Epione: Lightweight Contact Tracing with Strong Privacy

Ni Trieu* Kareem Shehata† Prateek Saxena‡ Reza Shokri‡ Dawn Song¶

Abstract

Contact tracing is an essential tool in containing infectious diseases such as COVID-19. Many states and research groups have launched or announced mobile phone apps to facilitate contact tracing by recording contacts between users with some privacy considerations. Most of the focus has been on using random tokens, which are exchanged during encounters and stored locally on users' phones. Prior systems allow users to search over released tokens in order to learn if they have recently been in the proximity of a user that has since been diagnosed with the disease. However, prior approaches do not provide end-to-end privacy in the generation, collection, and querying of tokens. In particular, these approaches neither protect against linkage attacks using spatial and temporal information about the tokens, nor prevent false reporting of encounters from users.

In this work, we introduce Epione, a lightweight system for contact tracing with strong privacy protections for users that alerts users directly if any of their contacts have since been diagnosed with the disease, without revealing any more information. Epione protects the privacy of users' contacts from both central services and other users, and provides protection against false reporting. As a key building block, we present a new cryptographic tool for secure two-party private set intersection cardinality (PSI-CA). We specifically tailor it to the case of large-scale contact tracing where clients have small input sets and the server's database of tokens is much larger.

1 Introduction

Contact tracing is an important method to curtail the spread of infectious diseases. The goal of such tracing is to identify individuals which may have come into contact with a person later diagnosed with the disease, so they can be isolated and tested and thus prevent the spread of the disease.

In the ongoing COVID-19 pandemic, recording of proximity data has been facilitated by mobile apps that detect nearby mobile phones using Bluetooth. Several countries have been developing contact tracing apps. Such large-scale collection of personal contact information is a significant concern for privacy [18, 41].

The main purpose of contact tracing applications—recording the fact that two or more individuals were near each other at a certain moment of time—seems to be at odds with their privacy, as the app must keep the information about the individual's personal contacts and should be able to reveal this information (possibly, on demand) to some authorities. Indeed, in a fully untrusted environment one should expect any participant to behave adversarially with the goal to exploit others' personal information. Further, both end users of tracing apps as well as the authorities using the collected data can become victims of security attacks, which can allow powerful adversaries to misuse the information collected by the app.

*UC Bekerley, nitrieu@bekerley.edu

†kareem@shehata.ca

‡National University of Singapore, {prateeks, reza}@comp.nus.edu.sg

¶UC Bekerley and Oasis Labs, dawnsong@bekerley.edu

Multiple ways have been proposed to protect the users' private proximity data, offering different privacy guarantees and coming at different implementation costs. For instance, in the recently released BlueTrace protocol used by the Singapore Government [2], users exchange time-dependent tokens when they are in close contact. The tokens are generated by government-controlled servers that appear random to users, but can be linked by the servers back to users' phone numbers.

When a user (say) Bob is diagnosed with the disease, he shares with the server all the tokens he has recently received, which can help the health authorities to identify Bob's encounters. The authorities can then follow up directly with users who may be at risk of contracting the disease. At the same time, however, this allows authorities to learn all of the social contacts of individuals diagnosed positive.

We consider a model where the government authorities do not store any identifying information (e.g. phone numbers and social contacts) of the users. Such sensitive information databases are a lucrative target for cyber-attackers, and in many countries, collection of contact information conflicts with privacy regulations and public concerns, which may create a barrier to the usage of the tracing service. In our model, the health authorities maintain a database of random tokens corresponding to users which have been diagnosed with the disease. The user's tracing app periodically checks an (untrusted) government-controlled server to check if the user is potentially at risk in a *privacy-preserving* manner. We propose a design that minimizes what users and government entities can infer from the information collected, limiting it to only the desired tracing functionality. This is important, as privacy concerns may hinder adoption in some jurisdictions and contact tracing is expected to be effective only when pariticipation is high (e.g. 60% or more of the population [25]).

Our model can also be contrasted to another baseline design proposed in several decentralized mobile contact tracing systems/protocols [3,4,15,18,41] as well as recently by Google and Apple [1]. These designs make public notifications and generally work as follows. Every time two individuals are in close proximity, a contact event is generated by exchanging tokens between the user's phones, typically via Bluetooth. These tokens are time-varying random numbers for each contact event and associated with an individual for a certain time period. All users locally keep a list of tokens the app has transmitted[1] as well as a separate list of tokens received from nearby phones. If an individual is diagnosed with the disease, they are requested to send the log of all contact tokens they have transmitted to a *public notification list*[2] with the user's consent. The app periodically compares the list of received tokens to the public list, and if there is a match, the user privately learns that they may be at risk. A trusted authority, such as a government authority, can cryptographically sign all tokens of users diagnosed positively to prevent false claims of infection (e.g. from compromised phones) which may trigger false notifications.

This public notification design can potentially address linkage attacks by the server. A user (say Bob) releases a list of his own transmitted tokens upon being diagnosed positive to the public server. Since the other users do not reveal their received tokens, the server cannot link them to other users' tokens. However, this design is susceptible to other avenues of attacks that link adversary's observations of the exchanged tokens with the released tokens on the public server, or that attack the integrity of tokens.

There are three general privacy attacks that existing proposed or launched systems are vulnerable to:

(1) *Linkage attacks by the server*: If a central database is used to collect both sent and received tokens as in [4], or it's possible to infer the source of a sent token as in the case of [7], then the

[1] or the seed of a psuedo-random generator that generated the tokens

[2] tokens received never leave the user's phone

operator of this server is able to deduce all of the social connections of a user that is reported positive for the disease, including when and for how long each contact was made.

(2) *Linkage attacks by users*: Alice can determine which publicly-posted token matches the log on her phone. This could reveal the time, for example, when Alice and the user diagnosed positive with the disease were in close proximity, enabling her to identify Bob. Such identification is undesirable as individuals have been reported to harass individuals suspecting to be source of exposure to the disease [6], leading to the so-called "vigilante" problem.

This linkage can also be used to track a user's movements via Bluetooth beacons. Bluetooth has protections against tracking users over time introduced in Bluetooth 4.0 Low Energy [42]. With solutions such as [1, 15] it is possible to link previously seen Bluetooth device (MAC) addresses despite these protections via exchanged contact tokens once the seeds used to generate the tokens are released[3], producing a similar attack to [11].

(3) *False claims by users*: Users may falsely claim to be diagnosed positive or may claim to be in contact with someone by modifying their app logs. For example, a user who has recovered after being diagnosed positive may threaten to retroactively add a user not currently diagnosed with the disease to his contact log, unless he is paid a ransom. Similarly, a user not diagnosed may falsely claim to be in contact with a user diagnosed positive whose tokens have been revealed, for example, to qualify for a diagnostic test.

1.1 Our Contribution

In this work, we introduce a new system for decentralized contact tracing (Epione) with stronger privacy protections than the strongest models currently found in related work. As a key primitive that enables Epione, we introduce a new private set intersection cardinality or PSI-CA primitive, which is used to check how many tokens of a user (client) match with those on the server. Formally, PSI-CA allows two parties, each holding a private set of tokens, to learn their intersection set size without revealing any additional information. Our PSI-CA primitive is designed to be efficient for a large server-side database and a small client-side database, as is the case for contact tracing applications. Our new PSI-CA constructions allow us to meet all our privacy goals. With several other optimizations in our system design, we show that PSI-CA can make privacy-preserving contact tracing practical.

