Bounding the Genus of a Graph: A Project Proposal

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Abstract—An embedding of a graph G on some surface Σ is a drawing of G on Σ such that the edges of G only intersect at their endpoints, namely the vertices. The genus of a surface Σ is the number of handles present on Σ, or the number of crosscaps in cases where Σ is a nonorientable surface. The genus of a graph is the minimum number of handles (or crosscaps, in cases of nonorientability) that mube be added to Σ in order for an embedding of G to be possible. The graph genus problem is given as follows: Given a graph G, find the (minimum) genus of G. In 1989, Thomassen proved that the graph genus problem is NP-Complete. However, in 1979 Filotti et al. gave a fixed paramater-tractable algorithm for the graph genus problem. The goal of this project is to investigate these papers and others to evaluate the current state of the graph genus problem. Approaches will then be developed to bound the genus of a graph G to a sufficiently small interval such that repeated execution of the algorithm in Filotti 1979 will be feasible. Therefore, the expected result of this project is a set of one or more heuristics that will reduce the search space for determining the genus of a graph.

Index Terms—Genus,Embedding,Surfaces

I. INTRODUCTION

Graph Embedding Problems lie at the intersection between Graph Theory and Topology. The problem in graph embedding of particular interest to this project is the graph genus problem, defined as follows: Given a Graph G = (V,E), find the minimum genus of G.

The remainder of this proposal is structured as follows: Section II gives a detailed definition of the graph genus problem. Section III constitutes a survey of literature relevant to the GGP. Next, section IV will discuss specific points of investigation. Section V will propose contributions related to those points, and section VI will detail the proposed programming components of this project.