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# An automatic report for the dataset : 01-airline

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## Abstract

This report was produced automatically by the Gaussian process structure search algorithm. See <http://arxiv.org/abs/1302.4922> for a preliminary paper and see <https://github.com/jamesrobertlloyd/gpss-research> for the latest source code.

## 1 Executive summary

The raw data and full model posterior with extrapolations are shown in figure 1.

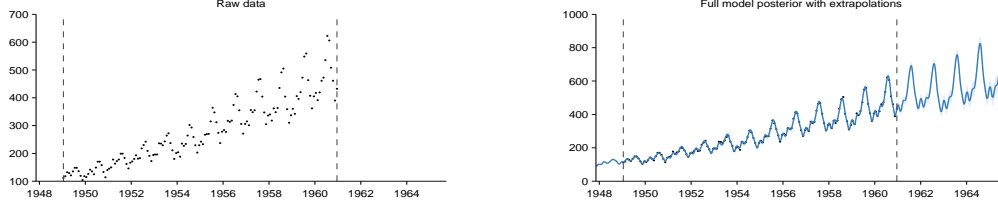


Figure 1: Raw data (left) and model posterior with extrapolation (right)

The structure search algorithm has identified four additive components in the data:

- A very smooth monotonically increasing function.
- An approximately periodic function with a period of 1.0 years and with approximately linearly increasing amplitude.
- An exactly periodic function with a period of 4.3 years but with linearly increasing amplitude.
- Uncorrelated noise with linearly increasing standard deviation.

| # | $R^2$ (%) | $\Delta R^2$ (%) | Residual $R^2$ (%) | Cross validated MAE | Reduction in MAE (%) |
|---|-----------|------------------|--------------------|---------------------|----------------------|
| - | -         | -                | -                  | 280.30              | -                    |
| 1 | 86.2      | 86.2             | 86.2               | 32.37               | 88.5                 |
| 2 | 99.3      | 13.1             | 95.0               | 8.45                | 73.9                 |
| 3 | 99.8      | 0.5              | 66.7               | 5.61                | 33.6                 |
| 4 | 100.0     | 0.2              | 100.0              | 5.61                | 0.0                  |

Table 1: Summary statistics for cumulative additive fits to the data. The residual coefficient of determination ( $R^2$ ) values are computed using the residuals from the previous fit as the target values; this measures how much of the residual variance is explained by each new component. The mean absolute error (MAE) is calculated using 10 fold cross validation with a contiguous block design; this measures the ability of the model to interpolate and extrapolate over moderate distances. The model is fit using the full data and the MAE values are calculated using this model; this double use of data means that the MAE values cannot be used reliably as an estimate of out-of-sample predictive performance.

Model checking statistics are summarised in table 2 in section 4. These statistics have not revealed any inconsistencies between the model and observed data.

The rest of the document is structured as follows. In section 2 the forms of the additive components are described and their posterior distributions are displayed. In section 3 the modelling assumptions of each component are discussed with reference to how this affects the extrapolations made by the model. Section 4 discusses model checking statistics, with plots showing the form of any detected discrepancies between the model and observed data. A glossary of terms is provided in section B.

## 2 Detailed discussion of additive components

### 2.1 Component 1

This component is a very smooth and monotonically increasing function.

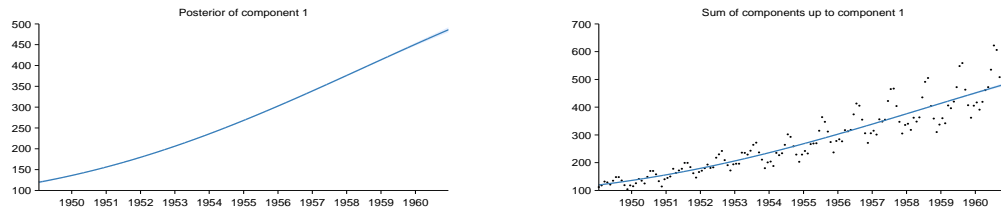


Figure 2: Pointwise posterior of component 1 (left) and the posterior of the cumulative sum of components with data (right)