In summary, we make the following contributions:

- We design Epione, an efficient high-performance contact tracing system that provides strong privacy guarantees, specifically that user contact information is not revealed in any meaningful way to any party, and that diagnosis status is revealed only to health authorities. The system prevents all important attacks (e.g. linkage attack, false-positive attack) which exist in current models underpinning related work.

- We propose a new semi-honest private set intersection cardinality (PSI-CA) for asymmetric set size. To the best of our knowledge, it is the first PSI-CA protocol which has communication complexity linear in the size of the smaller set (n), and logarithmic in the larger set size N. More precisely, if the set sizes are $n << N$, we achieve a communication overhead of $O(n \log N)$.

[3] As internet users have already pointed out https://twitter.com/moxie/status/1248707315626201088

System	System Req. Trusted Server	#	Privacy Protection Against Linkage Attack By User	By Server	False-positive Claim	Malicious User	Client Comm. Cost
TraceTogether	Yes	1	Yes	No	Yes	Yes	$O(1)$
Baseline*	**No**	1	No	No	Some	No	$O(N)$
Cho *et al.* [18]		3	No	Yes	No	No	
PACT [15]		1	Yes	No	No	No	$O(n+N)$
Simple Epione		1	**Yes**	No	No	**Yes**	$O(n\log(N))$
Epione		2		**Yes**	**Yes**		

Table 1: Comparison of contact tracing systems with respect to security, privacy, required computational infrastructure, and client's communication cost. **Baseline** systems include Private Kit [41], Covid-watch [4], CEN [3], DP-3 [5]. Some of these systems provide limited false-positive claim protection with additional server (or healthcare provider). N is the total number of encounter tokens from users diagnosed positive with the disease, n is the number of encounter tokens recorded by an average user that need to be checked for disease exposure (Note that $\frac{N}{n}$ is typically the number of positive diagnoses for infectious days, so $n << N$). Linkage attacks, false-positive claim, and malicious user are described in Section 2.2, and discussed in Section 6.1. "Simple Epione " refers to a model in which user diagnosed positive sends their tokens directly to that server, thus there is only one server in this model. The "Malicious user" column indicates systems where it is possible to prevent attacks from malicious users as described in Section 5.3.2.

Table 1 provides a brief comparison of different systems with respect to different security/privacy properties, required computational infrastructure and client's communication cost which are important for contact tracing in real-world challenges.

1.2 System Overview

Figure 1 shows the system overview in Epione. Users of Epione want to be notified if any of the people they have come in contact with are diagnosed with the disease. The contact tracing app helps notify any of the people they have been in contact with if they are eventually diagnosed positive themselves. They do *not* want to reveal to other users their identity, reveal whether they have been diagnosed positive, be tracked over time, or reveal their contacts to any other organization.

Every time when two individuals are within close range, we use a short-range network (e.g. Bluetooth) to detect close proximity and exchange a randomly generated "contact token". All of the sent and received contact tokens are stored securely on the user's phone in the "sent token list" and "received token list", respectively. The received token list never leaves the user's phone in a form that can be used by anyone else, and the sent token list is only revealed to a healthcare provider on a positive diagnosis and with the user's consent. In Section 4, we explain in detail how to generate and store the tokens.

In Epione, we assume that there is an untrusted service provider, called server, which can collect the history of transmitted contact event tokens from all users tested positive with the disease in a privacy-preserving manner. The server allows users to check whether they have received a token from a user who has since been diagnosed with the disease, without revealing to the server their contacts and without the server revealing any information to the user about the tokens of users diagnosed positive beyond the count of encounters tokens in common. We use secure computation, particularly PSI-CA, for private matching. This prevents the server from inferring linkages between users, as well as preventing users from inferring the diagnosis status of other users.

It is assumed that when a healthcare provider (such as a hospital) provides a positive diagnosis

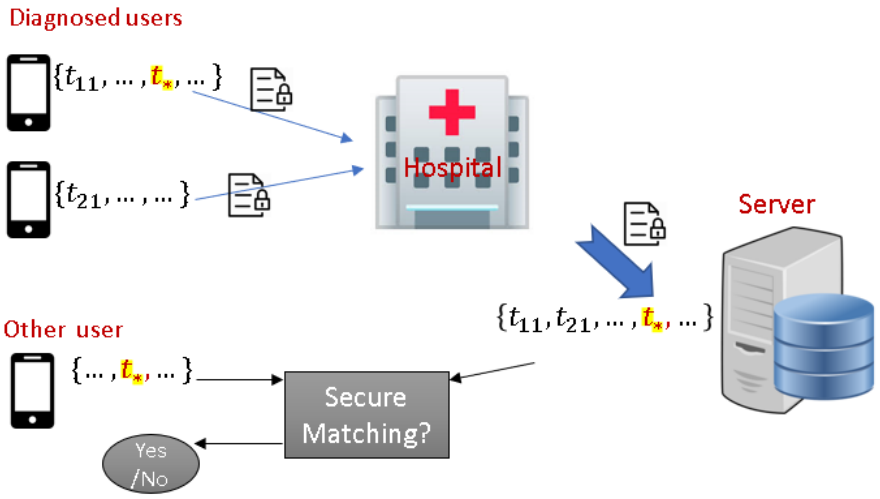

Figure 1: Overview of our Epione system. When a person is diagnosed with the disease, the hospital collects their encrypted sent tokens and forwards them to the server (actually, PRG seed is collected to reduce communication costs). The tokens are encrypted under the server's public key. Using his's secret key, the server decrypts the received ciphertexts from hospital, and obtains transmitted tokens of all patients diagnosed positive. Next, each phone and server securely compare their tokens' dataset via PSI-CA. If the intersection size is more than zero, then the user is alerted that they may have been exposed to the disease.

of the disease to a user that the healthcare provider is aware of the identity of the user. Thus, protecting the identity of the user diagnosed positive from the state is not considered in our system overview. We do consider a method of hiding this information in Section 4. It is also assumed that the healthcare provider has some kind of a data server, and will verify that the user does in fact have a legitimate positive diagnosis. The healthcare provider will then collect (with the user's consent) the list of "sent tokens" from the user's app, and send it to the server to be added to the list of tokens of users diagnosed positive.

Note that in this model the server does not know the identity of the user diagnosed positive. Even then, the sent tokens are not useful for identifying any contacts or any other private information. To establish a linkage, the server would need to know which users received those tokens. We will show later that the server never learns the received tokens of any user, and thus linkage attacks are not possible.

2 Problem Statement and Security Goal

2.1 Problem Definition

We define the problem of contact tracing based on token exchange as follows. Various clients communicate via a contact tracing service. The service is provided by one or multiple servers, each controls different administrative functions. The overall system consists of main interactive or randomized procedures as follows:

- $\texttt{Generate}(\kappa) \to t$: Client takes a security parameter κ as input, and uses the `Generate` function to generate random contact event number/tokens t.

- $\texttt{Exchange}(t, u_i) \rightarrow t$: Client uses the `Exchange` function to exchange his token t to user u_i, who will store this received token on his phone.
- $\texttt{Query}(\widetilde{T}, S) \rightarrow \widetilde{T}$: With a set $\widetilde{T}$ of received tokens from `Exchange`, client uses the `Query` function to generate a contact event matching request sent to the server(s) S.
- $\texttt{Answer}(u_i) \rightarrow \{0, 1\}$: Upon receiving matching request from user u_i, the server(s) use the `Answer` function to provide an answer of whether any tokens of their dataset is matched.

