∵ $x_i \sim \text{Binomial}(40, \theta)$ and IID

∴ $P_{x_i}(x|\theta) = \binom{40}{x} \theta^{x} (1-\theta)^{40-x}$

$P_{\underline{x}}(x|\theta) = (40!)^{8} (\prod x_i! (40-x_i)!)^{-1} (1-\theta)^{320} (\frac{\theta}{1-\theta})^{\sum x_i}$

we can find

$T(\underline{x}) = \sum x_i$ is sufficient statistic for θ

assume $\hat{\theta} = \frac{x}{n}$

then $E[\hat{\theta}] = E[\frac{x}{n}] = \theta$ unbiased

$Var[\hat{\theta}] = \frac{\theta(1-\theta)}{n}$

$\frac{\partial}{\partial \theta} \log L(\theta; x) = \frac{\partial}{\partial \theta} \log P_{\underline{x}}(x|\theta)$

$= \frac{\sum x_i}{\theta} - \frac{\sum(40-x_i)}{1-\theta}$

Fisher Information

$I(x) = -E[\frac{\partial}{\partial \theta}(\frac{\sum x_i}{\theta} - \frac{\sum(40-x_i)}{1-\theta})]$

$= \frac{320}{\theta(1-\theta)}$

∵ Cramer-Rao Lower Bound

∴ $E[(\hat{\theta}-\theta)^2] \geq \frac{1}{I(x)} = \frac{\theta(1-\theta)}{320} = Var(\hat{\theta})$

∴ $\hat{\theta} = \frac{x}{n}$ is the estimator which minizes MSE