Ad or Subtract?: An Analysis of Ad Frequency and Ad Tolerance on Twitter

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Abstract

In this project, I develop a novel probabilistic model, called TFAM, for the frequency of ads on a Twitter feed. By collecting data from my own feed, I simulate feeds under different parameters, finding that the empirical distribution of ads leads to nearly identical results to an assumed Poisson distribution. Furthermore, I find through simulation that for a given user ad tolerance, it is possible to identify an optimal frequency of ads which maximizes Twitter's ad revenue.

1 Introduction

Billions of people use social media every day, spending billions of hours scrolling through their "feed"–an algorithmized stream of text, images, and videos, noticeably interspersed with advertisements [Ahmed and Raziq, 2018]. In online advertising, there are a few main metrics that advertisers must consider. Cost per click (CPC), and its siblings cost per impression (CPI) and cost per action (CPA), measure the average amount owed by an advertiser to the ad network when a user clicks, sees, or generally engages with the ad [Hu et al., 2016]. Click-through-rate (CTR) is the percentage of viewers of the ad that actually click the ad or engage with it in a meaningful way.

These ads are the lifeblood of social media networks, and as such, the companies behind them pour massive resources into optimizing the targeting and placement of ads to maximize their revenue. One important tradeoff for these companies comes when deciding how frequntly to show ads to users. On the one hand, the more ads shown, the more impressions and clicks will be generated leading to more revenue. On the other hand, if *too* many ads are shown, users will either start ignoring the ads or altogether quitting their session on the platform due to annoyance or boredom.

Twitter (now known as X) is an excellent platform to analyze this tradeoff since its feed is a simple mixture of user posts, called tweets, with ads that appear visually similar to tweets interspersed between. In this project, I set out to

1. Propose a plausible model for the frequency of ad appearances in the Twitter algorithm
2. Discover patterns behind ad presentation on Twitter by collecting data from my own Twitter feed
3. Identify relationships between ad frequency, ad tolerance, and revenue generated

2 Model Description

In this section, I lay out the definitions and mechanisms behind the Tolerance-Frequency Ad Model (TFAM). We begin by defining the feed, which is a sequence of posts $x_i \in \{0, 1\}$; if $x_i = 0$, the ith post is a tweet, and otherwise it is an ad. Inspired by a Poisson process, we model the number of tweets between two consecutive ads as a Poisson random variable. Let $a_j \sim \text{Pois}(\mu)$ be the number of tweets between the jth ad and the $j+1$th, where μ is the average number of tweets between consecutive ads. Thus if, for example, $a_0 = 3, a_1 = 4, a_2 = 2$, the resulting feed would be $x = \{0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 0, 1\}$.

Let t_i be the time that the viewer spends viewing the ith post. We will assume this time follows an exponential distribution, a common distribution for modeling time spent on/between events. Specifically, we assume

$$t_i \sim \begin{cases} \text{Expo}(\lambda_t) & \textit{if } x_i = 0 \\ \text{Expo}(\lambda_a) & \textit{if } x_i = 1 \end{cases}$$

where $\lambda_t > \lambda_a$ since users spend more time on average looking at a tweet than an ad. We also define a threshold for interaction with an ad (i.e. a click, retweet, or follow). I use the inverse CDF of the $\text{Expo}(\lambda_a)$ distribution to find a time threshold over which we assume the user will interact with the ad, such that they interact at the CTR:

$$T_{\text{interact}} = -\log(\text{CTR}) \cdot \lambda_a \tag{1}$$

Finally, the model takes into account user frustration with ads. I assume that for a given user, there exists a threshold $N_{\text{tolerance}}$ which represents the ad tolerance of a user before they quit their session, thus cutting off any remaining revenue that might have been earned by Twitter. Specifically, I define $N_{\text{tolerance}}$ to be the maximum number of ads seen by a user in the last 60 seconds of browsing without quitting their session.

In this way, TFAM models two competing forces on Twitter's revenue. First, the more frequently users are shown ads, the more frequently they will interact with them and thus the more revenue Twitter will make. However, if ads are shown too frequently, users will end their sessions early, cutting Twitter's revenue short of its potential. To reflect this trade-off, the two main parameters I will be testing are μ and $N_{\text{tolerance}}$.

