第 13 考场使用！请确认你所在第 13 考场！

北京邮电大学 2019——2020 学年第二学期

Discrete Mathematics – Final Exam 13

考试注意事项	
考 试 注 意 事 项	一、学生参加考试须带学生证或学院证明，未带者不准进入考场。学生必须按照监考教师指定座位就坐。 二、书本、参考资料、书包等与考试无关的东西一律放到考场指定位置。 三、学生不得另行携带、使用稿纸，要遵守《北京邮电大学考场规则》，有考场违纪或作弊行为者，按相应规定严肃处理。 四、学生必须将答题内容做在试题答卷上，做在草稿纸上一律无效。

考试课程	离散数学	考试时间	2020 年 6 月 23 日

题号	一	二	三	四	五	六	七	八	九	十	十一	十二	十三	十四	总分
满分	5	10	10	10	10	5	6	6	6	6	6	8	8	4	
得分															
阅卷教师															

1. [5 points]

a) Which of these sentences are propositions? What are the truth values of those that are propositions?

i) $x + y = 100$. __________

ii) 80 is a perfect square. _________

iii) If you use a wrong test paper, then you will get an invalid score. _________

iv) Birds can fly, unless pigs can not fly. _________

v) To eliminate poverty in all poor counties and regions of China by 2020. _________

b) Let L(x, y) be the statement “x loves y,” and H(x,y) be the statement “ x hates y”, where the domain for both x and y consists of all people in the world. Translate each of these (1),(2) nested quantifications into a statement. And use quantifiers to express each of these statements (3),(4),(5).

（1） $\forall y\,(\neg\exists x\, H(y,x))$

（2） $\forall x\, \exists y L(x,y) \rightarrow \forall x\, \exists y H(x,y)$

（3） Everybody loves somebody and hates somebody

(4) Nobody loves everybody.

(5) There is somebody whom everybody loves and there is no one whom everybody hates.

姓名：　　班内序号：　　班级：　　装　订　线

2. [10 points] Show that $t\vee((r\to w)\wedge(r\to\neg s))$ and $(w\to s)\to(r\to t)$ are logically equivalent.

3. [10 points] Find the principal disjunctive normal form of (a) and (b).
   (a) $(\neg s \vee \neg t) \to (s \leftrightarrow \neg t)$

   (b) $(p\to (q \vee r)) \wedge (\neg p\to (\neg q \wedge \neg r))$

4. [10 points] Put the statement (a) and (b) in prenex normal form.
(a) $(\forall x P(x) \wedge \exists y Q(y)) \to \forall z W(z)$

(b) $\neg\exists x \exists y\, Q(x,y) \to (\exists z F(z) \vee R(x))$

5. [10 points] Show that the premises "There is someone in this class who has been to Guangdong," and " Everyone who goes to Guangdong visits Shenzhen" imply the conclusion "Someone in this class has visited Shenzhen."

6. [5 points] Prove that the equation $2x^3 + y^2 = 17$ has positive integer solution and explain the proof method you use.

7. [6 points] Find the Boolean product of A and B, where

$$A=\begin{bmatrix}1&0&1&0\\1&1&0&1\\0&0&1&1\end{bmatrix} \text{ and } B=\begin{bmatrix}1&0&1\\0&1&1\\1&1&0\\0&0&1\end{bmatrix}.$$

8. [6 points]
   a) Prove or disprove: if A, B and C are sets, then $(A - C) - (B - C) = A - B$.
   b) Give an example of a function f: $\mathbf{Z} \to \mathbf{N}$ that is both 1-1 and onto.

9. [6 points]Use Fermat's little theorem to evaluate 9^{20022} mod 13.

10. [6 points] Find all solutions, if any, to the system of congruences
    $x \equiv 3(mod\ 5), x \equiv 4(mod\ 7)$..

11. [6 points]List these functions so that each function is big-O of the next function in the list:
    $(100\log n)^2$, $n^2/10000$,$n^{1/2}$,$200 n^2 +100n + 1001$, 9^n, n^n, 2^n+c.

12. [8 points] Prove that the distributive law $B\cap(A_1 \cup\cdots\cup A_n) = (A_1\cap B) \cup\cdots\cup(A_n \cap B)$ is true for all $n\geq 2$

13. [8 points] How many ways can be made by distribute hands of 7 cards each to 3 players from a standard deck of 54 cards?

14. [4 points] INDISTINGUISHABLE OBJECTS AND INDISTINGUISHABLE BOXES Some counting problems can be solved by determining the number of ways to distribute indistinguishable objects into indistinguishable boxes. We illustrate this principle with an example.
Example： How many ways are there to pack six copies of the same book into four identical boxes, where a box can contain as many as six books?
Solution:We will enumerate all ways to pack the books. For each way to pack the books, we will list the number of books in the box with the largest number of books, followed by the numbers of books in each box containing at least one book, in order of decreasing number of books in a box. The ways we can pack the books are

    6
    5, 1
    4, 2
    4, 1, 1
    3, 3
    3, 2, 1
    3, 1, 1, 1
    2, 2, 2
    2, 2, 1, 1.

For example, 4, 1, 1 indicates that one box contains four books, a second box contains a single book, and a third box contains a single book (and the fourth box is empty). We conclude that there are nine allowable ways to pack the books, because we have listed them all.
Try to find how many ways there are to pack seven copies of the same book into three identical boxes, where a box can contain as many as seven books?