



Christ Church Grammar School

Semester Two Examination, 2021 Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

Section One: Calculator-free

WA student number: In figures

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In words

Your name

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Number of additional
answer booklets used
(if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	13	13	100	98	65
Total					100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.
2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
5. It is recommended that you do not use pencil, except in diagrams.
6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free**35% (52 Marks)**

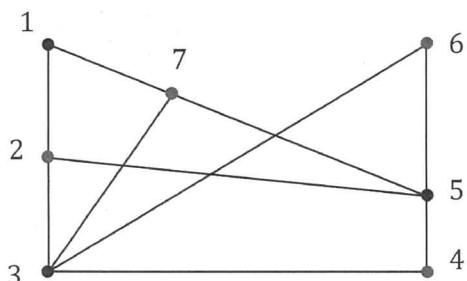
This section has **eight** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1**(6 marks)**

- (a) A connected planar graph has 17 edges and 12 faces. Determine the number of vertices this graph has. (2 marks)

- (b) The vertices in the following graph can be split into two distinct groups to demonstrate that the graph is bipartite. List the vertices in each group. (2 marks)



- (c) Determine the number of edges that must be added to a tree with 4 vertices so that it becomes a complete graph with 4 vertices. (2 marks)

Question 2**(5 marks)**

A sequence is defined as $T_{n+1} = T_n + a$ with $T_1 = b$, so that $T_4 = 40$ and $T_5 = 43$.

- (a) Determine the value of the constant a and the value of the constant b . (2 marks)

The sequence can also be written in the form $T_n = an + k$.

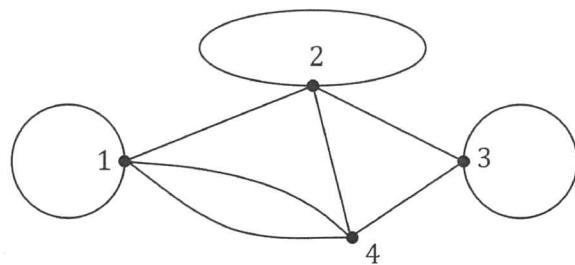
- (b) Determine the value of the constant k . (1 mark)

- (c) Determine the value of n so that $T_n = 250$. (2 marks)
-

Question 3

(7 marks)

A company runs sightseeing boat trips from several terminals throughout a harbour city. In the graph below, the numbered vertices represent terminals, and the edges represent trips either between the terminals or that start and finish at the same terminal.



- (a) State two reasons why the graph above is not simple. (2 marks)

- (b) Construct an adjacency matrix from the graph. (2 marks)

- (c) Describe two conditions necessary for the existence of a semi-Eulerian trail in a graph and state the length of such a trail in the above graph. (3 marks)