Through research, I found estimates of the CTR and CPC on Twitter. The exact values are arbitrary to the model formulation, but in my simulations, I adopt the following estimates: CTR of 1.64% [Hubspot, 2017] and CPC of $1.35 [AdsTargets, 2022]. Furthermore, for the purposes of simulation, I assume that on average, I look at tweets for 10 seconds and ads for 5, giving $\lambda_t = 10, \lambda_a = 5$. Plugging these values into equation (1) gives $T_{\text{interact}} = -\log(0.0164) \cdot 5 = 20.55$ seconds. For the remainder of this paper, these parameters are fixed at these values.

2.1 Data Collection & Analysis

In order to more accurately model a Twitter feed, I collected data on the number of tweets between ads shown on my feed. I did so by scrolling through my feed and recording the number of tweets between ads as a sequence of integers. One important note is that the distribution of tweets between subsequent ads did not depend on the speed of browsing, but rather remained constant across sessions. The dataset size is 225, from a total of 1263 viewed posts. The resulting data is visualized in Figure 1.

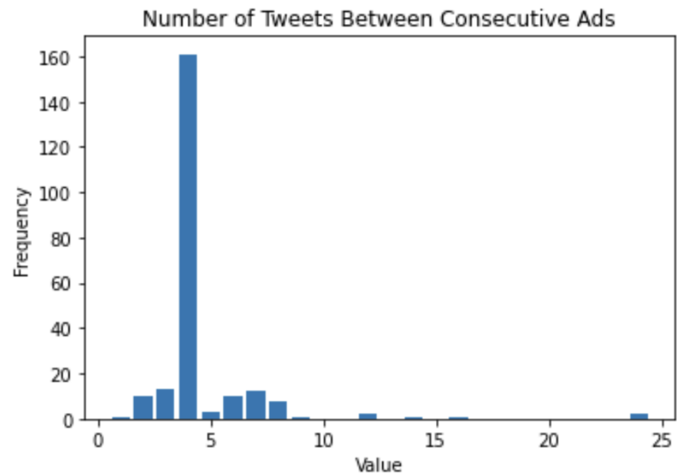

Figure 1: **Data overview.** Histogram of number of tweets between subsequent ads on my Twitter feed.

Surprisingly, it seems that Twitter is not using a very sophisticated algorithm for deciding when to show an ad. Rather, almost always, Twitter shows users 4 tweets followed by a single ad. Sometimes, Twitter will show a slightly fewer or more tweets between ads, and on rare occasions will show many more tweets between ads (the only values I encountered in data collection were 1, 2, 3, 4, 5, 6, 7, 8, 9, 12, 14, 16, and 24). The mean of this distribution is $\mu = 4.61$ and the standard deviation is 2.54.

2.2 Monte Carlo Simulation

Using Monte Carlo simulations, I conduct a variety of experiments to measure the impact of a variety of parameters on the revenue generated by Twitter. The experiments involve simulating individual Twitter sessions over and over again and averaging their results. In each session simulation, my code runs logic summarized in Algorithm 1.

In Experiment 1, I test the impact of the ad frequency distribution on the revenue-tolerance curve, defined as the relation between $N_{\text{tolerance}}$ and average revenue generated in a single session. For a range of ad tolerances (1-7) which likely cover most users, I first run Algorithm 1 for 5000 iterations. Next, I modify the algorithm such that instead of sampling a from a Poisson distribution, I sample from the empirical distribution illustrated in Figure 1. I run this modified algorithm for 5000 iterations.

In Experiment 2, I test the impact of average ad frequency on revenue, for a variety of ad tolerances. This is the main test of the model, in which I see how the competing forces of greater ad frequency and users quitting their sessions interact in practice. For a range of ad tolerances (2-5, narrowed down after the first experiment) and a range of μ (1-10), I run Algorithm 1 for 500 iterations.

