$



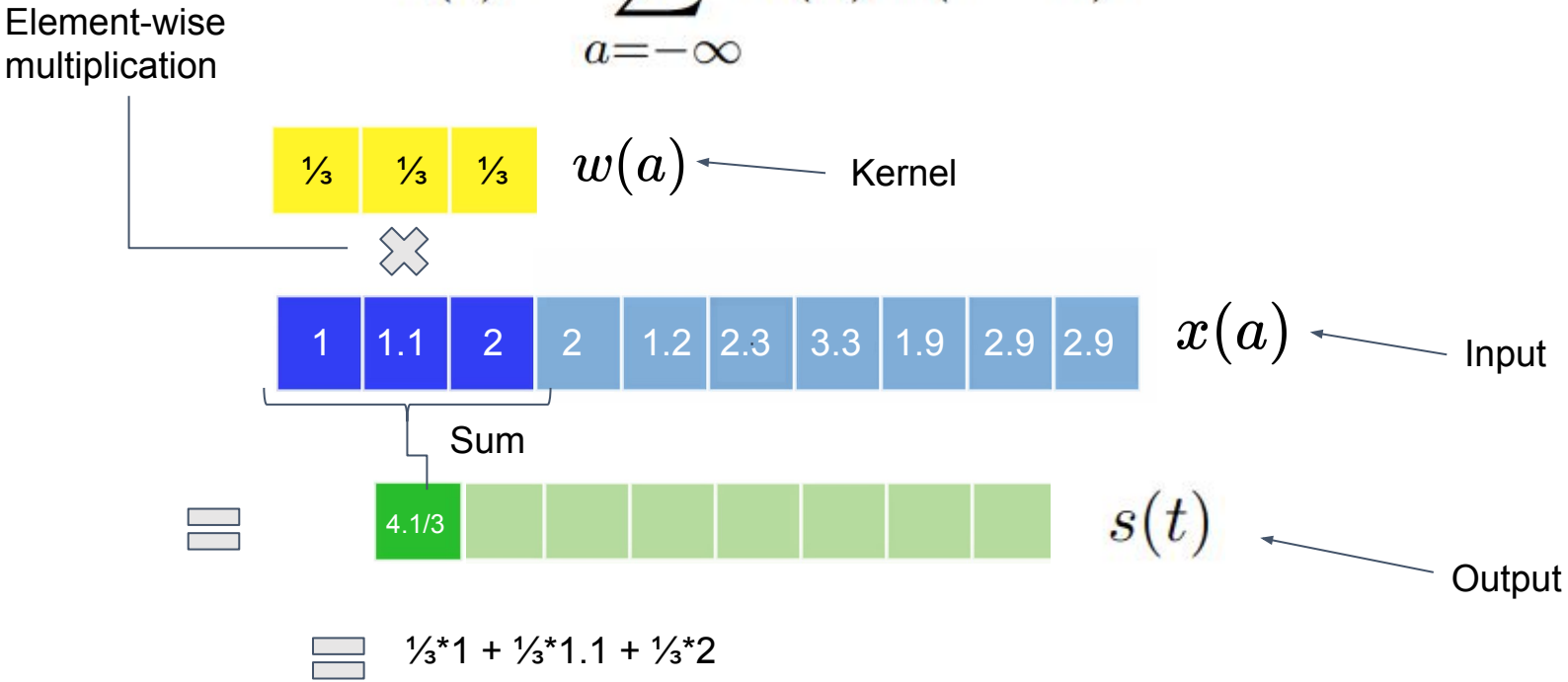
1D Convolution

$$s(t) = \sum_{a=-\infty}^{\infty} x(a)w(t-a).$$



1D Convolution

$$s(t) = \sum_{a=-\infty}^{\infty} x(a)w(t-a).$$



1D Convolution

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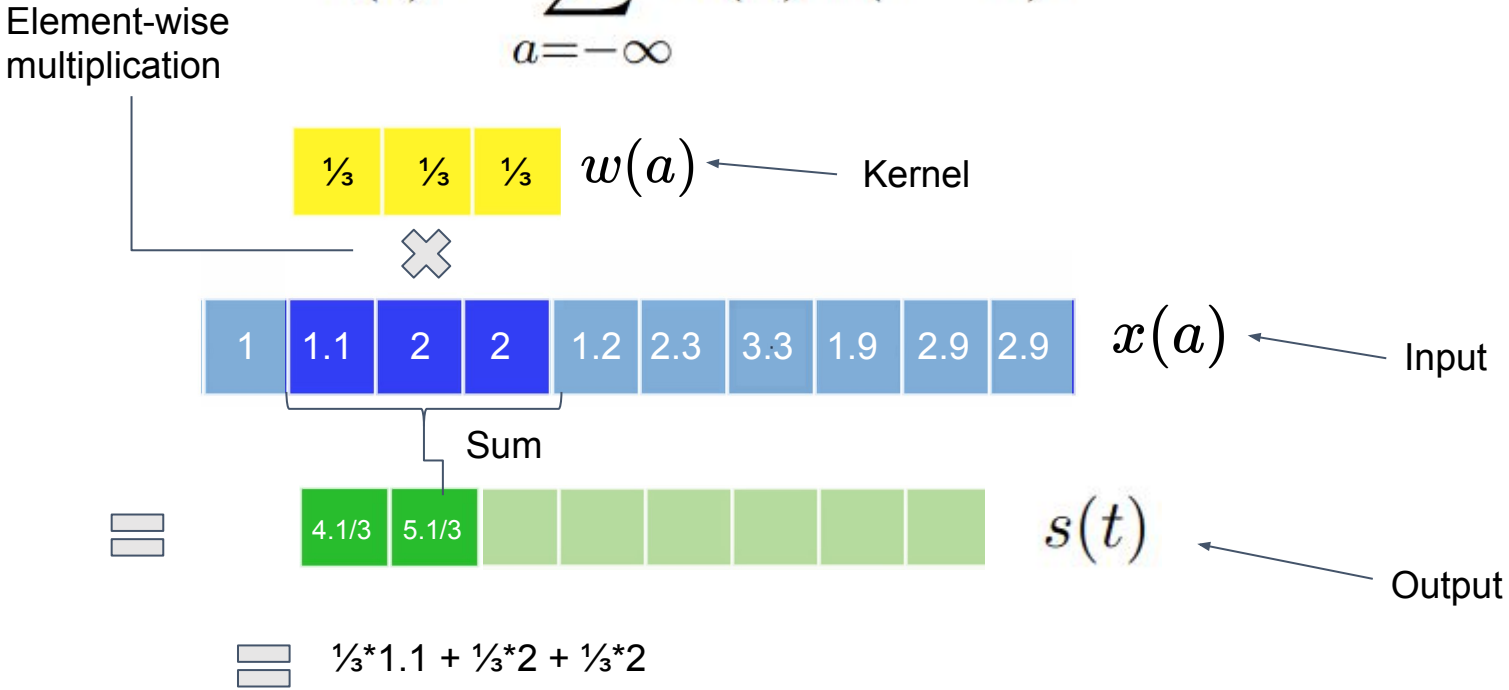

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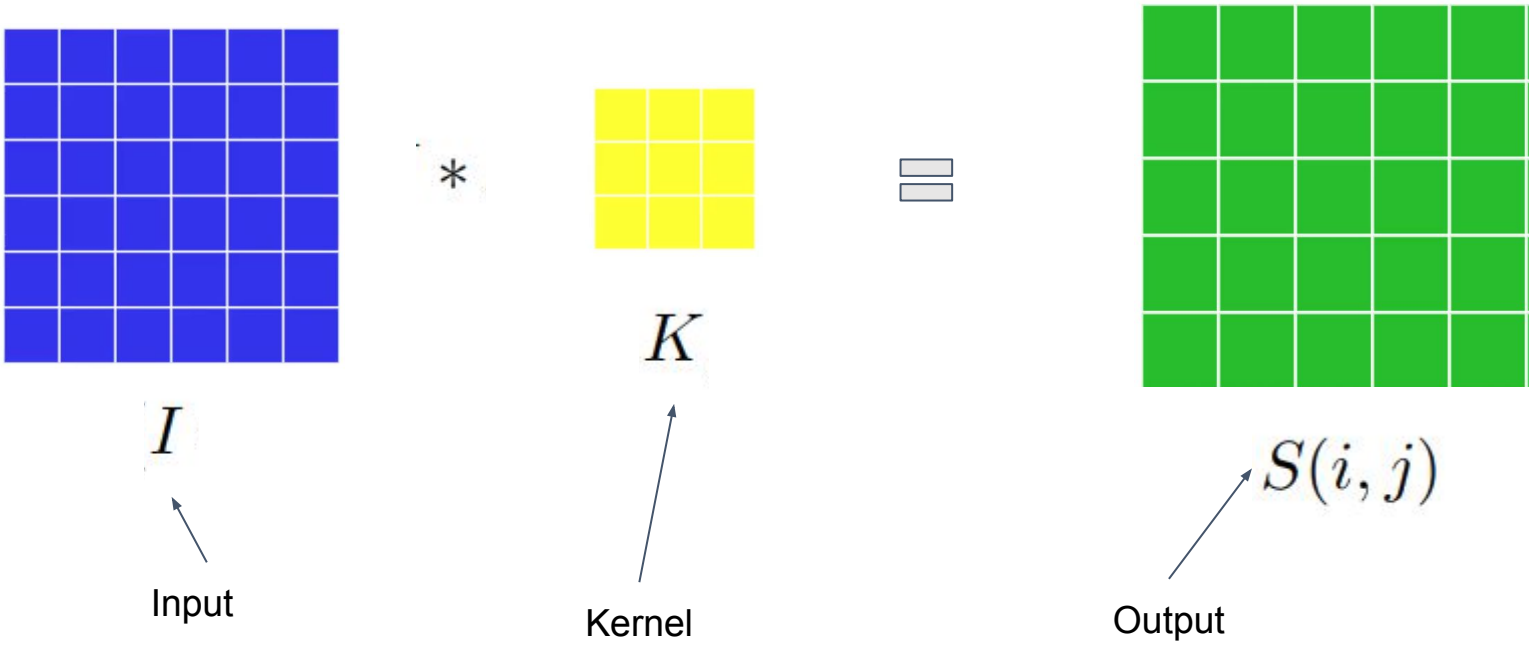
1D Convolution

2D Convolution

Now let’s look at a **2D convolution** animation:

2D Convolution

$$S(i,j) = (I * K)(i,j) = \sum_m \sum_n I(m,n) K(i-m, j-n).$$



2D Convolution

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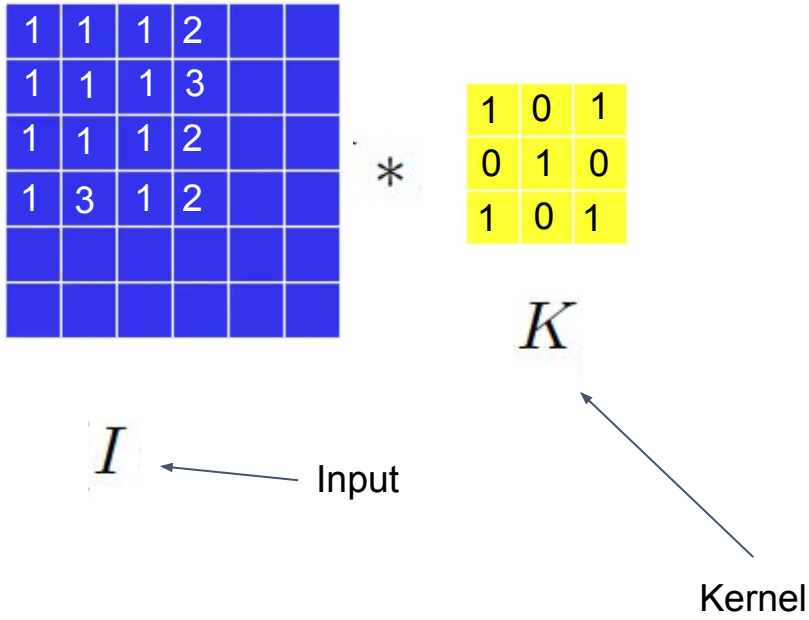

