7c

Plot the streamlines and velocity potential (on the same figure) for a variety of δ values and compare to the limiting case.

In [1]:
```
import matplotlib
import matplotlib.pyplot as plt
import numpy as np
```

Velocity potential (2D)

The velocity potential for a point source and a point sink equidistant from origin is given as

$$ \Phi(\mathbf{r}) = - \frac{m}{4\pi\sqrt{(x+d)^2+y^2}} + \frac{m}{4\pi\sqrt{(x-d)^2+y^2}} $$

Let $m = 1/d$:

In [2]:
```
def potential(x,y,d):
    Phi = -1/(d*4*np.pi*((x+d)**2+y**2)**(1/2)) + 1/(d*4*np.pi*((x-d)**2+y**2)**(1/2))
    return Phi
```

The velocity vector is then $\mathbf{u}=\nabla\Phi$ or

$$ \mathbf{u} = \begin{bmatrix} \frac{(x+d)m}{4\pi((x+d)^2+y^2)^{\frac{3}{2}}} - \frac{(x-d)m}{4\pi((x-d)^2+y^2)^{\frac{3}{2}}} \\ \frac{ym}{4\pi((x+d)^2+y^2)^{\frac{3}{2}}} - \frac{ym}{4\pi((x-d)^2+y^2)^{\frac{3}{2}}} \end{bmatrix} $$

In [3]:
```
def velocity(x,y,d):
    u = (x+d)/(d*4*np.pi*((x+d)**2+y**2)**(3/2)) - (x-d)/(d*4*np.pi*((x-d)**2+y**2)**(3/2))
    v = y/(d*4*np.pi*((x+d)**2+y**2)**(3/2)) - y/(d*4*np.pi*((x-d)**2+y**2)**(3/2))
    U = np.array([u,v])
    return U
```

In [4]:
```
velocity(0,0,1)
```

Out[4]:
```
array([0.15915494, 0.        ])
```

Let $d=1$

In [5]:
```
d = 1
```

Calculate velocity on gridded domain

In [6]:
```
x = np.linspace(-1.5,1.5,20)
y = np.linspace(-1.5,1.5,20)
X,Y = np.meshgrid(x,y)
```

In [7]:
```
U = velocity(X,Y,d)
```

In [8]:
```
u = U[0,:,:]
v = U[1,:,:]
```

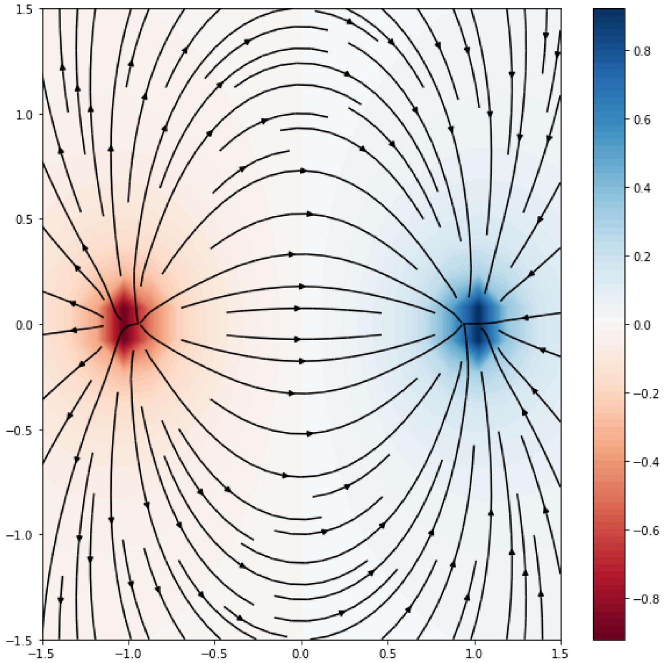
Calculate velocity potential

In [9]:
```
Phi = potential(X,Y,d)
```

Plot velocity potential and streamlines

In [10]:
```
fig = plt.figure(figsize=(10,10))
p = plt.contourf(X,Y,Phi,cmap='RdBu',levels=100)
plt.streamplot(X,Y,u,v,color='k');
plt.colorbar(p)
```