Algorithm 1 Twitter session simulation

```
t ← 0
n_interact ← 0
while t < T_session do
    a ∼ Pois(μ)                                          ▷ Sample # of tweets before next ad
    t_tweets ∼ Expo(λ_t)                                 ▷ Sample view times for all a tweets
    t_ad ∼ Expo(λ_a)                                     ▷ Sample view time for next ad
    if # of ads in last 60s > N_tolerance then
        return n_interact · CPC
    else if t_ad > T_interact then
        n_interact ← n_interact + 1
    end if
    t ← t + t_tweets + t_ad
end while
return n_interact · CPC
```

3 Results & Analysis

The results of experiment 1 are displayed in Figure 2. The experiment shows that in spite of major differences between the theoretical and empirical distributions, the resulting revenue curves are not significantly different. This implies that for a given average number of tweets between ads μ, the exact distribution does not have a large impact on revenue generation. However, we do see a significant relationship between ad tolerance and revenue that holds for both distributions. Users with low ad tolerances (1-3) generate lower revenue for Twitter; however, once a user's tolerance is $N_{\text{tolerance}} \geq 4$, there is not a noticeable relationship between that tolerance and revenue generated.

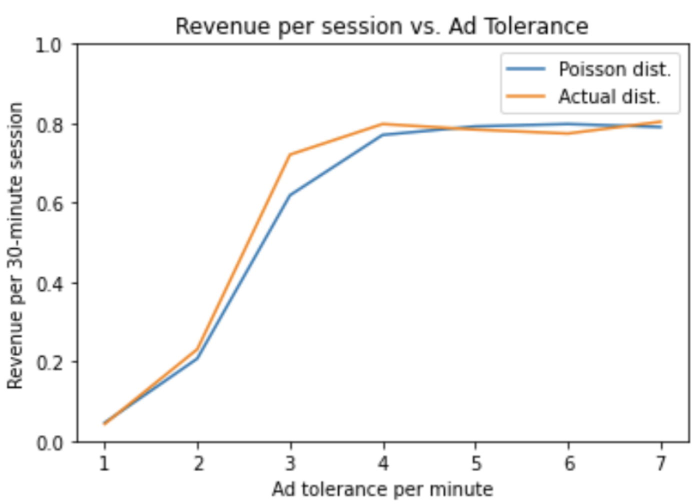

Figure 2: **Experiment 1 Results.** Plot of revenue versus ad tolerance, simulated with both Poisson distribution and empirical distribution.

Since the exact distribution has little effect, I proceed with experiment 2, in which I apply the Poisson distribution but with differing means μ. The results are shown in Figure 3. The plot demonstrates how the competing forces I discussed create concave

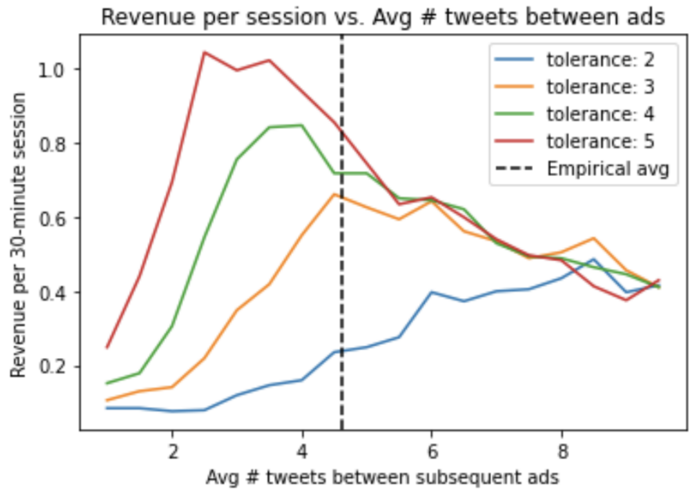

Figure 3: **Experiment 2 Results.** Plot of revenue versus mean # tweets between ads μ, for a range of ad tolerances. A vertical line has been drawn at the empirical μ.

curves for each ad tolerance, each peaking in revenue at a different μ. The pattern makes sense: for higher ad tolerances, showing ads more frequently results in the optimal revenue generation, while for lower ad tolerances, the inverse occurs.

Since the model simulates every post the user sees, we can also analyze the number of ads/tweets seen, the ratio of amount of time spent looking at ads vs. tweets, and the probablity a user ends their session early. These results are summarized in Table 1.

# Tweets	# Ads	# Ad interactions	Time on ads	Sessions quit early
131.0	28.4	0.46	10.4%	39.1%

Table 1: Mean simulation statistics for 30-minute sessions, with $\mu = 4.61, N_{\text{tolerance}} = 3$.

