



## YEAR 11 MATHEMATICS SPECIALIST

### TEST 5, 2018 (Matrices)

#### Section One: Calculator Free

Student's Name: \_\_\_\_\_

Total Marks: 15  
Time Allowed: 15 mins

#### MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST

*Standard Items:* Pens, pencils, eraser, ruler

*Special Items:* WACE Formula Sheet

#### INSTRUCTIONS TO STUDENTS

Do not open this paper until instructed to do so.

You are required to answer ALL questions.

Write answers in the spaces provided beneath each question.

Marks are shown with the questions.

**Show all working** clearly, in sufficient detail to allow your answers to be checked and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

It is recommended that students **do not use pencil**, except in diagrams.

**Question 1. [4, 1, 1 = 6 marks]**

Let  $\mathbf{A} = \begin{bmatrix} m & -3 \\ 4 & 7 \end{bmatrix}$ ,  $\mathbf{B} = \begin{bmatrix} 4 \\ 2 \end{bmatrix}$ ,  $\mathbf{C} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$  and  $\mathbf{D} = \begin{bmatrix} -3 & 0 \\ 0 & 1 \end{bmatrix}$

- (a) Evaluate each of the following where possible. If not possible, state this clearly.
- (i)  $2\mathbf{A} - \mathbf{D}$  (ii)  $\mathbf{CD}$

(iii)  $\mathbf{BD}$

(iv)  $|\mathbf{D}|$

- (b) What value(s) of  $m$  make the matrix  $\mathbf{A}$  singular?

- (c) If matrix  $\mathbf{E}$  is  $\begin{bmatrix} 2 & 3 \\ 4 & -7 \end{bmatrix}$  and  $\mathbf{AC} = \mathbf{E}$ , then determine the value of  $m$ .

Q2. [5 marks]

Find Matrix **A** if  $\mathbf{AB} + 2\mathbf{B} = 4\mathbf{I}$ , where **I** is the identity matrix and  $\mathbf{B} = \begin{bmatrix} 2 & -1 \\ -2 & 3 \end{bmatrix}$ .

Question 3 [1, 1, 1, 1 = 4 marks]

Let  $\mathbf{A} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$ . Find  $\mathbf{A}^2$ ,  $\mathbf{A}^3$ , and  $\mathbf{A}^4$ . Hence write down a formula for  $\mathbf{A}^n$ .



## YEAR 11 MATHEMATICS SPECIALIST

TEST 5, 2018  
(Matrices)

### Section Two: Calculator Assumed

Student's Name: \_\_\_\_\_

Total Marks: 25  
Time Allowed: 25 mins

### MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST

*Standard Items:* Pens, pencils, eraser, ruler

*Special Items:* Up to three approved calculators  
One page (unfolded A4 sheet) front and back of Notes  
WACE Formula Sheet

### INSTRUCTIONS TO STUDENTS

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Write answers in the spaces provided beneath each question.  
Marks are shown with the questions.

**Show all working** clearly, in sufficient detail to allow your answers to be checked and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

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**Question 1 [1, 1, 2 = 4 marks]**

Consider the system of equations

$$2x - 3y = 3$$

$$4x + y = 5$$

(a) Write this system of equations in matrix form, as  $AX = K$

(b) Re-write the above matrix equation in the form  $X = \dots$

(c) Hence solve the system of equations using your CAS.

**Question 2** [2, 2, 2, 2, 2, 3 = 15 marks]

Let  $A = \begin{bmatrix} 3 & 0 \\ 0 & 2 \end{bmatrix}$  and  $B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

- a) Describe the transformations represented matrix A
- b) Describe the transformations represented matrix B
- c) Object  $PQRS$  is transformed by matrix A to  $P'Q'R'S'$  and then by matrix B to  $P''Q''R''S''$ . Determine the single matrix that will transform  $PQRS$  directly to  $P''Q''R''S''$ .
- d) The **point**  $P'$  is  $(4, -3)$  determine the coordinates of  $P$ .

- e) Triangle ABC with coordinates A(1,4), B(5,4) and C(5,1) is transformed by matrix A. Determine the area of the image of triangle ABC after the transformation.

- f) Matrix C represents a rotation of  $\frac{\pi}{6}$  clockwise about the origin. Determine Matrix C.

- g) Matrix D represents a reflection in the line  $y = \frac{x}{\sqrt{3}}$ . Determine Matrix D.

Question 3 [4, 2 = 6 marks]

Suppose that **A** and **B** are  $2 \times 2$  matrices with  $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ , and  $B = \begin{bmatrix} e & f \\ g & h \end{bmatrix}$ .

(a) Prove that  $\det(\mathbf{AB}) = \det(\mathbf{A}) \det(\mathbf{B})$

(b) Hence prove that if both **A** and **B** are invertible, then **AB** is invertible.