2.2 Security Definition and Goal

We consider a set of parties who have agreed upon a single functionality f (e.g. contact tracing) to compute and have also consented to give the f's final result to some particular party. At the end of the computation, nothing is revealed by the computational process except the final output. In the real-world execution, the parties often execute the protocol in the presence of an adversary $\mathcal{A}$ who corrupts a subset of the parties. In the ideal execution, the parties interact with a trusted party that evaluates the function f in the presence of a simulator Sim that corrupts the same subset of parties. There are two classical security models.

- Colluding model: This is modeled by considering a single monolithic adversary that captures the possibility of collusion between the dishonest parties. The protocol is secure if the joint distribution of those views can be simulated.
- Non-colluding model: This is modeled by considering independent adversaries, each captures the view of each independent dishonest party. The protocol is secure if the individual distribution of each view can be simulated.

There are also two adversarial models.

- Semi-honest model (or honest-but-curious): The adversary is assumed to follow the protocol, but attempts to obtain extra information from the execution transcript.
- Malicious model: The adversary may follow any arbitrary polynomial-time strategy to deviate from the protocol, such as supplying inconsistent inputs, or executing different computation than expected.

For simplicity, we assume there is an authenticated secure channels between each client-client, and client-server pair (e.g., with TLS). In this work, we consider a model with non-colluding servers. We formally present the security definition considered in this work, which follows the definition of [32, 36].

Real-world execution. The real-world execution of protocol Π takes place between users $(P_1, \ldots, P_n)$, servers $(P_{n+1}, \ldots, P_N)$ and adversaries $(\mathcal{A}_1, \ldots, \mathcal{A}_m)$, where $m < N$. Let $H \subseteq [n]$ denote the honest parties, $I \subseteq [n]$ denote the set of corrupted and non-colluding parties and $C \in [n]$ denote the set of corrupted and colluding parties.

At the beginning of the execution, each user $P_{i\in[n]}$ receives its input x_i, an auxiliary input z_i and random tape r_i, while each server $P_{i\in[n+1,N]}$ receives only an auxiliary input z_i and random tape r_i. Each adversary $\mathcal{A}_{i\in[m-1]}$ receives an index $i \in I$ that indicates the party it corrupts, while adversary $\mathcal{A}_m$ receives C indicating the set of parties it corrupts.

For all $i \in H$, let OUT_i denote the output of honest party P_i and, let OUT'_i denote the view of corrupted party P_i for $i \in I \cup C$ during the execution of Π. The i^{th} partial output of a real-world execution of Π between parties $(P_1, \ldots, P_N)$ in the presence of adversaries $\mathcal{A} = (\mathcal{A}_1, \ldots, \mathcal{A}_m)$ is defined as

$$\mathsf{REAL}^{i}_{\Pi,\mathcal{A},I,C,z_i,r_i}(k, x_i) \stackrel{\mathsf{def}}{=} \{\mathsf{OUT}_j \mid j \in H\} \cup \mathsf{OUT}'_i$$

Ideal-world execution. In the ideal-world execution, all the parties interact with a trusted party that evaluates a function f. Similar to the real-world execution, the ideal execution begins with each user $P_{i\in[n]}$ receiving its input x_i, an auxiliary input z_i, and random tape r_i. Each server $P_{i\in[n+1,N]}$ receives only an auxiliary input z_i and random tape r_i.

Each user $P_{i\in[n]}$ sends x'_i to the trusted party, where x'_i is equal to x_i if this user is semi-honest, and is an arbitrary value if he is malicious. If any semi-honest server sends an abort message ($\perp$), the trusted party returns $\perp$ to all users. The trusted party then returns $f(x'_1, \ldots, x'_n)$ to all the parties.

For all $i \in H$, let OUT_i denote the output returned to the honest party P_i by the trusted party, and let OUT'_i denote some value output by corrupted party P_i for $i \in I \cup C$. The i^{th} partial output of a ideal-world execution of Π between parties $(P_1, \ldots, P_N)$ in the presence of independent simulators $\mathcal{S} = (\mathcal{S}_1, \ldots, \mathcal{S}_m)$ is defined as

$$\mathsf{IDEAL}^{i}_{\Pi,\mathcal{S},I,C,z_i,r_i}(k, x_i) \stackrel{\text{def}}{=} \{\mathsf{OUT}_j \mid j \in H\} \cup \mathsf{OUT}'_i$$

Definition 1. *(Security) Suppose f is a deterministic-time n-party functionality, and Π is the protocol. Let x_i be the parties' respective private inputs to the protocol. Let $I \in [N]$ denote the set of corrupted and non-colluding parties and $C \in [N]$ denote the set of corrupted and colluding parties. We say that protocol $\Pi(I, C)$ securely computes deterministic functionality f with abort in the presence of adversaries $\mathcal{A} = (\mathcal{A}_1, \ldots, \mathcal{A}_m)$ if there exist probabilistic polynomial-time simulators $\mathsf{Sim}_{i\in m}$ for $m < n$ such that for all $\bar{x}, \bar{z}, \bar{r} \leftarrow \{0,1\}^\star$, and for all $i \in [m]$,*

$$\{\mathit{REAL}^{i}_{\Pi,\mathcal{A},I,C,\bar{z},\bar{r}}(k, \bar{x}) \stackrel{c}{=} \{\mathit{IDEAL}^{i}_{\Pi,\mathsf{Sim},I,C,\bar{z},\bar{r}}(k, \bar{x})\}$$

Where $\mathcal{S} = (\mathcal{S}_1, \ldots, \mathcal{S}_m)$ and $\mathcal{S} = \mathsf{Sim}_i(\mathcal{A}_i)$

Desirable Security/Privacy Properties. A desirable contract tracing system would make an honest user's actions perfectly indistinguishable from actions of all other honest users as well as servers. Thus, an ideal security system property would guarantee that executing the system in the real model is equivalent to executing this system in an ideal model with a trusted party as presented in the above definition 1.

Based on the above security definition, we consider following attacks, especially in the context of contract tracing.

- Linkage attack: An attack attempts to match anonymized records with non-anonymized records in a different dataset [24]. For contract tracing there are two types of linkage attacks: by users and by the server.

  The adversarial server aims to link users and re-identify their contact history by observing tokens it receives. For example, if the server is able to deduce that Alice and Bob had come in contact, regardless of frequency or duration, that is a linkage attack.

  On the user side, most users are aware of who they are in contact with for at least some amount of time, thus linkage to tokens is not useful. Instead, we consider any other use of the anonymized information, such as finding out about other users' contacts, the infection status of other users, or the source of their own exposure to the disease. Note that if a user was only near to one individual during infection period, and if she gets an alert of having been in contact with a confirmed case, then she knows who it was. This case cannot be avoided while providing the functionality of the application.

PARAMETERS: Two parties: server and client; and upper bound on the input set size.
FUNCTIONALITY:

- Wait for input set X from the server
- Wait for input set Y from the client
- Give server nothing
- Give client $|X \cap Y|$

Figure 2: The PSI-CA Functionality.

- False-positive claim: A malicious user may claim to have been diagnosed with the disease when in reality, they are not. This would spread false information and panic other users, and reduce trust in the system. This nefarious action must been caught with high probability.

As we will demonstrate in the following sections, Epione provably provides all of the functions of contact tracing while protecting against the attacks above.

3 Preliminaries

We now introduce notations and cryptographic primitives used in later sections.

3.1 Notation

In this work, the computational and statistical security parameters are denoted by κ, λ, respectively. For $n \in \mathbb{N}$, we write $[n]$ to denote the set of integers $\{1, \ldots, n\}$. We use '||' notation to refer to a string concatenation. We use party to refer to either server or user in the system.