Out[10]:
```
<matplotlib.colorbar.Colorbar at 0x1d7edb0b520>
```



Let $d=0.5$

In [11]:
```
d = 0.5
```

Calculate velocity on gridded domain

In [12]:
```
x = np.linspace(-1.5,1.5,20)
y = np.linspace(-1.5,1.5,20)
X,Y = np.meshgrid(x,y)
```

In [13]:
```
U = velocity(X,Y,d)
```

In [14]:
```
u = U[0,:,:]
v = U[1,:,:]
```

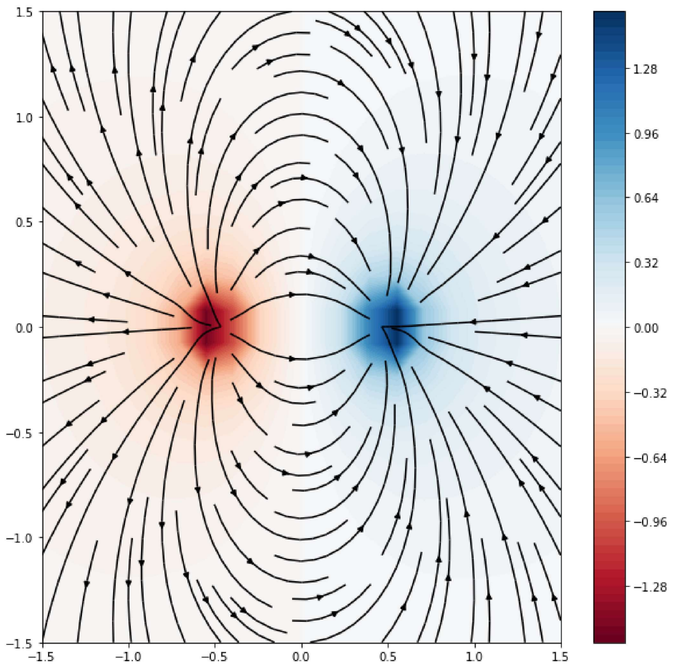
Calculate velocity potential

In [15]:
```
Phi = potential(X,Y,d)
```

Plot velocity potential and streamlines

In [16]:
```
fig = plt.figure(figsize=(10,10))
p = plt.contourf(X,Y,Phi,cmap='RdBu',levels=100)
plt.streamplot(X,Y,u,v,color='k');
plt.colorbar(p)
```

Out[16]:
```
<matplotlib.colorbar.Colorbar at 0x1d7edff5e20>
```



Let $d=0.1$

In [17]:
```
d = 0.1
```

Calculate velocity on gridded domain

In [18]:
```
x = np.linspace(-1.5,1.5,20)
y = np.linspace(-1.5,1.5,20)
X,Y = np.meshgrid(x,y)
```

In [19]:
```
U = velocity(X,Y,d)
```

In [20]:
```
u = U[0,:,:]
v = U[1,:,:]
```

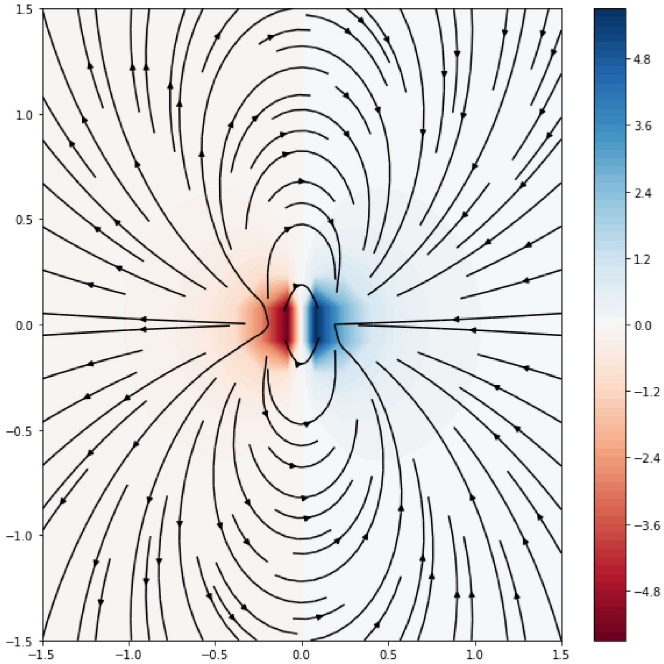
Calculate velocity potential

In [21]:
```
Phi = potential(X,Y,d)
```

Plot velocity potential and streamlines

In [22]:
```
fig = plt.figure(figsize=(10,10))
p = plt.contourf(X,Y,Phi,cmap='RdBu',levels=100)
plt.streamplot(X,Y,u,v,color='k');
plt.colorbar(p)
```

