

JOHN CURTIN  
*College of the Arts*  
COLLEGE OF THE ARTS

Semester One Examination, 2018

Question/Answer booklet

**MATHEMATICS  
SPECIALIST  
UNIT 1**

**Section One:  
Calculator-free**

Student number: In figures

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In words

Your name

**Time allowed for this section**

Reading time before commencing work: five minutes

Working time: fifty minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

**Important note to candidates**

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

**Structure of this paper**

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	97	65
<b>Total</b>					100

**Instructions to candidates**

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet.
3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.
4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
6. It is recommended that you do not use pencil, except in diagrams.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

**Section One: Calculator-free****35% (53 Marks)**

This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

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**Question 1****(5 marks)**

Relative to the origin  $O$ , points  $A$  and  $B$  have position vectors  $\mathbf{i} - 4\mathbf{j}$  and  $-5\mathbf{i} - \mathbf{j}$  respectively.

(a) Determine the unit vector  $\hat{\mathbf{c}}$ , where  $\mathbf{c} = \overrightarrow{BA}$ . (3 marks)

(b) Vector  $\mathbf{d}$  has magnitude  $2\sqrt{5}$ , is parallel to  $\mathbf{c}$  and in the opposite direction. Determine  $\mathbf{d}$ . (2 marks)

**Question 2****(5 marks)**

Let the displacement vectors  $\mathbf{a}$ ,  $\mathbf{b}$  and  $\mathbf{c}$  be  $(14, -5)$ ,  $(10, 11)$  and  $(4, n)$  respectively, where  $n$  is a constant.

(a) Determine the vector  $2\mathbf{a} + 4\mathbf{b}$ . (2 marks)

(b) Given that  $|\mathbf{a} + \mathbf{b} + m\mathbf{c}| = 0$ , determine the values of  $m$  and  $n$ . (3 marks)

## Question 3

(6 marks)

- (a) Determine the value of the constant  $n$ , given that the vectors  $5\mathbf{i} + 3\mathbf{j}$  and  $-8\mathbf{i} + n\mathbf{j}$  are perpendicular. (2 marks)

- (b) The vectors  $\mathbf{a}$  and  $\mathbf{b}$  are such that  $|\mathbf{a}| = 17$ ,  $|\mathbf{b}| = 13$  and  $\mathbf{a} \cdot \mathbf{b} = -21$ . Evaluate

(i)  $-2\mathbf{a} \cdot 5\mathbf{b}$ . (1 mark)

(ii)  $(\mathbf{b} + \mathbf{a}) \cdot (\mathbf{a} - \mathbf{b})$ . (3 marks)

**Question 4****(8 marks)**

Consider the following statement about a simple (no edges that cross) polygon:

*If it has an interior angle sum of  $360^\circ$ , then it is a square.*

(a) Use a counter-example to explain why the statement is false. (2 marks)

(b) Write the converse statement and state whether it is always, sometimes or never true. (2 marks)

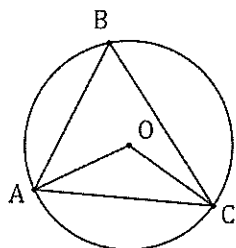
(c) Write the inverse statement and state whether it is always, sometimes or never true. (2 marks)

(d) Write the contrapositive statement and state whether it is always, sometimes or never true. (2 marks)

## Question 5

(7 marks)

- (a) In the diagram below, the vertices of triangle  $ABC$  lie on a circle with centre  $O$ . Given that  $\angle ABC = 47^\circ$ , determine the values of  $\angle AOC$  and  $\angle OAC$ . (2 marks)



- (b) Prove, assuming only basic axioms and properties of triangles, that the size of the angle subtended by an arc at the centre of a circle is twice the size of the angle subtended by the same arc at any point on the circumference. (5 marks)

**Question 6****(6 marks)**

A drone leaves point  $P$  and travels 74 m on bearing of  $290^\circ$  to  $Q$ , then 40 m on bearing  $020^\circ$  to  $R$  and finally 114 m on bearing  $110^\circ$  to  $S$ .