Question 4**(10 marks)**

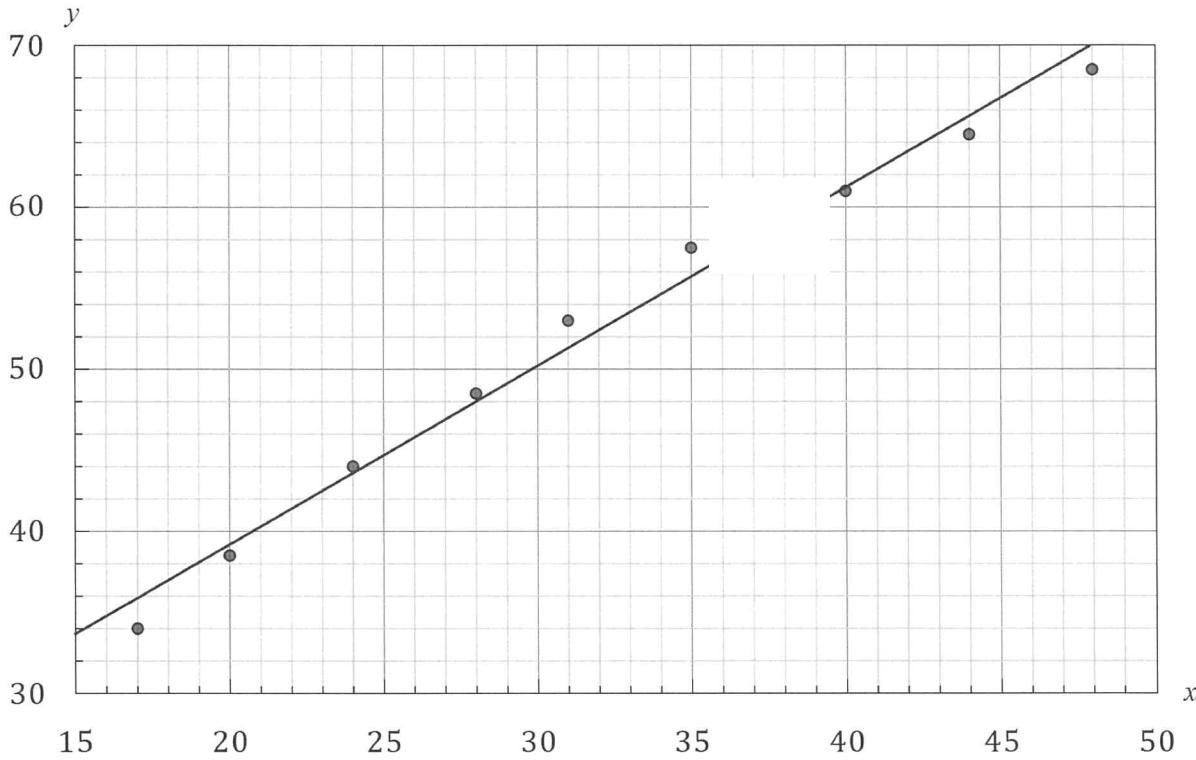
An analyst collected data from a sample of ten trees in a plantation, as shown in the table below.

The variables x and y are the diameter of a tree in centimetres and the daily water use of the tree in litres respectively.

\hat{y} is the predicted daily water use in litres, calculated using the least-squares line $\hat{y} = 1.1x + 17.1$, and R is the residual.

x	17	20	24	28	31	35	37	40	44	48
y	34.0	38.5	44.0	48.5	53.0	57.5	59.0	61.0	64.5	68.5
\hat{y}	35.8	39.1	43.5	47.9	51.2	55.6	57.8	-	65.5	69.9
R	-1.8	-0.6	0.5	0.6	-	1.9	1.2	-	-1.0	-1.4

Nine of the data points (x, y) are shown on this scatterplot:



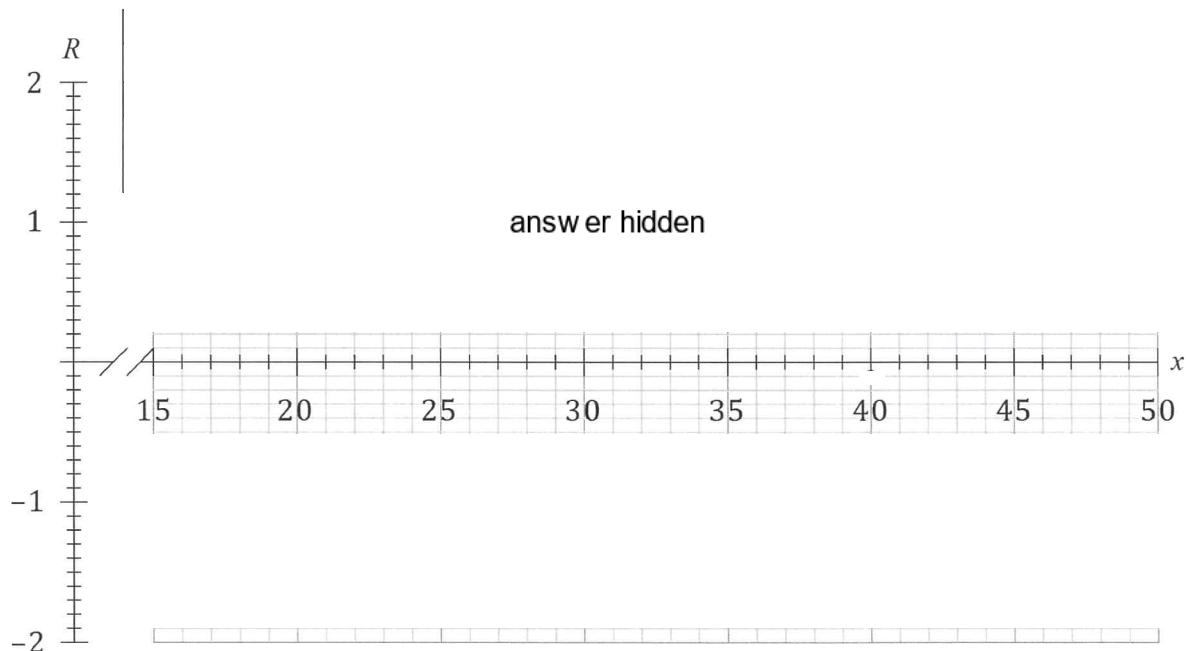
- (a) Add the missing point to the scatterplot. (1 mark)
-
- (b) Explain how information from the table can be used to draw the least-squares line on the scatterplot and hence draw this line. (2 marks)