3.2 Cryptographic building blocks

3.2.1 Decisional Diffie–Hellman

Definition 2. *[22] Let $\mathcal{G}(\kappa)$ be a group family parameterized by security parameter λ. For every probabilistic adversary $\mathcal{A}$ that runs in time polynomial in λ, we define the advantage of $\mathcal{A}$ to be:*

$$|\Pr[\mathcal{A}(g, g^a, g^b, g^{ab}) = 1] - \Pr[\mathcal{A}(g, g^a, g^b, g^c) = 1]|$$

Where the probability is over a random choice G from $\mathcal{G}(\lambda)$, random generator g of G, random $a, b, c \in [|G|]$ and the randomness of $\mathcal{A}$. We say that the Decisional Diffie–Hellman assumption holds for G if for every such $\mathcal{A}$, there exists a negligible function ϵ such that the advantage of $\mathcal{A}$ is bounded by $\epsilon(\lambda)$.

3.3 Discrete Log Zero-Knowledge Proof

Discrete Log Zero-Knowledge Proof (DLZK) is a cryptographic protocol, which allows Alice to convince Bob that she has k for known $y = g^k$ in the cyclic group $\mathbb{G} = \langle g \rangle$ without revealing the value of k. One of the simplest and frequently used proofs of knowledge for discrete log is Schnorr protocol, which incurs communication of 2 group elements, and computation of 3 modular exponentiations in a cyclic group.

3.3.1 Garbled Circuits for MPC

Garbled Circuit (GC) is currently the most common generic technique for two-party secure computation (2PC), which were first introduced by Yao [43] and Goldreich et al. [27]. The ideal functionality GC is to take the parties inputs x and y respectively, and computes f on them. We denote this garbled circuit by $z \leftarrow \mathcal{GC}(x, y, f)$. GC has seen dramatic improvements in recent years. Modern GC protocols evaluate two million AND gates per second on a 1Gbps LAN. In our protocols, we use "subtraction" and "less than" circuit.

3.4 Private Information Retrieval

It allows a client to query information from one or multiple servers in a such way that the servers do not know which information the client requested. When one first thinks about PIR, a trivial solution is to let servers send the whole database to the client, who locally performs his query. Unfortunately, this is extremely inefficient for large databases. Recent PIR [8,10,14,23,26] reduces communication to logarithmic in the database size.

In PIR, the server(s) hold a database DB of N strings, and the client wishes to read item $DB[i]$ without revealing i. In general, 1-server PIR construction [9,10] consists of the following algorithms:

- $\mathsf{PIR.Gen}$: takes a security parameter, and generates an additively homomorphic public and secret key pair (pk, sk).
- $\mathsf{PIR.Query}$: a randomized algorithm that takes index $i \in [N]$ and public key pk as input and outputs a (short and unexpanded) key k of size $O(\log(N))$.
- $\mathsf{PIR.Expand}$: takes a short key k and public key pk as input and outputs a long *expanded key* $K \in \{0,1\}^N$.
- $\mathsf{PIR.Answer}$: takes an expanded key K, public key pk, and a database DB as input, returns an answer d under an encrypted form.
- $\mathsf{PIR.Extract}$: takes a secret key sk and answer d as input, returns $DB[i]$.

With domain size N, the PIR's client can compute $(k) \leftarrow \mathsf{PIR.Gen}(i)$, and send it to the server. Server can expand $K = \mathsf{PIR.Expand}(k)$ and compute the following inner product: $d \stackrel{\text{def}}{=} \bigoplus_{j=1}^{N} K[j] \cdot DB[j]$. The server then sends $\mathsf{PIR.Answer}$'s output to client, who can then reconstruct $DB[i]$ by computing $\mathsf{PIR.Extract}(sk, d)$.

The correctness property of a PIR is that, if $(k) \leftarrow \mathsf{PIR.Gen}(i)$ then $\mathsf{PIR.Expand}(k)$ is an encrypted string of zero everywhere except for a 1 in the ith bit. Thus, d is an encrypted value of $DB[i]$ under the public key pk.

While the communication cost is $O(\log(N))$ bits, the computation requires $O(N)$ additive homomorphic operations, which is costly. In order to reduce the computational overhead on the server's side, most efficient PIR schemes use multiple servers with the assumption that not all of them collude.

With two servers, the scheme does not require additively homomorphic encryption. It contains only AES and bit operations. In summary, 2-server PIR [12,13] works as follows. The PIR's client can compute $\mathsf{PIR.Gen}(i)$ *without using any public key*, which instead outputs two (short) keys k_1, k_2, each of size $O(\log(N))$. The client sends k_1 to server 1, k_2 to server 2. Each server $i \in \{0,1\}$ can expand k_i as $K_i = \mathsf{PIR.Expand}(k_i)$ *without using any public key.* They locally compute the inner product $K_i \cdot DB \stackrel{\text{def}}{=} \bigoplus_{j=1}^{N} K_i[j] \cdot DB[j]$, and then send the result to the client. The client can then

reconstruct as $(K_1 \cdot DB) \oplus (K_2 \cdot DB) = (K_1 \oplus K_2) \cdot DB = DB[i]$ since $K_1 \oplus K_2$ is zero everywhere except in position i.

In 2-server PIR, the communication cost is $O(\log(N))$ bits and the computation requires $O(N)$ symmetric key operations which is much faster than additive homomorphic operation.

Chor, et al. [19] defined a variant of PIR called keyword PIR, in which the client has an item x, the server has a set S, and the client learns whether $x \in S$. This variant of PIR has been used for password checkup problem [9], where a client aims to check whether their password is contained in breached data, without revealing the information queried. The keyword PIR can be implemented by classical PIR using Cuckoo hashing with a constant overhead (approximately 3×). In this paper, we are interested in Keyword PIR based on both 1-server PIR [9, 10] and 2-server PIR [12, 13] .

3.5 Private Set Intersection Cardinality

Private set intersection cardinality (PSI-CA) is a two-party protocol that allows party learn the intersection size of their private sets without revealing any additional information. The PSI-CA functionality is presented in Figure 2.

4 Our Epione System

We now present the Epione system in detail, the construction of which closely follows the high-level overview presented in the second part of Section 1.2. Recall, Epione aims to alert any users of the systems who have, within the infection window (e.g. 14 days for COVID-19), come into contact with an individual diagnosed positive with an infectious disease.

4.1 System Phases

Epione 's design combines several different cryptographic primitives. To explain the design clearly, in this section we only present the functionality of gadgets and how to use them. Section 3.2, and Section 5 discuss how Epione implements them. The Epione system consists of four phases as follows.

4.1.1 Agreement and Setup Phase

The first phase requires all parties (including users and servers) to agree to perform the objective function (e.g. contact tracing using our Epione) over their joint data, and security parameters for MPC. The parties should also agree to release the computed result to each user. This agreement might happen before user initializes Epione on their phone.

Server takes a security parameter λ as input, and outputs a public-private key pair (pk, sk), and shares the public key with every user. Each user/phone u_i generates a random PRG seed s_i which uses to generate random tokens in the next phase. As long as the server' configuration does not change, this phase does not need to be re-run more than once. Whenever a new user registers Epione, they only needs to generate his own PRG seed.

4.1.2 Contact

Similar to most recent contract tracing systems [3, 4, 18], we use Bluetooth to exchange a randomly generated contact event token whenever two users are in close proximity. The `Generate` function is used to generate the n $\kappa-$bits tokens $\{t_{i,1}, \ldots, t_{i,n}\}$ using a PRG as $t_{i,1}|| \ldots ||t_{i,n} = \mathsf{PRG}(s_i||d)$, where s_i is the user's secret seed PRG, d is the current day, and n is an upper bound number of tokens needed for that day. For example, each user might locally execute the `Generate` function

How large is n?