2D Convolution

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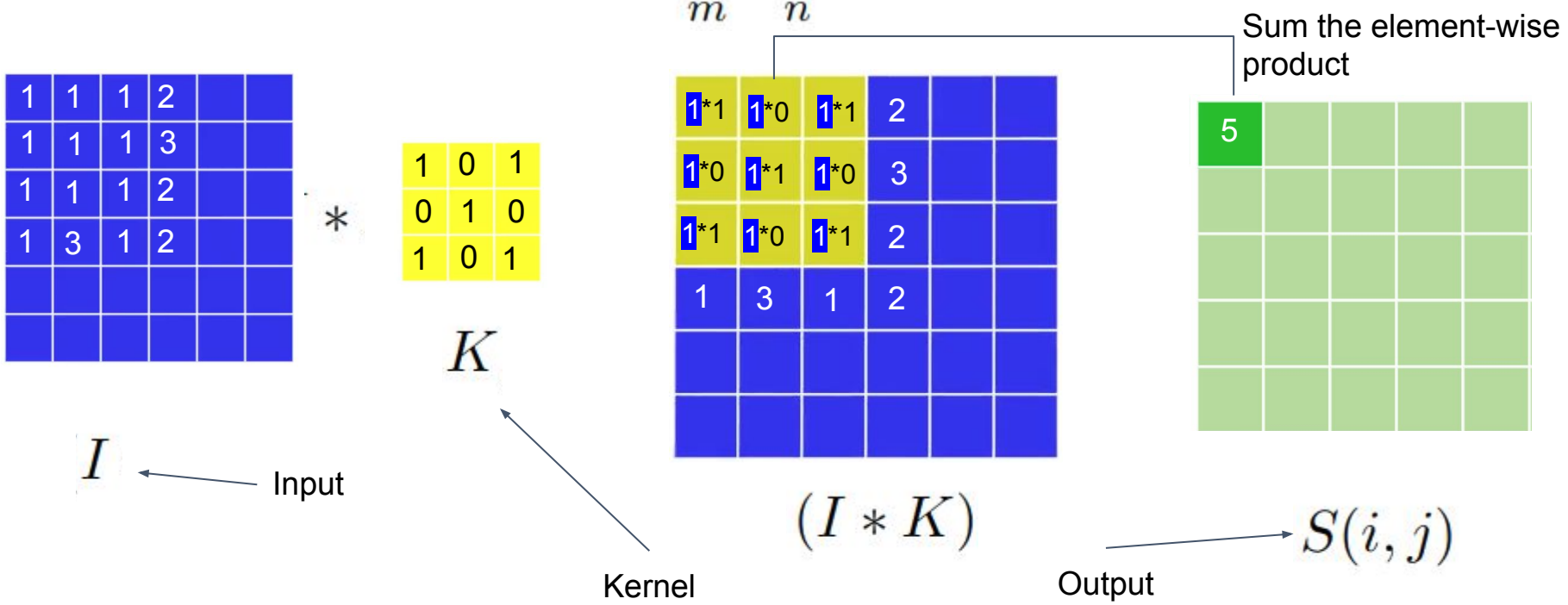

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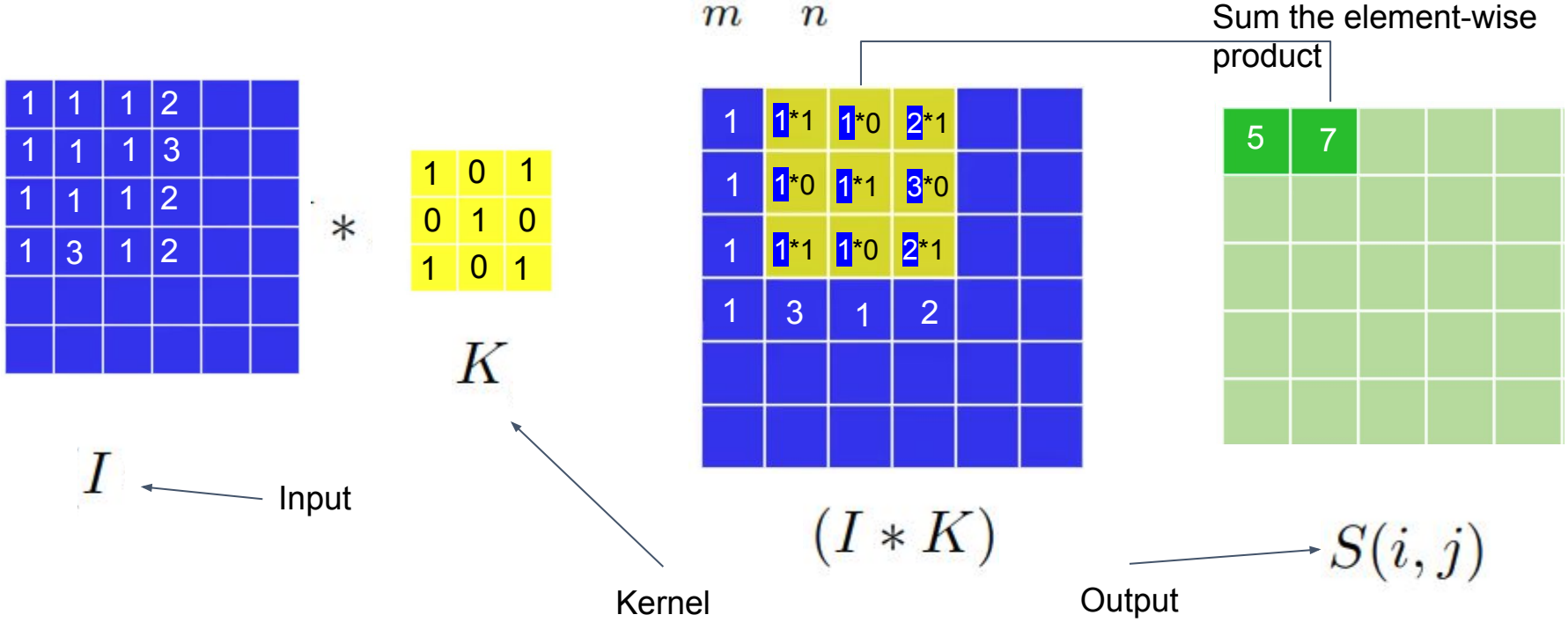

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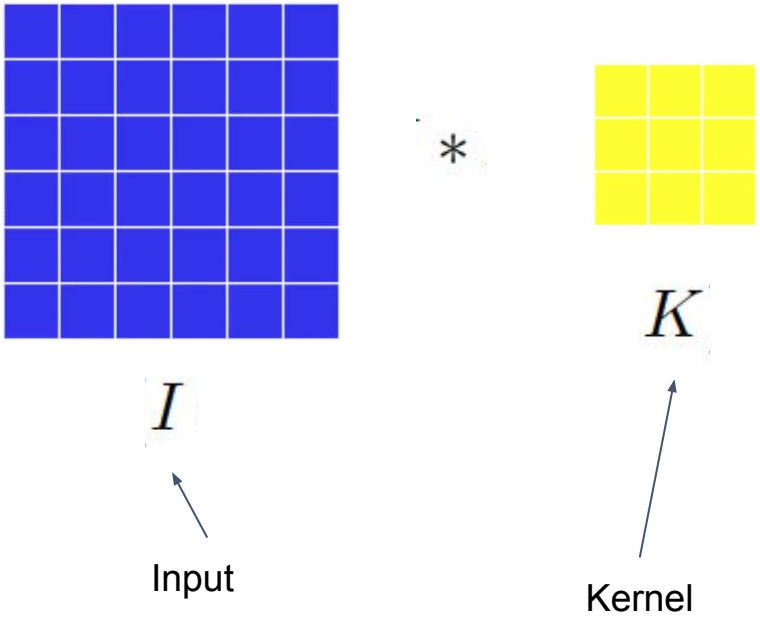
2D Convolution

$$S(i,j) = (I * K)(i,j) = \sum_m \sum_n I(m,n) K(i-m, j-n).$$

2D Convolution

The idea is that the network can **learn** the optimal parameters of the **kernel** to solve a given task.



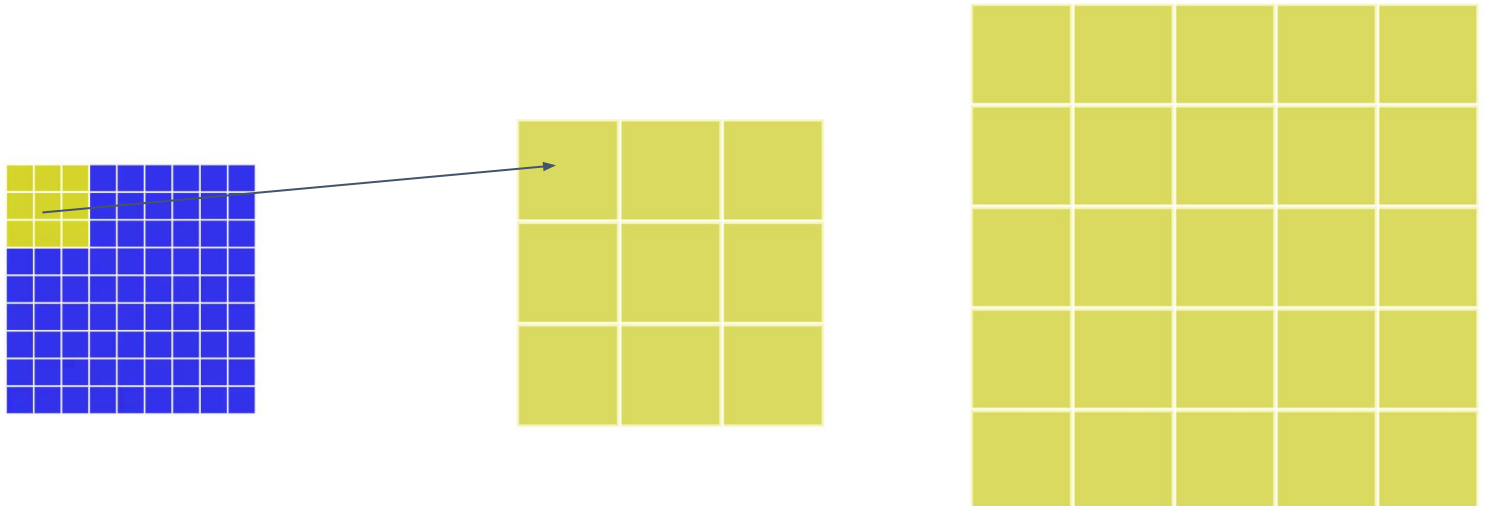
Pedantic timeout

Technically, convolutions in neural networks are really implemented as **cross-correlations** (the kernels are not flipped as they should be).

https://xkcd.com/1475/

Spatial Extent

The size of the kernel, or filter, usually denoted *F,* references how many pixels the kernel spans. Kernels are typically square in size.

