# College Algebra Lecture Notes

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Andrew Cousino

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# Part I.

# Equations and Inequalities

# 1. Solutions Bound in Sets

**New Things**

## Definitions

**Set** a collection of items

**Element of set $A$**
An item inside the set $A$. If $x$ is an element of the set $A$, then we write $x \in A$. Otherwise, we write $x \notin A$ to say that the item $x$ is not an element of $A$.

**Empty Set**
the unique set which has nothing inside it, i.e. no elements. We write $\emptyset$ to mean the empty set.

**Real Numbers**
the set which contains all integers, fractions, and irrational numbers. We write $\mathbb{R}$ to mean the real numbers, or more properly, the set of real numbers.

## Notation

- Roster Method for describing a set
- Set-Builder Method for describing a set

## Sets Contain Solutions

### Definition

A **set** is simply a container, albeit an abstract container, of various items. When we talk about a specific set, lets just use an arbitrary set $A$ as an example, the items contained inside this set $A$ are called the **elements** of this set. When talking about a specific item, say $x$ for example, we will write $x \in A$ to mean that the item $x$ is an element of the set $A$. Likewise, we write $x \notin A$ to mean that $x$ is not an element of $A$.

### Naming Sets

We need to have some examples of the kind of sets that we'll be working with in this class. There are two ways of naming, or describing, a set.

#### Roster Method

Here is one such example.

$$\{2, 3, 5, 7\}$$

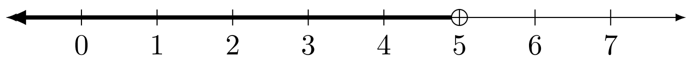
This is the set that contains the numbers 2, 3, 5, and 7, and no other elements. The method of writing a set in this way is known as the **roster method**. It's most convenient when dealing with sets that have a small number of elements.

### Set-Builder Method

But we will be dealing with sets that have infinitely many members. For these sets, we describe them using **Set-Builder method**.

$$\{x \mid x < 5\}\ \text{read: the set of all element } x \text{ such that } x \text{ is less than } 5$$

It starts, and ends, with the same curly braces $\{\}$, describes the type of elements it contains, e.g. a single number represented by $x$, and gives a condition which determine whether or not an item is a member in the set, e.g. $x < 5$. For our set above, if any single number, represented by $x$, is less than 5, then $x$ is an element of this. And every element of our set must be numbers less than 5. Another way to represent this set is with a number line.




## Sets in College Algebra

The other common types of sets we will encounter are the set of all real numbers $\mathbb{R}$, and the empty set $\emptyset$, which is the set that contains nothing at all.

# 2. Linear Equations and Inequalities

**New Things**

## Definitions

**Linear Equation**
An equation which can be put in the form $ax + b = 0$.

**Contradiction, an equation which is a**
An equation which has no solution, like $x + 1 = x$, or is outright false, like $0 = 1$.

**Identity, an equation which is an**
An equation which is true regardless of the value of the variable(s), for example $(x+1)^2 - (x-1)^2 = 4x$.

**Conditional Equation**
An equation which is true when the variables have certain values, and is false otherwise. These are the equations you know and love/hate.

## Linear Equation Models

**Example 2-1**

The cost, in dollars, to rent a storage unit for $t$ months is given by $C := 150 + 52.50t$. If you have \$1,200 budgeted for storage, how long can you rent such a unit?

**WANT** Time $t$ when cost is \$1,200.

**KNOW** $C = \$1,200$.

$$\begin{aligned} C &= 1200 \\ 150 + 52.50t &= 1200 \\ 52.50t &= 1050 \\ t &= 20 \end{aligned}$$

So you can rent the unit for up to 20 months while staying within budget.

**Example 2-2**

How much of a 4% acid solution should be mixed with 200 mL of a 12% to make a 9% solution?

**WANT** Amount of 4% solution, $x$, to add.

| | **4% Solution** | **12% Solution** | **9% Solution** |
|---|---|---|---|
| **Total Amount** | $x$ | 200 | x + 200 |
| **Amount of Acid** | $0.04x$ | $0.12 \cdot 200$ | $0.09\,(x + 200)$ |

$$\begin{aligned} 0.04x + 0.12 \cdot 200 &= 0.09\,(x + 200) \\ 0.04x + 24 &= 0.09x + 18 \\ 6 &= 0.05x \\ x &= 120 \end{aligned}$$

Mix 120 mL of 4% solution with 200 mL of 12% solution to get 320 mL of 9%.