(c) Determine the value of the residual R when:

(i) $x = 31$. (1 mark)

(ii) $x = 40$. (2 marks)

(d) Construct a residual plot on the axes below. (2 marks)



(e) Comment on the appropriateness of fitting a linear model to the data. Justify your answer. (2 marks)

Question 5

(5 marks)

A relay team consists of four cadets who must each be assigned to one of the four sections of an assault course in order to minimise their overall time.

The table shows the least time, in minutes, that each cadet has previously taken to complete the different sections.

	Section			
	1	2	3	4
Drew	33	26	32	34
Eve	27	26	29	28
Faye	30	27	30	32
Gem	31	30	27	30

- (a) Show use of the Hungarian algorithm to determine the optimum assignment of cadets, writing the assignment in the table below. (4 marks)

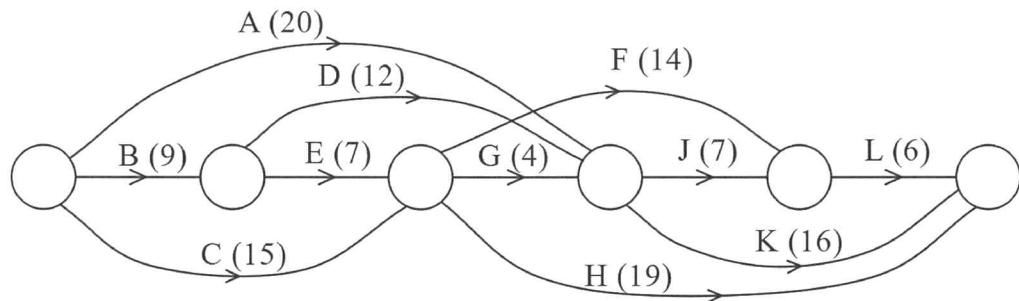
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Section	1	2	3	4
Cadet	-			

- (b) State the least overall time for the team to complete the relay. (1 mark)

Question 6**(6 marks)**

The network below represents the durations and interdependencies of the 11 activities required to complete a project. For example, activity H has a duration of 19 days and cannot commence until activities C and E are complete.



- (a) Determine the minimum completion time for the project. (2 marks)

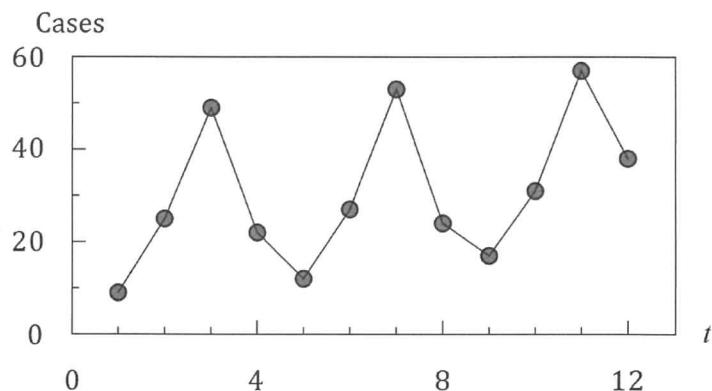
- (b) Determine which of the non-critical activities has the greatest float time and state the earliest start time and latest start time for this activity. (2 marks)