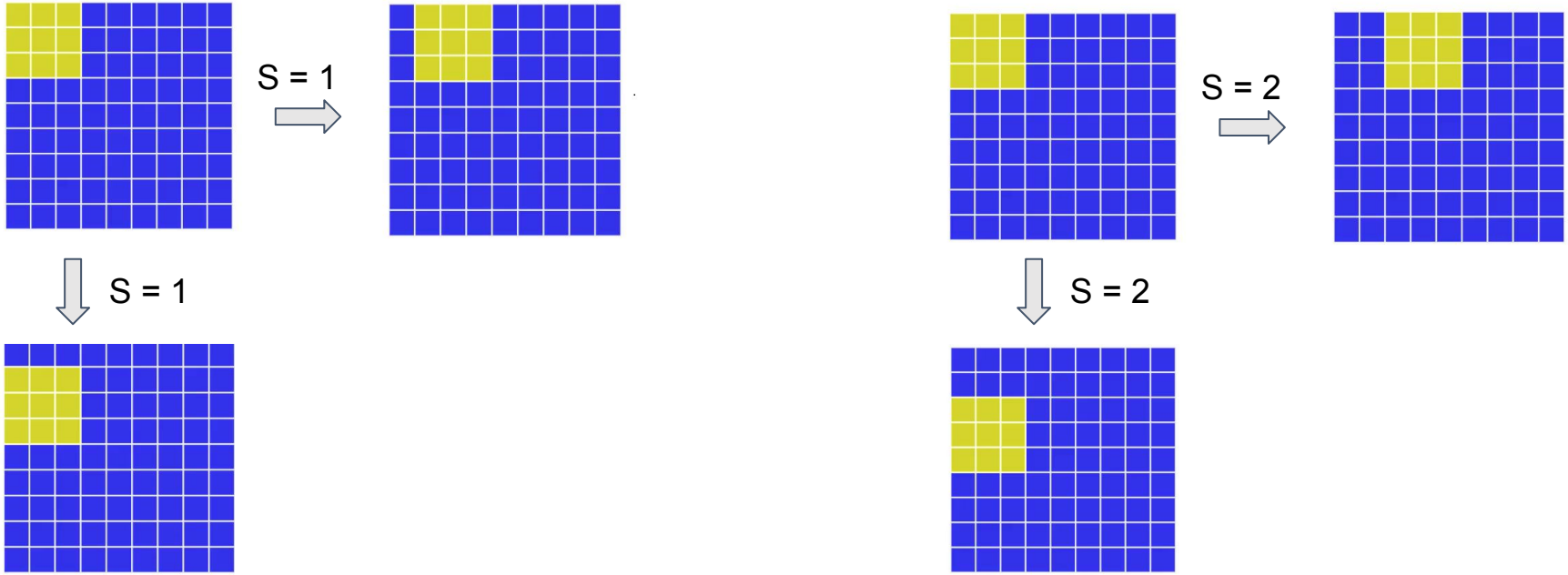


F = 3

F = 5

Stride

The **stride**, often denoted as *S,* represents the number of pixels to move in (x,y) before each operation in the convolution. Typical values used are either *S=1* or *S=2.*



Stride

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Stride = 1 in both x and y

Stride = 2 in both x and y

Stride

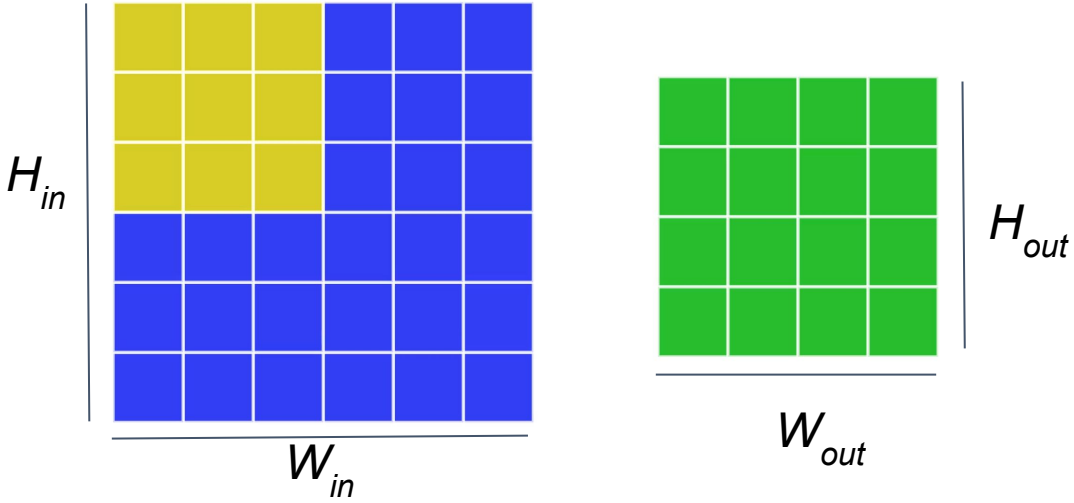
The **stride**, often denoted as *S,* represents the number of pixels to move in (x,y) before each operation in the convolution. Typical values used are either *S=1* or *S=2.*

Stride = 2 in both x and y

2D Convolution Arithmetic

In general, if we have an input of size W_{in} x H_{in}, a stride S and a spatial extent F, our output will have dimensions

$$W_{out} = (W_{in} - F) / S + 1$$
$$H_{out} = (H_{out} - F) / S + 1$$



In this example:

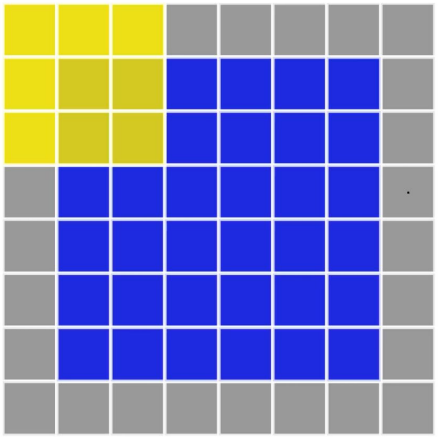
$S = 1, F = 3,$

$W_{in} = H_{in} = 6$

$W_{out} = H_{out} = (6 - 3) / 1 + 1 = 4$

Padding

You may have noticed that until now the convolution output is **smaller** than the input. **Padding** uses some strategy to pad the borders of our inputs to artificially increase the input size (gray). The number of pixels with which we pad is denoted *P.*



Example of padding with *P*=1

Padding

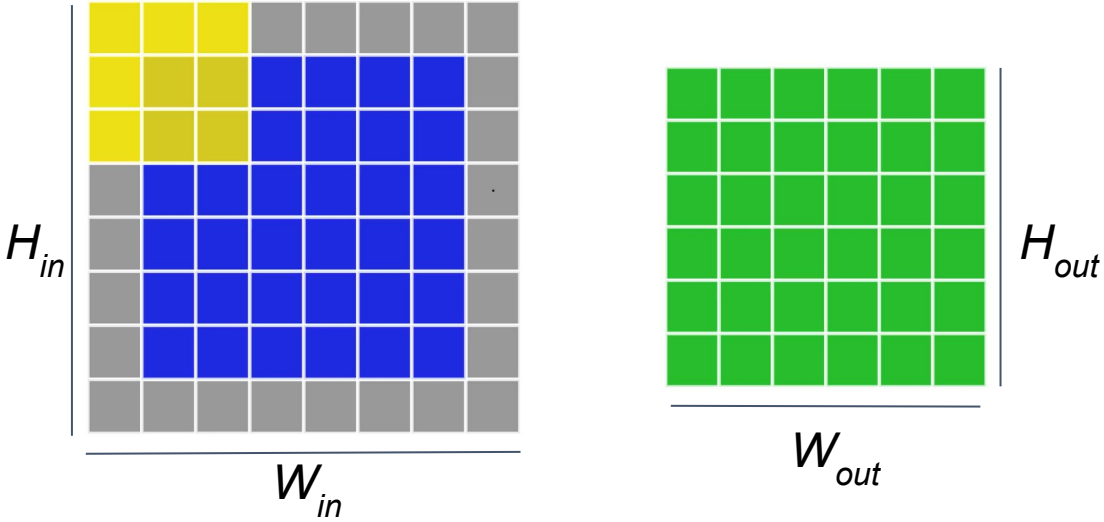
You may have noticed that until now the convolution output is **smaller** than the input. **Padding** uses some strategy to pad the borders of our inputs to artificially increase the input size (gray). The number of pixels with which we pad is denoted *P.*

Example of padding with *P*=1

Padding

By denoting *P* as the number of pixels with which we pad our borders, we get:

$$W_{out} = (W_{in} - F + 2P) / S + 1$$
$$H_{out} = (H_{out} - F + 2P) / S + 1$$



In this example:

$S = 1, D = 1, F = 3, P=1$

$W_{in} = H_{in} = 6$

$W_{out} = H_{out} = (6 - 3 + 2*1) / 1 + 1 = 6$

Padding

In practice, there are 3 **ways** padding can be used to achieve different output sizes:

Valid convolution

The kernel is allowed to visit only positions where the kernel is contained entirely within the input (i.e. no padding)

Padding

In practice, there are 3 **ways** padding can be used to achieve different output sizes:

Same convolution

Just enough padding is added to keep the size of the output equal to the size of the input

Padding

In practice, there are 3 **ways** padding can be used to achieve different output sizes:

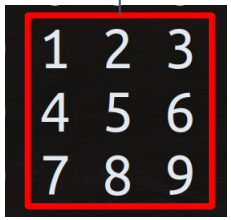
Full convolution

Enough padding is added for every pixel to be visited exactly *M* times in each direction.

Padding Strategies

There are different strategies for padding. The most common one is **zero padding**, in which zeros are simply added at the borders.

```
t = tf.constant([[1, 2, 3], [4, 5, 6], [7,8,9]])
```



https://www.tensorflow.org/api_docs/python/tf/pad

Padding Strategies

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```
t = tf.constant([[1, 2, 3], [4, 5, 6], [7,8,9]])
paddings = tf.constant([[2, 2,], [2, 2]])
out = tf.pad(t, paddings, "CONSTANT")
```

Specify how much to pad along each dimension

```
[[0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0]
 [0 0 1 2 3 0 0]
 [0 0 4 5 6 0 0]
 [0 0 7 8 9 0 0]
 [0 0 0 0 0 0 0]
 [0 0 0 0 0 0 0]]
```

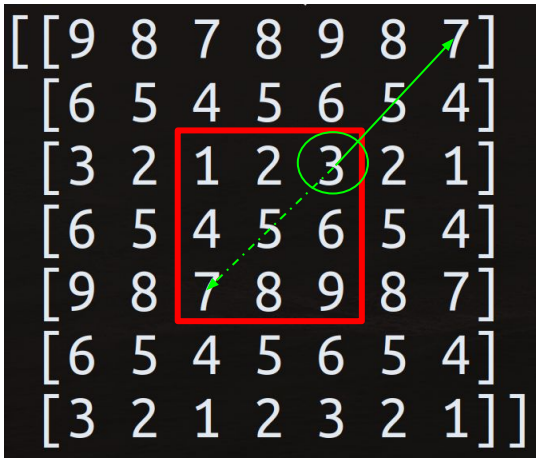
https://www.tensorflow.org/api_docs/python/tf/pad

Padding Strategies

There are different strategies for padding. The most common one is **zero padding**, in which zeros are simply added at the borders. Other strategies include **reflective** and symmetric padding.

