Since $Y_i \sim \text{Binomial}(40, \theta)$

We know $E[Y_i] = n \cdot \theta$ and $var[Y_i] = n\theta(1-\theta)$

Therefore, $\check{\theta} = \frac{Y}{40}$ is an unbiased estimator for θ

say $\hat{\theta} = E[\check{\theta} \mid T(Y)]$, where $T(Y) = \sum_{i=0}^{7} Y_i$, Y_i represents observations of each row.

So let's prove $T(Y)$ is complete statistics. If this holds, we can then say $\hat{\theta} = E[\check{\theta} \mid T(Y)]$ is minimum variance unbiased estimator since $E[\hat{\theta}] = \theta = \frac{1}{8} \cdot 8 \cdot \theta$

And for $T(Y)$, since $Y_i \sim \text{Binomial}(40, \theta)$, $i \in [0, 7]$.

$T(Y) = \sum_{i=0}^{7} Y_i$ is also a binomial $(320, \theta)$

From the textbook, we can know $T(Y)$ should be a complete statistic.

Proof: Let g be a function such that $E_\theta g(T) = 0$

$$\sum_{t=0}^{320} g(t) \binom{320}{t} \theta^t (1-\theta)^{320-t} = 0$$

since $\binom{320}{t} > 0$, $\theta^t > 0$, $(1-\theta)^{320-t} > 0$,

then $g(t) = 0$ must hold for $\forall t \in [0, 320]$ and $t \in \mathbb{Z}$

which implies $P_\theta(g(T) = 0) = 1$ for all θ.

So T is a complete statistic.

So $\hat{\theta} = E[\check{\theta} \mid T(Y)] = \frac{1}{8}\left[\sum_{i=0}^{7} \frac{Y_i}{40}\right] = \frac{1}{320} \sum_{i=0}^{7} Y_i$ is a minimum variance unbiased estimator.

So at this time MSE is minimized.