1 Week 1 - Time Value of Money

1.1 Time Value of Money (TVM)

1.1.1 The Essence of Decision-Making

- Virtually every decision involves time and uncertainty
- Very important to understand the impact of just the passage of time on a decision
- First assume no uncertainty to internalize TVM

1.1.2 Some Terminology

PV (or P)= Present Value ($)

FV= Future Value ($)

n or $t = \#$ of Periods ($\#$) depends on the nature of the problem

$r =$Interest Rate (%) > 0 (assumption)

Book: Theory of Interest - Irving Fisher (1930s)

1.2 Simple Future Value (FV)

1.2.1 Importance of Time Lines

We should be able to take a problem and put it on a timeline.

At point 0, we call the value a Present Value (PV), r is the interest rate that applies to one period. We will take PV and try to relate it to FV.

- A dollar today is worth more than a dollar tomorrow

Example

Suppose a bank pays a 10% interest rate per year and you are given a choice between two plans

A: Receive $100 today

B: Receive $100 one year from now

Which would you prefer, why?

Probably the first one

Future Value problem

$100 0————1 A

0————1 $100 B

You can't compare these two at time 0, but you can either bring the FV down to present or carry PV forward.

We'll carry the FV forward first, easier to understand.

so, if $r = 10\%$ with A, the $FV = \$110$

1.2.2 Future Value (Concept)

In general, **Future Value of a Single Cash Flow Invested for n Periods**

$$FV = \text{Initial Payment} + \text{Accumulated Interest}$$

$$\mathbf{FV = PV + r \cdot PV = (1 + r)^n \cdot PV}$$

So, for the example

$$FV = \$100 + 10\% \cdot (\$100)$$

1.3 Simple FV

1.3.1 Future Value (Multiple Periods, Example)

What is future value of $100 two years from now?

0———1———-2 FV?

0: $100

1: $110 = $100(1.10)

2: $121 = $110(1.10)

1.3.2 Future Value: Example

Suppose you invest $500 in the bank at an interest rate of 7%. How much will you have at the end of 10 years?

$$FV = 500 \cdot (1 + 0.07)^{10} = 983.576$$

1.4 Simple FV: Example

What are the future values of investing $100 at 10% versus 5% for 100 years?

$$FV_{100,5\%} = \$100 \cdot (1 + 0.05)^{100} = \$13150.1$$

$$FV_{100,10\%} = \$100 \cdot (1 + 0.1)^{10} = \$1378060$$

5% is closer to a bond

10% is closer to a stock market rate

1.5 Simple FV: Exmaple 2

Peter Minuit bought the Manhattan Island for $24 in 1626. Suppose that Native Americans could have earned 6% on their investment all these years. How much would they have today?

$$FV = 24 \cdot (1 + 0.06)^{386} = \$1.40694 * 10^{11}$$

In Excel, FV function is fv()

1.6 Simple Present Value (PV)

What is the present value of receiving $110 one year from now if the interest rate is 10%?

$100 0————1 $110

at 10%

PV 0————-1 FV

if

$$FV = PV \cdot (1+r)^n$$

The process is called **discounting**

Present Value of a Single Cash Flow Received n Periods from Now

$$\mathbf{PV} = \frac{\mathbf{FV}}{\mathbf{(1+r)^n}}$$

for $n = 1$

so for the example

$$PV = \frac{\$110}{(1+0.1)}$$

Ex.:

Suppose you will inherit $121,000$ two years from now and the interest rate is $r = 10\%$. What is the value today to you?

$2 \rightarrow 1$

$\frac{121000}{1.1} = 110000$

$1 \rightarrow 2$

$\frac{\frac{121000}{1.1}}{1.1} = 100000$

1.6.1 Present Value (Concept & Formula)

$$PV = \frac{FV}{(1+r)^n}$$

In Excel, PV function is pv()