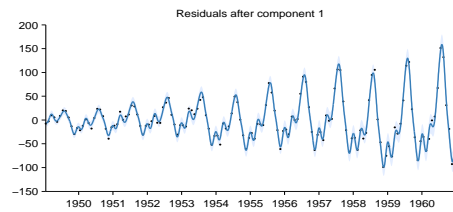


Figure 3: Pointwise posterior of residuals after adding component 1

## 2.2 Component 2

This component is approximately periodic with a period of 1.0 years and varying amplitude. Across periods the shape of this function varies very smoothly. The amplitude of the function increases approximately linearly. The shape of this function within each period has a typical lengthscale of 6.0 weeks.

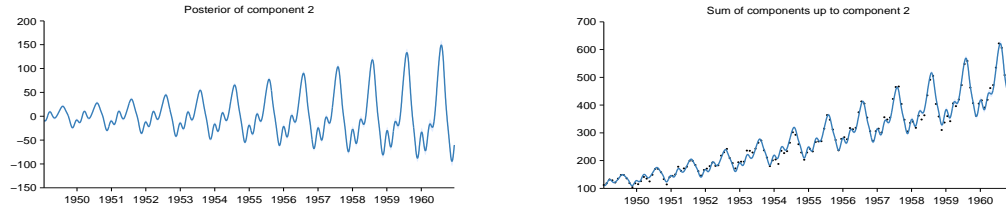


Figure 4: Pointwise posterior of component 2 (left) and the posterior of the cumulative sum of components with data (right)

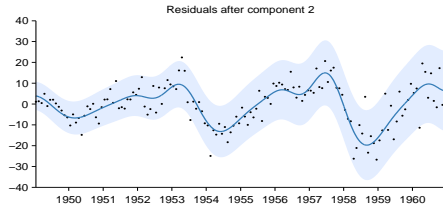


Figure 5: Pointwise posterior of residuals after adding component 2

### 2.3 Component 3

This component is exactly periodic with a period of 4.3 years but with varying amplitude. The amplitude of the function increases linearly. The shape of this function within each period has a typical lengthscale of 7.4 months.

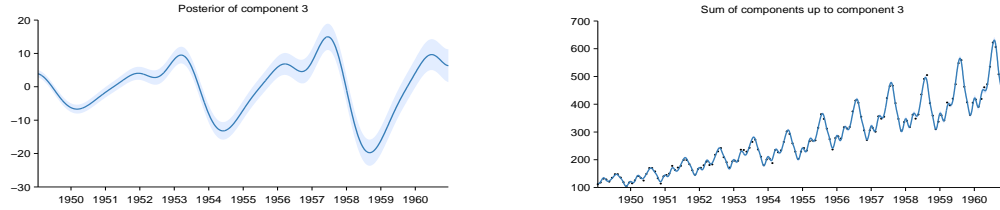


Figure 6: Pointwise posterior of component 3 (left) and the posterior of the cumulative sum of components with data (right)

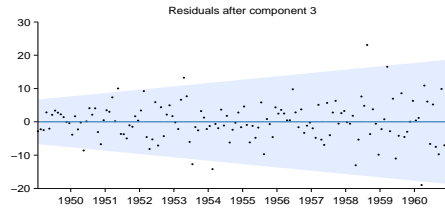


Figure 7: Pointwise posterior of residuals after adding component 3

## 2.4 Component 4

This component models uncorrelated noise. The standard deviation of the noise increases linearly.

