# EP 307 Assignment 6

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## Problem 2

To prove: $\langle n \,|\, T \,|\, n\rangle = \langle n \,|\, V \,|\, n\rangle$

Firstly, we note that $\hat{x} = \sqrt{\frac{\hbar}{2m\omega}}(\hat{a} + \hat{a}^\dagger), \hat{p} = i\sqrt{\frac{m\omega\hbar}{2}}(\hat{a}^\dagger - \hat{a})$

$$\begin{aligned}
\therefore \langle n \,|\, T \,|\, n\rangle &= \left\langle n \,\middle|\, \frac{1}{2m}\hat{p}^2 \,\middle|\, n \right\rangle \\
&= -\frac{\omega\hbar}{4}\left\langle n \,\middle|\, (\hat{a}^\dagger - \hat{a})^2 \,\middle|\, n \right\rangle \\
\text{And} \quad \langle n \,|\, V \,|\, n\rangle &= \left\langle n \,\middle|\, \frac{1}{2}m\omega^2\hat{x}^2 \,\middle|\, n \right\rangle \\
&= \frac{\omega\hbar}{4}\left\langle n \,\middle|\, (\hat{a}^\dagger + \hat{a})^2 \,\middle|\, n \right\rangle \\
\therefore \langle n \,|\, V \,|\, n\rangle - \langle n \,|\, T \,|\, n\rangle &= \frac{\omega\hbar}{4}\left\langle n \,\middle|\, (\hat{a}^\dagger + \hat{a})^2 \,\middle|\, n \right\rangle + \frac{\omega\hbar}{4}\left\langle n \,\middle|\, (\hat{a}^\dagger - \hat{a})^2 \,\middle|\, n \right\rangle \\
&= \frac{\omega\hbar}{2}\left\langle n \,\middle|\, \hat{a}^{\dagger 2} + \hat{a}^2 \,\middle|\, n \right\rangle \\
&= 0
\end{aligned}$$

since both $\hat{a}^\dagger$ will move the state to $|n+2\rangle$, which is orthogonal to $|n\rangle$, and $\hat{a}$ will move it to $|n-2\rangle$ or $0\,|0\rangle$ (if $n < 2$). In both cases the net result is zero

Thus $\langle n \,|\, T \,|\, n\rangle = \langle n \,|\, V \,|\, n\rangle$

Also,

## Problem 3

## Problem 4

## Problem 5

## Problem 6