- (a) Sketch a neat diagram to show the path of the drone. (2 marks)

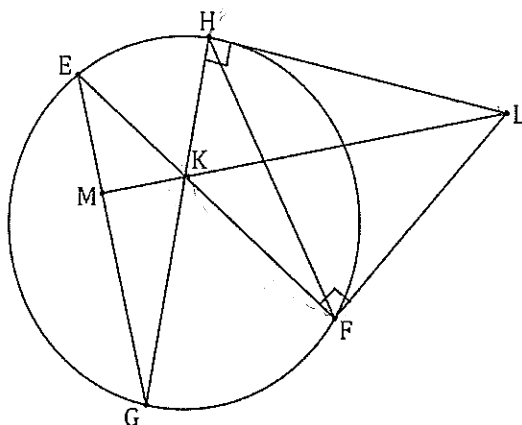
- (b) The drone is to return directly from  $S$  to  $P$ . Determine the distance it must fly and on what bearing. (4 marks)



Question 7

(7 marks)

In the diagram below, two chords of a circle,  $EF$  and  $GH$ , intersect at  $K$ .  $LF$  is perpendicular to  $EF$  at  $F$  and  $LH$  is perpendicular to  $GH$  at  $H$ . The line  $LK$  intersects chord  $EG$  at  $M$ .



(a) Explain why  $KHLF$  is a cyclic quadrilateral. (1 mark)

(b) Prove that  $\angle HLK = \angle EGH$ . (3 marks)

(c) Prove that  $LM$  is perpendicular to  $EG$ . (3 marks)

## Question 8

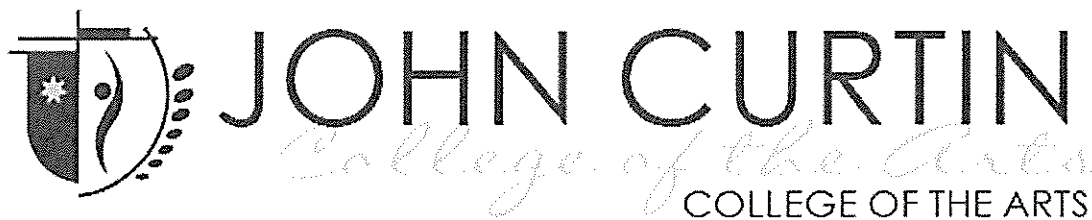
(9 marks)

(a) Evaluate  ${}^{21}P_{14} \div {}^{19}P_{15}$ .

(3 marks)

(b) Express  $7! + 6! + 5!$  in the form  $a^2b!$ , where  $a$  and  $b$  are positive integers. (3 marks)

(c) Show that for  $n \in \mathbb{Z}, n \geq 2$ , the sum  $(n+1)! + n! + (n-1)!$  can always be expressed in the form  $a^2b!$  where  $a$  and  $b$  are positive integers. (3 marks)



**Semester One Examination, 2018**

**Question/Answer booklet**

**MATHEMATICS  
SPECIALIST  
UNIT 1**

**Section Two:**

**Calculator-assumed**

Student number: In figures

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In words

Your name

**Time allowed for this section**

Reading time before commencing work: ten minutes  
Working time: one hundred minutes

**Materials required/recommended for this section**

***To be provided by the supervisor***

This Question/Answer booklet  
Formula sheet (retained from Section One)

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

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**Section Two: Calculator-assumed****65% (97 Marks)**

This section has **thirteen (13)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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**Question 9****(7 marks)**

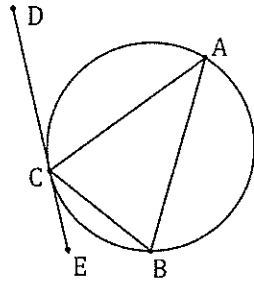
- (a) A body travels with a velocity  $12\mathbf{i} - 5\mathbf{j} \text{ ms}^{-1}$ . Determine its speed and the bearing on which it is moving, assuming the positive  $y$ -axis to be due north. (3 marks)

- (b) Given that  $\lambda(5\mathbf{i} - 2\mathbf{j}) + \mu(-7\mathbf{i} + 4\mathbf{j}) = 25\mathbf{i} - 13\mathbf{j}$ , determine the values of  $\lambda$  and  $\mu$ . (4 marks)

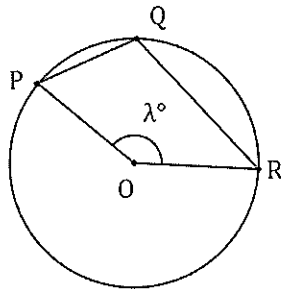
## Question 10

(7 marks)

- (a) In the diagram below, points  $A$  and  $B$  lie on a circle,  $DE$  is a tangent to the circle at  $C$ ,  $\angle DCA = 79^\circ$  and  $\angle CAB = 32^\circ$ . Determine the sizes of  $\angle ABC$ ,  $\angle BCE$  and  $\angle BCA$ . (3 marks)



- (b) In the next diagram,  $P, Q$  and  $R$  lie on a circle with centre  $O$  and  $\angle POR = \lambda^\circ$ . Determine, with reasons, the size of  $\angle PQR$  in terms of  $\lambda$ . (4 marks)



**Question 11****(6 marks)**

The largest Australian family recently met with the largest English family. Between them, these two families had 37 children.

