**MATHEMATICS METHODS**

**MAWA Semester 2 (Unit 3&4) Examination 2019**

**Calculator-assumed**

# Marking Key

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The release date for this exam and marking scheme is

* **the end of week 1 of term 4, Fri October 18th 2019**

**Section Two: Calculator-assumed (103 Marks)**

**Question 10 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| i.e. 5.42% received this invitation. | |
| Mathematical behaviours | Marks |
| * states the correct percentage | 1 |

**Question 10 (b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * States the correct length | 1 |

**Question 10 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| P(B ≥ 7) | |
| Mathematical behaviours | Marks |
| * Determines probability one player can kick longer than 45 m * Associates this question to a Binomial Distribution * Determines the correct probability | 1  1  1 |

**Question 11 (a) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Uing CAS,  Alternatively, sketch graph and proceed without CAS…    The *x-*intercepts are (or 0.707, 1.581)  Let a = , b =  The area for the enclosed region...  = + ( ) +  =  = 8·125 units2 | |
| Mathematical behaviours | Marks |
| * states an integral expression for the area with correct limits * states correct area rounded to 3 decimal places   Alternatively,   * shows integration based on three separate areas * uses correct bounds on integration * determines solution, correctly to three decimal places | 2  1  1  1  1 |

**Question 11 (b) (i) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Area of P = 18 units2  Area of Q = 7.33 units2 | |
| Mathematical behaviours | Marks |
| * Area of P * Area of Q | 1  1 |

**Question 11 (b) (ii) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * finds *x* value for intersection of functions  in terms of *a.* * determines equation (in terms of integrals) showing region P is half the area of region Q * anti-differentiates both integrals * solves equation to determine the value of *a* | 1  1  1  1 |

**Question 12(a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| *P*(up to 600 vehicles pass through in one hour) = | |
| Mathematical behaviours | Marks |
| * states probability | 1 |

**Question 12(b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| |  |  |  | | --- | --- | --- | |  | 0 | 1 | |  |  |  | | |
| Mathematical behaviours | Marks |
| * completes P(Y=0) correctly * completes P(Y=1) correctly | 1  1 |

**Question 12(c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Bernoulli, | |
| Mathematical behaviours | Marks |
| * identifies the distribution as ‘Bernoulli’ * states the variance | 1  1 |

**Question 12(d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| Let  be the number of times that Mel faces congestion in one week | |
| Mathematical behaviours | Marks |
| * indicates a binomial distribution * states both parameters correctly * determines probability | 1  1  1 |

**Question 12(e) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| P(congestion occurs once in first 3 days)  P(congestion occurs on Thursday) | |
| Mathematical behaviours | Marks |
| * states expression showing that congestion occurs exactly once in the first 1st   three days   * identifies that congestion occurs on 4th day * calculates probability | 1  1  1 |

**Question 13(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| - axis intercept, | |
| Mathematical behaviours | Marks |
| * states * states - axis intercept | 1  1 |

**Question 13(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * locates and identifies- axis intercept * labels and shows asymptote * correct shape including containing | 1  1  1 |

**Question 13(c) (3 mark)**

|  |  |
| --- | --- |
| Solution | |
| is a horizontal translation of , 2 units to the right.  If  then  is the root of  translated 2 units to the right. Hence | |
| Mathematical behaviours | Marks |
| * rearrange to determine *g(p)* * solves algebraically for * describes that  represents the axis intercept (root) for the translated   function | 1  1  1 |

**Question 14 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Confidence interval is  So | |
| Mathematical behaviours | Marks |
| * answers correctly | 1 |

**Question 14 (b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Confidence interval is  So | |
| Mathematical behaviours | Marks |
| * answers correctly | 1 |

**Question 14 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| i.e.  Solving for gives and so the sample size was (approximately) | |
| Mathematical behaviours | Marks |
| * uses * solves for and rounds | 1  1 |

**Question 14 (d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| For this interval  and so (\*)  and so  So and hence the confidence level is approximately | |
| Mathematical behaviours | Marks |
| * obtains equation for (\*) * solves for * obtains correct answer | 1  1  1 |

**Question 14 (e) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| The sample provides strong evidence that a majority opposes the plan, but it is hardly compelling because the 95% confidence interval extends into the region | |
| Mathematical behaviours | Marks |
| * gives a sensible answer * provides a good reason | 1  1 |

**Question 15(a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Hence distilled water is neutral | |
| Mathematical behaviours | Marks |
| * demonstrates use of law, * evaluates  and draws conclusion | 1  1 |

**Question 15(b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| Hence the concentration of hydrogen ions is  moles per litre. | |
| Mathematical behaviours | Marks |
| * states solution with unit | 1 |

**Question 15(c) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states correct expression | 1 |

**Question 15(d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| From part (c)    ie The number of hydrogen ions in Black Coffee is  times the number of hydrogen ions in Lemon Juice. | |
| Mathematical behaviours | Marks |
| * substitutes into formula * rewrites logarithmic equation as an exponential and states ratio. | 1  1 |

**Question 16(a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines first derivative * equates  and solves, rejecting * evaluates  coordinate and justifies minimum * equates 2nd derivative to 0 and locates point of inflection | 1  1  1  1 |

