DEPED - GENERAL SANTOS CITY
SIMPLIFIED
SELF-LEARNING MODULE
(SSLM)

GENSAN's ProTUNA
General Santos City's Pro-Active Response in Turbulent, Uncertain, New and Ambiguous Environment

GENERAL PHYSICS 1 - GRADE 12

Name: ______________________________ Date: ____________________

Grade: ______________________________ Section: ____________________

Quarter: **1** **Week:** **5** **SSLM No.** **5** **MELC(s):** Calculate the dot or scalar product of vectors (STEM_GP12WE-If-40); Determine the work done by a force acting on a system (STEM_GP12WE-If-41); Define work as a scalar or dot product of force and displacement (STEM_GP12WE-If-42); and Interpret the work done by a force in one-dimension as an area under a Force vs. Position Curve (STEM_GP12WE-If-43).

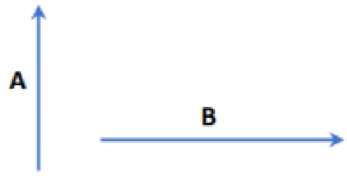
Title of Textbook/LM to Study: General Physics 1: Dot Product of Vectors & Work

Lesson 1. Dot Product of Vectors

- **Objectives:**
  1. Describe the idea of dot product;
  2. Calculate the magnitude of vectors; and
  3. Find angle between two vectors.

Let Us Discover

Let's consider the two vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$ as shown. So, we could be representing the dot product of vector A and vector B as $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$



Now, how do we get the dot or scalar product of these vectors?

Supposing, we have vector $\vec{\mathbf{A}}$ with the components (a,b,c) and vector $\vec{\mathbf{B}}$ with components (1,2,3), could be written as

$\vec{\mathbf{A}} = (a,b,c)$

$\vec{\mathbf{B}} = (1,2,3)$

Remember that *a* and *1* are the x components of each of the vectors. Thus, *b* and *2* as y components, and *c* and *3* as z components of both vectors. Simply multiply *a* by *1* for x components, plus *b* by *2* for y components, plus *c* by *3* for z component.

So, its dot product is $\vec{\mathbf{A}} \cdot \vec{\mathbf{B}}$ = **a1+b2+c3**

> **Note**: *i, j,* and *k* are universal unit vector notations signifying the x, y, and z planes, respectively.

For example, if we have two vectors

$\vec{\mathbf{M}}$ = 4i+6j+8k and

$\vec{\mathbf{N}}$ = 5i+3j+k

A. How do we find the dot product of vectors $\vec{\mathbf{M}}$ and $\vec{\mathbf{N}}$?

Applying the previous principle, we could have $\vec{\mathbf{M}} \cdot \vec{\mathbf{N}}$ = (4 x 5)+(6 x 3)+(8 x 1)

then,

|M·N| = 20+18+8 , accordingly

|M·N| = **46**

B. What is the angle between vectors $\vec{\mathbf{M}}$ and $\vec{\mathbf{N}}$?

Applying the Theorem of the Dot Product, which states that the dot product of two vectors is equal to the product of the magnitudes of the vectors times cosine of the angle between them, or

$$\vec{\mathbf{M}} \cdot \vec{\mathbf{N}} = |\mathrm{M}||\mathrm{N}| \cos(\theta)$$

where,

- $\vec{\mathbf{M}} \cdot \vec{\mathbf{N}}$ – the dot product of vectors $\vec{\mathbf{M}}$ and $\vec{\mathbf{N}}$
- |**M**| - the magnitude of vector $\vec{\mathbf{M}}$
- |**N**| - the magnitude of vector $\vec{\mathbf{N}}$
- θ - the angle between vectors $\vec{\mathbf{M}}$ and $\vec{\mathbf{N}}$

finding θ could be $$\mathbf{\cos\theta = \frac{M.N}{/M//N/}}$$

Please recall that |M·N| = 46

Finding |M| and |N|:

|M| $= \sqrt{(4)^2+(6)^2+(8)^2}$

$= \sqrt{16 + 36 + 64}$

$= \sqrt{116}$

|M| **= 10.7 units**

For |N| $= \sqrt{(5)2+(3)^2+(1)^2}$

$= \sqrt{25 + 9 + 1}$

|N| **= 5.9 units**

Substituting the values, $$\mathbf{\cos\theta = \frac{M.N}{/M//N/}}$$

Note:
M = 4i+6j+8k
N = 5i+3j+k

$= \dfrac{46}{(10.7)\,(5.9)}$

$= \dfrac{46}{63.13}$

$= \cos^{-1}(0.728655156)$

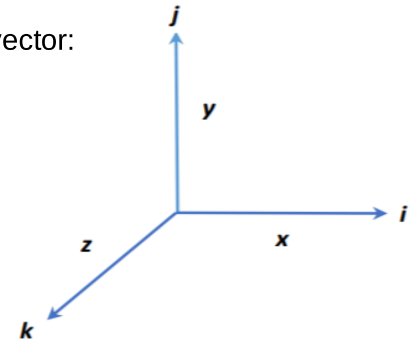
= 43.226 or

ϴ = 43°

Let Us Try

Read and understand the process in solving for the magnitude of the given vector.