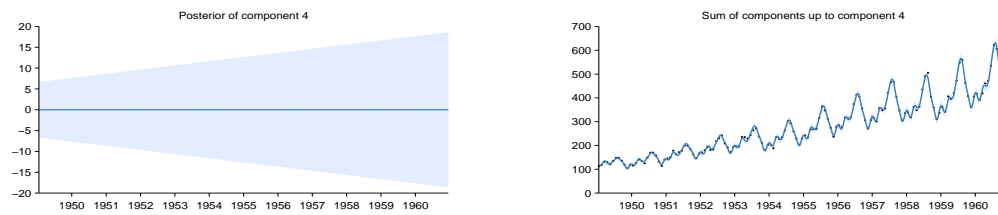


Figure 8: Pointwise posterior of component 4 (left) and the posterior of the cumulative sum of components with data (right)

### 3 Extrapolation

Summaries of the posterior distribution of the full model are shown in figure 9. The plot on the left displays the mean of the posterior together with pointwise variance. The plot on the right displays three random samples from the posterior.

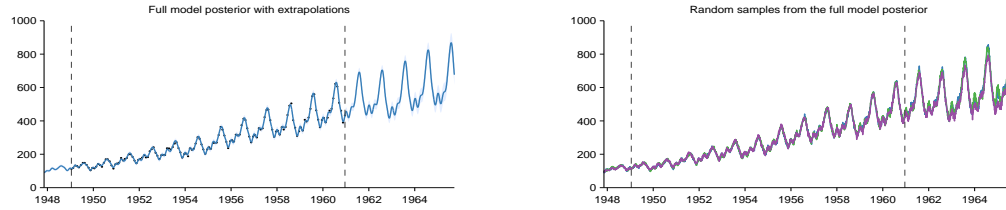


Figure 9: Full model posterior with extrapolation. Mean and pointwise variance (left) and three random samples (right)

Below are descriptions of the modelling assumptions associated with each additive component and how they affect the predictive posterior. Plots of the pointwise posterior and samples from the posterior are also presented, showing extrapolations from each component and the cumulative sum of components.

### 3.1 Component 1 : A very smooth monotonically increasing function

This component is assumed to continue very smoothly but is also assumed to be stationary so its distribution will eventually return to the prior. The prior distribution places mass on smooth functions with a marginal mean of zero and a typical lengthscale of 14.3 years. [This is a placeholder for a description of how quickly the posterior will start to resemble the prior].

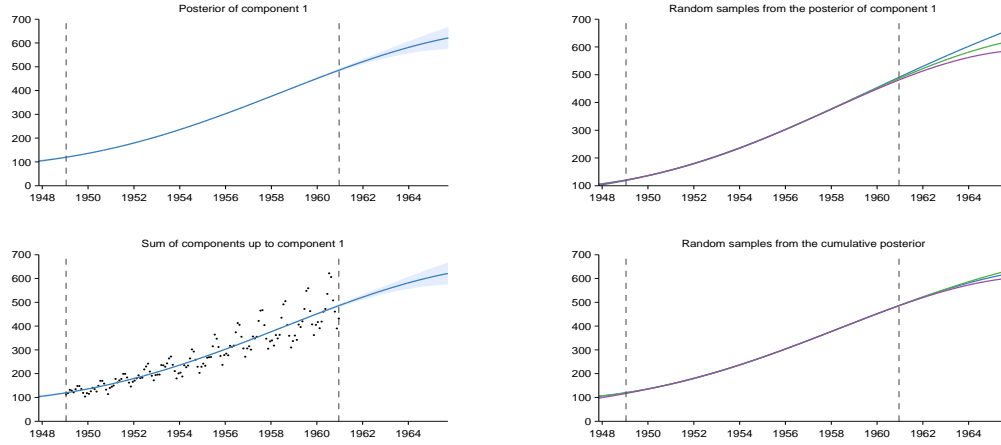


Figure 10: Posterior of component 1 (top) and cumulative sum of components (bottom) with extrapolation. Mean and pointwise variance (left) and three random samples from the posterior distribution (right).