- (c) Proposed changes to the project will decrease the duration of activity D by 4 days. Determine the impact this will have on the critical path of the project and its minimum completion time. (2 marks)

Question 7**(6 marks)**

A clinic recorded the number of cases of influenza that presented each quarter, and an extract from the data is shown in the table and graph below.

Year	Cases per quarter			
	1	2	3	4
2015	9	25	49	22
2016	12	27	53	24
2017	17	31	57	38



- (a) Describe the trend and seasonality of the data.

(2 marks)

- (b) Calculate the 4-point centred moving average for the number of cases that presented in the second quarter of 2017. (2 marks)

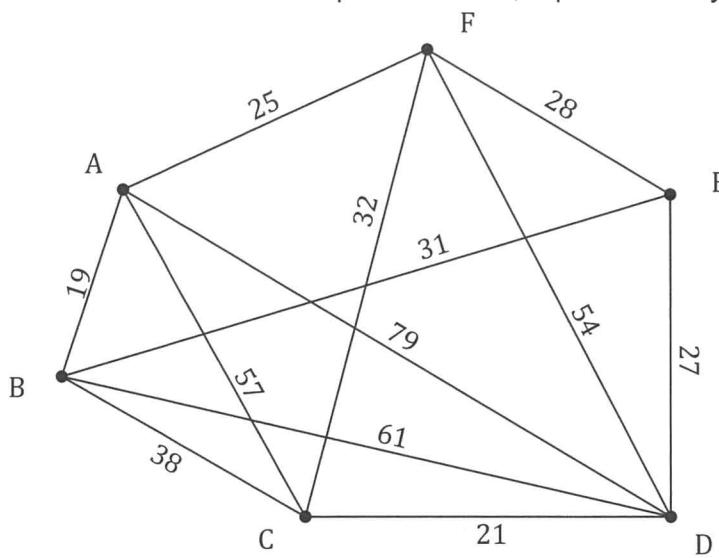
The 3-point moving average for the number of cases that presented in the first quarter of 2015 is 18.

- (c) Determine the number of cases that presented in the fourth quarter of 2014. (2 marks)

Question 8

(7 marks)

The edge weights on the graph below represent the time, in milliseconds, to send a data packet between routers on a computer network, represented by the vertices.



- (a) Determine the minimum time to send a data packet from router A to router D and state, in order, the routers on this path. (3 marks)
- (b) Explain, with justification, why the graph in this question is Hamiltonian. (2 marks)
- (c) State, with reasoning, the least number of edges that must be removed from the graph so that it is no longer Hamiltonian. (2 marks)



Christ Church Grammar School

Semester Two Examination, 2021 Question/Answer booklet

MATHEMATICS APPLICATIONS UNITS 3&4

Section Two: Calculator-assumed

WA student number: In figures

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In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes

Number of additional
answer booklets used
(if applicable):

Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet

Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

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Section Two: Calculator-assumed**65% (98 Marks)**

This section has **thirteen** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9**(5 marks)**

The table below shows the duration and immediate predecessors for all the activities required to complete a project.

Activity	A	B	C	D	E	F
Duration (minutes)	17	39	22	12	14	17
Immediate predecessors	—	A	A	B	C	B, E

- (a) Construct a project network to show all the above information. (3 marks)
-

- (b) Determine the minimum completion time for the project and list, in order, the activities that lie on the critical path. (2 marks)
-

Question 10**(7 marks)**

A builder bought a scissor lift for \$58 000 and for accounting purposes will depreciate its value by 23% annually.