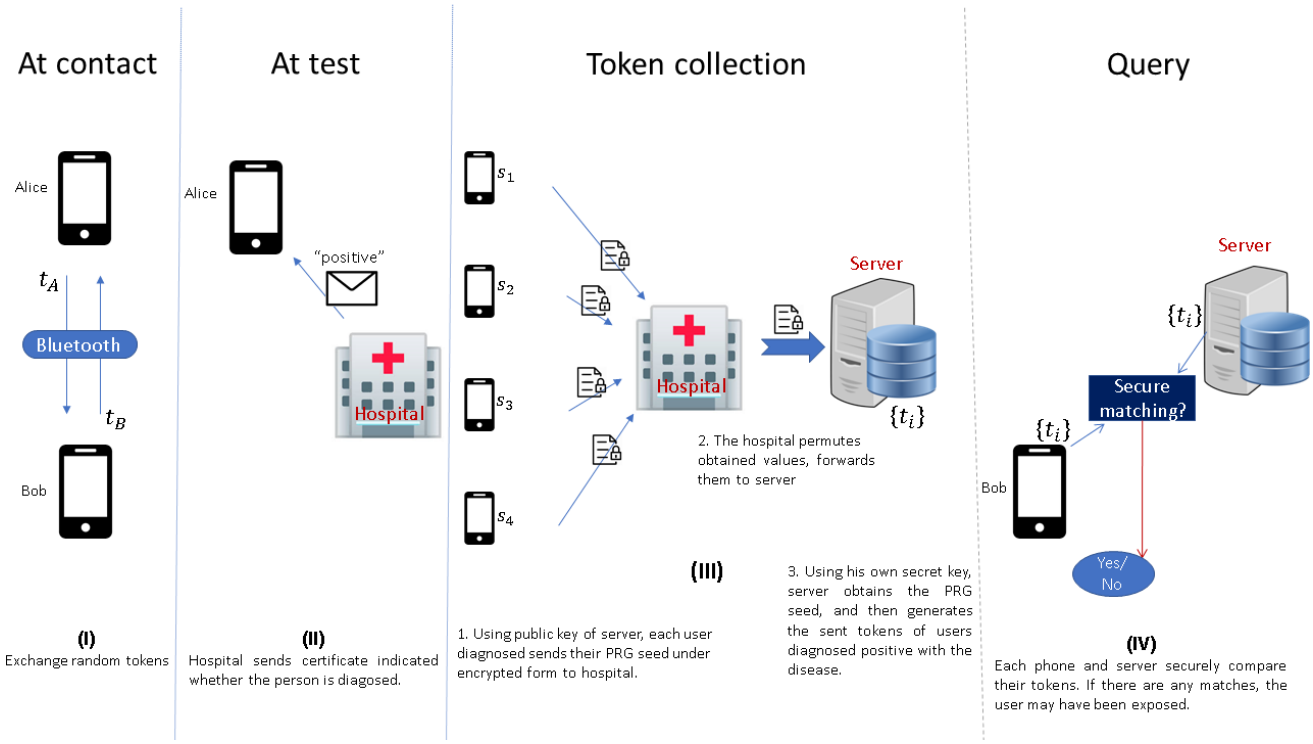

Figure 3: Epione System Design without Agreement and Setup Phase

at the beginning of the d^{th} day, and during that day he releases/transmits them in order whenever another user is detected in close range (e.g. function `Exchange`() is triggered). He also locally stores received tokens $\widetilde{T}_i = \{t_{1,i}, \ldots, t_{1i}\}$ on his phone. Figure (3, I) illustrates the "exchanging random token" process of this phase.

4.1.3 Positive Diagnosis and Token Collection

When a user u_i is diagnosed with the disease, the healthcare provider, with the user's consent, collects the user's secret PRG seed under an encrypted form (e.g. the user encrypts the PRG seed using the public key of the server), and transmits it over a secure channel to the server. Using his own secret key, the server decrypts the received encrypted values, and obtains the secret PRG seed of users diagnosed. The server can generate the sent tokens $\mathbf{T}$ of users diagnosed positive with the disease. Figure (3, II&III) presents the 'token collection" process of this phase.

4.1.4 Query

Recall that each user u_i holds the received tokens $\widetilde{T}_i$ from the "contact" phase. The "query" phase aims to securely compare $\mathbf{T}$ to $\widetilde{T}_i$. If there are any matches, the user u_i has come into contact with an individual diagnosed positive within the infection window, and should be notified that they are at risk of having contracted the disease. To do this comparison, we use PSI-CA, a special problem of MPC as a cryptographic gadget of our design. Note that revealing the intersection size is acceptable in the contact tracing application we consider. However, it is possible hide the intersection size as we discussed in Section 5.3.1. The PSI-CA protocol is presented in Section 5.

Each user calls a `Query` function for sending a contact event matching request to server via MPC. Upon receiving the request, the server jointly performing PSI-CA with that user, and uses

the `Answer` function to response an answer. Figure (3, IV) illustrates the phase.

4.2 Security Discussion

The security of our Epione follows in a straightforward way from the security of its building block (e.g. PSI-CA) and the encryption scheme. Thus, we omit the proof of the following theorem.

Theorem 1. *The Epione construction securely implements the contact tracing functionality defined in Section 2.1 in semi-honest setting, given the PSI-CA primitive, encryption scheme described in Figure 2, Section 3.2, respectively.*

Malicious User Queries. A malicious user, Mallory, can deviate from the protocol by submitting arbitrary queries to the server. Mallory can record the time and place at which tokens from other phones were received and then later use this information for re-identifying other users. Mallory legitimately has access to a set of tokens, so it can use arbitrary *subsets* of these tokens in its queries to the server.

There are several ways to mitigate the threat of arbitrary queries. First, we could require that users submit a cryptographic hash (e.g. by computing a Merkle root[4]) of their local token list periodically to the server, say once every day. When Mallory invokes the server, his query includes the cryptographic hash of the set used in the query. The is hash ensures that the entire set of tokens obtained by Mallory at a particular point in time is used in the query, and not some chosen subset. Note that the tracing functionality itself reveals whether there is a match within Mallory's full set of tokens. This is not a linkage attack, rather an direct implication of the information given as part of the desired functionality of the application. Mallory, or any benign user, can have singleton received token sets, in which case they learn that their received token matches with the server's database. This is fundamentally unavoidable as it is part of the benign functionality.

Periodic commitment to cryptographic hashes of a user's token set mitigates the threat of false claims as well. Users cannot retroactively add tokens to their local lists without being detected by the server.

A second complementary mitigation is rate limiting users to a few queries per day, and requiring a minimum number of tokens per query, but that would only slow Mallory down, not prevent the attack completely. With enough queries, Mallory will eventually deduce with high probability the source of her exposure.

Lastly, using application DRM protocols such as Android SafetyNet and Apple DeviceCheck will make it much harder for Mallory to submit such queries and is highly recommended. When combined with rate limits and a minimum token set size, these protocols make crafting queries to find the exposure source impractical, though again not impossible. We will not cover protection from these attacks in this paper, as they cannot be distinguished from actual user queries and the protections above are the best possible defence.

[4]The following details ensure that the committed hash value is randomized and defeats any dictionary attacks by the server: We first permute the local tokens randomly, add a dummy random value in the list, and then compute a Merkle tree. The Merkle root is committed as the cryptographic hash of the list. The adversary does not know the dummy value or the permuted order and hence is unable to forge a Merkle proof.

