Black hole radiation, an overview of quasi-normal modes

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Abstract

In this review, I describe the literature on quasi-normal modes, including the theory and methods to calculate them with the purpose of evaluating my Fortran implementation of these techniques and discussing my vision of current and future research in this area.

Keywords: General relativity, perturbation theory, quasi-normal modes

I. INTRODUCTION

Quasi-normal modes (QNMs) are a set of complex frequencies which describe damped oscillations of a field characterized by its spin. This behavior may be induced from perturbations to the geometry which could induce GWs. This makes tensor field perturbations of great interest.

Perturbation theory has had great achievements in the field of GR since its beginning (Regge & Wheeler,1957). Much of this work has been done in relation to BHs (Ruffini & Press, 1971), (Detweiler, 1975), (Press & Teukolsky, 1973), now reaffirmed by the successful detection of GWs, however, the new era of GW astronomy promises to integrate with diverse astrophysical sources; this makes relativistic stars and supernovae of interest (Thorne, 1969), (Seidel & Myra, 1988), (Thorne, 1998).

In the case of black holes, the modes themselves depend only on no-hair characteristics thus making QNMs suitable for ringdown parameter estimation. The perturbation evolves according to the Regge-Wheeler equation (or Teukolsky's in a more general case):

$$-\frac{\partial^2\varphi}{\partial t^2}+\frac{\partial^2\varphi}{\partial r_*^2}+V(r)\varphi=0\,, \tag{1}$$

where the exact form of $V(r)$ depends on the type of perturbation. In section II I briefly describe the major methods for finding these modes for the Schwarzschild black hole, then, in section III modes obtained in Fortran are compared to their literature counterparts, finally in section IV I discuss my vision on the subject and its future.

II. SCHWARZSCHILD QUASI-NORMAL MODES

Computation of QNMs has historically relied on numerical techniques (Chandrasekar & Detweiler, 1975), (Detweiler, 1979), (Leaver, 1985). Semianalytical methods also exist and are based on the WKB technique by similarity to Schrodinger's equation (Schutz, 1985), (Iyer, 1987) with better precision for higher angular index l. Either approach is vastly benefited by computer implementations of them.

Leaver's method is based on a continued fraction whose roots fulfill the ingoing and outgoing wave boundary conditions and are thus QNMs. This can be implemented in Fortran by declaring a recursive function of the frequency ω, then scanning the complex plane for regions where the function is small, and lastly finding the root with a local method like Mueller's.

The WKB method is based on solving the fourier transform of equation 1 by approximating the function $-Q(r)\equiv-w^2+V(r)$, where $V(r)=F(r)\left(l(l+1)/r^2-6/r^3\right)$, near its peak. The mode may then be found from a relation involving derivatives w.r.t r_* of V(r). Fortran implementation involves finding the maximum of the potential by Newton-Raphson, for example, then applying the formula according to the method's order.

III. RESULTS

As suggested, QNMs have been computed vastly in the literature, so in order to review the Fortran implementation I estimate accuracy by a quotient between the values obtained and those already known, taking into account computational cost. This is done in table 1 for Leaver's method and table 2 for WKB of 2nd order.

Table 1. Single core Fortran QNM ω_F vs ω_L from literature (Leaver, 1986).

l,m	$\frac{\lvert\omega_F\rvert}{\lvert\omega_L\rvert}-1$	Time (s)
2,2	-4×10^{-8}	0.7
3,2	-2×10^{-6}	63.2
4,2	-2×10^{-6}	63.5

Table 2. Single core Fortran QNM ω_F vs ω_S from literature (Schutz, 1985).

l,n	$\frac{\lvert\omega_F\rvert}{\lvert\omega_S\rvert}-1$	Time (s)
2,0	1×10^{-4}	5×10^{-4}
3,0	-6×10^{-5}	6×10^{-4}
4,0	8×10^{-6}	2×10^{-3}

IV. DISCUSSION

The Fortran implementation of both methods is accurate, however, finding the biggest mode with Leaver's approach takes about a minute. This may be improved with insight from optimization problems where minimizing a function is desired, or by assigning sections of the complex plane to different cores.

Today, computation of QNMs is mainly focused on improving templates and understanding signals better. For instance, considering higher order modes in ringdown modeling (London, Shoemaker & Healy, 2014), or filtering out some QNMs in simulations to find secondary effects (Sizheng Ma, et. al, 2022). Also, as some researchers focus on testing GR, their attention turns to ringdown (H. O. Silva, et. al, 2022), (J. L. Blázquez-Salcedo, et. al, 2016) using QNMs to test the no-hair theorem or exotic theories. I believe that these examples show that as detectors become more sensitive and NR more accurate perturbation theory will find ways to stay relevant, as we are only scratching the surface of its applications.