EP 222 FORMULA SHEET

September 5, 2013

- Action $\mathcal{S}$ over a path γ is $\mathcal{S} = \int_\gamma \mathcal{L}[\{q_i\}, \{\dot{q}_i\}, t]\mathrm{d}t$.
- Euler-Lagrange equations: $\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}_i}\right) - \frac{\partial \mathcal{L}}{\partial q_i} = 0$.
- For a system with no velocity dependant forces, we have $\mathcal{L} = T - V$. For an electromagnetic system, the corresponding contribution to the potential V is $q\phi - q\mathbf{v}\cdot\mathbf{A}$.
- Generalized momentum for a coordinate q_i is defined by $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$. Generalized force is $Q_i = \frac{\mathrm{d}p_i}{\mathrm{d}t}$. From D'Alembert's principle, we also get $Q_i = \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\partial T}{\partial \dot{q}_i}\right) - \frac{\partial T}{\partial q_i}$.
- Hamiltonian equations: $\frac{\partial \mathcal{H}}{\partial q_i} = -\dot{p}_i$, $\frac{\partial \mathcal{H}}{\partial p_i} = \dot{q}_i$, $\frac{\mathrm{d}\mathcal{H}}{\mathrm{d}t} = -\frac{\partial \mathcal{L}}{\partial t}$. $\mathcal{H}$ is defined as $\sum_j \frac{\partial L}{\partial \dot{q}_j}\dot{q}_j - \mathcal{L}$. $\mathcal{H} = T + V$ for velocity independent system.
- Gauge symmetry: If F is a velocity independent field, then $\mathcal{L}' = \mathcal{L} + \frac{\mathrm{d}F}{\mathrm{d}t}$ is also a valid Lagrangian.
- Noether's theorem:
  - If q_i is a cyclic coordinate, then p_i is conserved.
  - For a more general symmetry, in one coordinate, if $\frac{\mathrm{d}}{\mathrm{d}s}\mathcal{L}[Q(s,t), \dot{Q}(s,t), t] = 0$, the conserved quantity is $p\frac{\mathrm{d}Q}{\mathrm{d}s}\Big|_{s=0}$.
  - In case of multiple symmetries, if $\frac{\mathrm{d}}{\mathrm{d}s_k}L[Q_1(s_1, s_2, ..., t), ..., \dot{Q}_1(s_1, s_2, ..., t), ..., t] = 0\ \forall\ k$, then the conserved quantities are $\Lambda_k(q_1, q_2, ..., \dot{q}_1, \dot{q}_2, ..., t) = \sum_j p_j \frac{\mathrm{d}Q_j}{\mathrm{d}s_k}\Big|_{s_k=0}$
- Central force: For the potential $V = -\frac{\alpha}{r}$, the conic section obtained is $r = \frac{M}{L\cos\theta + \frac{\alpha}{2M}}$ where $L = \sqrt{E + \frac{\alpha^2}{4M^2}}$ (Compare this with $r = \frac{p}{1+\epsilon\cos\theta}$). For a generalized central force, if $u = \frac{1}{r}$, $F(u) = \frac{\ell^2}{\mu^2}\left(u^2\frac{\mathrm{d}^2u}{\mathrm{d}\theta^2} + u^3\right)$. The term contributed to V_{eff} by the centrifugal force is $\frac{\ell^2}{2\mu r^2}$.