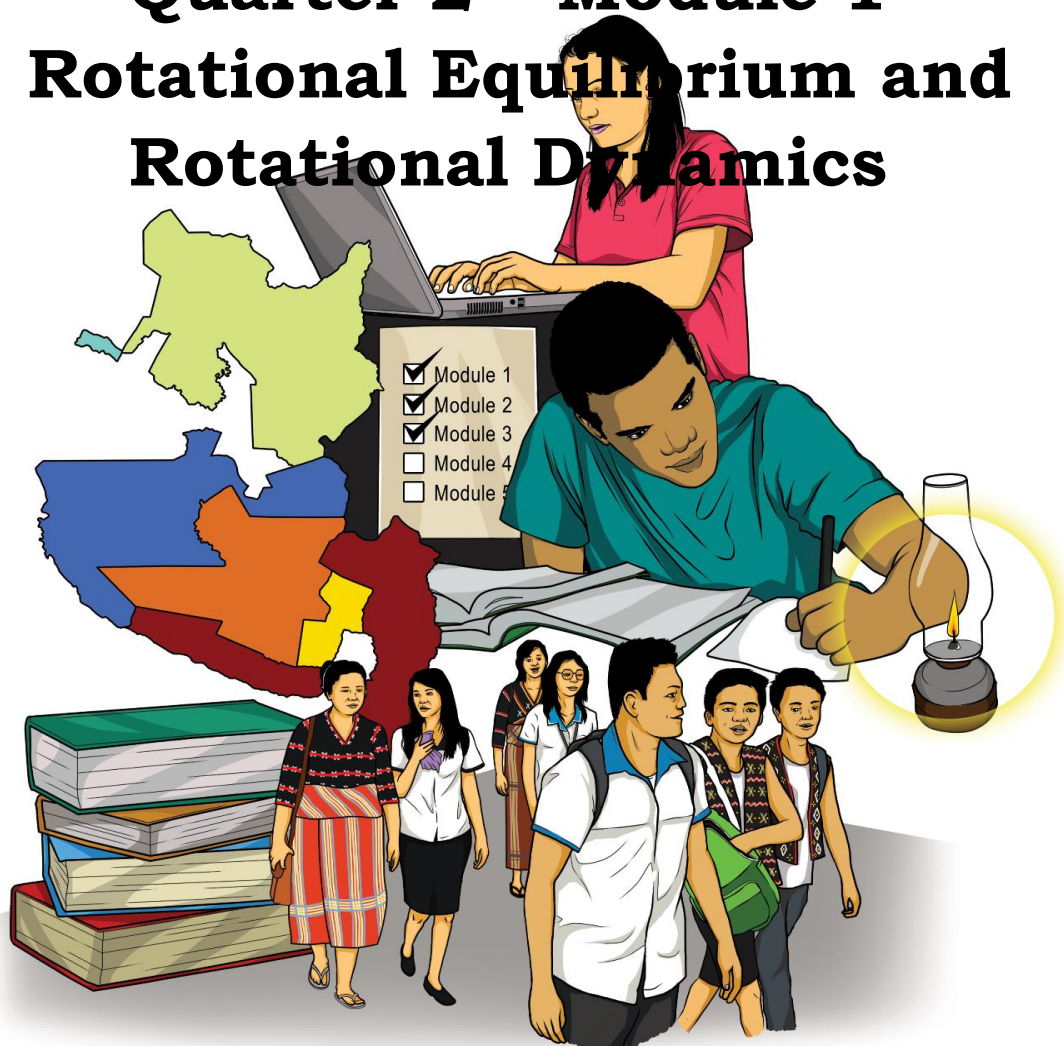




# General Physics 1

## Quarter 2 – Module 1

### Rotational Equilibrium and Rotational Dynamics



SELF-LEARNING MODULE



DEPARTMENT OF EDUCATION - SOCCSKSARGEN

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**Subject Area – Grade Level: General Physics 1 – Grade 12**  
**Self-Learning Module (SLM)**  
**Quarter 2 – Module 1: Rotational Equilibrium and Rotational Dynamics**  
**First Edition, 2020**

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**Development Team of the Module**

<b>Writers:</b> Emma T. Suritta	Judie E. Dela Cruz	Jerold F. Villalobos
John F. Villalobos	Anna Liza C. Suello	Jessa J. Jakil
Ligaya D. Maneja	Catherine S. Panaligan	

**Editors:** Retchie Joy B. PISAÑA

**Reviewers:** Blessy Mae M. Cabayao, Murdy F. Bautista, Jay Sheen A. Molina

**Illustrator:** Merbin M. Sulit

**Layout Artists:** Solomon P. Lebeco Jr.

**Cover Art Designer:** Ian Caesar E. Frondoza

**Management Team:**

Allan G. Farnazo, CESO IV – Regional Director  
Atty. Fiel Y. Almendra, CESO V – Assistant Regional Director  
Ruth L. Estacio PhD, CESO VI – OIC-Schools Division Superintendent  
Jasmin P. Isla - Assistant Schools Division Superintendent  
Gilbert B. Barrera – Chief, CLMD  
Arturo D. Tingson Jr. – REPS, LRMS  
Peter Van C. Ang-ug – REPS, ADM and Science  
Lalaine SJ. Manuntag PhD - CID Chief  
Nelida A. Castillo – EPS, LRMS  
Marichu Jean R. Dela Cruz -EPS – Science, ADM