```
t = tf.constant([[1, 2, 3], [4, 5, 6], [7,8,9]])
paddings = tf.constant([[2, 2,], [2, 2]])
out = tf.pad(t, paddings, "REFLECT")
```

Specifies how much to pad along each dimension



https://www.tensorflow.org/api_docs/python/tf/pad

Padding Strategies

There are different strategies for padding. The most common one is **zero padding**, in which zeros are simply added at the borders. Other strategies include reflective and **symmetric** padding.

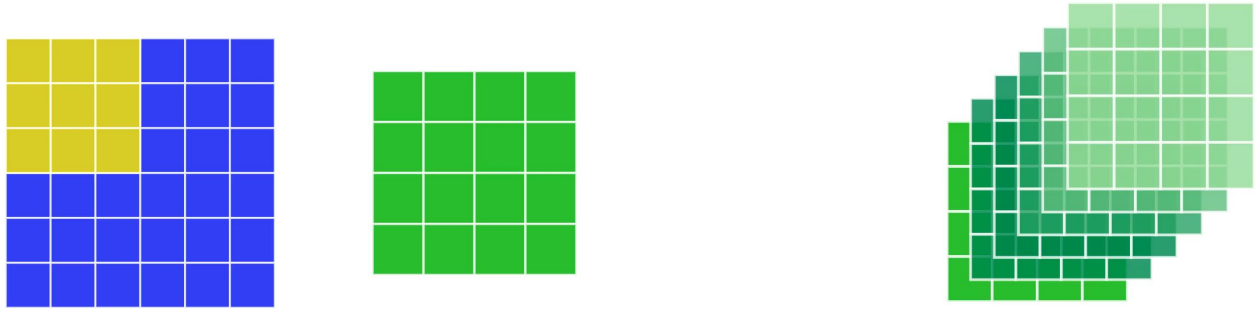
```
t = tf.constant([[1, 2, 3], [4, 5, 6], [7,8,9]])
paddings = tf.constant([[2, 2,], [2, 2]])
out = tf.pad(t, paddings, "SYMMETRIC")
```

Specifies how much to pad along each dimension

https://www.tensorflow.org/api_docs/python/tf/pad

Depth

We can use more than one kernel to compute many feature maps. The **depth**, or number of kernels, denoted as *D,* represents how many kernels will be used and will determine the **depth** of the volume (number of channels) of the output. The resulting feature maps are stacked to produce a volume.



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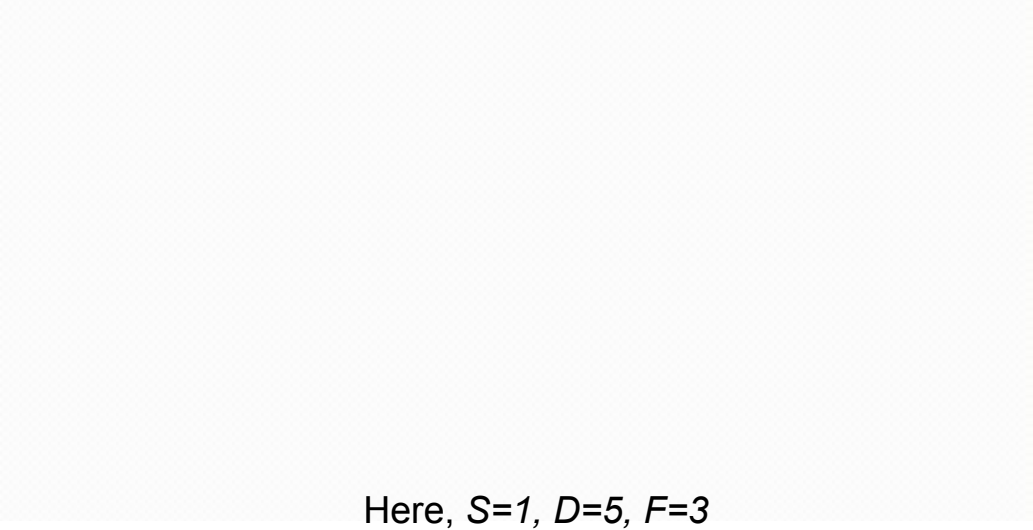


Here, *S=1, D=5, F=3*

Bias

We add a constant **bias** *b* to every depth layer in our network. The value of the bias is learned by the network. For a depth of *D*, we require *D* biases *b,* one per kernel.

$$\theta = (W, b)$$

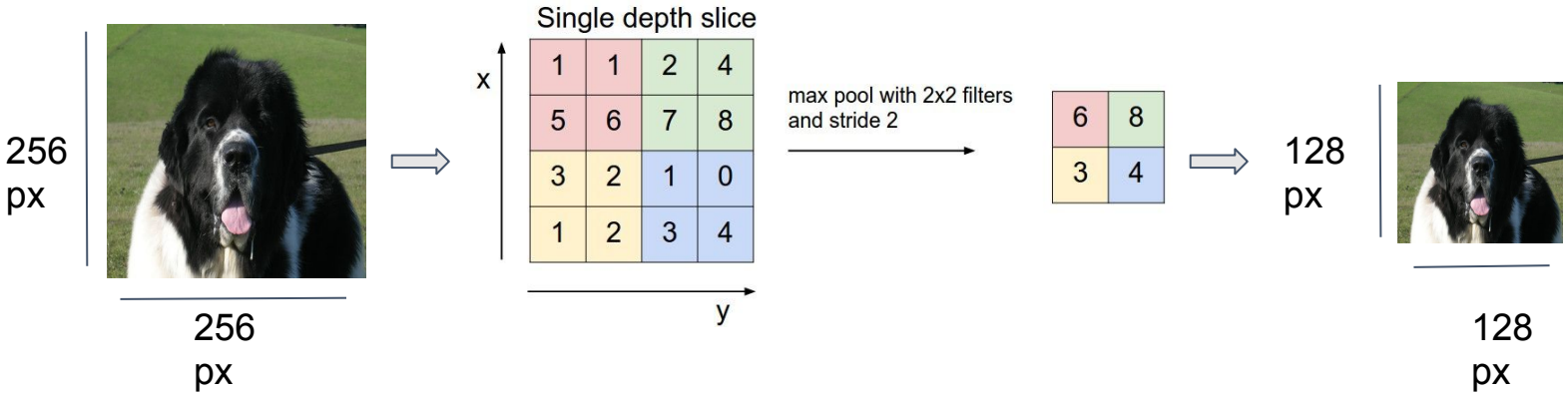
$$y = Wx + b \qquad y = W * x + b$$

MLP: Matrix multiplication

CNN: Convolution

Pooling

Pooling is used to downsample an image. The most common type of pooling is **max-pooling**. Pooling makes the network **less sensitive** to fine details.



http://cs231n.github.io/convolutional-networks/
https://farm3.static.flickr.com/2313/2133789553_e37c0ccf1b.jpg

Pooling

Notice how we increment our kernel with a stride of 2 in both the x and y direction. This allows us to downsample by a factor of 2 in every dimension.

2x2 max-pooling with stride 2

Dilated Convolution

Another way to downsample our input is by using **dilated convolutions** (or *atrous* convolution). The **dilation coefficient** *d* determines pixel spacing between convolutions.

Dilated convolution with $d = 2$

Transposed Convolution

Transposed convolutions are used to **increase** the resolution of a **feature map**. It can be thought of as an interpolation, or a convolution on the output. It is often used in the context of **upsampling** the output of a network.