**Example 2-3**

Two companies, company A and company B, both make yard signs among other things. Company A charges \$1.20 per sign, while B charges a flat fee of \$15.90 for the design on top of \$1.10 per sign. First write out a model for the cost of ordering $x$ signs from both companies. Obviously, Company A is cheaper if you're only going to make a few signs; but if you're going to make a lot of signs then eventually Company B becomes cheaper. How many signs need to be ordered so that the costs from both companies become equal?

$$\begin{aligned} A &= 1.2x \\ B &= 1.1x + 15.9 \end{aligned}$$

**WANT** Number of signs needed, $x$, for $A = B$.

$$\begin{aligned} A &= B \\ 1.2x &= 1.1x + 15.9 \\ 0.1x &= 15.9 \\ x &= 159 \end{aligned}$$

So ordering up to 159 signs, company A is cheaper. But when ordering over 159 signs, company B is cheaper.

## Contradictions and Identities

An equation involving the variable $x$ that is true when $x$ is one value but false when $x$ is another value is called a **conditional equation**. There are equations which are true regardless of what the value of $x$ is. We call these kinds of equations **identities** or tautologies. Finally, the third type of equation is one which is never true for any value of $x$. These are **contradictions**.

**Example 2-4**

Identify each equation as either as being conditional, a contradiction, or an identity. Also find the solution set of each.

1. $4x + 1 - x = 6x - 2$
2. $2(-5x - 1) = 2x - 12x + 6$
3. $2(3x - 1) = 6(x + 1) - 8$

1.

$$\begin{aligned} 4x + 1 - x &= 6x - 2 \\ 3x + 1 &= 6x - 2 \\ 3 &= 3x \\ 1 &= x \\ x &= 1 \end{aligned}$$

If $x = 1$, then the original equation will be true. So this is a conditional equation whose solution set is $\{1\}$.

2.

$$\begin{aligned} 2(-5x - 1) &= 2x - 12x + 6 \\ -10x - 2 &= -10x + 6 \\ 0 &= 8 \end{aligned}$$

This is clearly false. So the original equation must always be false, regardless of the value of $x$. This is a contraction whose solution set is the empty set $\emptyset$.

3.

$$\begin{aligned} 2(3x - 1) &= 6(x + 1) - 8 \\ 6x - 2 &= 6x + 6 - 8 \\ 6x - 2 &= 6x - 2 \\ 0 &= 0 \end{aligned}$$

This is obviously true. The original equation must always be true. So we have an identity with the solution set being the set of all real numbers $\mathbb{R}$.

## Linear Inequalties

### Example 2-5

Donovan has offers for two different sales jobs. Job A pays a base salary of \$25,000 plus 10% commission on all sales. Job B pays a base salary of \$30,000 plus 8% commission. How much would Donovan have to sell for the salary from Job A to exceed that of Job B?

$$
\begin{aligned}
A &:= 25000 + 0.10x \\
B &:= 30000 + 0.08x \\
A &> B \\
25000 + 0.10x &> 30000 + 0.08x \\
0.02x &> 5000 \\
x &> 250000
\end{aligned}
$$

So the solution set is the interval $(250,000, +\infty)$. This means that Donovan must sell at least \$250,000 in order for Job A to pay better than Job B.

### Example 2-6

Body temperature usually between 36.5 °Cand 37.5 °C. Given that $C = \frac{5}{9}(F - 32)$ converts from $F$ °Fto $C$ °C, find the typical range for body temperature in °F.

$$
\begin{aligned}
36.5 < C < 37.5 && (2.1)\\
36.5 < \frac{5}{9}(F - 32) < 37.5 && (2.2)\\
4.5 < \frac{5}{9}F - \frac{160}{9} < 5.5 && (2.3)\\
\frac{977}{18} < \frac{5}{9}F < \frac{995}{18} && (2.4)\\
\frac{977}{10} < F < \frac{199}{2} && (2.5)\\
97.7 < F < 99.5 && (2.6)\\
&& (2.7)
\end{aligned}
$$

In Fahrenheit, typical body temperature ranges between 97.5–99.5 °F.

**Example 2-7**

For what values of $x$ will the following expressions avoid taking square roots of negative numbers.

1. $\sqrt{x-6}$
2. $\sqrt{6-x}$

1. $$\begin{aligned} x-6 &\geq 0 \\ x &\geq 6 \end{aligned}$$

   The values of $x$ that avoid square roots of negative numbers are the values of the interval $[6,+\infty)$.

