Convolutional Coding and Viterbi Decoding

Anrui Liang

UIN: 726007240

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Abstract

This paper simulates the process of sending message using phones. The message is translated to bits and modulated with 16 Quadrature Amplitude Modulation (16QAM) method. After being transmitted in the white Gaussian channel, the signal is demodulated and translated to message. For finding and correcting error bits, I use Convolutional Channel Coding to code the message and use Viterbi Algorithm to decode the signal. This paper compares the error rate of system with different signal noise ratio and different length of binary code.

Key Words: Viterbi Algorithm

1. Introduction

Sending message is a common method for people to communicate with each others. If I type message in phones and send message, the message will be translated to bits and modulated as the high frequency signal. After passing many different kinds of channels, the new signal is demodulated as digital data. Finally, the phones decode this digital data and our friends can read the message.

However, after modulation and demodulation, there are some error bits in the message and they should be decreased. Channel Coding is a common method to decrease error bits. This method makes the original code can find and correct error bits.

2. Convolutional Coding and Viterbi Algorithm

2.1 Convolutional Coding

Convolutional Coding is called so, because it performs convolution of input stream with encoder's impulse responses.

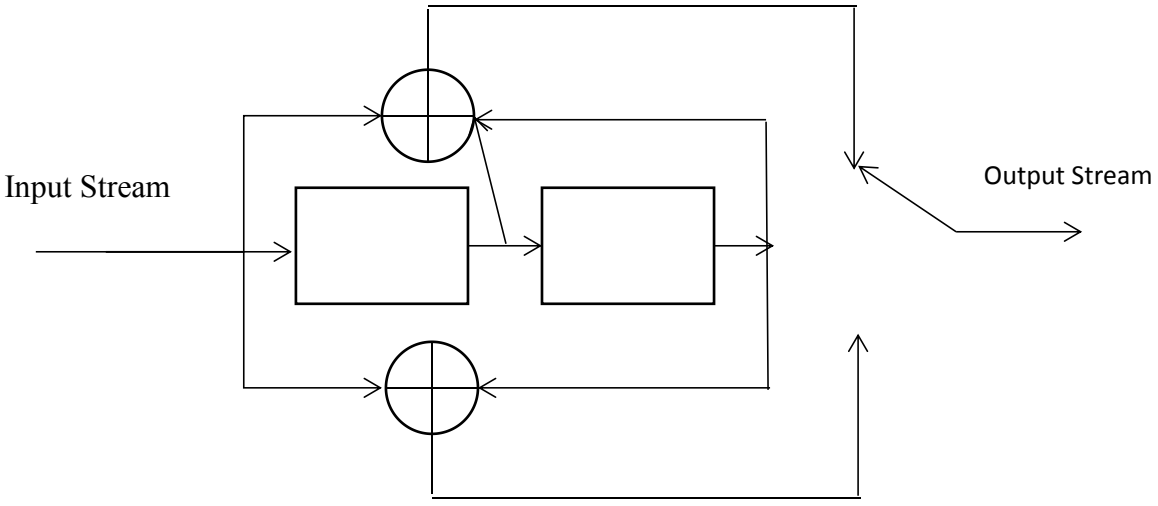

Figure 1. (2,1,3)Convolution Encoder

Convolutional Coding improves accuracy of the system. However, because of convolution, the complexity of the system increases and it is hard to decode the convolutional code. As the maximum likelihood, I need to find a coding method to minimize error probability. If I assume 0 and 1 come out with the same probability. And then, I need to find

$$\tilde{c} = \arg \max_{c \in C} \{P(c/y)\} \tag{1}$$

$\tilde{c}$ is the bit stream after recovery, **c** is input stream and **y** is output stream. Because of Bayes rule and equal probability of input 0 and 1, the Maximum a posteriori (MAP)estimation is equal to Maximum Likelihood estimation. So the question is changed.

$$\tilde{c} = \arg \max_{c \in C} \{\log P(y/c)\} \tag{2}$$

If I assume the length of the stream is L and the number of error bits in the stream is d, I can get new equation.

$$\begin{aligned} \log P(y/c) &= \sum_{0}^{L-1} \log P(y_l / c_l) \\ &= d(y,c) \log \frac{P}{1-P} + L \log(1-P) \end{aligned} \tag{3}$$

P is the error probability of each bit. Because P is less than half, the Maximum Likelihood estimation is changed.

$$\tilde{c} = \arg \max_{c \in C} \{\log P(y/c)\} = \arg \min_{c \in C} \{\sum_{0}^{L-1} d(y_l, c_l)\} \tag{4}$$

The purpose is to minimize the sum of the error bits(distance) in the stream.