5 Cryptographic Gadgets

5.1 PSI cardinality (PSI-CA) for asymmetric set sizes

In this section, we present PSI-CA construction which functionality is described in Figure 2 and used as a core component of our Epione.

5.1.1 Our technique

We start with a classical PSI functionality in semi-honest setting, where two parties want to learn the intersection of their private set, and nothing else. The earliest protocols for PSI were based on Diffie–Hellman (DH) assumption in cyclic groups. Currently, DH-based PSI protocols [35] are still preferable in many real-world applications[5], due to their extremely low communication cost.

Classical PSI Assume that server has input $X = \{x_1, \ldots, x_N\}$ and client has input $Y = \{y_1, \ldots, y_n\}$. Given a random oracle $H : \{0,1\}^* \to G$, and a cyclic group G in which the DDH assumption holds, the basic DH-based PSI protocol is shown in Figure 4.

PARAMETERS: cyclic group G of order p; random oracle H.

INPUTS: Server has input $X = \{x_1, \ldots, x_N\}$; client has input $Y = \{y_1, \ldots, y_n\}$.

PROTOCOL:

1. Client chooses $r \leftarrow \mathbb{Z}_p$ for each $y_i \in Y$ and sends $m_i = H(y_i)^r$ to the server.
2. Server chooses $k \leftarrow \mathbb{Z}_p$ and for each $i \in [n]$, sends $m'_i = m_i^k$ to the receiver in a randomly permuted order.
3. For each $i \in [n]$, the client computes $v_i = (m'_i)^{1/r}$.
4. For each $x_i \in X$, the server computes $u_i = H(x_i)^k$ and sends $U = \{u_i \mid x_i \in X\}$ (randomly permuted) to the client.
5. [PSI-CA output] The client outputs $|\{i \in [n] \mid v_i \in U\}|$.

Figure 4: Basic PSI-based protocol and extension to PSI-CA with changes highlighted.

Intuitively, the client sends $\{H(y_i)^r\}_{y_i \in Y}$ for some random, secret exponent r. The server raises each of these values to the k power, and the client can then raise these results to the $1/r$ power to obtain $\{H(y_i)^k\}_{y_i \in Y}$ as desired[6].

From classical PSI into PSI cardinality (PSI-CA) It is possible to use different r exponents for each item. In other words, the client sends $\{H(y_i)^{r_i}\}_{y_i \in Y}$ and then later raises the responses to the correct $1/r_i$ power. This assumes that the server preserves the order of items when raising to the k power.

[5] https://security.googleblog.com/2019/06/helping-organizations-do-more-without-collecting-more-data.html

[6] Alternatively, the client can raise each $H(x_i)^k$ to the r power and compare to the $H(y_i)^{kr}$ values. However, the variant where the client raises $H(y_i)^{kr}$ to the $1/r$ power is compatible with further optimizations.

However, using the same r for every item is important in some extensions of PSI like computing only the PSI-CA (e.g. intersection set size). This observation was suggested by [29] and recently is incorporated into private intersection sum [30], which allows two parties to compute the sum of payloads associated with the intersection set of two private datasets, without revealing any additional information. Clearly, PSI-CA is a special case of private intersection sum, where the payload is constant and equal to 1.

Figure 4 also shows PSI-CA protocol with changes highlighted. The key idea to transform PSI into PSI-CA is that instead of sending m'_i in step 2 of Figure 4 in order, the server shuffles the set in a randomly permuted order. Shuffling means the client can count how many items are in the intersection (PSI-CA) by checking whether $v_i \in U$, but learns nothing about which specific item was in common (e.g. which v_i corresponds to the item y_j). Thus, the intersection set is not reveled.

From PSI-CA into PSI-CA for asymmetric set sizes In contact tracing applications, the two parties have sets of extremely different sizes. A typical client has less than 1000 new tokens per day, while the server may have tens of millions of tokens from users diagnosed with the disease in its input set. In classical PSI, most work is optimized for the case where two parties have sets of similar size, and as such their communication and computation costs scale with the size of the larger set. For contact tracing, it is crucial that the client's effort (especially communication cost) be sub-linear in the server's set size. More practically, we aim for communication of at most a few megabytes in a setting where the client is a mobile device. There is a small handful of works [16,17] focused on classical PSI for asymmetric set sizes. However, to the best of our knowledge, ours is the first PSI-CA protocol which has communication complexity linear in the client's set (n), and logarithmic in the server's set (N).

We observe that last two steps of Figure 4 is similar to the function performed by keyword PIR, which is communication-efficient in the conventional client-server setting. Keyword PIR allows clients to check whether their item is contained in a set held by a server, without revealing the actual item to the server. Therefore, step 4 and 5 of Figure 4 can be replaced by keyword PIR. Concretely, after step 3, the client has an input set $V = \{v_1, \ldots, v_n\}$ and the server has input set $U = \{u_1, \ldots, u_N\}$. The client sends a multi-query keyword PIR request with all of the elements in V to be queried against U on the server. From the PIR response, the client can count the number of $v_i \in U$ to find the set size, without revealing to the server the actual values in V and without the client learning anything more information about U.

5.1.2 Protocol

Our semi-honest PSI-CA protocol is presented in Figure 5, following closely the description in the previous section. Client runs keyword PIR searches each time for $v_{i\in[n]}$ in a set U. The protocol is correct as long as there are no spurious collisions among that set. Thus, we can limit the overall probability of a spurious collision to 2^{λ} by truncating both u_i and v_i to length $\lambda + log(N)$.

Theorem 2. *The PSI-CA construction of Figure 5 securely implements the PSI-CA functionality defined in Figure 2 in semi-honest setting, given the Multi-query Keyword PIR described in Section 3.4.*

Sketch of proof:

The security of the PSI-CA protocol follows from the fact that $\{H(z_1)^k, \ldots, H(z_n)^k\}$ are indistinguishable from random, for *distinct* z_i, if H is a random oracle and the DDH assumption holds in G. To see why, consider a simulator that receives a DDH challenge $g^k, g^{a_1}, \ldots, g^{a_n}, g^{c_1}, \ldots, g^{c_n}$

PARAMETERS: cyclic group G of order p; random oracle H, Multi-query Keyword PIR primitive (Section 3.4), a truncate function $\tau(z)$ takes z as input and returns first $\lambda + log(N)$ bits of z.

INPUTS: Server has input $X = \{x_1, \ldots, x_N\}$; client has input $Y = \{y_1, \ldots, y_n\}$; Helper has no input

PROTOCOL:

1. Server chooses $k \leftarrow \mathbb{Z}_p$, and compute dataset $U = \{\tau\big(H(x_i)^k\big) \mid i \in [N]\}$. Server sends U to Helper
2. Client chooses $r \leftarrow \mathbb{Z}_p$ for each $y_i \in Y$ and sends $m_i = H(y_i)^r$ to the server.
3. Server chooses a random permutation $\pi : [n] \rightarrow [n]$. For each $i \in [n]$, sends $m'_i = (m_{\pi(i)})^k$ to the client.
4. For each $i \in [n]$, the client computes $v_i = \tau\big((m'_i)^{1/r}\big)$.
5. Parties invoke Multi-query Keyword-PIR with 2 servers:
   - Server acts as Keyword-PIR's server 1 with dataset U
   - Helper acts as Keyword-PIR's server 2 with dataset U
   - Client acts as Keyword-PIR's client with $V = \{v_i \mid i \in [n]\}$
   - Client learns whether $v_i \in U, \forall i \in [n]$
6. Client outputs $\sigma = |V \cap U|$

Figure 5: Our semi-honest setting PSI-CA protocol for asymmetric set sizes, and extension to 2-server PIR based PSI-CA with changes highlighted

where each c_i is either random or $c_i = a_i k$. The simulator programs the random oracle so that $H(z_i) = g^{a_i}$ and then simulates the outputs as $\{g^{c_1}, \ldots, g^{c_n}\}$. It is easy to see that these messages are distributed as specified by F if the c_i's are distributed as $a_i k$, but are distributed uniformly otherwise, with the difference being indistinguishable by the DDH assumption.