Out[22]:
```
<matplotlib.colorbar.Colorbar at 0x1d7ee4cd130>
```



Velocity potential (2D) - limiting case

The velocity potential for a point source and a point sink at the origin is given as

$$ \Phi(\mathbf{r}) = \frac{\mu x}{4\pi(x^2+y^2)^{3/2}} $$

In [23]:
```
def potential(x,y,mu):
    Phi = mu*x/(4*np.pi*(x**2+y**2)**(3/2))
    return Phi
```

The velocity vector is then $\mathbf{u}=\nabla\Phi$ or

$$ \mathbf{u} = \begin{bmatrix} \frac{\mu}{4\pi(x^2+y^2)^{3/2}} - \frac{3\mu x^2}{4\pi(x^2+y^2)^{5/2}} \\ -\frac{3\mu xy}{4\pi(x^2+y^2)^{5/2}} \end{bmatrix} $$

In [24]:
```
def velocity(x,y,mu):
    u = mu/(4*np.pi*(x**2+y**2)**(3/2)) - 3*mu*x**2/(4*np.pi*(x**2+y**2)**(5/2))
    v = -3*mu*x*y/(4*np.pi*(x**2+y**2)**(5/2))
    U = np.array([u,v])
    return U
```

In [25]:
```
velocity(0,1,1)
```

Out[25]:
```
array([0.07957747, 0.        ])
```

Let $\mu=10$

In [26]:
```
mu = 10
```

Calculate velocity on gridded domain

In [27]:
```
x = np.linspace(-1.5,1.5,20)
y = np.linspace(-1.5,1.5,20)
X,Y = np.meshgrid(x,y)
```

In [28]:
```
U = velocity(X,Y,mu)
```

In [29]:
```
u = U[0,:,:]
v = U[1,:,:]
```

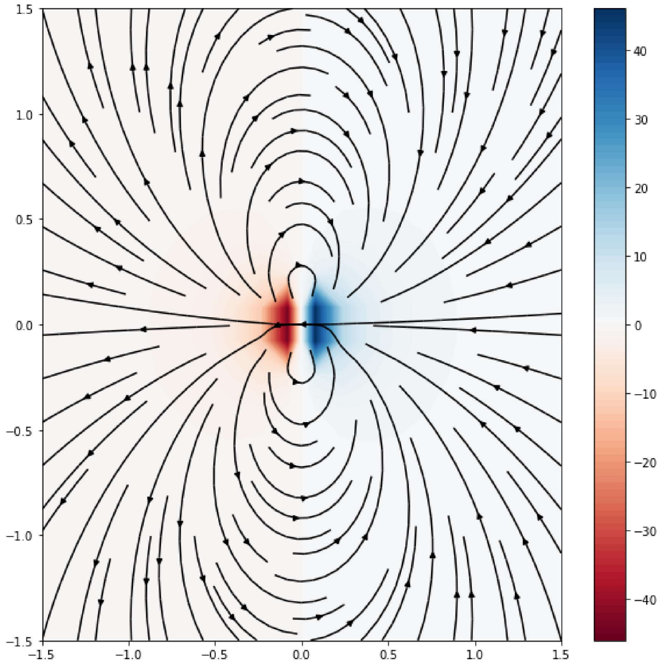
Calculate velocity potential

In [30]:
```
Phi = potential(X,Y,mu)
```

Plot velocity potential

In [31]:
```
fig = plt.figure(figsize=(10,10))
p = plt.contourf(X,Y,Phi,cmap='RdBu',levels=100)
plt.streamplot(X,Y,u,v,color='k');
plt.colorbar(p)
```

Out[31]:
```
<matplotlib.colorbar.Colorbar at 0x1d7ee943400>
```



In [ ]: