Based on the given assumptions we know

- The prior $f_\Theta(\theta)$ is a Beta distribution so $f_\Theta(\theta;\alpha,\beta) = \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\mathrm{B}(\alpha,\beta)}$.
- The likelihood is the product distribution of n iid Binomial distributions for $\boldsymbol{x} = (x_1, x_2, \cdots, x_n)$ so

$$f_{\boldsymbol{X}|\Theta=\theta}(\boldsymbol{x};N) = \prod_{i=1}^{n} \binom{N}{x_i} \theta^{x_i}(1-\theta)^{N-x_i} = \left(\prod_{i=1}^{n} \binom{N}{x_i}\right) \theta^{\sum_{i=1}^{n} x_i}(1-\theta)^{nN-\sum_{i=1}^{n} x_i}.$$

- The posterior probability distribution is proportional to the product of prior and likelihood and hence,

$$\begin{aligned} f_{\Theta|\boldsymbol{X}=\boldsymbol{x}}(\theta) &\propto f_{\boldsymbol{X}|\Theta=\theta}(\boldsymbol{x};N) f_\Theta(\theta;\alpha,\beta) \\ &= \left(\prod_{i=1}^{n} \binom{N}{x_i}\right) \theta^{\sum_{i=1}^{n} x_i}(1-\theta)^{nN-\sum_{i=1}^{n} x_i} \frac{\theta^{\alpha-1}(1-\theta)^{\beta-1}}{\mathrm{B}(\alpha,\beta)} \\ &= \frac{\prod_{i=1}^{n} \binom{N}{x_i}}{\mathrm{B}(\alpha,\beta)} \theta^{\alpha-1+\sum_{i=1}^{n} x_i}(1-\theta)^{\beta-1+nN-\sum_{i=1}^{n} x_i} = c\theta^{a-1}(1-\theta)^{b-1}, \end{aligned}$$

  where,

$$a = \alpha + \sum_{i=1}^{n} x_i, \quad b = \beta + nN - \sum_{i=1}^{n} x_i, \quad c = \frac{\prod_{i=1}^{n} \binom{N}{x_i}}{\mathrm{B}(\alpha,\beta)}.$$

  Therefore the posterior distribution is

$$f_{\Theta|\boldsymbol{X}=\boldsymbol{x}}(\theta) = \frac{c\theta^{a-1}(1-\theta)^{b-1}}{\int_0^1 c\theta^{a-1}(1-\theta)^{b-1}\mathrm{d}\theta} = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \theta^{a-1}(1-\theta)^{b-1},$$

  since $\int_0^1 \theta^{a-1}(1-\theta)^{b-1}\mathrm{d}\theta = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)}$ (the Euler integral of the first kind).

Now, in the Bayesian setting the minimum mean square error estimator is the expected value of the posterior, i.e. for $\theta \in [0,1]$ we have

$$\begin{aligned} \hat{\theta}(\boldsymbol{x}) = \mathrm{E}[\theta|\boldsymbol{x}] &= \int_0^1 f_{\Theta|\boldsymbol{X}=\boldsymbol{x}}(\theta)\theta\mathrm{d}\theta = \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \int_0^1 \theta^{a+1-1}(1-\theta)^{b-1}\mathrm{d}\theta \\ &= \frac{\Gamma(a+b)}{\Gamma(a)\Gamma(b)} \frac{\Gamma(a+1)\Gamma(b)}{\Gamma(a+1+b)} = \frac{\Gamma(a+1)\Gamma(a+b)}{\Gamma(a)\Gamma(a+b+1)}, \end{aligned}$$

using the property $\Gamma(a+1) = a\Gamma(a)$ we have

$$\hat{\theta}(\boldsymbol{x}) = \frac{a\Gamma(a)\Gamma(a+b)}{\Gamma(a)(a+b)\Gamma(a+b)} = \frac{a}{a+b} = \frac{\alpha + \sum_{i=1}^{n} x_i}{\alpha + \sum_{i=1}^{n} x_i + \beta + nN - \sum_{i=1}^{n} x_i} \implies$$

$$\boxed{\hat{\theta}(\boldsymbol{x}) = \frac{\alpha + \sum_{i=1}^{n} x_i}{\alpha + \beta + nN}.}$$

For the given parameters $(\alpha = 2, \beta = 5, n = 8, N = 40)$ we have

$$\boxed{\hat{\theta}(\boldsymbol{x}) = \frac{2 + \sum_{i=1}^{n} x_i}{327}.}$$