http://deeplearning.net/software/theano/tutorial/conv_arithmetic.html

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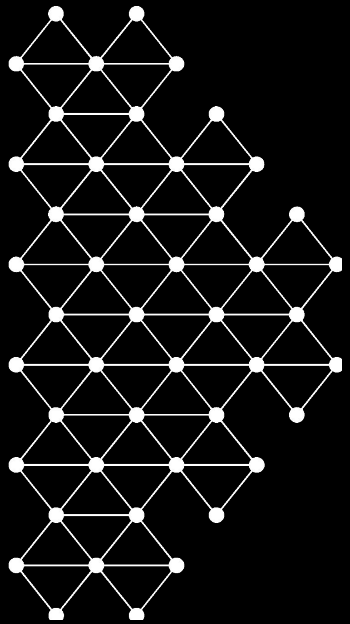
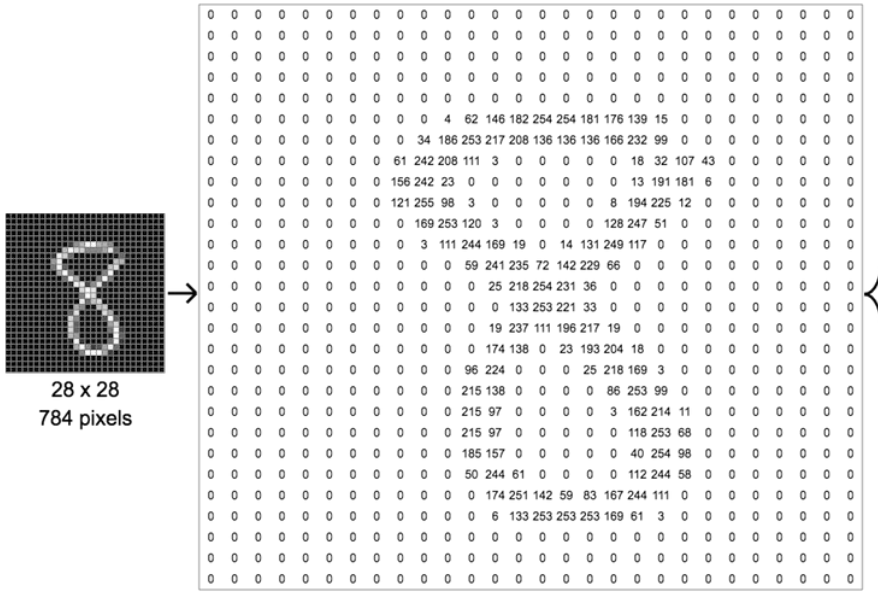


Image Representations

Image representations

- An image on a screen can be defined as an **array of pixels**.
- A grid of pixel intensities is referred to as a **channel**.
- A **grayscale** image varies in white and black intensity, typically in the range [0, 255].



https://cdn-images-1.medium.com/max/1600/1*av47vApmzuM0AN21VaIcSA.png

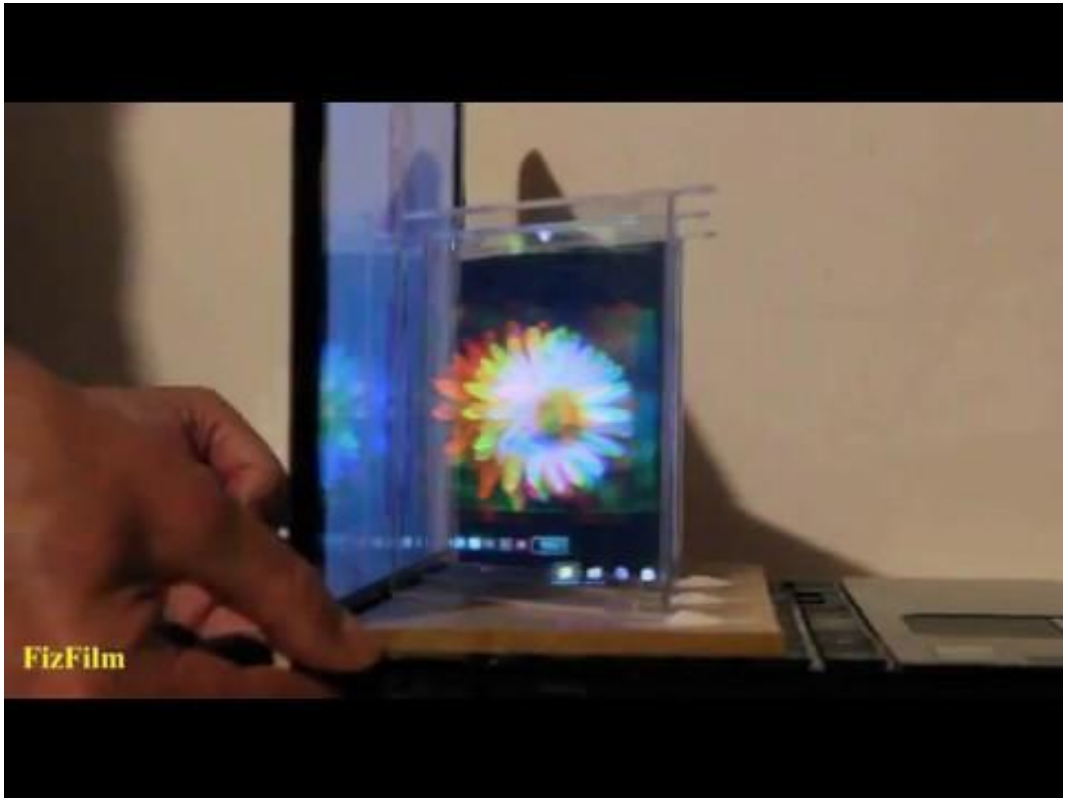
2D Convolution on a grayscale image

Image representations

To display colours, we use can use an **RGB** representation. **R**ed, **G**reen, and **B**lue **channels** are used to define the different **intensities** in each and combined to give all the colours our eyes can see.

https://en.wikipedia.org/wiki/RGB_color_model#/media/File:Rgb-compose-Alim_Khan.jpg

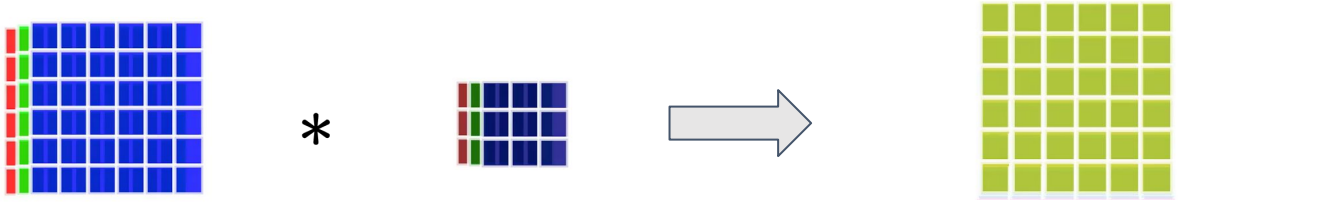
RGB Image Demo



Link to the demo

2D Convolution on an RGB Image

2D convolutions layers typically deal with volumes as **input**. Each kernel will also be a **volume**. For **each** kernel, we will have a different single-channel output.



Input (image)
Volume : W_{in} x H_{in} x C_{in}

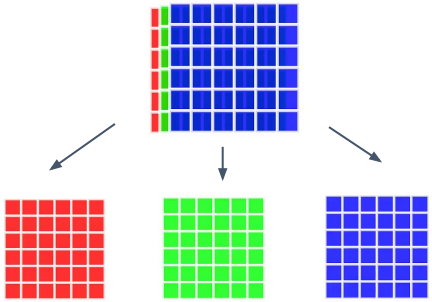
Kernel
Volume : F x F x C_{in}

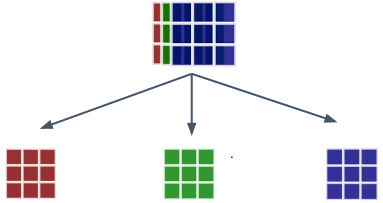
Output (feature map)
Volume: W_{out} x H_{out} x 1.

Additional material

2D Convolution

Separate the inputs and kernels by channel

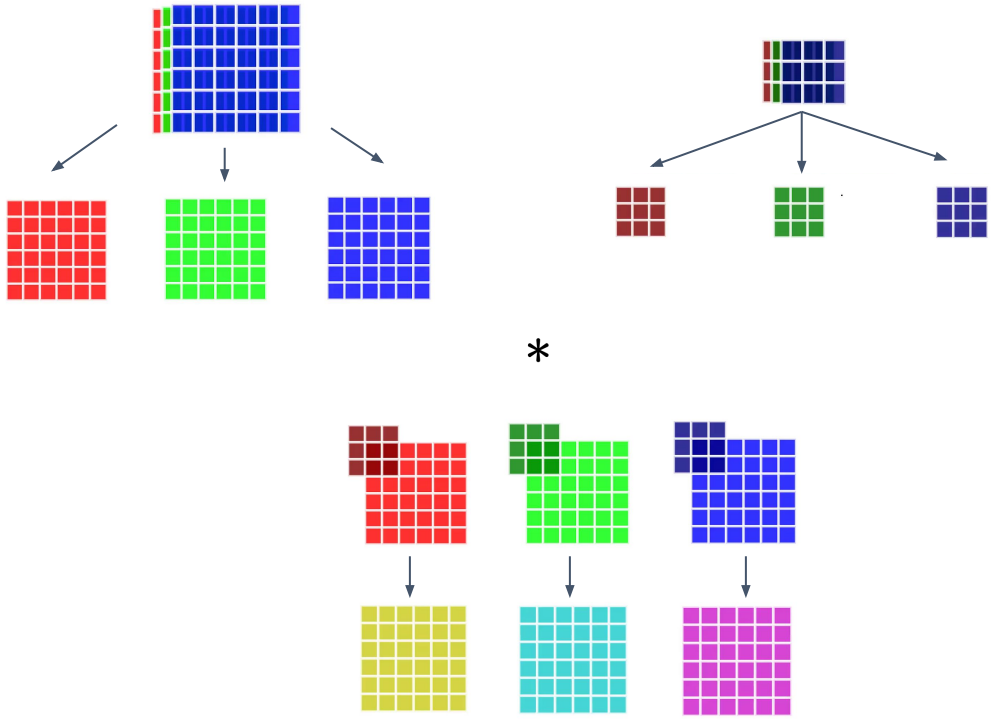




2D Convolution

Separate the inputs and kernels by channel

Perform 2D convolution on each channel

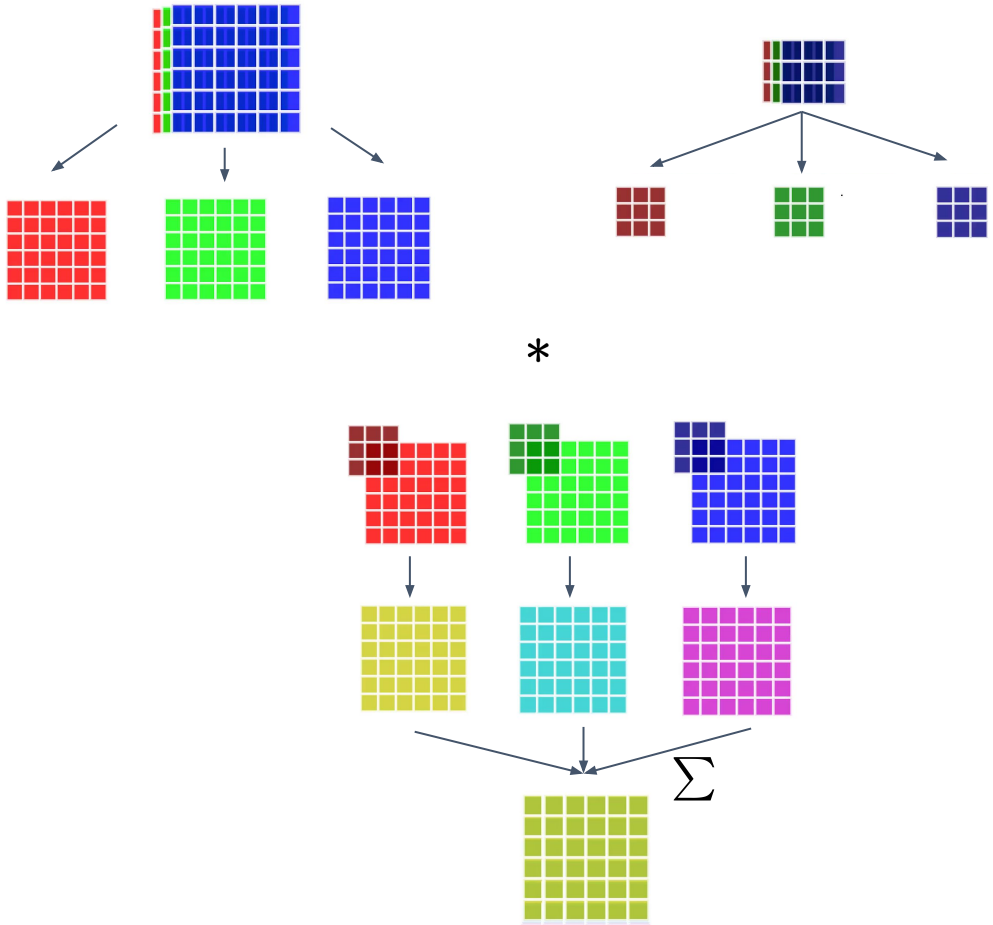


2D Convolution

Separate the inputs and kernels by channel

Perform 2D convolution on each channel

Sum the results to get a $W_{out} \times H_{out} \times 1$ feature map



2D Convolution on an RGB Image

Here is an animation showing the entire procedure of 2D convolution using a single kernel:

Additional material

From volume to volume

To create depth, we simply use **multiple kernels**. Here, we are using D=4 kernels, each of volume 3x3x3. We are going from an *6x6x3* RGB image to an 6x6x4 feature map. The number of kernels determine the depth, D, or C_{out}.