- (a) Three of the children were chosen at random to feature in a TV documentary about the two families. Determine the number of different selections of three children that were possible. (1 mark)

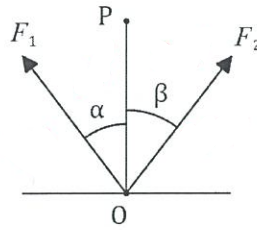
- (b) Prove that at least four of the children were born in the same month of the year. (2 marks)

There were more children in the English family than the Australian family and the English children all had blue, brown, hazel or grey coloured eyes.

- (c) Show that at least five English children had the same eye colour. (3 marks)

**Question 12****(8 marks)**

Two forces,  $F_1 = 415 \text{ N}$  and  $F_2 = 355 \text{ N}$ , act on a body at  $O$ , and make angles of  $\alpha = 35^\circ$ , and  $\beta = 24^\circ$  respectively with the vertical  $OP$ , as shown in the diagram below.



- (a) Determine the magnitude of the resultant force and the angle it makes with the vertical. (5 marks)

- (b) The size of angle  $\alpha$  is to be adjusted so that the direction of the resultant is vertical. Determine the required value of  $\alpha$ , given  $0 \leq \alpha \leq 90^\circ$ . (3 marks)



## Question 13

(8 marks)

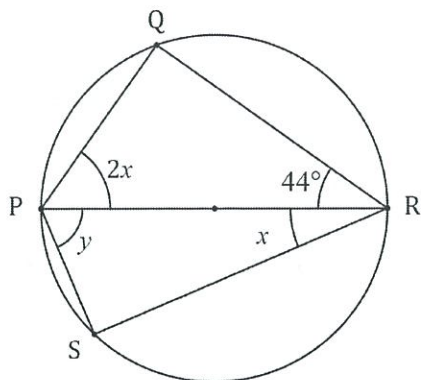
- (a) Simplify  $(4\mathbf{a} - 2\mathbf{b}) \cdot (\mathbf{a} - 3\mathbf{b})$  given that  $|\mathbf{a}| = 5$ ,  $|\mathbf{b}| = 3$  and vector  $\mathbf{a}$  is parallel and in the opposite direction to vector  $\mathbf{b}$ . (4 marks)

- (b) Using  $\overrightarrow{OA} = \mathbf{a}$  and  $\overrightarrow{OC} = \mathbf{c}$ , demonstrate a vector method to show that if the diagonals  $\overrightarrow{OB}$  and  $\overrightarrow{AC}$  of parallelogram  $OABC$  are perpendicular, then the parallelogram is a rhombus. (4 marks)

## Question 14

(8 marks)

- (a) Determine the size of angles  $x$  and  $y$  in the diagram below, where  $Q$  and  $S$  lie on the circumference of the circle with diameter  $PR$ . (3 marks)



- (b) Triangle  $ABC$  has sides of length  $AB = 4$  cm,  $BC = 8$  cm and  $AC = 7$  cm. Prove, using the method of contradiction, that if  $BC$  is a diameter of a circle then  $A$  does not lie on the circumference of the circle. (5 marks)

**Question 15****(9 marks)**

- (a) Determine the number of integers between 1 and 500 that are divisible by 6 or 7. (4 marks)

- (b) A pigeon fancier has 3 Fantail, 5 Carrier, 6 Archangel and 8 Dragoon pigeons and must choose four of them to enter in a local show. Determine the number of different ways the four pigeons can be chosen if

- (i) there are no restrictions. (1 mark)

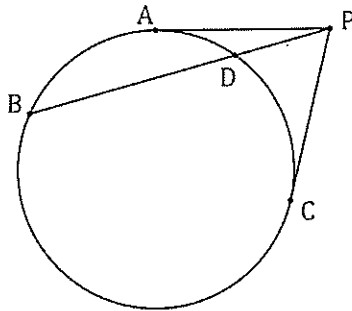
- (ii) the fancier decides to take one of each breed. (2 marks)

- (iii) the fancier decides to take at least three Carrier pigeons. (2 marks)