### 3.2 Component 2 : An approximately periodic function with a period of 1.0 years and with approximately linearly increasing amplitude

This component is assumed to continue to be approximately periodic. The shape of the function is assumed to vary very smoothly between periods but will eventually return to the prior. The amplitude of the function is assumed to continue to increase approximately linearly. The prior is entirely uncertain about the phase of the periodic function. Consequently the pointwise posterior will appear to lose its periodicity, but this merely reflects the uncertainty in the shape and phase of the function. [This is a placeholder for a description of how quickly the posterior will start to resemble the prior].

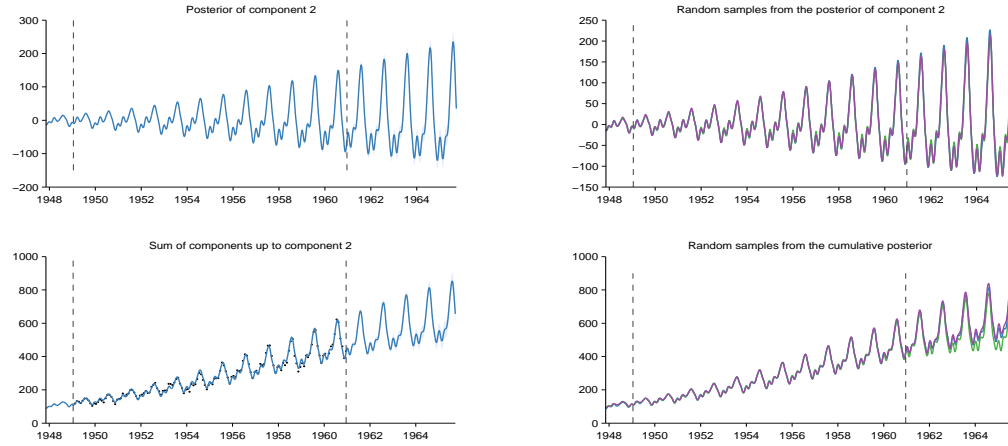


Figure 11: Posterior of component 2 (top) and cumulative sum of components (bottom) with extrapolation. Mean and pointwise variance (left) and three random samples from the posterior distribution (right).

### 3.3 Component 3 : An exactly periodic function with a period of 4.3 years but with linearly increasing amplitude

This component is assumed to continue exactly periodically with a period of 4.3 years but with varying amplitude. The amplitude of the function is assumed to continue to increase linearly.

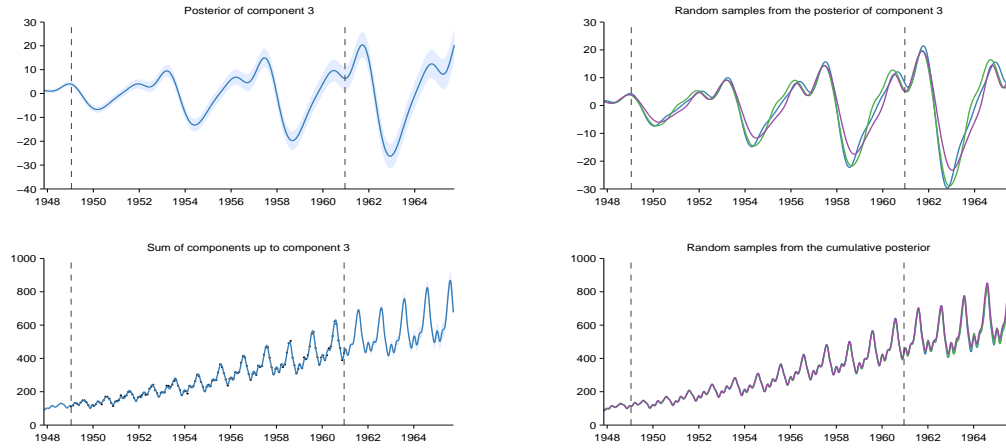


Figure 12: Posterior of component 3 (top) and cumulative sum of components (bottom) with extrapolation. Mean and pointwise variance (left) and three random samples from the posterior distribution (right).

### 3.4 Component 4 : Uncorrelated noise with linearly increasing standard deviation

This component assumes the uncorrelated noise will continue indefinitely. The standard deviation of the noise is assumed to continue to increase linearly.