We consider two cases, corresponding to each party being corrupt.

- A corrupt server first sees $\{H(y_i)^r\}_{y_i \in Y}$. By our observation above, these values are pseudo-random. A corrupt server also sees PIR transcripts. Because pseudorandomness guarantees of PIR, the client's message to the server can be simulated as a random message.
- A corrupt client sees $\{H(y_{\pi(i)})^{rk} = (H(y_{\pi(i)})^k)^r\}_{y_{\pi(i)} \in Y}$ and PIR response (as the extra PSI message), along with its private randomness r and any random oracle queries/responses that it made. Consider modifying this view, replacing each $H(z)^k$ term with an independently random group element for each $z \in X \cup Y$ (each distinct, by definition). This change will be indistinguishable, by the reasoning above. Now it is not necessary to know the identities of $x_i \in X \setminus Y$ as well as $x_i \in X \cup Y$, as their corresponding $H(x_i)^k$ values have been replaced with random group elements that are independent of everything else, and the secret permutation π hides the common items. In other words, this is a distribution that can be generated by the simulator, with knowledge of only Y and $|X \cap Y|$.

□

5.1.3 PSI-CA Costs

In our PSI-CA protocol, the communication cost is $O(n\log(N))$ while the computation cost is $O(nN)$ (only on the server's side). Concretely,

- The server and client must communicate (1) $O(n)$ group elements, (2) $O(n\log(N))$ homomorphic encryption ciphertext for keyword PIR.
- The client's computation cost of the PSI-CA protocol consists of: (1) $O(n)$ group elements, (2) $O(n)$ homomorphic encryption for keyword PIR to encoding the PIR queries
- The server's computation cost of the PSI-CA protocol consists of: (1) $O(n)$ group elements, (2) $O(nN)$ additive homomorphic encryption for keyword PIR for PIR's answers

In order to speed up the computation on the server's side, we can use two-server PIR model, which can avoid $O(nN)$ additive homomorphic encryption computation.

5.2 PSI-CA with 2-server PIR (server-aided)

Recall that client and server invoke keyword-PIR in step 5 of Figure 5. To speed up the computational overhead on the server's side, we introduce a (server) helper who will acts as another server during PIR computation. Concretely, after computing the dataset U, the server sends it a helper. Since u_i is random, the helper learns nothing about the item x_i.

Client sends PIR's query with keyword v_i to two PIR's servers (i.e. helper and server), and learns whether $v_i \in U$ and nothing else. The PIR's servers learn nothing about client's query as long as two PIR's servers (i.e. helper and server) do not collude.

With 2-server PIR, the computation cost of keyword PIR contains only symmetric-key operations. Concretely, it invokes roughly $2N$ PRF calls. Figure 5 also shows 2-server PIR based PSI-CA protocol with changes highlighted.

5.3 PSI-CA extension and its discussion

Revealing the intersection size is acceptable in the contact tracing application we consider. However, it is possible that in other settings, knowing the size of the intersection is undesirable leakage.

5.3.1 Potential approach from PSI-CA into threshold PSI-CA (t-PSI)

In general, threshold PSI-CA is an extension of PSI where parties learn the intersection size (or even the intersection items) if it is greater than a given threshold. In our t-PSI definition, the client learns whether two input sets has any common items and nothing else (e.g. $t = 0$).

A simple solution is to pad the input set with "dummy elements". Concretely, the two parties decide on a pseudo-random generator function (PRG) and seed s which are used to generate common dummy elements. Each party (e.g. client and server) randomly chooses a number of dummy elements, n' and N' for client and server respectively such that $n' > N'$. This step can be done by 1) parties randomly choose n' and N'; 2) parties invoke garbled circuit to check whether $n' > N'$; 3) they repeat this process until the "if" condition is true. Next step is that the client and server use the agreed PRG and seed s to generate n' and N' fake items, respectively. The resulting intersection over the original and the common dummy elements will be $\sigma = |X \cap Y| + N'$, which is known by the client.

To securely remove the term N' in σ and check whether $|X \cap Y| > t$, parties invoke a garbled circuit where the server's input is N', the client's input is σ. The circuit takes the parties inputs, computes $f = \big(\sigma - N'\big) > t?$ and returns the result to the client.

There remains an important concern in this approach: how to choose n' and N', so that σ informationally hides the actual intersection size. Since the range of intersection size is from 0 to n, the bound of information leakage is $O(\log(n))$ bits. Therefore, it is sufficient to choose n' and N' to be $O(2^{\log(n)})$, which is essentially $O(n)$. However, it is not clear which coefficient value behind the big O. Further analysis needs to be done to prove if this method provides sufficient bits to prevent information leakage.

5.3.2 Potential approach for extending to malicious client

In the context of contact tracing, a malicious client seeks to obtain information about the server's database (set X in this case). Thus, they attempt to compute $m_i = H(y_i)^r$ incorrectly, since in step 3 of Figure 5 the server returns $m'_i = (m_{\pi(i)})^k$ and it may be possible to learn part or all of the value of k. With that, the client can determine which values of v_i map to which values of u_j, and thus learn which items from its set exist on the server.

One solution to prevent this is to augment the protocol with a zero-knowledge proof [21,31] that the m_i's were computed correctly. This adds the following step to the protocol:

(2a) Client performs a zero-knowledge proof of knowledge of r such that $\forall i \in [n] : m_i = (y_i)^r$. The server aborts if the proof does not verify.

This modification is enough to guarantee security against a malicious client for the classical DH-based PSI protocol.

Achieving a PSI-CA or t-PSI protocol resilient to a malicious client requires more work because the construction of the client's message involves more building blocks, namely keyword PIR and garbled circuit. The solution is to use versions of these building blocks resilient to such one-side malicious attacks. Moreover, to verify that the client uses the correct values for v_i for step 5 of Figure 5 in the keyword PIR query, we employ a consistent check such as MACs from the SPDZ protocol [20]. Using all of these techniques we achieve security from a malicious client. We will explore more detail in this direction.

6 Related Work

6.1 Contact Tracing

Due to the rapid spread of the COVID-19 pandemic and the importance of contact tracing, many research groups have been developing tools to improve contact tracing. Most schemes either (1) rely on and expose data to a trusted third-party, such as TraceTogether [2], or (2) use an untrusted server such as in COVID-Watch [4], which doesn't have explicit knowledge of users' identity but is still able to infer linkages between users, or (3) uses a decentralized/public list approach such as PACT [15] or Google/Apple [1] that allows users to infer linkages such as exposure sources. We divide existing Contact Tracing system into Trusted Central Authority and Untrusted Centralized / Distributed approaches.

6.1.1 Trusted Third-Party (Centralized) Approaches

The primary example of a trusted third party approach is TraceTogether by the Singaporean Government, released on March 20, 2020 [7]. Based on the Bluetrace Specification, the protocol works

as follows. Let Alice and Bob be users of the app, and let Grace be the government server (or other central authority).

1. **Setup.** Alice and Bob both install the app on their smartphones. During the setup process, both Alice and Bob register a phone number with Grace and are each assigned a unique identifier (i.e., ID_A for Alice, and ID_B for Bob). Grace stores the phone numbers and ID numbers registered in a database.