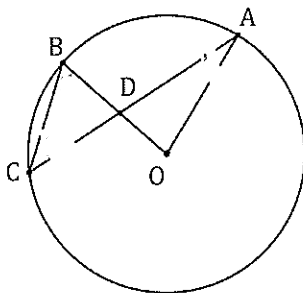
## Question 16

(9 marks)

- (a) In the diagram below,  $PA$  and  $PC$  are tangents to the circle, with  $PA = 34$  cm. Secant  $PB$  cuts the circle at  $D$ , so that  $PD = 16$  cm. Determine the lengths of  $PC$  and  $BD$ . (4 marks)



- (b) In the diagram below,  $A$ ,  $B$  and  $C$  lie on the circumference of the circle with centre  $O$ , with  $AC$  intersecting  $OB$  at  $D$ . Prove that  $\angle DBC = \angle DAO + \angle DCB$ . (5 marks)



**Question 17****(6 marks)**

Three vectors are  $\mathbf{u} = a\mathbf{i} + b\mathbf{j}$ ,  $\mathbf{v} = \mathbf{i} + 5\mathbf{j}$  and  $\mathbf{w} = 2\mathbf{i} - 3\mathbf{j}$ .

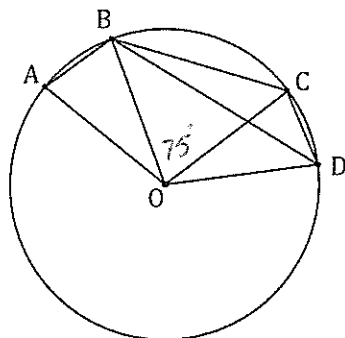
- (a) Determine the vector projection of  $\mathbf{w}$  on  $\mathbf{v}$  in exact form. (2 marks)

- (b) If  $\mathbf{u}$  is perpendicular to  $\mathbf{v}$  and has the same magnitude as  $\mathbf{w}$ , determine the exact values of the coefficients  $a$  and  $b$ . (4 marks)

## Question 18

(7 marks)

- (a) In the diagram below, points  $B$  and  $C$  lie on the minor arc  $AD$  of the circle with centre  $O$ . The lengths of chords  $AB$  and  $CD$  are congruent,  $\angle BOC = 75^\circ$  and  $\angle AOD = 151^\circ$ . Determine the size of  $\angle CBD$ . (3 marks)



- (b) Line segment  $AC$  intersects line segment  $BD$  at  $N$ . Given that  $AC$  and  $BD$  are non-parallel and the lengths  $AN, AC, BN$  and  $BD$  are 33, 43, 21 and 37 cm respectively, explain whether the points  $A, B, C$  and  $D$  are concyclic. (4 marks)

**Question 19****(7 marks)**

A small boat leaves jetty *A* to travel to jetty *B*, 285 m away on a bearing of  $320^\circ$ . A steady current of  $2.2 \text{ ms}^{-1}$  runs in the river between the jetties on a bearing  $080^\circ$ . If the small boat travels at a constant speed of  $5.8 \text{ ms}^{-1}$ , determine the bearing it should steer to reach jetty *B* and how long the journey will take.

## Question 20

(8 marks)

- (a) Triangle  $ABC$  has vertices with position vectors  $A(-4, 3)$ ,  $B(11, 13)$  and  $C(15, -5)$ . Point  $P$  lies on side  $AB$  so that  $\overrightarrow{AP} = 4\overrightarrow{PB}$ . Determine the vector  $\overrightarrow{CP}$ . (4 marks)

- (b)  $OPQR$  is a parallelogram. Point  $M$  is the midpoint of side  $QR$  and point  $N$  is on side  $PQ$  so that  $PN = \frac{3}{4}PQ$ . If  $\overrightarrow{OP} = \mathbf{p}$  and  $\overrightarrow{OR} = \mathbf{r}$ , determine  $\overrightarrow{NM}$  in terms of  $\mathbf{p}$  and  $\mathbf{r}$ . (4 marks)



**Question 21****(7 marks)**

A child is playing with thirteen coloured cubes, all the same size. There are six pink cubes, three navy and one each of red, blue, orange and green.

- (a) If the child stacks cubes one on top of another to make a column, determine the number of different coloured columns that can be made using
- (i) all the red, blue and green cubes. (1 mark)
  - (ii) all the pink, red and orange cubes. (2 marks)
  - (iii) all the cubes. (2 marks)
- (b) If all but one of the cubes are used to make a column, determine the number of different coloured columns that can now be made. Justify your answer. (2 marks)