JOHN CURTIN
COLLEGE OF THE ARTS

YEAR 11 MATHEMATICS SPECIALIST

TEST 6, 2018
(Proofs and Complex Numbers)

Section One: Calculator Free

Student's Name: _____

Total Marks: 45

Time Allowed: 45 mins

MATERIAL REQUIRED/RECOMMENDED FOR THIS TEST

Standard Items: Pens, pencils, eraser, ruler

Special Items: WACE Formula Sheet

INSTRUCTIONS TO STUDENTS

Do not open this paper until instructed to do so.
You are required to answer ALL questions.
Write answers in the spaces provided beneath each question.
Marks are shown with the questions.

Show all working clearly, in sufficient detail to allow your answers to be checked and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks.

It is recommended that students **do not use pencil**, except in diagrams.

1. [6 marks]

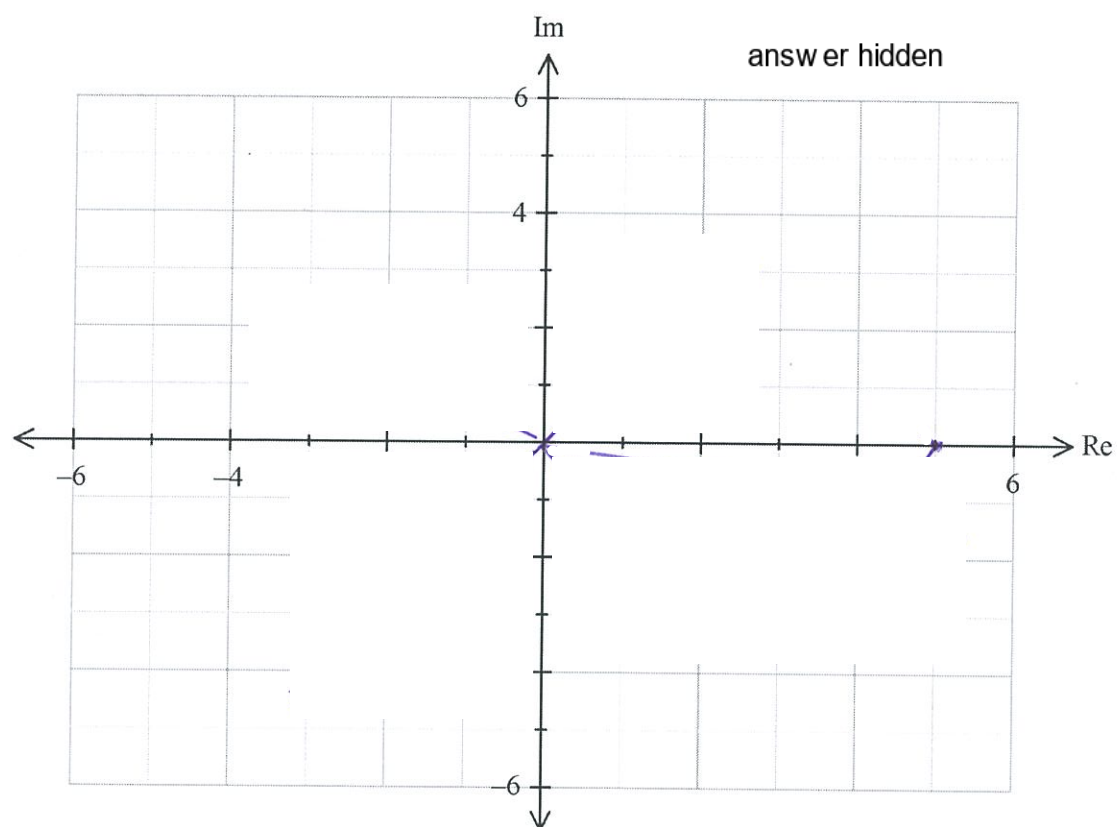Plot the following complex numbers on the Argand diagram.

a) $z_1 = 2 + 3i$

b) $z_2 = -3 - 4i$

c) $z_3 = \overline{z_1}$

d) $z_4 = z_1 i$

e) $z_5 = |z_2|$

f) $z_6 = z_1 - \overline{z_2}$

answer hidden

2. [3, 5 = 8 marks]

a) Prove that the square of an odd number add 11 is a multiple of 4.

b) Prove that the product of 3 consecutive even numbers is divisible by 24.

3. [8 marks]

Given that $z = 2 + 3i$ and $w = 4 - i$ determine:

a) zw

b) $\bar{z} \times \bar{w}$

c) $\dfrac{w}{z}$

d) $Re(z) + Im(w)$

4. [7 marks]

Prove by mathematical induction that, for $n \in \mathbb{Z}^+$,

$$1 + 2\left(\frac{1}{2}\right) + 3\left(\frac{1}{2}\right)^2 + 4\left(\frac{1}{2}\right)^3 + \ldots + n\left(\frac{1}{2}\right)^{n-1} = 4 - \frac{n+2}{2^{n-1}}\,.$$

5. [2, 4 = 6 marks]

Consider the complex numbers $z = 1 + 2i$ and $w = 2 + ai$, where $a \in \mathbb{R}$. Find a when:

(a) $|w| = 2|z|$;

(b)   Re $(zw) = 2$ Im(zw).

6. [5 marks]

Use the method of proof by contradiction to prove that $\sqrt{6}$ is irrational.