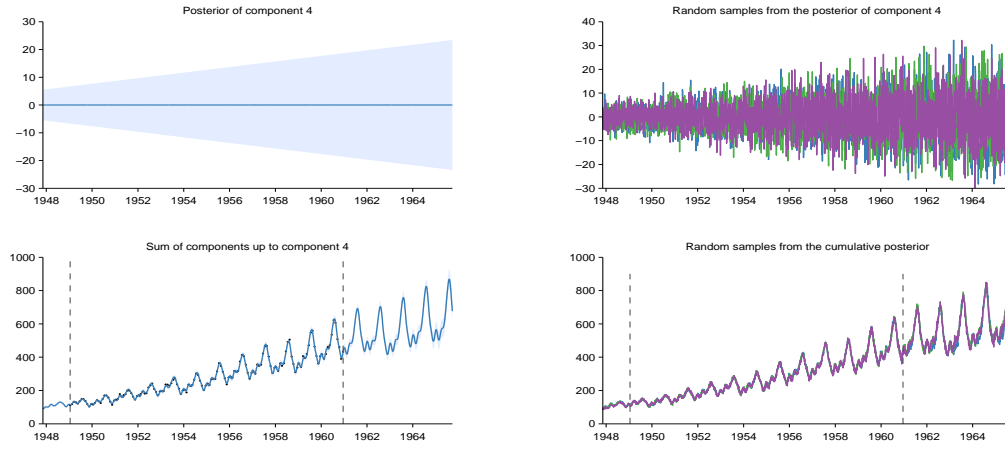


Figure 13: Posterior of component 4 (top) and cumulative sum of components (bottom) with extrapolation. Mean and pointwise variance (left) and three random samples from the posterior distribution (right).

## 4 Model checking

Several posterior predictive checks have been performed to assess how well the model describes the observed data. These tests take the form of comparing statistics evaluated on samples from the prior and posterior distributions for each additive component. The statistics are derived from autocorrelation function (ACF) estimates, periodograms and quantile-quantile (qq) plots.

Table 2 displays cumulative probability and  $p$ -value estimates for these quantities. Cumulative probabilities near 0/1 indicate that the test statistic was lower/higher under the posterior compared to the prior unexpectedly often i.e. they contain the same information as a  $p$ -value for a two-tailed test and they also express if the test statistic was higher or lower than expected.  $p$ -values near 0 indicate that the test statistic was larger in magnitude under the posterior compared to the prior unexpectedly often.

| # | ACF   |         | Periodogram |         | QQ    |       |
|---|-------|---------|-------------|---------|-------|-------|
|   | min   | min loc | max         | max loc | max   | min   |
| 1 | 0.340 | 0.707   | 0.526       | 0.360   | 0.249 | 0.842 |
| 2 | 0.607 | 0.420   | 0.495       | 0.344   | 0.512 | 0.991 |
| 3 | 0.484 | 0.393   | 0.624       | 0.370   | 0.865 | 0.509 |
| 4 | 0.240 | 0.255   | 0.878       | 0.669   | 0.221 | 0.487 |

Table 2: Model checking statistics for each component. Cumulative probabilities for minimum of autocorrelation function (ACF) and its location. Cumulative probabilities for maximum of periodogram and its location.  $p$ -values for maximum and minimum deviations of QQ-plot from straight line.

No statistically significant discrepancies between the data and model have been detected but model checking plots for each component are presented below.