**Question 16(b) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * function undefined for * point of inflection, minimum and  axis intercept identified (labelled) * correct shape | 1  1  1 |

**Question 17 (a) (3 marks)**

|  |  |
| --- | --- |
|  | |
| Area of rectangle is ()  Area of triangle half base height (  So total area is | |
| Mathematical behaviours | Marks |
| * gives correct area of rectangle * gives correct area of triangle and sums to give total area | 1  1+1 |

**Question 17 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| So  and so (\*) | |
| Mathematical behaviours | Marks |
| * correct expression for perimeter * correct (one-variable) expression for area (\*) | 1  1 |

**Question 17 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| So and  when  Since  has a maximum when  So  So the maximum total area is () | |
| Mathematical behaviours | Marks |
| * draws a sketch of *A(y*) as found in part (b) – or states that uses a calculator sketch of *A(y*) * indicates the maximum area as the *y-*value of the TP * provides this value correctly rounded   Alternatively,   * differentiates correctly * obtains critical point * obtains correct answer to the required level of accuracy | 1  1  1  1  1  1 |

**Question 18 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| (\*)  from a calculator | |
| Mathematical behaviours | Marks |
| * standardises (\*) * obtains correct answer | 1  1 |

**Question 18 (b)**  **(2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * obtains correct indefinite integral (\*) * evaluates correctly | 1  1 |

**Question 18 (c) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
| from 12(b)  So | |
| Mathematical behaviours | Marks |
| * correct answer | 1 |

**Question 18 (d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| For the Bernoulli random variable and  So (\*) | |
| Mathematical behaviours | Marks |
| * uses correct values for and * obtains (\*) * obtains correct answer | 1  1  1 |

**Question 19 (a) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines the proportion | 1 |

**Question 19 (b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines the standard deviation | 1 |

**Question 19 (c) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Determine the relevant z | |
| Mathematical behaviours | Marks |
| * determines the z value for 85% confidence level * calculates the margin of error | 1  1 |

**Question 19 (d) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| Graph approaches the shape of a binomial distribution.  For large sample sizes it begins to approach the shape of a normal distribution  The distribution is centred on 0.3 | |
| Mathematical behaviours | Marks |
| * uses one of the descriptors above. * uses another one of the descriptors above. | 1  1 |

**Question 20 (a) (5 marks)**

|  |  |
| --- | --- |
| Solution | |
| Since when  the first zero (after occurs when  So and so  Since (\*)  and so and  when  So  i.e. | |
| Mathematical behaviours | Marks |
| * obtains correct value for * obtains correct formula for (\*) * obtains correct value for * obtains correct value for * rounds *a*, *b* and *c* to 3 decimal places | 1  1  1  1  1 |

**Question 20 (b) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * shows decaying oscillatory nature * shows maximum at | 1  1 |

**Question 20 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
| In the first 15 seconds the mass travels i.e. cm. (\*)  The second turning point occurs when (\*\*)  So in the second 15 second period the mass travels So the total distance travelled is  Alternatively, could use CAS to find absolute value of the velocity function. | |
| Mathematical behaviours | Marks |
| * obtains correct distance for first seconds (\*) * obtains second turning point (\*\*) * obtains correct answer   If uses CAS –   * states the function to be integrated with correct limits * states correct appropriately rounded answer. | 1  1  1  2  1 |

**Question 21 (a) (4 marks)**

|  |  |
| --- | --- |
| Solution | |
| 2019 Sample =  Historically p = 0.35  Standard Deviation s =  i.e.  i.e. Difference in terms of standard deviations  Given the difference between the long-term proportion and sample proportion exceeds three standard deviations, it is unlikely that this 2019 prediction is correct. Whilst it could occur, the Principal is correct to say that this is extremely unlikely. | |
| Mathematical behaviours | Marks |
| * determines the sample proportion for 2019. * calculates the standard deviation based on *n* = 225. * calculates the difference between the two proportions and connects this result to the standard deviation. * Recognises that the Principal was justified. | 1  1  1  1 |

**Question 21 (b) (i) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| This method may be biased for the following reasons  - only one car park was chosen (of a possible five) and the "drop off" zones were ignored.  - sample was small.  - the car park sample probably eliminated parents. | |
| Mathematical behaviours | Marks |
| * indicates a valid reason for bias. * indicates a second valid reason for bias. | 1  1 |

**Question 21 (b) (ii) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
| This method may be biased for the following reasons  - only interested members are sampled.  - not all community members may be on emails. eg one parent of two may be on email. | |
| Mathematical behaviours | Marks |
| * indicates a valid reason for bias. * indicates a second valid reason for bias. | 1  1 |

**Question 22 (a) (2 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * states the integral equal to one * determine the value of *a* | 1  1 |

**Question 22 (b) (1 mark)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines the correct probability | 1 |

**Question 22 (c) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines the probability of arriving after 11 am * determines the probability of arriving before 11.45 am and after 11 am * determines the conditional probability | 1  1  1 |

**Question 22 (d) (3 marks)**

|  |  |
| --- | --- |
| Solution | |
|  | |
| Mathematical behaviours | Marks |
| * determines the mean * determines the variance * determines the variance for *2T - 1* | 1  1  1 |