4 Discussion

These results reveal that the trade-off between increasing ad frequency and decreasing user engagement is very real, and can have a dramatic impact on the revenue generated by Twitter. As Figure 3 shows, the difference between optimal frequency and suboptimal frequency is stark; for example, for users with a higher ad tolerance of 5, showing ads at the optimal rate of every 2 tweets leads to double the revenue of showing ads at a suboptimal rate of every 8 tweets.

Further, I can use this simulation to infer Twitter's assumption for user ad tolerance, and the resulting revenue per user generated. Notice that the vertical dotted line in Figure 3, which marks the empirical value of $\mu = 4.61$, almost perfectly intersects the orange curve, representing the revenue-tolerance curve for $N_{\text{tolerance}} = 3$. This implies that, under the assumptions made, and *assuming that Twitter successfully optimizes its own revenue under these competing forces*, the average user ad tolerance is 3, leading to an average revenue per session of \$0.66.

One noticeable aspect of Figure 3 is that as μ increases, all of the ad tolerance curves seem to converge. This can be explained since if ads are shown very rarely, almost no users, no matter their ad tolerance, will be ending sessions early due to seeing too many ads, and thus these usersr will have experience the same results.

As Table 1 indicates, simulated users spend around 10% of the time viewing ads. However, a limitation of my project is that adjusting this ratio, i.e. adjusting λ_t and λ_a, would impact the ratio of time spent on ads, as well as the proportion of sessions ended early. This is because if a user spends less time viewing each ad, a greater proportion of time will be spent looking at tweets, leading to fewer ads per 60-second interval. I ran follow-up experiments in which I altered λ_a while keeping λ_t constant. Interestingly, as Figure 4 shows, lowering the time users spend viewing ads does not cause a major change in the optimal revenue earned for any given user tolerance.

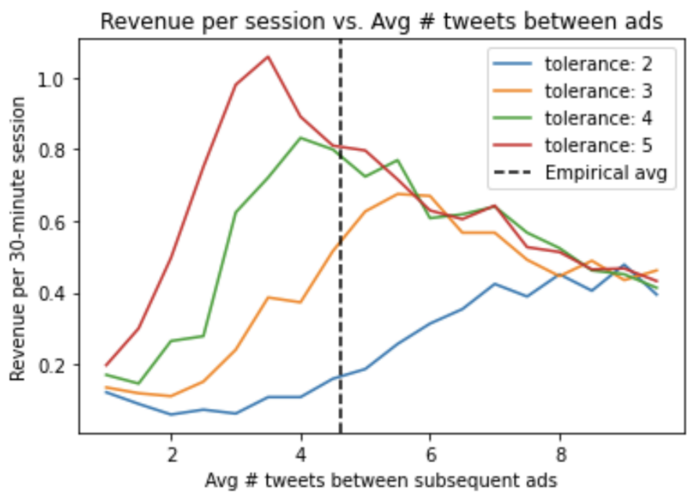

Figure 4: **Further experimentation.** Running experiment 2 again with $\lambda_a = 2$ yields similar looking curves, shifted slightly rightward.

The peaks of the curves shift rightward, indicating the optimal ad frequency should decrease, reflecting the higher rate of sessions ending early. However, further research is recommended into the relation between viewing habits and ad engagement in order to robustify this conclusion.

5 Conclusion

In this project, I presented the Tolerance-Frequency Ad Model (TFAM), a probabilistic model of ads in a Twitter feed, with the goals of estimating statistics and revenue generation from my own feed data and modeling the opposing interaction of ad tolerance and ad frequency. I apply Monte Carlo simulation to show that a Poisson process for ad frequency approximates the revenue generation results of the empirical distribution. Furthermore, I find that fixing most parameters other than ad tolerance and ad frequency reveals a concave revenue vs. frequency curve, showing that there exists an optimal frequency which maximizes Twitter's revenue.

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This project was my own personal endeavor, but I did consult a few other students for advice and help throughout the project. Aurelia Balkanski was very helpful in giving me the insight to use my own feed as a data source when I was struggling to find relevant data. Luke Stevens was essential in lending his knowledge of the mechanics of online advertising. Finally, I occasionally used ChatGPT for python coding assistance. I would give ChatGPT a simple prompt, such as "how to draw a vertical line on a plot using matplotlib," and would modify the resulting output to fit my existing code. ChatGPT assisted with the efficiency of simple coding tasks such as visualizations so I could spend more time thinking about the definition and development of the model.

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