## 4.1 Model checking plots for components without statistically significant discrepancies

### 4.1.1 Component 1 : A very smooth monotonically increasing function

No discrepancies between the prior and posterior of this component have been detected

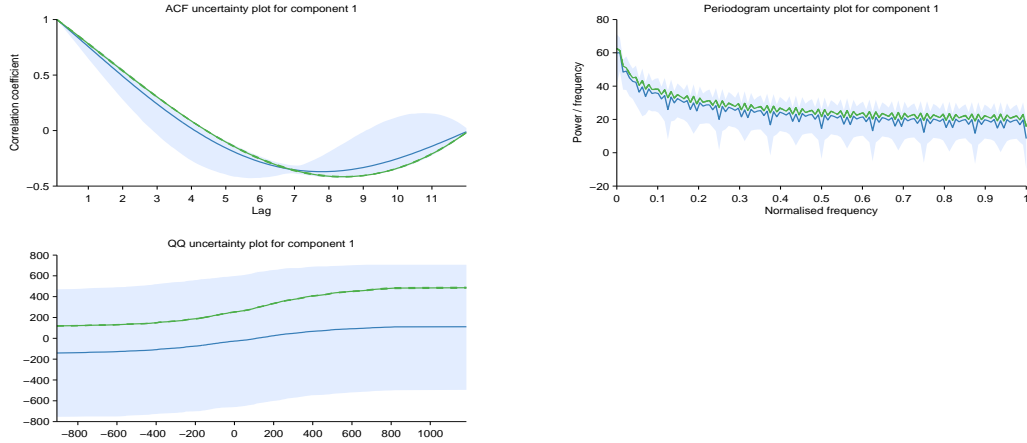


Figure 14: ACF (top left), periodogram (top right) and quantile-quantile (bottom left) uncertainty plots. The blue line and shading are the pointwise mean and 90% confidence interval of the plots under the prior distribution for component 1. The green line and green dashed lines are the corresponding quantities under the posterior.

### 4.1.2 Component 2 : An approximately periodic function with a period of 1.0 years and with approximately linearly increasing amplitude

No discrepancies between the prior and posterior of this component have been detected

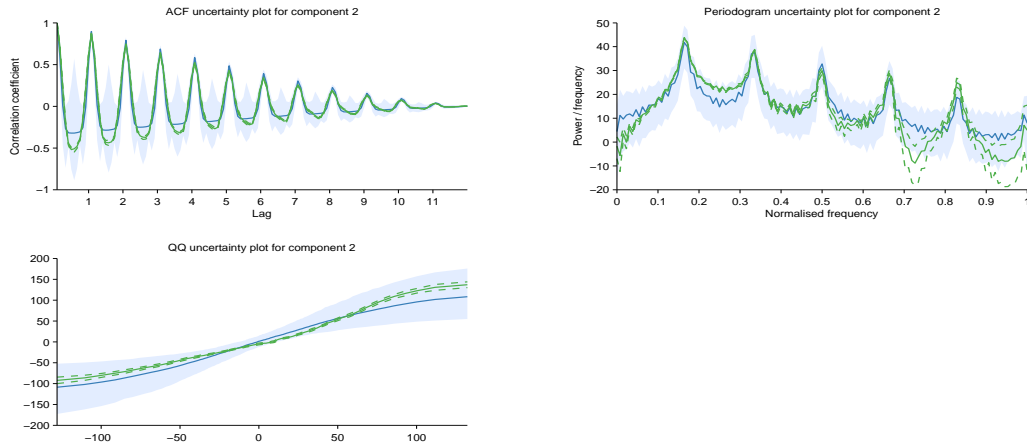


Figure 15: ACF (top left), periodogram (top right) and quantile-quantile (bottom left) uncertainty plots. The blue line and shading are the pointwise mean and 90% confidence interval of the plots under the prior distribution for component 2. The green line and green dashed lines are the corresponding quantities under the posterior.