- (a) Show that the value of the lift after one year will be \$44 660. (1 mark)

- (b) Write a recursive rule to calculate the value of the lift, T_n in dollars, after n years. (2 marks)

- (c) Calculate the value of the lift after 7 years. (1 mark)

- (d) Deduce the n^{th} term rule for the value of the lift after n years. (1 mark)

- (e) Determine the least number of years that must pass for the lift to depreciate by more than 95% of its original value. (2 marks)

Question 11**(9 marks)**

The body length and eye diameter of eight tropical fish of the same species are shown in the table below.

Body length, L cm	6.3	11.9	3.8	8.2	9.1	10.5	5.2	7.5
Eye diameter, D mm	9.3	8.6	9.9	9.1	9.0	8.7	9.8	9.4

- (a) Use your calculator to graph the above data and hence describe the direction and strength of the linear association between the variables. (1 mark)
- (b) Determine the coefficient of determination for the linear association and interpret its value in context. (2 marks)
- (c) Determine the equation of the least-squares line to predict D from L and interpret the slope of the line in context. (3 marks)
- (d) Predict the eye diameter of another tropical fish of the same species that has a body length of 5.6 cm and give two reasons that support the validity of this prediction. (3 marks)

Question 12**(8 marks)**

A researcher observed a large number of mice, noting for each one its gender and the paw that it used to react to a stimulus placed in front of it. The numbers in each category are shown in the table below.

Mice	Left-pawed	Right-pawed
Male	39	63
Female	23	60

The researcher is interested in whether there is an association between the variables.

- (a) Name one of the variables in the study and classify it as numerical or categorical.

(2 marks)

- (b) Determine the percentage of left-pawed mice that were male.

(2 marks)

- (c) Use the above data to complete the following table so that it shows column percentages rounded to the nearest whole number.

(2 marks)

Mice (%)	Left-pawed	Right-pawed
Male		
Female		

- (d) Explain whether the percentaged table suggests the presence of an association between the variables.

(2 marks)

Question 13

(7 marks)

A reducing balance loan is defined by the recurrence relation $T_{n+1} = k \times T_n - m$, $T_1 = p$, where T_n is the balance of the loan in dollars at the start of month n . The relation was used to create the following spreadsheet.

Month n	Balance of loan at start of month n	Monthly interest	Monthly repayment	Loan balance carried forward
1	12 500.00	92.50	395.00	12 197.50
2	12 197.50	90.26	395.00	11 892.76
3	11 892.76	88.01	395.00	Q
4		R		S

- (a) Determine the annual percentage interest rate that applies to the loan. (2 marks)

- (b) State the value of each of the constants k , m and p in the recurrence relation. (2 marks)

- (c) Determine the value of Q , the value of R and the value of S shown in the spreadsheet. (3 marks)

Question 14

(9 marks)

The table below shows the number of new clients attracted by a business during the first three weeks of an advertising campaign, together with some derived values to assist in the analysis of the time series data.

Week	Day (n)		New Clients	Weekly mean	Percentage of weekly mean
1	Mon	1	49	A	98.4
	Tue	2	53		106.4
	Wed	3	53		106.4
	Thu	4	49		B
	Fri	5	45		90.4
2	Mon	6	58	60.6	95.7
	Tue	7	66		108.9
	Wed	8	62		102.3
	Thu	9	63		104.0
	Fri	10	54		89.1
3	Mon	11	68	70.2	96.9
	Tue	12	C		106.8
	Wed	13	72		102.6
	Thu	14	71		101.1
	Fri	15	65		92.6

- (a) Calculate the value of A , the value of B and the value of C in the table. (3 marks)