Input (RGB image),
W_{in} x H_{in} x 3
(6x6x3)

Kernels
D x F x F x C_{in}=
(***4*** x 3x3x3)

Feature Map
W_{out} x H_{out} x D =
(6x6x**4**)

From volume to volume

To create depth, we simply use **multiple kernels**. Here, we are using D=8 kernels, each of volume 3x3x4. We are going from an 6x6x4 feature map to an 6x6x8 feature map. The number of kernels determine the depth, D, or C_{out}.

Input (feature map),
W_{in} x H_{in} x 3
(6x6x4)

Kernels
D x F x F x C_{in}=
(***8*** x 3x3x4)

Feature Map
W_{out} x H_{out} x D =
(6x6x**8**)

1x1 Convolutions

1x1 convolutions are useful in the context of **non-linear** channel reduction. They allow us to preserve the width and height and reduce on the depth. They also require few parameters.

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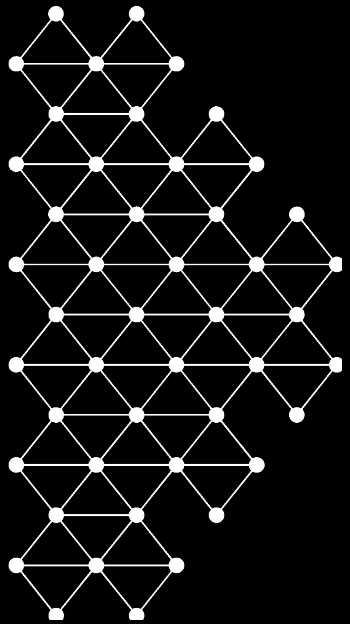
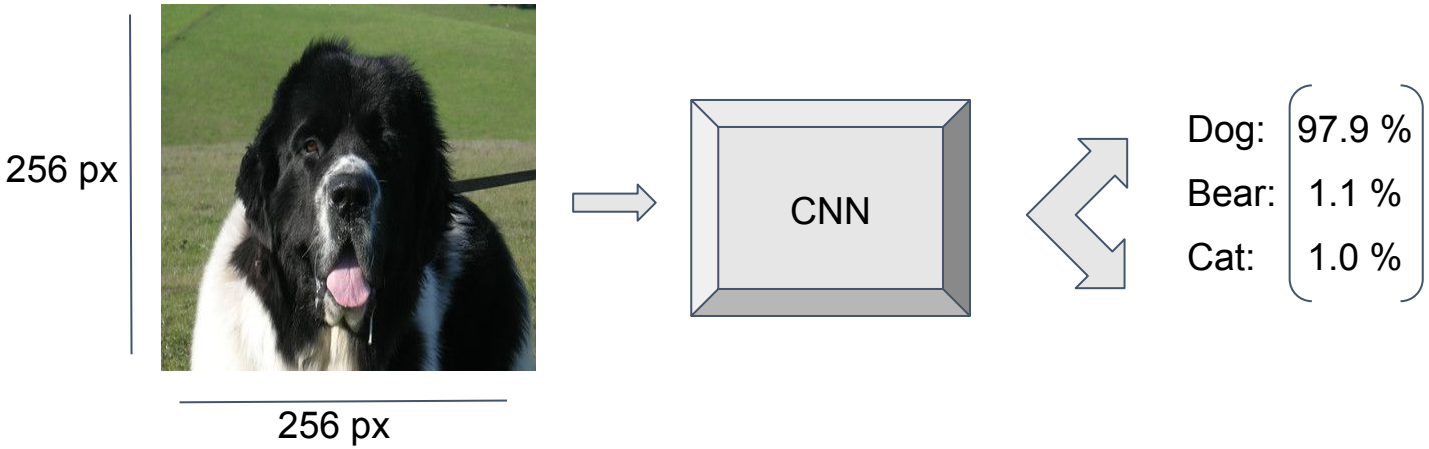


CNN Building Blocks

Building Blocks

A CNN can be used to map an **input** (i.e. an image) to a **probability distribution**.

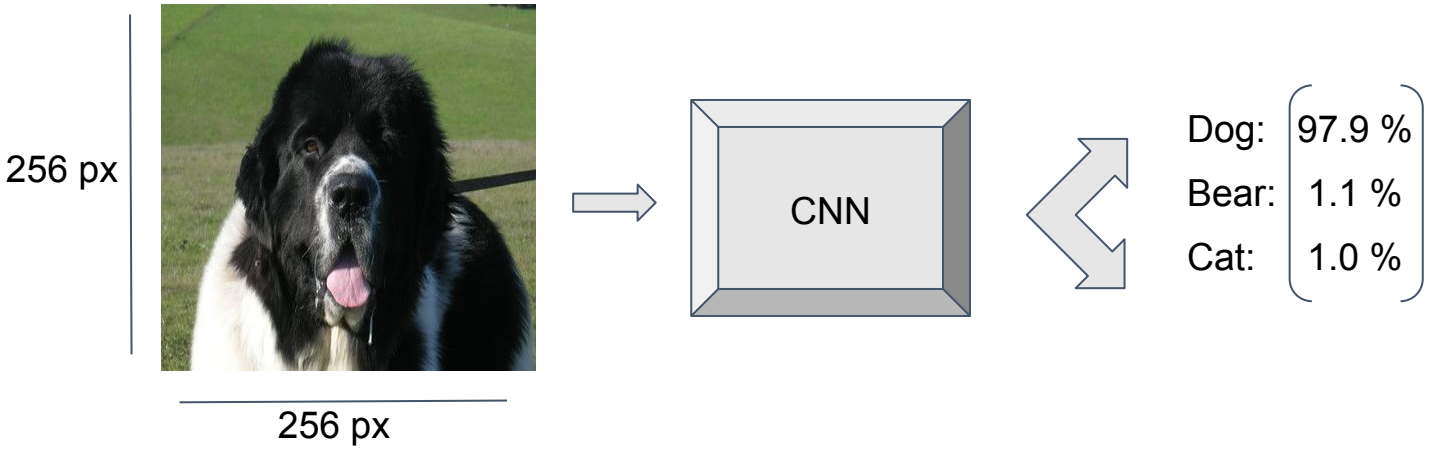


Building Blocks

Consider a colour image of 256x256 pixels. We have a total of:

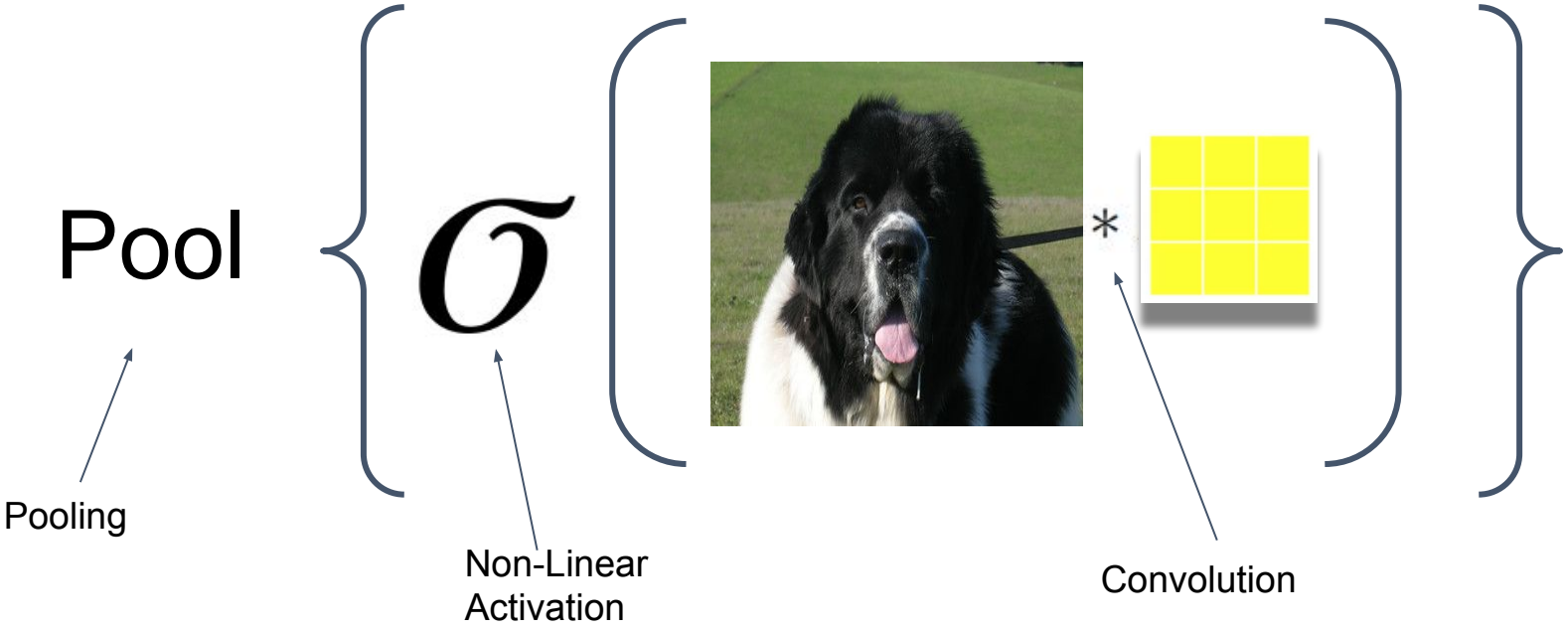
256 pixels *256 pixels *3 channels = 196 608 inputs.

We want to **reduce** it to a vector of size 1x*M*, containing the probability of belonging to each of the M classes.



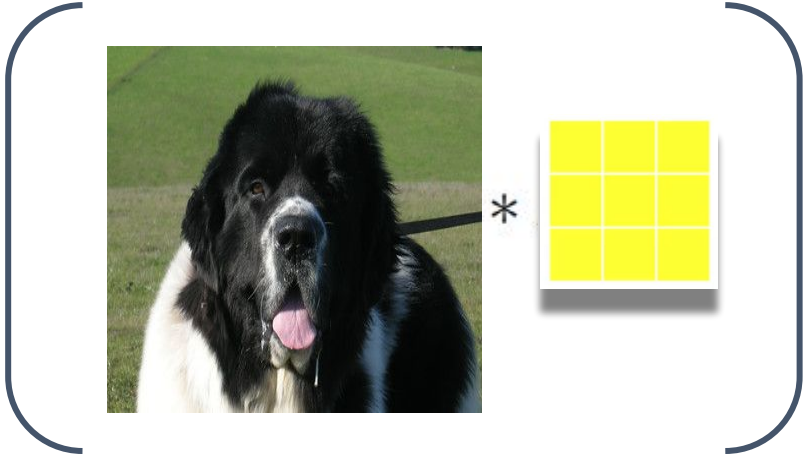
Building Blocks

To do so, we typically use a series of building blocks. A common building block for a CNN is shown here:



Building Blocks

Step 1: **Convolution** - Apply the convolution operation to our input with learned kernels



Building Blocks

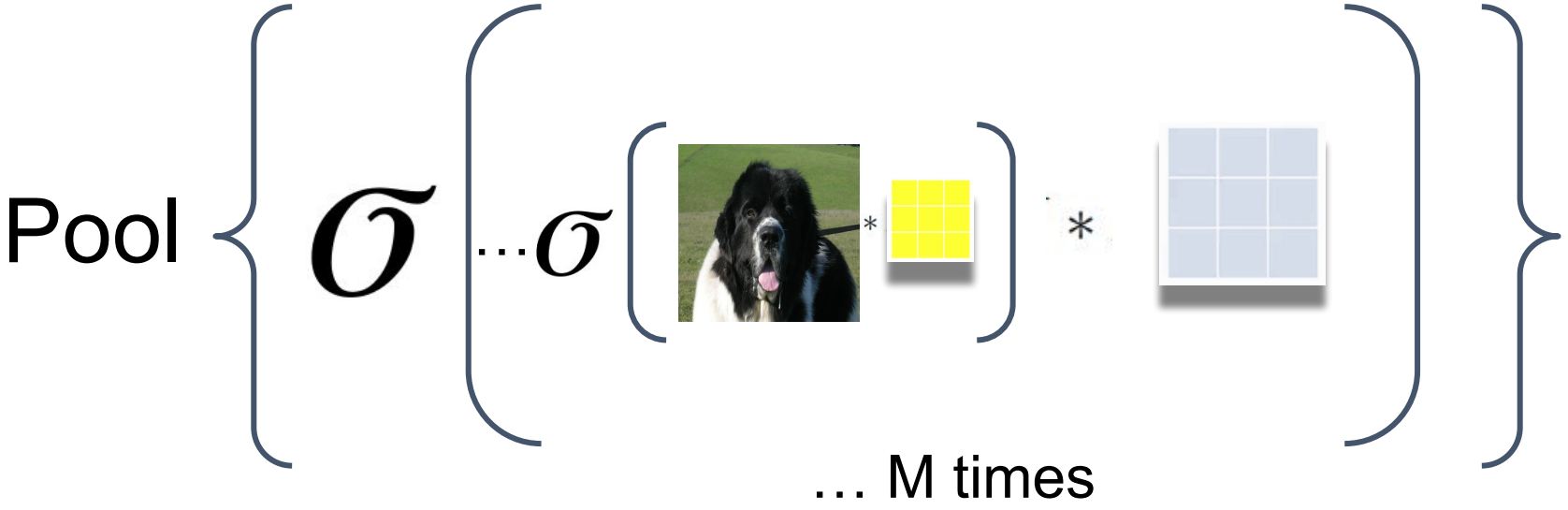
Step 2 : **Activation** - Apply a non-linear activation function element-wise to the output of the convolution. This can be i.e. a ReLU(x) or tanh(x).

Building Blocks

Step 3: **Pooling** - Apply the pooling operation to the output of the activation function. Max Pooling is commonly used in modern architectures.

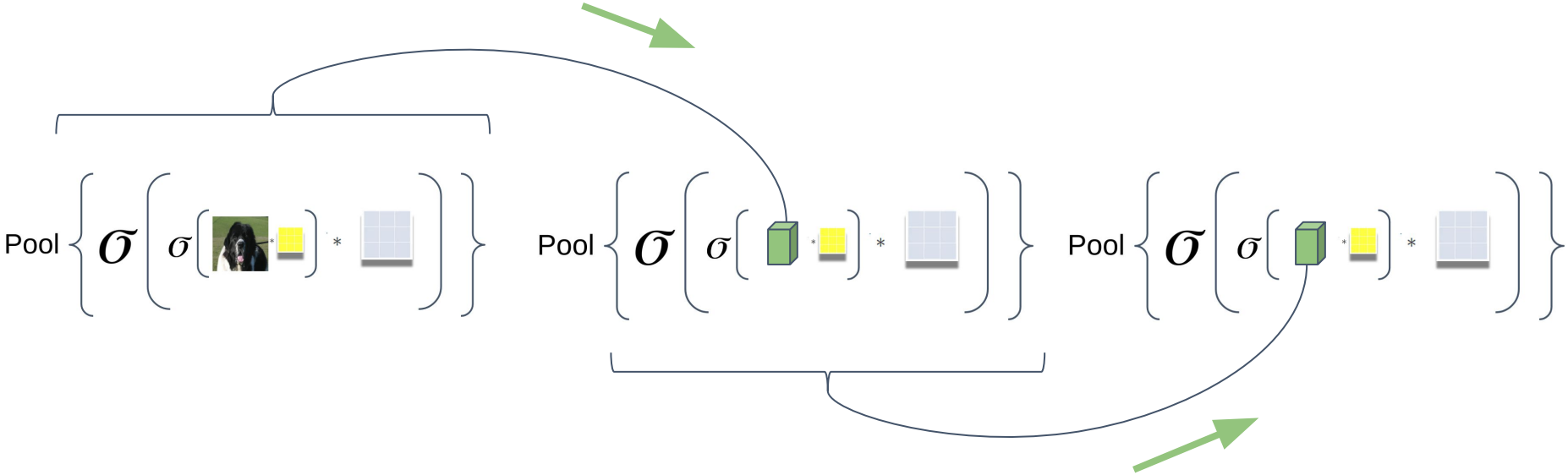
Building Blocks

We can have many *Convolutions* + *Activations* chained before pooling:



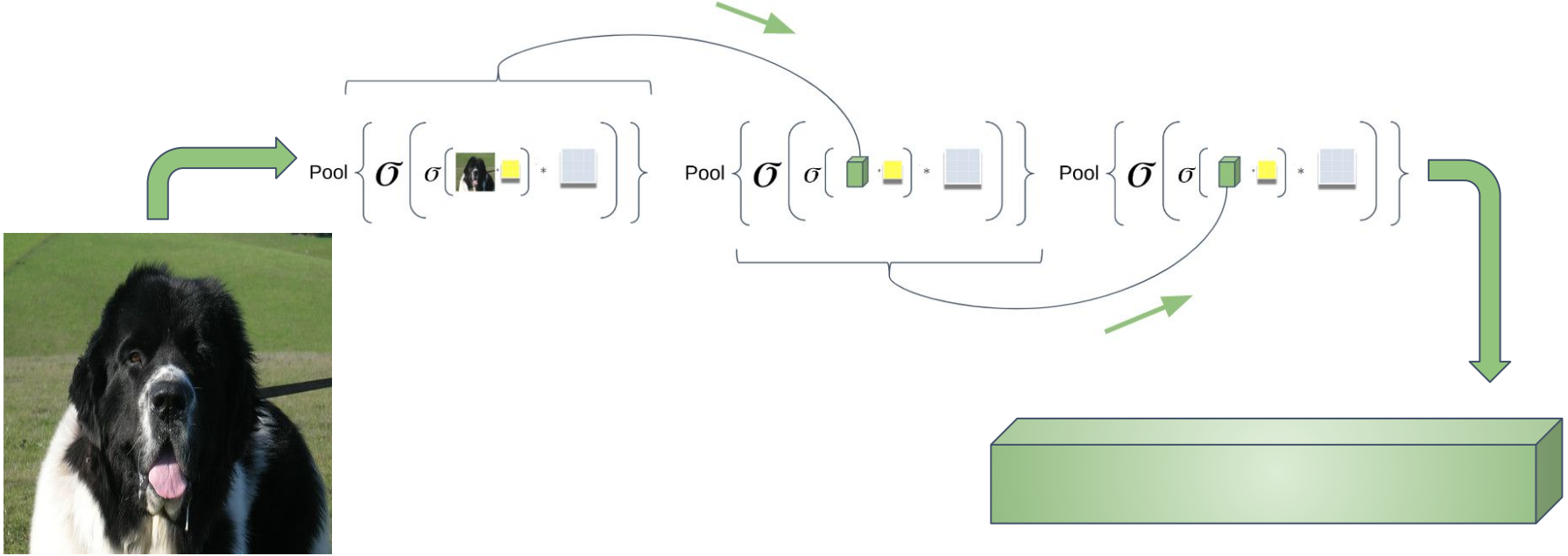
Building Blocks

… And we can have many of these blocks chained together



Building Blocks

These blocks allow us to map from one volume to another of arbitrary dimensions



