Automatic construction and description of nonparametric models

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This analysis was automatically generated

The raw data and full model posterior with extrapolations are shown in figure 1.

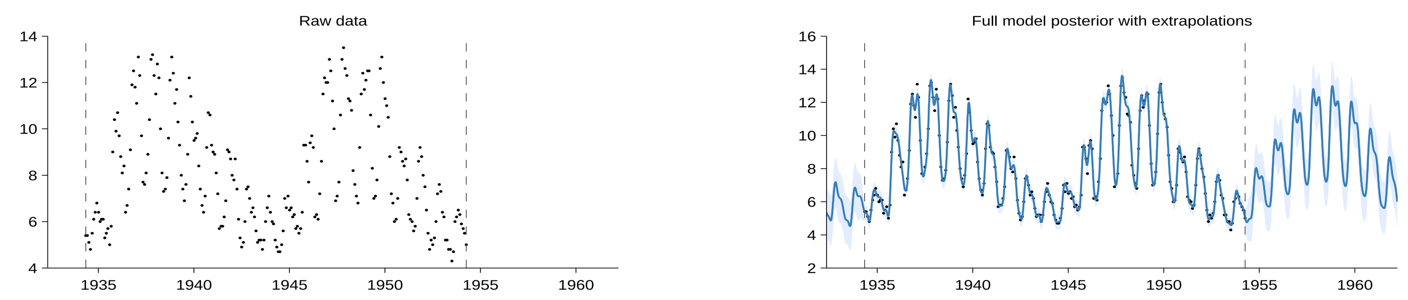


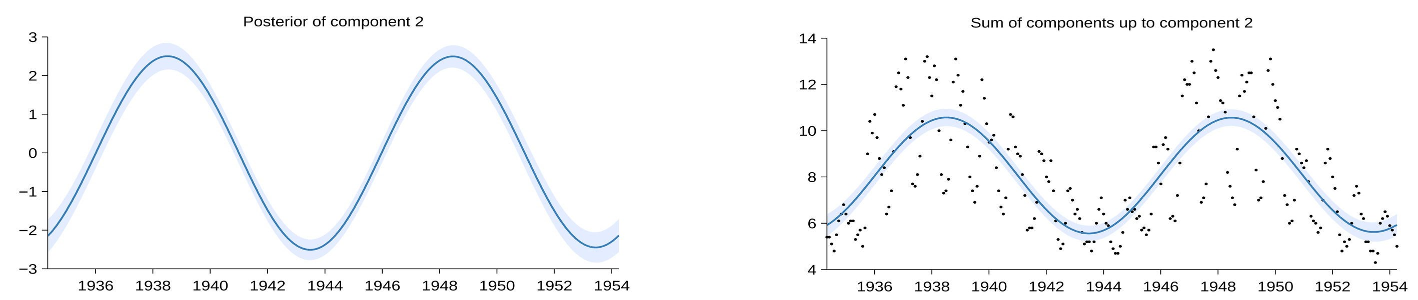
Figure 1: Raw data (left) and model posterior with extrapolation (right)

The structure search algorithm has identified six additive components in the data:

- A constant.
- An approximately sinusoidal function with a period of 9.9 years.
- An exactly periodic function with a period of 1.0 years.
- An approximate product of a periodic function and a sinusoid.
- A smooth function.
- A very approximately sinusoidal function with a period of 9.9 years.

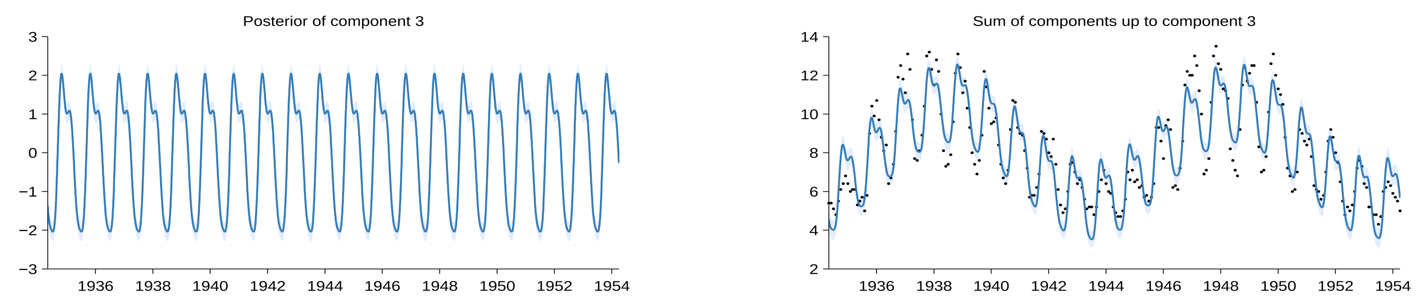
2.2 Component 2

This component is approximately sinusoidal with a period of 9.9 years. Across periods the shape of the function varies very smoothly.