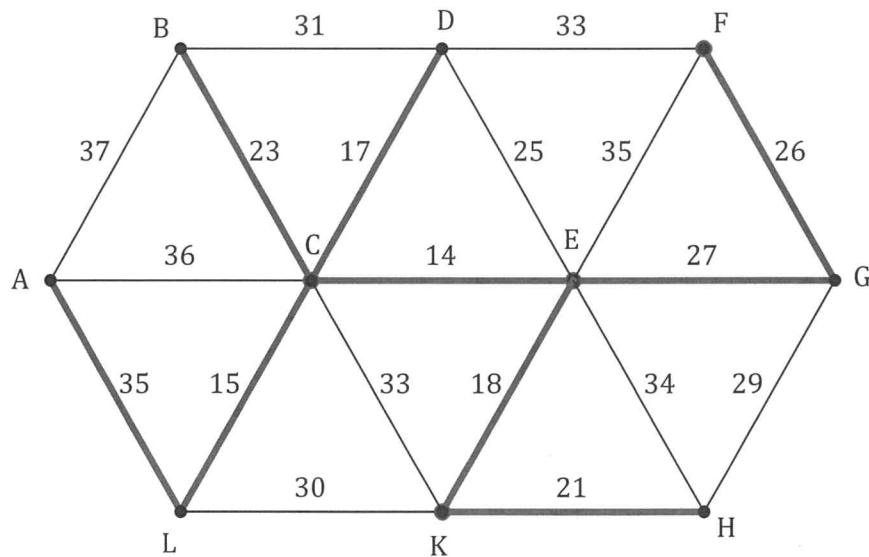
- (b) Determine the seasonal index for Mondays. (2 marks)

- (c) The least-squares line to predict the deseasonalised number of new clients c from the day is $c = 1.83n + 45.6$. Stating any assumptions made, determine the best estimate for the number of new clients expected on Friday of Week 4. (4 marks)

Question 15**(8 marks)**

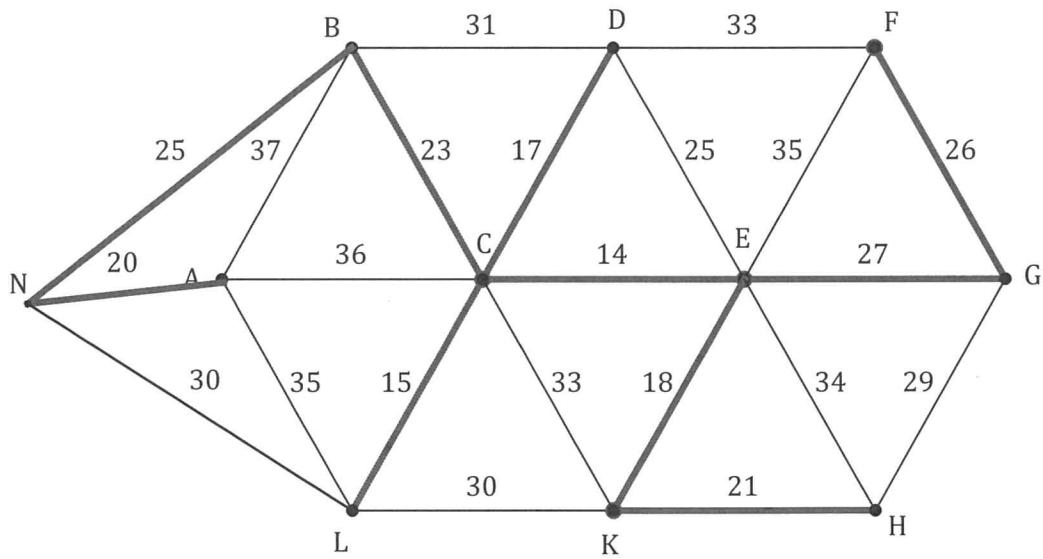
Ten mains-powered smoke alarms must be installed in a building. The edge weights in the graph below represent the length of cable, in metres, required between adjacent alarms.

couldn't hide the answer for this



- (a) Clearly identify the minimum spanning tree on the graph above. _____ (3 marks)
- (b) Determine the cost of installing the cabling between the alarms using the minimum spanning tree, given that each metre of cabling will cost \$21.50. _____ (2 marks)

- (c) Explain how your answer to part (b) will change if smoke alarm N is added to the system with cable lengths of 20, 25 and 30 metres to alarms A , B and L respectively. (A copy of the graph from the previous page is shown below if you wish to use it.) (3 marks)