$$V_{in} = W_{in} \times H_{in} \times D_{in}$$

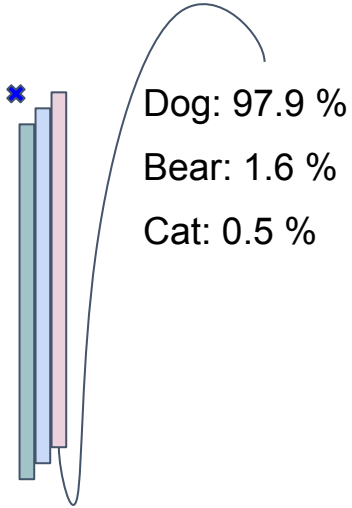
$$V_{out} = W_{out} \times H_{out} \times D_{out}$$

Building Blocks

To get to a probability distribution, we need to obtain a 1D vector of dimension 1xM (M categories). One approach is to **flatten** out the feature map volume and then use a dense **fully connected layer** followed by a **softmax** to obtain a **probability distribution** that sums to 1.

$V_{in} = W_{in} \times H_{in} \times D_{in}$

$V_{out} = W_{out} \times H_{out} \times D_{out}$



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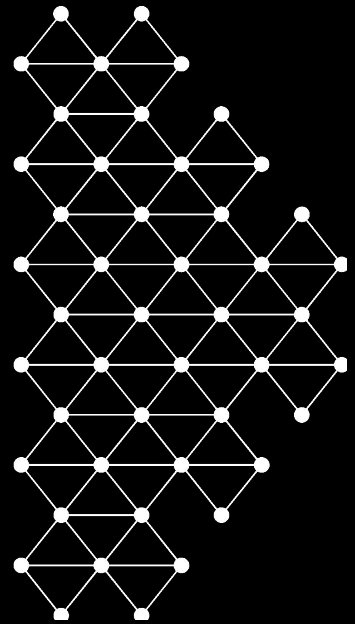
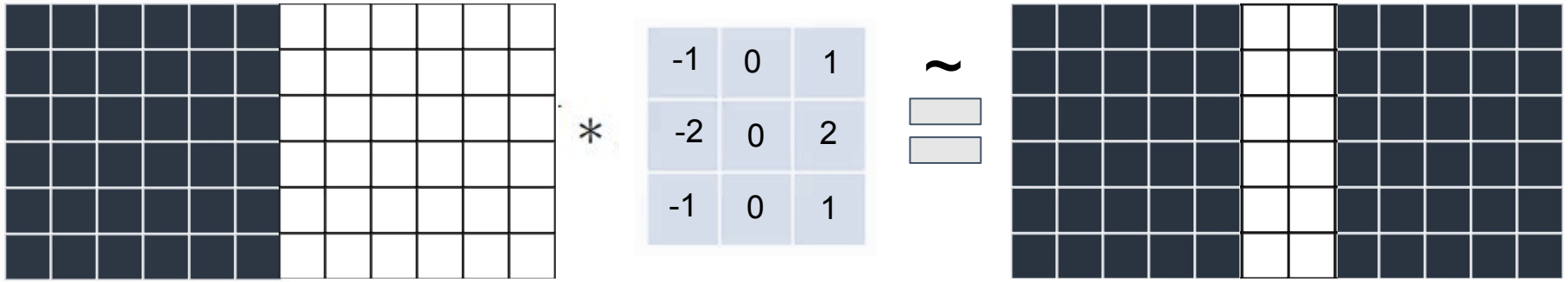


Motivations for CNNs

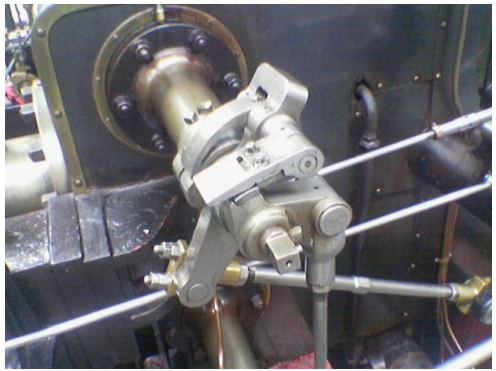
Motivations for CNNs

- CNNs can **learn** useful features from images automatically
- Classically, these features were handcrafted. For example, a **Sobel filter** was designed to extract edges. Only at the border will the output of the convolution be non-zero.



Motivations for Convolutions

- Here, we combine a horizontal and vertical Sobel filter to extract edges from the image.

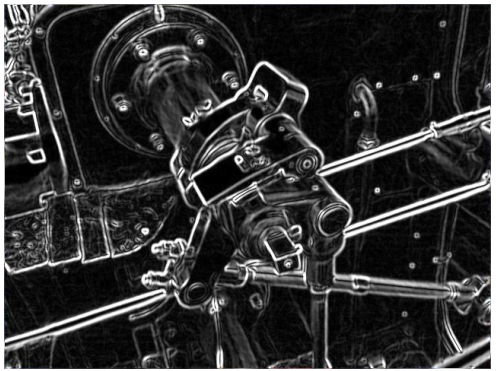


$*$

-1	0	1
-2	0	2
-1	0	1

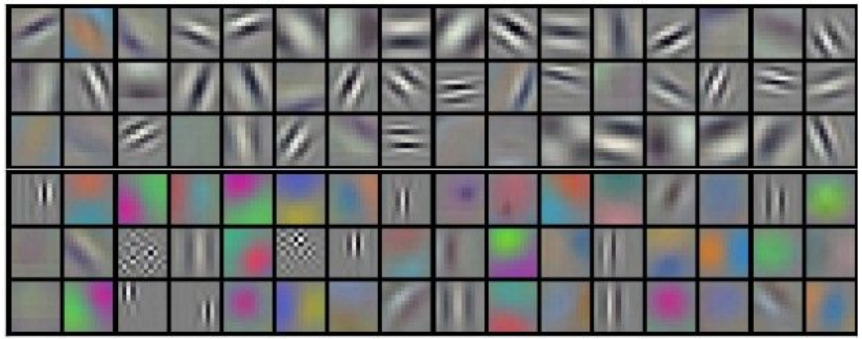
-1	-2	-1
0	0	0
1	2	1

$=$



Motivations for Convolutions

- It can be very hard and time consuming to handcraft these filters. Instead, we use deep learning to **learn** the best filters.
- By applying convolutions to **images**, we can extract some high-level features describing the images but it can be very challenging and limiting
- By applying convolutions to **features**, we can extract features of features at different levels of abstraction.

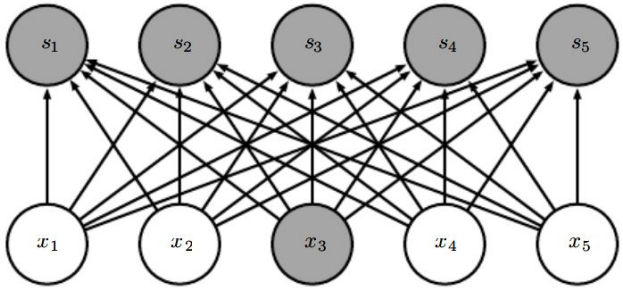


http://cs231n.github.io/assets/cnn/weights.jpeg

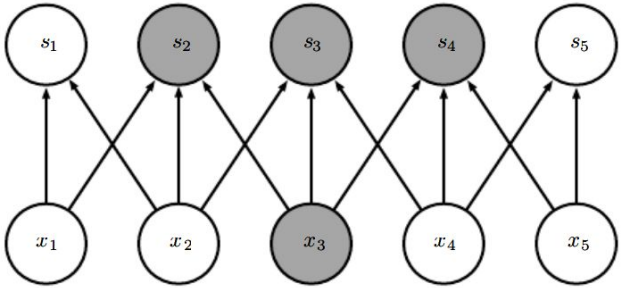
Filters learned by AlexNet on ImageNet.

Motivations for Convolutions

In MLPs, each input can in theory contribute equally to a signal. CNNs introduce a way to guide the neural network in focusing **hierarchically** at local features of an image.



MLP

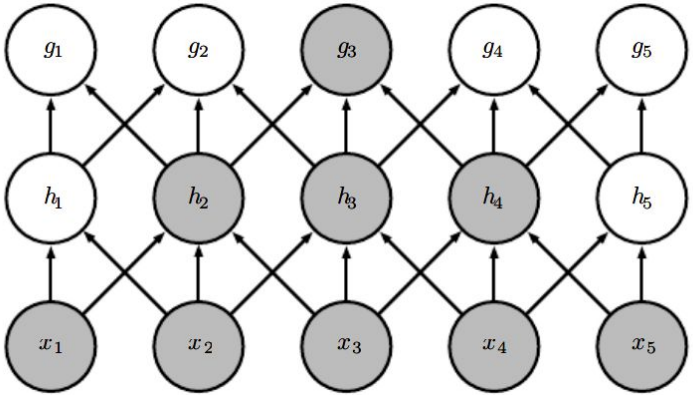


CNN

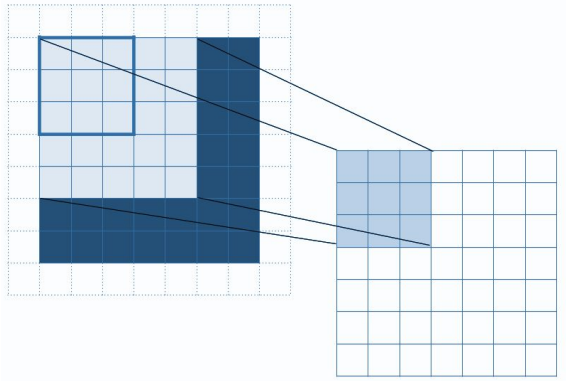
https://www.deeplearningbook.org/contents/convnets.html

Motivations for Convolutions