2. $$\begin{aligned} 6-x &\geq 0 \\ 6 &\geq x \\ x &\leq 6 \end{aligned}$$

   The desired values of $x$ are exactly those in the interval $(-\infty,6]$.

**Example 2-8**

Solve the following inequalities.

1. $3\,(2x+1)+4 \leq 6x+2$
2. $9+4c > 3\,(c+1)+c$

1. $$\begin{aligned} 3\,(2x+1)+4 &\leq 6x+2 \\ 6x+3+4 &\leq 6x+2 \\ 7 &\leq 2 \end{aligned}$$

   This is a contradiction. So the original inequality is a contradiction and has no solution. The solution set is empty $\emptyset$.

2.

$$
\begin{aligned}
9+4c &> 3(c+1)+c \\
9+4c &> 3c+3+c \\
9+4c &> 4c+3 \\
6 &> 0
\end{aligned}
$$

This is always true. So the original inequality is an identity and every $x$-value is a solution. The solution set is all reals $\mathbb{R}$.

## Interval Notation

Most of the sets that we work with in this class will be written in a style known as interval notations. As the name suggests, this notation represents intervals on the number line. There are 9 basic types of intervals which are listed in table 2.1 along with their meaning.

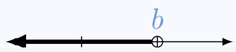
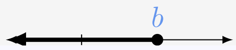
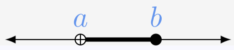
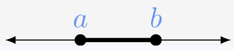
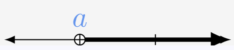
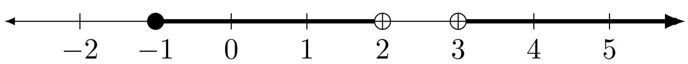
| Interval Notation | Set Notation | Number Line | Meaning |
|---|---|---|---|
| $(-\infty, b)$ | $\{x \mid x < b\}$ |  | all #s $< b$ |
| $(-\infty, b]$ | $\{x \mid x \leq b\}$ |  | all #s $\leq b$ |
| $(a, b)$ | $\{x \mid a < x < b\}$ |  | all #s btwn $a$ & $b$, excl. both endpoints |
| $(a, b]$ | $\{x \mid a < x \leq b\}$ |  | all #s btwn $a$ & $b$, excl. $a$ |
| $[a, b)$ | $\{x \mid a \leq x < b\}$ |  | all #s btwn $a$ & $b$ excl. $b$ |
| $[a, b]$ | $\{x \mid a \leq x \leq b\}$ |  | all #s btwn $a$ & $b$ |
| $[a, +\infty)$ | $\{x \mid x \geq a\}$ |  | all numbers $\geq a$ |
| $(a, +\infty)$ | $\{x \mid x > a\}$ |  | all numbers $> a$ |
| $(-\infty, +\infty)$ | $\mathbb{R}$ |  | all real numbers |

Table 2.1.: Table of Interval Notation

## Joining Intervals Together

We will also want to join two intervals together. To join two intervals, say for example $[-1, 2)$ and $(3, +\infty)$, we write $[-1, 2) \cup (3, +\infty)$.



# 3. Complex Numbers

# 4. Quadratic Equations

**New Things**

## Definitions

**Quadratic Equation**

An equation which can be put in the form $ax^2 + bx + c = 0$.

**Standard Form of a quadratic**

$ax^2 + bx + c$

**Vertex Form of a quadratic**

$a(x-h)^2 + k$

## Procedures

**Completeing the Square**

A process that allows us to rewrite a quadratic from standard form into vertex form.

## Factoring

## Completing the Square

## Quadratic Formula

# 5. Polynomial and Rational Equations

# 6. Polynomial and Rational Inequalities

**New Things**

## Definitions

**Polynomial Expression**

**Polynomial Inequality**

**Rational Expression**

**Rational Inequalities**

# Part II.

# Functions and Relations

# 7. Graphs and Circles

# 8. Functions and Their Graphs

# 9. Transformations of Functions

# 10. Arithmetic of Functions

# 11. Inverse Functions

# Part III.

# Library of Functions

# 12. Linear Functions

# 13. Quadratic Functions

# 14. Polynomial Functions

# 15. Dividing Polynomials

# 16. Zeros of Polynomial Functions

# 17. Rational Functions

# 18. Exponential Functions

# 19. Logarithmic Functions

# Part IV.

# Linear Systems

# 20. System of Linear Equations

# 21. Introduction to Matrices

# 22. Inconsistent and Dependent Systems

# 23. Arithmetic with Matrices

# 24. Inverse Matrices and Matrix Equations