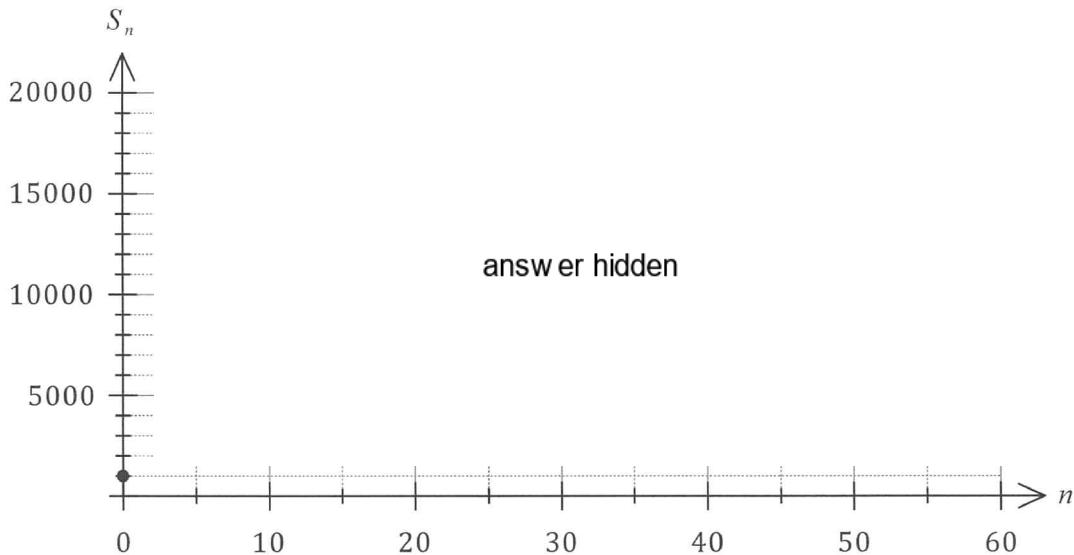
Question 16**(8 marks)**

Saltwater flows steadily into a tank, where it is mixed with existing water. An overflow spout on the tank allows excess water to flow out. The salt concentration in the tank can be modelled by $S_{n+1} = 0.95S_n + 950$, $S_0 = 1000$, where S_n is the concentration, in parts per million, after saltwater has been flowing into the tank for n minutes.

- (a) Determine S_5 . (1 mark)

- (b) Determine the value of n for S_n to first exceed 10 000 ppm. (1 mark)

- (c) Plot points to show the salt concentration at 10 minute intervals on the axes below, after first adding an appropriate scale to the vertical axis. (3 marks)



- (d) Describe the feature of the plotted points that indicates the salt concentration will eventually reach a steady-state. (1 mark)

- (e) Determine, with justification, the steady-state salt concentration. (2 marks)

Question 17**(7 marks)**

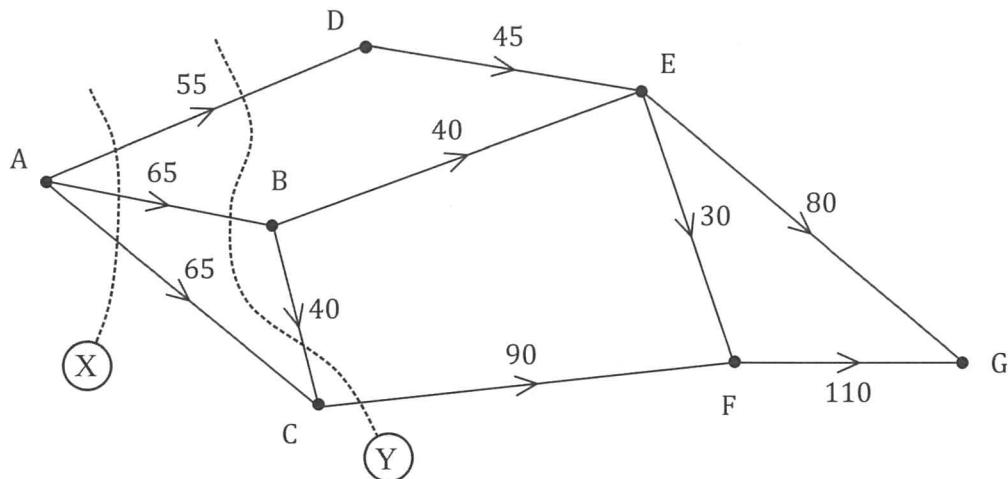
Loans are offered by lender A at a rate of 10.32% per annum compounded quarterly and from lender B at a rate of 10.22% per annum compounded daily.