1. Find the magnitude of the given vector:
$\vec{\mathbf{V}}$ = -4j+8k



Solution:

$$|V| = \sqrt{(0)^2 + (-4)^2 + (8)^2}$$
$$= \sqrt{0 + 16 + 64}$$
$$|V| = \sqrt{80}$$
$$|V| = 8.9 \text{ units}$$

Do what is asked.

2. Find the magnitude of the given vectors:
a. $\vec{\mathbf{P}}$ = 2i+5j-3k
b. $\vec{\mathbf{Q}}$ = 4i+6k

Let Us Do

Apply what you have learned on this lesson. Write your answer in an extra sheet of paper.

1. What does dot product of vectors will give you? (Please refer to the example involving vectors M and N).

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2. Is the dot product of vectors a scalar or a vector quantity? Discuss your answer.

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3. Given the two vectors $\vec{\mathbf{a}}$ = ⟨4i, 2j, 5k⟩ and $\vec{\mathbf{b}}$ = ⟨2i, 7j, 3k⟩
a. What is the dot product?
b. What is the angle between vectors $\vec{\mathbf{a}}$ and $\vec{\mathbf{b}}$?

Let Us Apply

Solve completely.

Find the approximate angle between vectors: $\vec{\mathbf{S}}$ = (2, 3, 5) and $\vec{\mathbf{T}}$ = (1, 6, -4).

Lesson 2. Work

- **Objectives:**
  1. Calculate the work done by a force acting on a system;
  2. Describe work as a scalar or dot product of force and displacement; and.
  3. Infer the work done by a force in one - dimension as an area under a Force vs. Position curve.

Let Us Discover

In physics, work is defined as a force acting upon an object, which results in a displacement of the object. Work is a familiar everyday concept. For example, it takes work to push a stalled car, lift a book above the table, or open a door. Force and displacement are the two essential elements of work. Mathematically, the work done on an object by a constant force (constant in both magnitude and direction) is defined as the product of the magnitude of the displacement times the component of the force in the direction of the displacement.

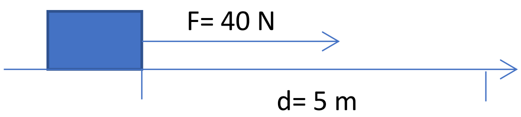
Work = Fnet Δd cos θ, where Fnet is the magnitude of the constant force, Δd is the magnitude of the displacement of the object, and θ is the angle between the direction of the force and the displacement. Note that if the displacement is zero, the work is zero, even if a force is applied. The θ factor is present because Fnetcos θ is the component of the force in the direction of the displacement. If the angle between the force and the displacement is zero, cos θ = 1.

Work is a scalar quantity—therefore, it has only a magnitude. The SI unit of work is a Newton-meter called the Joule (J), in honour of James Prescott Joule (1818–1889). However, note that work can be positive or negative, depending on whether work is gained or lost by the system.

Let Us Try

Consider the given problems involving work.

1. A 2.0-kg block is pulled over a 5.0-m distance by an applied force of 40.0 N which is directed in parallel to the displacement. How much work is done on the block by the force it moves at a constant speed?



Remember that the mass is a component of weight and that is influenced by the gravitational force which is directed vertically or in y-axis and not influencing the block's motion along the horizontal plane. Moreover, ***F*** *is parallel with* ***d****, thus,*

W = Fdcos θ

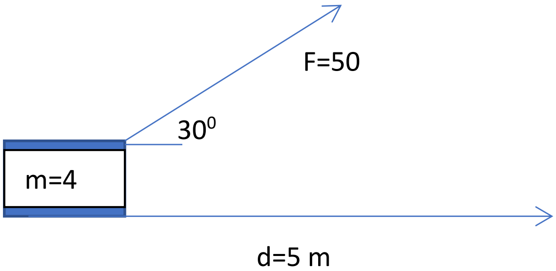
so that, = 40 N (5 m) cos 0^0

= 200 Nm (1)