#### 4.1.3 Component 3 : An exactly periodic function with a period of 4.3 years but with linearly increasing amplitude

No discrepancies between the prior and posterior of this component have been detected

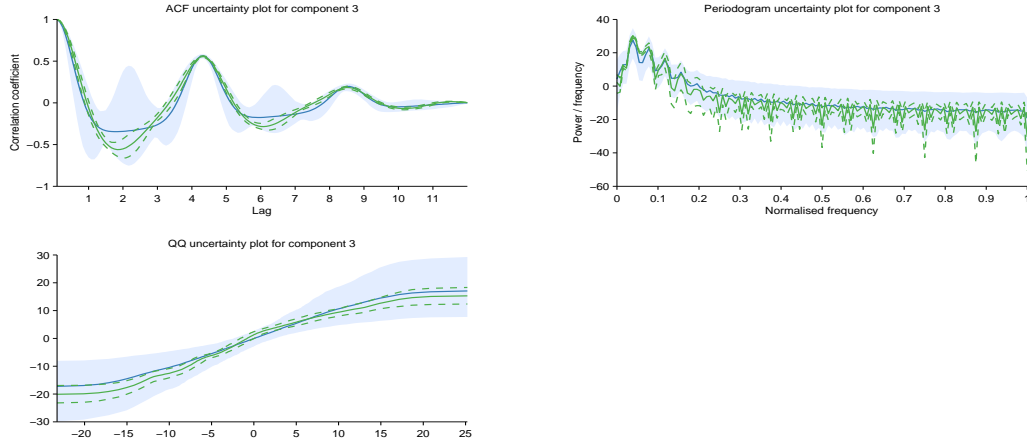


Figure 16: ACF (top left), periodogram (top right) and quantile-quantile (bottom left) uncertainty plots. The blue line and shading are the pointwise mean and 90% confidence interval of the plots under the prior distribution for component 3. The green line and green dashed lines are the corresponding quantities under the posterior.

#### 4.1.4 Component 4 : Uncorrelated noise with linearly increasing standard deviation

No discrepancies between the prior and posterior of this component have been detected

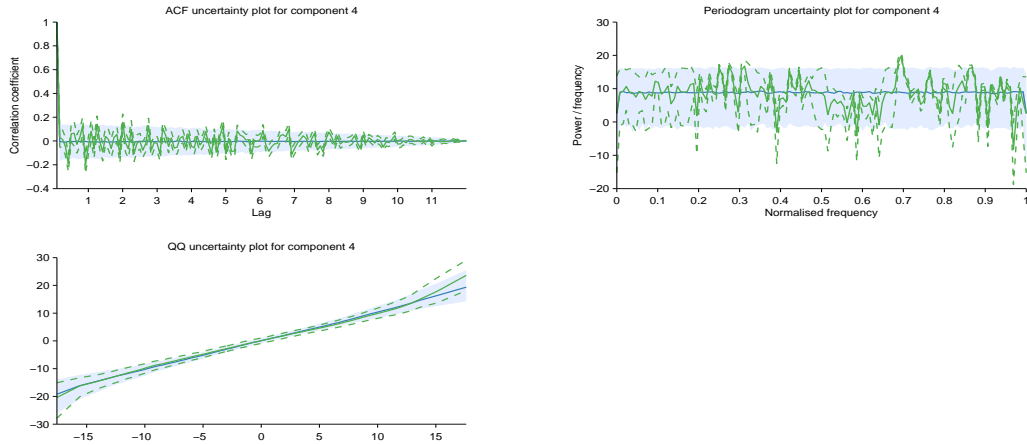


Figure 17: ACF (top left), periodogram (top right) and quantile-quantile (bottom left) uncertainty plots. The blue line and shading are the pointwise mean and 90% confidence interval of the plots under the prior distribution for component 4. The green line and green dashed lines are the corresponding quantities under the posterior.

## **A Residual style quantities**

This appendix contains plots of residual-like quantities. Their utility is still being investigated so there are currently no explanations of their calculation or interpretation.