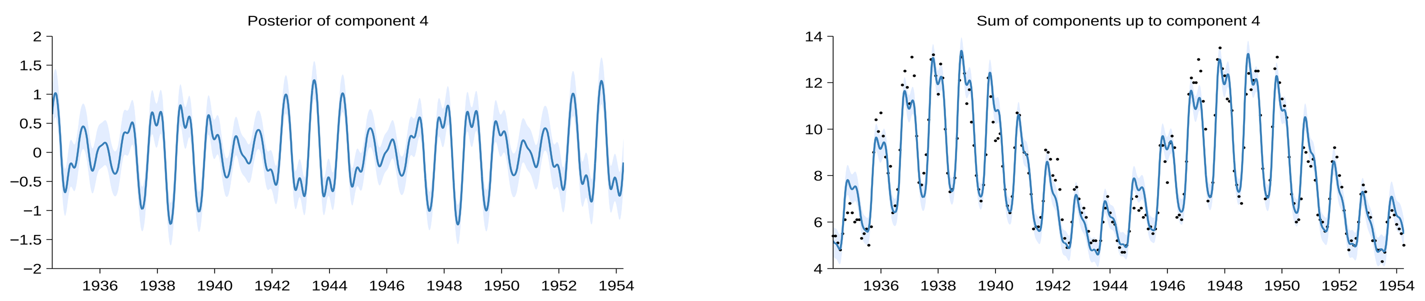
2.3 Component 3

This component is exactly periodic with a period of 1.0 years. The shape of the function within each period has a typical lengthscale of 3.3 months.



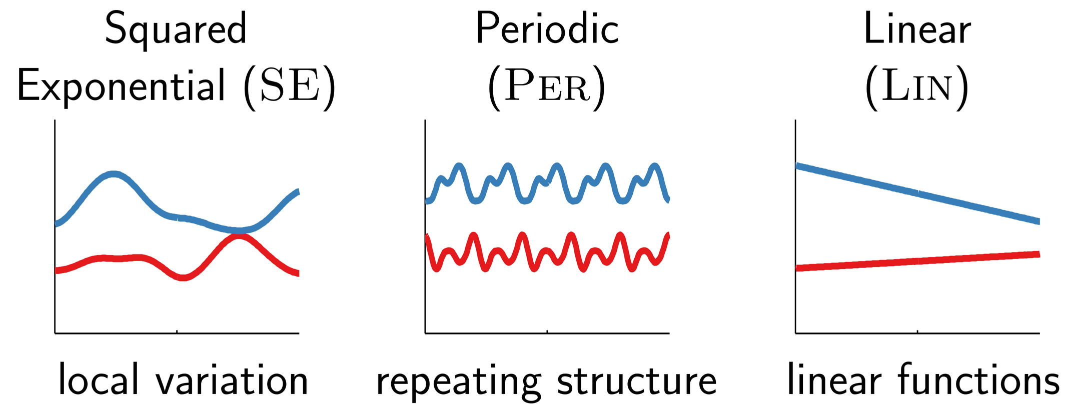
2.4 Component 4

This component is a product of a periodic function and a sinusoid. Across periods the shape of the function varies smoothly with a typical lengthscale of 44.5 years. The periodic function has a period of 1.0 years. The shape of this function within each period has a typical lengthscale of 3.3 months. The sinusoid has a period of 9.9 years.