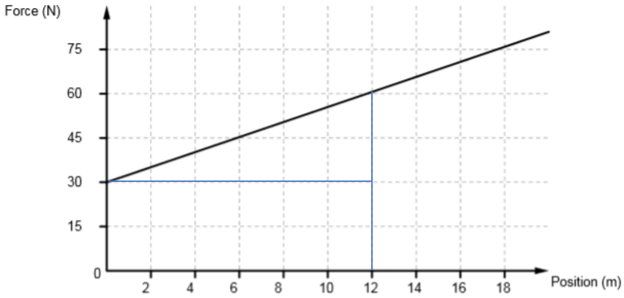
W = 200 Joules

2. A 50.0-N force is applied at an angle of 30^0 to the horizontal to move a 4.0-kg object at a constant speed for a horizontal distance of 5.0 m as shown. Find the work done by the applied force.

$W = Fd\cos\theta$
$= 50\text{ N }(5\text{ m})\cos 30^o$
$= 250\text{ Nm }(0.866)$
$= 216.506\text{ Nm}$
W = 217 Nm



3. How much work is done by a varying force that increases at a constant rate from 30 N to 60 N over a displacement of 12 m?



In calculating for the area under the s-curve, we need to consider the parts – the triangle and the rectangle.

First, computing the area of the rectangle is

$\boldsymbol{A_R = hb}$
$= (30-0)(12-0)$
$= (30)(12)$
$A_R = 360$

Next is for the area of the triangle,

$\boldsymbol{A_T = ½bh}$ where: b is for the base, and h is for the height
$= ½(12-0)(60-30)$
$= ½\,(12)(30)$
$= ½\,(360)$
$A_T = 180$

So that, the total area is

$\boldsymbol{A = A_R + A_T}$
$= 360 + 180$
$A = 540$

Since,

A = W

Then.

W = 540 Joules

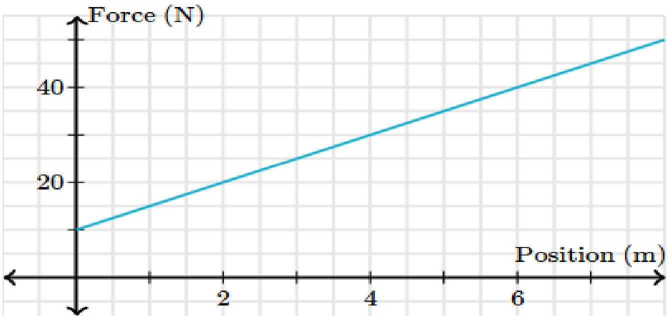
Let Us Do

Answer the given problems.

1. A box moves 10 m horizontally when force **F**=20 N is applied at an angle $\boldsymbol{\theta}$=120°. What is the work done on the box by the force during the displacement?

2. A box moves 10 m horizontally as a force F=20N is applied downward. What is the work done on the box by the force during the displacement?

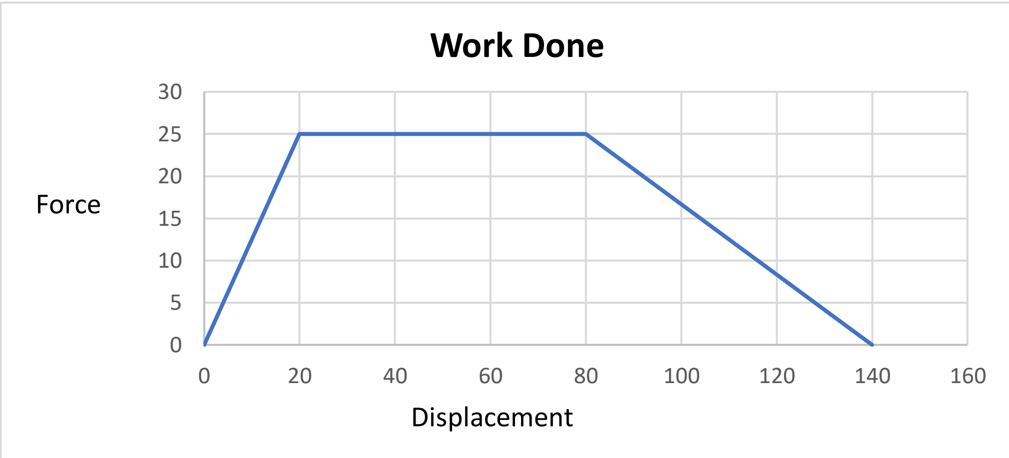
3. The net horizontal force on a box ***F*** as a function of the horizontal position ***P*** is shown below.



What is the work done on the box from 0 m to 5.0 m?

Let Us Apply

Find the total work done as shown in the graph. Show your complete solution.



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