2. **Token Generation** Grace assigns to Alice and Bob a set of encounter tokens[7] that are to be used at different times. Tokens are generated by concatenating the user's ID, the start time of the token, and the end time of the token, and then encrypting with AES-256-GCM with a key known only to Grace. Tokens are thus expressed as: $T_i = E_{AES}(k, ID||t_{start}||t_{end})$. Tokens are sent in batches along with validity times to users' devices in case they are not able to fetch new tokens on demand [8].

3. **Contact.** When Alice and Bob meet, they exchange tokens for the current epoch. Both devices keep a list of received and sent tokens.

4. **Diagnosis.** Some time later, Bob is confirmed to have contracted the disease. Bob sends his received tokens list (i.e. the list that contains T_A received from Alice) to Grace. Grace verifies that Bob does in fact have a positive test result (this can be done either by matching Bob in a separate database of test results, or by providing Bob with a passcode that validates his positive result).

5. **Follow-up** Grace then decrypts each token received from Bob using her key. This reveals the user ID and time of each contact along with other metadata. Grace can then look up each user's (e.g. Alice's) phone number and directly follow up with them.

Clearly, this system places a lot of trust in the government Health Authority (Grace). As soon as Bob is diagnosed with the disease and he submits his tokens, Grace learns all of Bob's social contacts, down to when and for how long they were in contact. If Grace's key is ever compromised, whether by a state actor or an attacker, this key could be used to track all users via Bluetooth.

By Singaporean law, anyone diagnosed with an infectious disease such as COVID-19 is required to submit full details of all of their contacts and travel history to the Ministry of Health [**?**], therefore this approach is consistent with current practices there. In other jurisdictions however, this may be met with reluctance by users, or may not be allowed by privacy regulations.

6.1.2 Untrusted Third-Party or Decentralized Approaches

Nearly all of the proposed or launched contact tracing schemes that do not rely on a trusted authority use a scheme as follows, with only minor variations:

1. Alice and Bob, two users of the contact tracing scheme, download and install the app

2. Alice and Bob both generate encounter tokens that rotate over time and cannot be used to reveal their identity directly or track them over time.

[7] In Bluetrace, tokens are called "TempIDs". We have kept the term token here for comparison with other systems.

[8] Note that tokens are generated by Grace mainly to reduce computational load on client devices. This scheme is equivalent to having clients produce tokens by encrypting with a public key for Grace with the same information.

3. When Alice and Bob meet, they exchange encounter tokens, for example via Bluetooth[9]. Alice and Bob both keep a list of received tokens and sent tokens.

4. Bob is later diagnosed with the disease. Bob submits his tokens to an untrusted server. This can be either the list of received tokens, the list of sent tokens, or both. Alternatively, Bob can submit the secret used to generate tokens from his sent list.

5. A list of tokens from users diagnosed with the disease is then maintained either in a private database to which users can submit queries or published in a public list so that users can check for intersections on their device.

6. If a user finds they have tokens in common with the public list, or via querying a central database, they may have been exposed to the disease and should be notified to seek appropriate next steps.

Covid-watch [4], Private Kit [41][10], PACT's baseline design [15], and Google/Apple [1] are all variations on this design. Some use pseudo-random number generators, and upload seeds for the sent token lists to reduce communication and storage costs at the expense of greater cost for comparisons, but this has no impact on privacy implications. All of these designs are susceptible to linkage attack by either users, the server, or both. Some offer protection against false positive claims.

In all of the above systems, each phone has to compare the publicly posted encounter tokens against their own history, which requires to them download all public tokens. This requires significant bandwidth and places a burden on mobile devices.

6.1.3 Privacy Improvements

Private Messaging To reduce the linkage information learned by server, [18] proposes to use two or more non-colluding servers operating as a private messaging network between users and the central server operated by the government (Grace). Concretely, assume that Grace stores a collection of mailboxes, one for each token that Alice and Bob exchange, and there are two non-colluding communication servers Frank and Fred. Frank forwards messages to/from Fred, and Fred forwards messages to/from Grace, all in such a way that Grace cannot know the source or destination of any messages. When Alice and Bob are in close contract, they exchange tokens. At fixed time points, both parties send a message which contains their current diagnosis status to each other via Frank, Fred, and Grace. For example, Bob addresses the message to Alice encrypted using Alice's public key, and gives the message to Grace (through Fred, who is received a forwarding messages from Frank), who puts it in Alice's mailbox. Alice checks all of the mailboxes through Frank and Fred to learn whether she has been exposed to the virus. Since Alice sees Bob's message in her mailbox, Alice might be able to infer who Bob is based on the time they are nearby. Moreover, Grace needs to maintain all tokens (messages) of all users, which requires storage.

Re-randomization of tokens. An alternative approach presented in PACT [15] aims to prevent a linkage attack by users by re-randomizing tokens. Their proposed solution is based on the DDH assumption, and works with the following changes compared to baseline system:

- Alice generates a token in the form $T_a = (g^{r_i}, g^{r_i s_A})$, where s_A is a key that Alice never shares, and r_i is a nonce used only for this token

[9] in GPS or geolocation based systems, Alice and Bob independently generate tokens as a function of location and time, for example by hashing a grid square and time quantum.

[10] PrivateKit claims that in V3 they will introduce strong privacy protections, but as of writing this paper the protocol to do so has not been announced.

- When Alice and Bob meet, Alice gives T_a to Bob
- When Bob is diagnosed positive, he chooses a random r'. If $T_a = (x, y)$, he submits the pair $T'_a = (x^{r'}, y^{r'})$ to the server
- Alice can determine whether she is at risk by checking all of the tokens in the public list to find one that satisfies $y = x^{s_A}$

While generating tokens is almost free in our Epione, PACT requires two group elements for each token's generation. Moreover, as mentioned by the authors, the privacy benefit inherently relies on each user re-using the same secret key (s_A in this case), and they cannot force a malicious user to comply. Using a different s_A for different encounters allows Alice to determine which encounter caused her exposure to the disease. In contrast, this malicious action does not happen in our Epione.

6.2 Secure Computation and Private Set Intersection

Private set intersection (PSI) refers to a cryptographic protocol that allows two parties holding private datasets, to compute the intersection of these sets without either party learning any additional information about the other's dataset. PSI has been motivated by many privacy-sensitive real-world applications. Consequently, there has been a long list of work on efficient secure PSI computation [16,17,28,33,34,37,38]. However, PSI only allows the computation of the intersection itself. In many scenarios (for example, contact tracing), it is preferable to compute some function of the intersection (e.g. whether intersection size is more than a given threshold) rather than reveal the elements in the intersection. Limited work focused on this so-called f-PSI problem, and most assumes sets of comparable size that can be communicated in their entirety to the other party. In this section we focus on protocols [28,30,38–40] that support f-PSI as well as PSI-CA.

GC-based PSI. Huang, Katz, and Evans [28] presented techniques for using the generic garbled-circuit approach for PJC, which is based on their efficient sort-compare-shuffle circuit construction. Later Pinkas et al [38–40] improved a circuit-PSI using several hashing techniques.

The main bottleneck in the existing circuit-based protocols is the number of string comparisons and computing the statistics (e.g, count) of the associated values that are done inside a generic circuit-based secure two-party computation, which is communication-expensive.

HE-based PSI. The Diffie-Hellman Homomorphic encryption approach of [30] has by far the lowest communication complexity of f-PSI protocols. However, the protocol of [30] has communication complexity linear in set size, which is still communication-expensive in client-server settings where the server's dataset size is large.

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