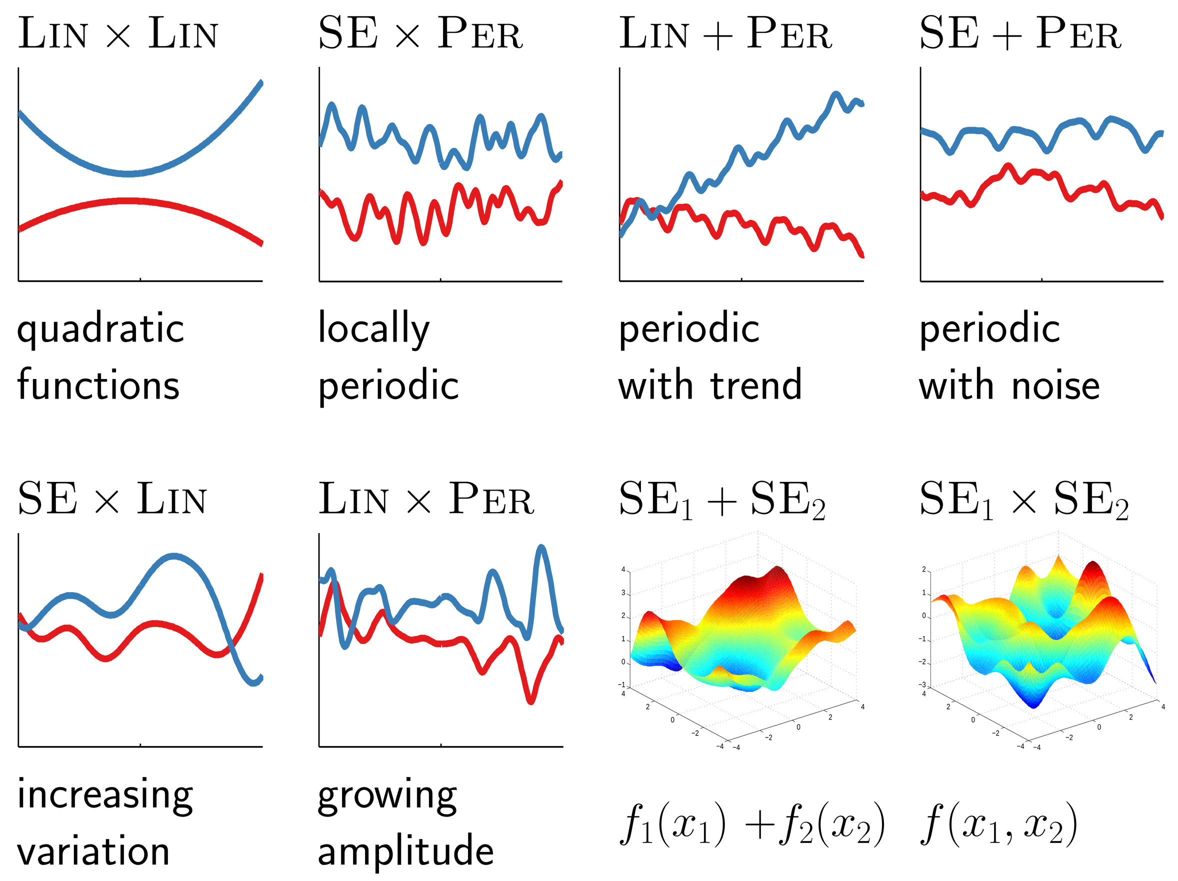


Gaussian processes model structure through kernels

- For Gaussian process models, the prior — and hence, the pattern of generalisation — is determined by a kernel function. Common examples include:

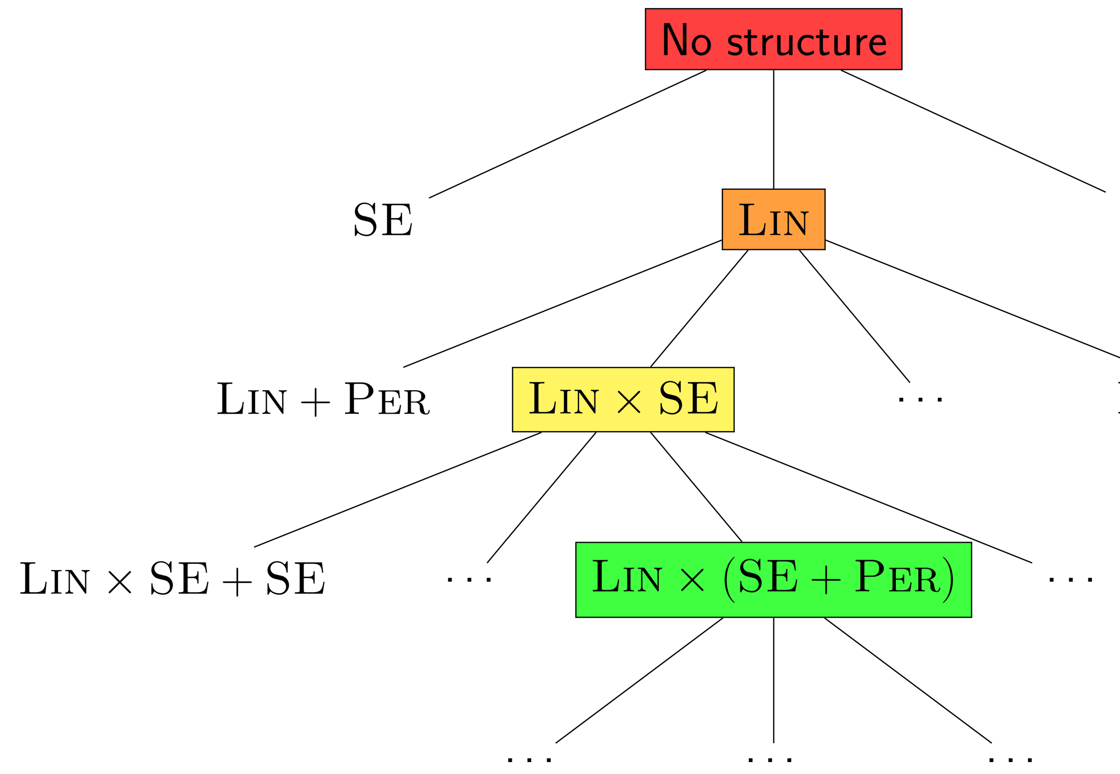


- Composite kernels can express many types of structure



- Building composite kernels has often required much human ingenuity

We build models via a greedy search



Automatically describing model properties

How to automatically describe arbitrarily complex kernels

1. Break each kernel into a sum of products.
2. For each product, look up the properties of each kernel in that product.
3. Combine the properties into one sentence.
4. Plot contribution of this compoenent to the model.

Kernels can be distributed into a sum of products

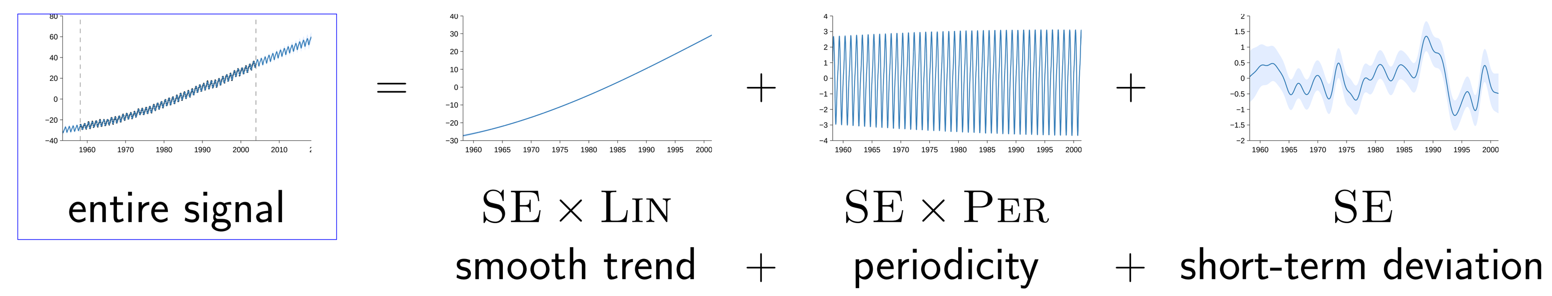
For example:

$$\text{SE} \times (\text{LIN} + \text{PER} + \text{SE})$$

becomes

$$(\text{SE} \times \text{LIN}) + (\text{SE} \times \text{PER}) + (\text{SE}).$$

Sums of kernels correspond to sums of functions



If $f_1(x) \sim \text{GP}(0, k_1)$ and $f_2(x) \sim \text{GP}(0, k_2)$ then $f_1(x)+f_2(x) \sim \text{GP}(0, k_1+k_2)$. Therefore, a sum of kernels can be described as a sum of independent functions.

Each kernel in a product roughly corresponds to an adjective

Kernel	How it modifies the prior
SE	functions change smoothly
PER	functions repeat
LIN	amplitude increases linearly

On its own, each kernel simply modifyies the constant function $f(x) = c$.

Example short descriptions

Product of Kernels	Description
PER	An exactly periodic function
PER × SE	An approximately periodic function
PER × SE × LIN	An approximately periodic function with linearly varying amplitude
LIN	A linear function
LIN × LIN	A quadratic function
PER × LIN × LIN	An exactly periodic function with quadratically varying amplitude

Code available at `github.com/jamesrobertlloyd/gpss-research`