Tutorial 2

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Problem 1

(i) $\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}$ is a tensor for all kinds of transformations. $\frac{\mathrm{d}x'^\mu}{\mathrm{d}\tau} = \frac{\mathrm{d}}{\mathrm{d}\tau}\left(x^\nu \frac{\mathrm{d}x'^\mu}{\mathrm{d}x^\nu}\right) = \frac{\mathrm{d}x'^\mu}{\mathrm{d}x^\nu}\frac{\mathrm{d}x^\nu}{\mathrm{d}\tau}$, and thus we get a contravariant tensor

(ii) $\frac{\mathrm{d}^2x^\mu}{\mathrm{d}\tau^2}$ is also a tensor because it is the derivative of a tensor (Use the property proved in (i) by replacing x^μ with $\frac{\mathrm{d}x^\mu}{\mathrm{d}\tau}$). This tensor is also contravariant.

(iii) $\partial_\mu\phi$ is a tensor. $\partial'_\mu\phi = \frac{\partial\phi}{\partial x_\nu \frac{\mathrm{d}x'_\mu}{\mathrm{d}x_\nu}} = \frac{\mathrm{d}x_\nu}{\mathrm{d}x'_\mu}\frac{\partial\phi}{\partial x_\nu} = \frac{\mathrm{d}x_\nu}{\mathrm{d}x'_\mu}\partial_\mu\phi$, making it a covariant tensor.

(iv) This is only a tensor when special relativistic constraints are present. We know that the covariant derivative of the tensor F, $F_{\mu\nu;\rho} = \partial_\rho F_{\mu\nu} - \Gamma^\sigma_{\nu\rho}A_{\mu\sigma} - \Gamma^\sigma_{\mu\rho}A_{\nu\sigma}$ is a tensor itself. However, $\Gamma^\sigma_{\nu\rho}F_{\mu\sigma}, \Gamma^\sigma_{\mu\rho}F_{\nu\sigma}$ are not tensors in the general case as proved in class, so $\partial_\rho F_{\mu\nu}$ is not. However, in special relativity the Christoffel symbols are zero, so the remaining portion of the equation must be a tensor too.

Problem 2

We first calculate the velocity and then γ_v of the particle in the frame:

$$
\begin{aligned}
v^2 &= \dot{x}^2 + \dot{y}^2 + \dot{x^2} \\
&= (a + b\omega \cos \omega t)^2 + (-b\omega \sin \omega t)^2 \\
&= a^2 + b^2\omega^2 + 2ab\omega \cos \omega t \\
\therefore \gamma_v &= \frac{1}{\sqrt{1 - \frac{a^2+b^2\omega^2+2ab\omega \cos \omega t}{c^2}}}
\end{aligned}
$$

From this, we get $w^\mu = \frac{1}{\sqrt{1-\frac{a^2+b^2\omega^2+2ab\omega\cos\omega t}{c^2}}} \begin{bmatrix} a + b\omega \cos \omega t \\ -b\omega \sin \omega t \\ 0 \end{bmatrix}$