- **Receptive field** - Image patches contain information from larger patches of previous inputs. The resulting 3x3 patch seen here contains information from the previous 5x5 patch. The deeper you go, the wider the receptive field.



Field of view for 1D system



Field of view for 2D system

https://www.deeplearningbook.org/contents/convnets.html

Motivations for Convolutions

- **Parameter sharing**
  - weights are used at different areas of a same image which requires significantly less memory for models.
  - kernels can extract similar features at multiple places in an image
- Convolutions are **equivariant to translation** - i.e. translating an input in space will result in a translated convolution output
- Useful features which have been learned can be **reused** at multiple regions in an image.

https://www.deeplearningbook.org/contents/convnets.html

Hello (CNN) World

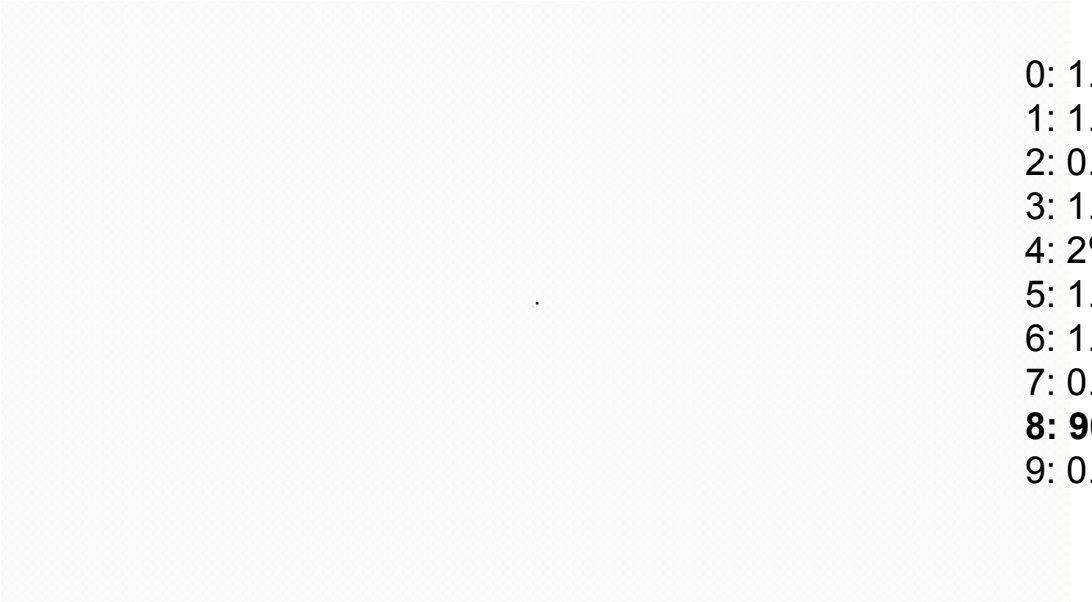
Let's now consider a real example of a CNN that has been around for a long time: **LeNet**. It was designed by Yann Lecun et al. to classify handwritten digits (0-9) from the **MNIST** dataset.

The MNIST dataset contains approximately 60 000 handwritten digits with their associated label.

https://commons.wikimedia.org/wiki/File:MnistExamples.png

MNIST and LeNet

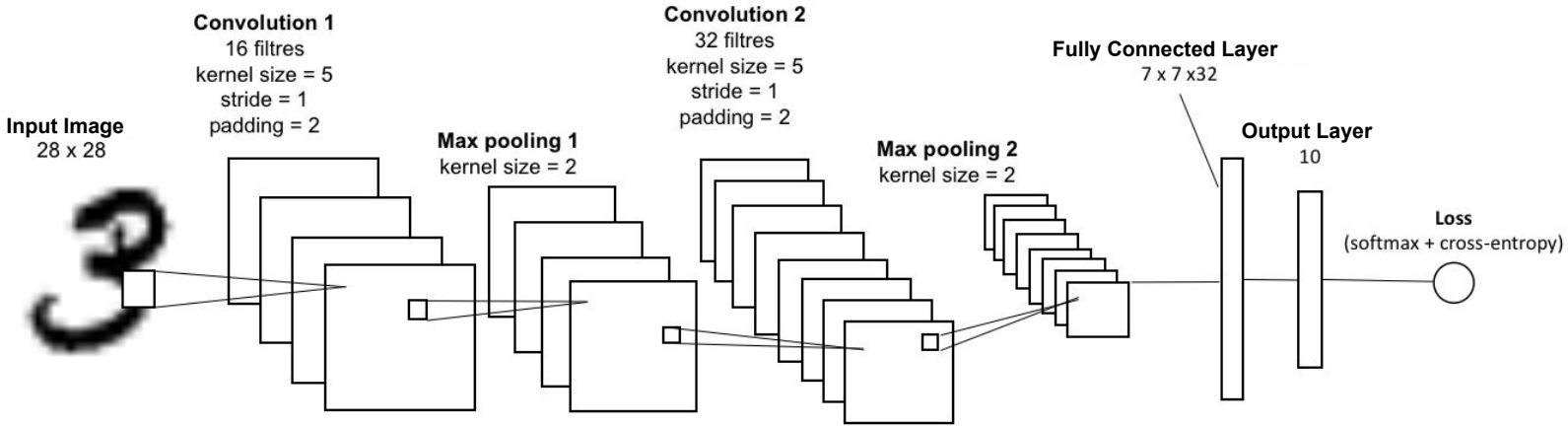
Problem statement : Given an **input** grayscale image, we want to **output** the **probability** of it being a certain digit.



0: 1.6 %
1: 1.2 %
2: 0.1 %
3: 1.25%
4: 2%
5: 1.3%
6: 1.2 %
7: 0.3%
8: 90.95%
9: 0.1%

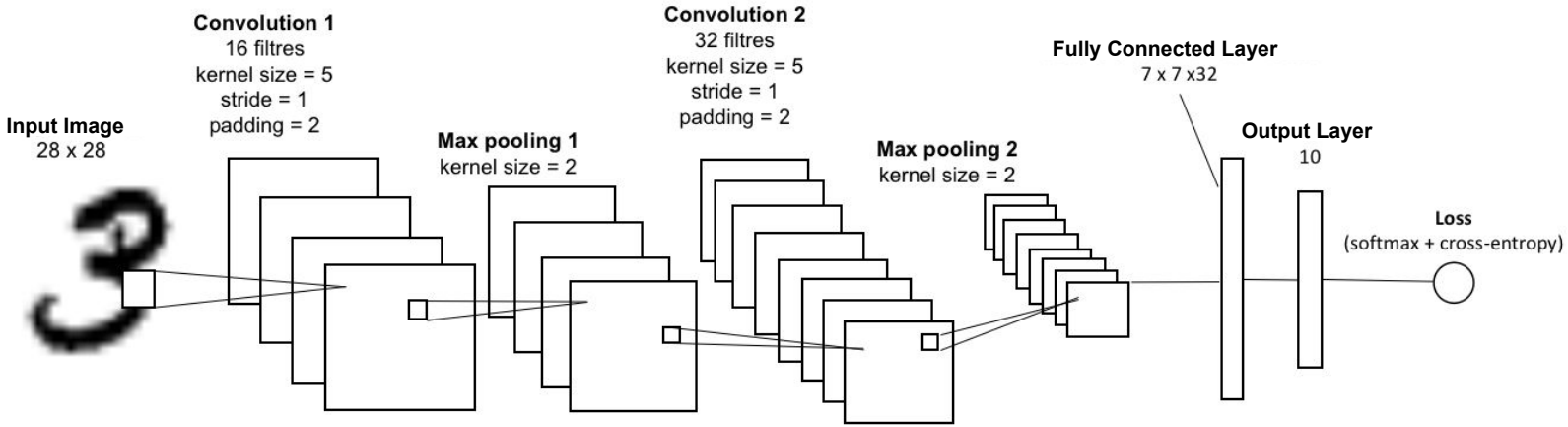
LeNet

Here is a version of LeNet that will be implemented in the tutorials later today. LeNet is considered a “simple” architecture. We will look in depth at modern architectures later today.



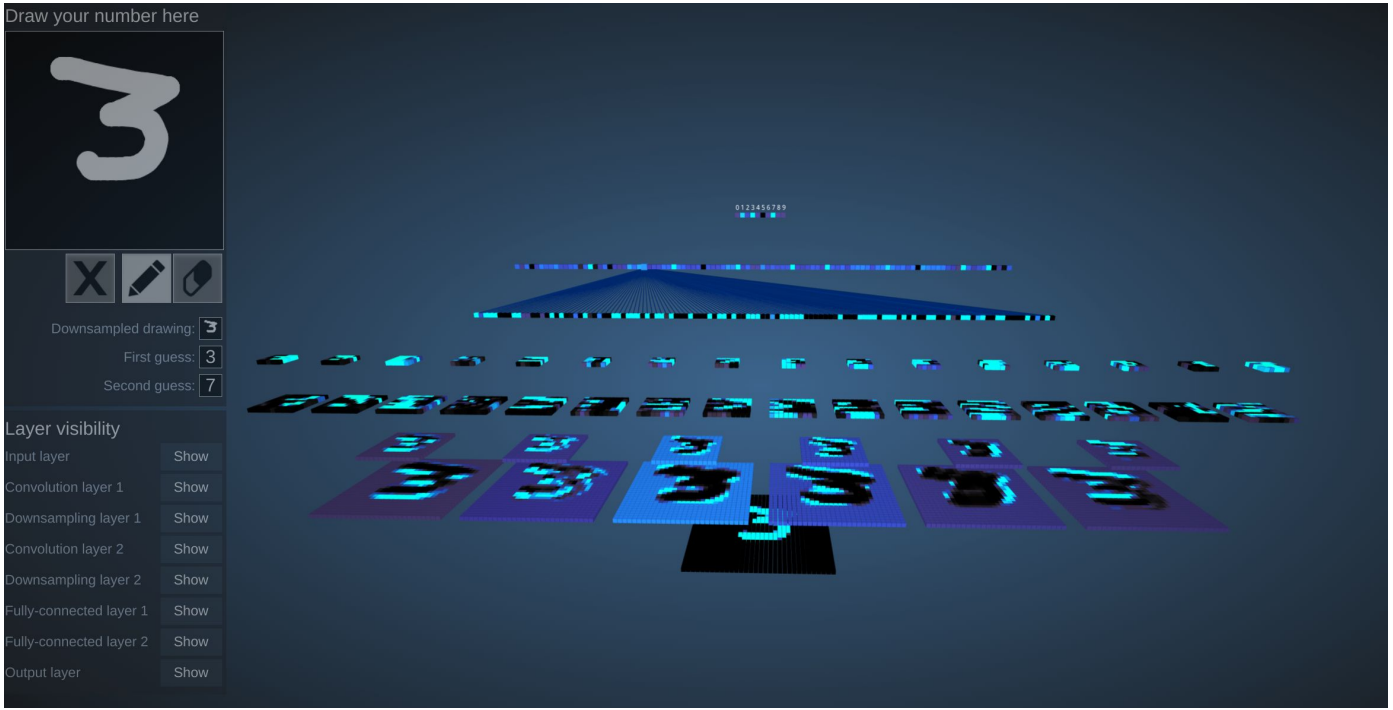
LeNet

Notice that the network is comprised of convolutions, pooling and fully connected layers.



Demo

[Here is a cool link]to visualize what is happening in LeNet:



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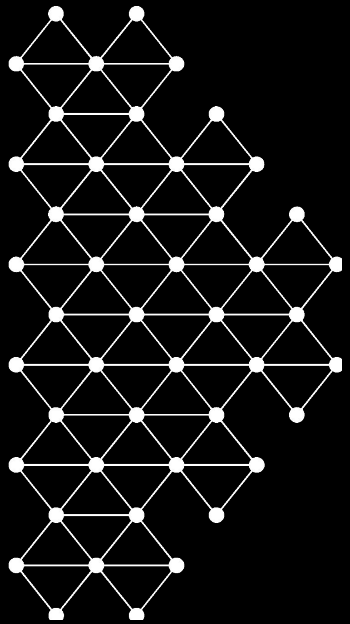

Questions?

Global Average Pooling

Another method to flatten the volume is **Global Average Pooling**. It transforms a 3D volume to a 1D vector. It takes each 2D feature map and transforms them to singular values by taking the average of the feature map and concatenates them depth-wise.

https://arxiv.org/pdf/1312.4400v3.pdf