MA 214 Short notes

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1 Framework

1. A method approximates to t sig digits if $\left|\frac{x-x*}{x}\right| \leq 5 \times 10^{-t}$
2. Relative error $\left|\frac{x-x*}{x}\right| \approx \left|\frac{x-x*}{x*}\right|$
3. Loss of sig digits is caused by subtraction of nearly equal quantities, or division by small numbers
4. Condition: Sensitivity to changes in x, $\frac{\Delta f/f}{\Delta x/x} \approx \frac{f'(x)x}{f(x)}$
5. IVT for functions: $f[a,b] \to \mathbb{R}$, $\exists \xi \in [a,b] \mid \sum f(x_i)g_i = f(\xi) \sim g_i$
6. $\alpha_n = \mathcal{O}(\beta_n)$ if $\alpha_n \leq k\beta_n$ for large n
7. nested multiplication ... less round off error
8. Instability: Sensitivity to digital round off errors

2 Interpolation

1. Weierstrauß approximation: $\forall\ \epsilon\ \exists\ P(x)$ such that $|f(x) - p(x)| < \epsilon$
2. Lagrange polynomial: $l_k(x) = \prod\limits_{i \neq k} \frac{x-x_i}{x_k-x_i}$; $p_n(x) = \sum l_k(x)f(x_k)$
3. Divided diff: $P_n(x) = P_{n-1}(x) + a_n(x-x_0)(x-x_1)(...)$, $f[x_0, x_1, ...x_n] = a_n$
4. $f[x_0...x_k] = \frac{f[x_1...x_k] - f[x_0]...x_{k-1}]}{x_k - x_0}$

5. Error in estimating $f(\bar{x})$ is $e_n(\bar{x}) = f[x_0, x_1 ... x_n, \bar{x}] \prod(\bar{x} - x_j)$
6. Osculatory: If derivatives at x_0 come n times, repeat x_0, n times and say that $f[x_0, x_0 ... n \text{ times}] = f^{(n)}(x)$
7. $\exists\ \xi \in [x_0, x_k] \mid f[x_0 ... x_k] = \frac{f^{(k)}(\xi)}{k!}$
8. hermite (piecewise cubic) interpolation: Same as osculatory, but piecewise
9. Cubic spline: ?
10. $f[x_0 ... x_k, x_{k+1}] = f[x_0 ... x_{k+1}] + f[x_0, ...]$

3 Integration

1. Error:
   (a) If ψ_k is of one sign in interval, $E(f) = \frac{f^{(k+1)}(\xi)}{(k+1)!} \int \psi_k(x)dx$ where $\psi_k(x) = \prod(x - x_j)$
   (b) If the integral is zero, $E(f) = \frac{f^{(k+2)}\xi)}{(k+2)!} \int \psi_{k+1}(x)dx$
2. Midpoint rule $I \approx (b-a)f\left(\frac{a+b}{2}\right)$, error is $f''(\xi)\frac{(b-a)^3}{24}$
3. Trapezoidal: $\frac{1}{2}(b-a)(f(a)+f(b)$, error is $-\frac{f''(\xi)(b-a)^3}{12}$
4. Simpson: $\frac{b-a}{6}\left(f(a) + 4f\left(\frac{a+b}{2}\right) + f(b)\right)$, $E = \frac{f^{4)}(\xi)(b-a)^5}{2^5}$

4 Matrices

4.1 LU factorization

To solve $Ax = b$, we first write $A = LU$, where the two matrices are l. triangular and u. triangular respectively. As there are ∞ solutions, set diagonal of L to 1 before proceeding.

Now solve $L(Ux) = b \implies Ly = b$, then solve $Ux = y$.