- (a) Calculate the total amount that must be repaid if \$1500 is borrowed from lender A for nine months. (2 marks)
- (b) Calculate the total interest that will be charged on a loan of \$90 000 from lender B for 30 days. (2 marks)
- (c) Minimising loan interest is the primary goal for a borrower. Calculate the effective interest rate for each lender and hence recommend which should be chosen. (3 marks)

Question 18

(7 marks)

When a city bypass is closed, traffic that would normally use it is forced to flow through main roads in the city. The edge weights on the directed graph below show the maximum number of vehicles per minute that can travel between junctions (represented by vertices) without causing congestion in the city.



- (a) Determine the value of cut X and the value of cut Y . (2 marks)
- (b) Determine the maximum flow of vehicles per hour from A to G . (3 marks)
- (c) City engineers recommend taking steps to improve traffic flow between junctions D and E . Determine, with reasoning, the maximum increase in the hourly flow of vehicles from A to G that their plan could achieve. (2 marks)

Question 19**(8 marks)**

A fund with a balance of \$475 000 is used to create an annuity, from which regular withdrawals of \$9800 are to be made at the end of each quarter. Interest at a rate of 7.2% per annum is added to the fund quarterly, just before each withdrawal.

- (a) Use one or more calculations to show that the balance of the fund after one withdrawal is \$473 750. (2 marks)

- (b) Write a recurrence relation to calculate the balance T_n after the n^{th} withdrawal. (2 marks)

- (c) Determine the total interest earned by the fund during its first year of operation. (2 marks)

- (d) The amount of the regular withdrawal from the fund can be modified so that from the outset, the fund is a perpetuity rather than an annuity. Determine the withdrawal required for this to occur. (2 marks)

Question 20**(7 marks)**

An electronics store advertises a TV for sale at a price of \$5950. Rather than pay this amount in full, the store offers customers a no deposit reducible balance loan with 24 monthly payments of \$281.20. The first repayment is due one month after the customer makes the purchase, just after interest for the month is added to the loan balance.

(a) Determine the total interest paid by customers who buy the TV using the loan. (2 marks)

(b) Determine the annual percentage interest rate that applies to the loan. (2 marks)

A customer decides to buy the TV using a similar reducible balance loan to that offered by the store but financed by their bank at an annual interest rate of 13.98% and over 36 months.

(c) Determine their monthly repayment. (2 marks)

(d) Determine the total interest that the bank will charge on the loan over the 36 months. (1 mark)

Question 21**(8 marks)**

For each quarter in the 12 years from 2006 to 2017, a city library calculated its mean number of users per day and used seasonal indices to deseasonalise the data. A snapshot of the data for the year 2015 is shown in the table below.

Quarter	1	2	3	4
Mean daily users	415.4	465.4	462.6	407.0
Deseasonalised mean daily users	436.5	442.7	431.7	439.8

The trend line for the deseasonalised mean daily users is $\hat{y} = 722 - 8.1t$ where t is the quarter and $t = 1$ corresponds to the first quarter of 2006.

- (a) State, with justification, whether the mean number of users per day was increasing or decreasing over the 12 years. (1 mark)
- (b) Calculate the seasonal indices and enter them in the following table. (2 marks)
- | Quarter | 1 | 2 | 3 | 4 |
|----------------|---|---|---|---|
| Seasonal Index | | | | |
- (c) State, with justification, in which quarter the library was usually busiest during the period that data was collected. (2 marks)
- (d) Determine an estimate, to the nearest whole number, for the mean daily users the library experienced in the first quarter of 2017. (3 marks)