2.2 Viterbi Algorithm

Because output stream is created by 2 streams(shown in figure 1), I can assume every bits combination of 2 stream as a status. So I have 4 kinds of statuses(00, 01, 10, 11) of an original bit. Because of convolution, after coding, every status has the relation with former statuses. So I use Viterbi algorithm to decode the output stream.

Viterbi algorithm is used to calculate the shortest path to the final status. Every status can only be changed to 2 status. For example, 00 can only change to 00 or 01 because next bit is either 0 or 1. And then, the algorithm needs to compare next status with possible status and calculate that, if next status is false, how many error bits may be in the status. For example, former status is 00 and next status is 11. And then, there may be 2 error bits in 11 and correct status is 00, or 1 error bit in 11 and correct status is 01. The error bit is named as distance. And then, the algorithm stores the less probable status and the path from former status to next status. The algorithm stores the shortest path of 4 status and the distances of the shortest paths in every step. Finally, after comparing all statuses, the algorithm chooses the shortest path as the real path and decodes the stream with shortest path.

3. Experiment

I get the message from the keyboard and translate it to binary code. However, I need to add 2 zero bits behind the binary code. There are two reasons. First, I use 16QAM as modulation method, so the length of the message is even number. Second, 2 zero bits behind the binary code can make sure last 2 status and makes the last status

is 00. And then, I compare the error rate of the system with convolutional coding or not.

And then, I modulate the binary code with 16 QAM and transmit signal on white Gaussian channel. After demodulation, I decode the stream with Viterbi algorithm and translate it to message.

Finally, I compare error rate of the system with convolutional coding or not with different signal noise ratio(SNR) and different length of code.

4. **Result and Discussion**

Table 1. The error rate of system, N=1600

SNR	2	3	4	5	6	7	8
With convolutional coding	0. 0693	0. 0343	0. 01	0. 0031	0. 0012	0. 0012	0
Without convolutional coding	0. 1077	0. 0824	0. 0643	0. 0456	0. 035	0. 0162	0. 0106

Table 2. The error rate of system, SNR=5

Length of code	1600	3200	4800	6400
With convolutional coding	0. 0031	0. 0047	0. 01	0. 0052
Without convolutional coding	0. 0468	0. 0471	0. 0511	0. 0448

As shown in table 1, convolutional coding improves the accuracy of the system. With increase of the SNR, the error rate of the system with convolutional coding decreases more rapidly than the error rate of the system without convolutional coding. When SNR is bigger or equal 8, the error rate of the system with convolutional coding is zero. Although error bits actually may not be deleted in a system, the result of the table 1 confirms that convolutional coding can dramatically increase the accuracy of the system.

As shown in table 2, the error rates of 2 systems do not change very much. However, I cannot draw a conclusion that convolutional coding is suitable to transmit a large number of data, because the length of binary code changes too little to get the conclusion. However, because of the limitation of time and computer, I cannot run the program with long length binary code. It will be a new goal to achieve.

5. Conclusion

In this paper, I need to find a method to improve the accuracy of the system. With maximum likelihood decoding, I need to find a kind of code to minimize the error bit (distance). And then, I decide to use convolutional coding to code the binary code and use Viterbi algorithm to decode output stream. Viterbi algorithm can decode output stream with the convolution of the code and find the best path as the final message. I find that convolutional coding can decrease error rate very efficiently and, with the increase of the SNR, the error rate of the system with convolutional coding can decrease rapidly even to zero for short length data. I cannot find the relation of error rate of system with length of binary code.

6. Reference

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