### A.1 Leave one out

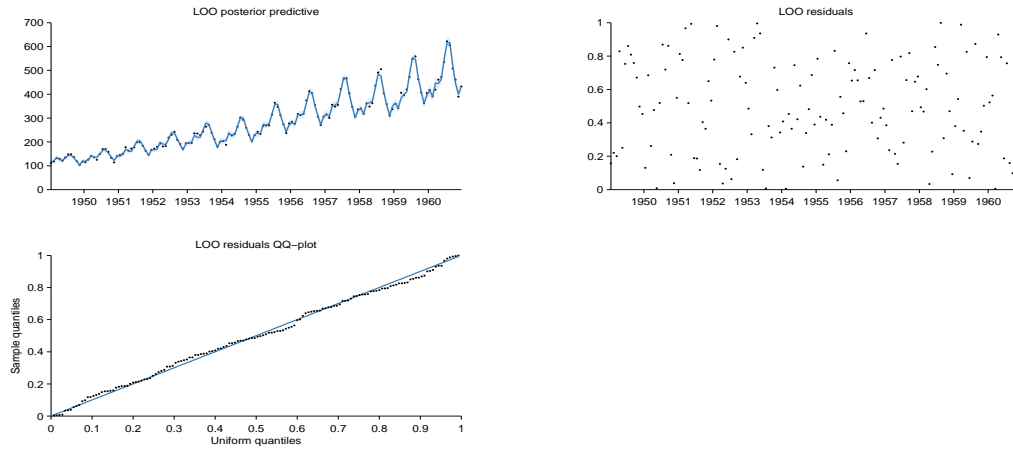


Figure 18: LOO posterior predictive. Distribution (left), standardised residuals (right) and qq-plot (below)

### A.2 Leave chunk out

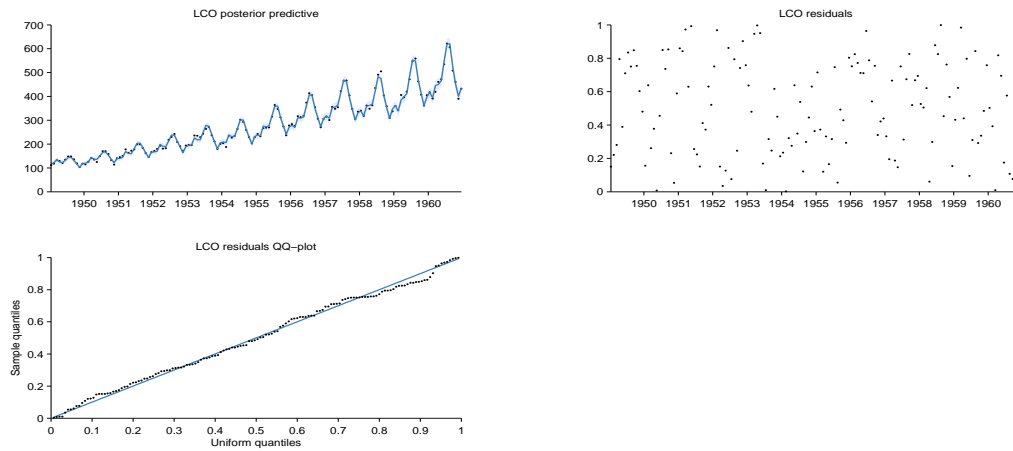


Figure 19: LCO posterior predictive. Distribution (left), standardised residuals (right) and qq-plot (below)

### A.3 Next data point

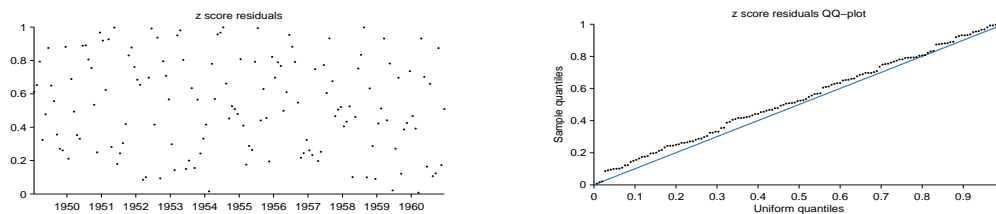


Figure 20: Inverse Cholesky thing. Standardised values (left) and qq-plot (right)



## **B Glossary of terms**

- *lengthscale* - A description of what a lengthscale is