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Office Address: Regional Center, Brgy. Carpenter Hill, City of Koronadal

Telefax: (083) 2288825/ (083) 2281893

E-mail Address: region12@deped.gov.ph

# Introductory Message

For the facilitator:

Welcome to the **General Physics 1 – Self - Learning Module (SLM)** on **Rotational Equilibrium and Rotational Dynamics**.

This module was collaboratively designed, developed and reviewed by educators both from public and private institutions to assist you, the teacher or facilitator in helping the learners meet the standards set by the K to 12 Curriculum while overcoming their personal, social, and economic constraints in schooling.

This learning resource hopes to engage the learners into guided and independent learning activities at their own pace and time. Furthermore, this also aims to help learners acquire the needed 21st century skills while taking into consideration their needs and circumstances.

In addition to the material in the main text, you will also see this box in the body of the module:



## ***Notes to the Teacher***

This contains helpful tips or strategies that will help you in guiding the learners.

As a facilitator you are expected to orient the learners on how to use this module. You also need to keep track of the learners' progress while allowing them to manage their own learning. Furthermore, you are expected to encourage and assist the learners as they do the tasks included in the module.

For the learner:

Welcome to the **General Physics 1** Self – Learning Module (SLM) on **Rotational Equilibrium and Rotational Dynamics**.

The hand is one of the most symbolized part of the human body. It is often used to depict skill, action and purpose. Through our hands we may learn, create and accomplish. Hence, the hand in this learning resource signifies that you as a learner is capable and empowered to successfully achieve the relevant competencies and skills at your own pace and time. Your academic success lies in your own hands!

This module was designed to provide you with fun and meaningful opportunities for guided and independent learning at your own pace and time. You will be enabled to process the contents of the learning resource while being an active learner.

This module has the following parts and corresponding icons:



***What I Need to Know***

This will give you an idea of the skills or competencies you are expected to learn in the module.



***What I Know***

This part includes an activity that aims to check what you already know about the lesson to take. If you get all the answers correct (100%), you may decide to skip this module.



***What's In***

This is a brief drill or review to help you link the current lesson with the previous one.



***What's New***

In this portion, the new lesson will be introduced to you in various ways such as a story, a song, a poem, a problem opener, an activity or a situation.



***What is It***

This section provides a brief discussion of the lesson. This aims to help you discover and understand new concepts and skills.



***What's More***

This comprises activities for independent practice to solidify your understanding and skills of the topic. You may check the answers to the exercises using the Answer Key at the end of the module.



***What I Have Learned***

This includes questions or blank sentence/paragraph to be filled in to process what you learned from the lesson.



### ***What I Can Do***

This section provides an activity which will help you transfer your new knowledge or skill into real life situations or concerns.



### ***Assessment***

This is a task which aims to evaluate your level of mastery in achieving the learning competency.



### ***Additional Activities***

In this portion, another activity will be given to you to enrich your knowledge or skill of the lesson learned. This also tends retention of learned concepts.



### ***Answer Key***

This contains answers to all activities in the module.

At the end of this module you will also find:

### ***References***

This is a list of all sources used in developing this module.

The following are some reminders in using this module:

1. Use the module with care. Do not put unnecessary mark/s on any part of the module. Use a separate sheet of paper in answering the exercises.
2. Don't forget to answer *What I Know* before moving on to the other activities included in the module.
3. Read the instruction carefully before doing each task.
4. Observe honesty and integrity in doing the tasks and checking your answers.
5. Finish the task at hand before proceeding to the next.
6. Return this module to your teacher/facilitator once you are through with it.

If you encounter any difficulty in answering the tasks in this module, do not hesitate to consult your teacher or facilitator. Always bear in mind that you are not alone.

We hope that through this material, you will experience meaningful learning and gain deep understanding of the relevant competencies. You can



## ***What I Need to Know***

This module contains three (3) lessons which are:

Lesson 1: Moment of Inertia

Lesson 2: Torque

Lesson 3: Static Equilibrium, Rotational Kinematics, and Work Done by a Torque

Learning Objectives:

1. Define and describe operationally moment of inertia;
2. Calculate the magnitude of moment inertia in a given system;
3. Define and describe torque;
4. Calculate the magnitude and direction of torque using the definition of torque as a cross product;
5. Describe rotational quantities using vectors;
6. Determine whether a system is in static equilibrium or not; and
7. Apply the rotational kinematic relations for systems with constant angular acceleration.



## ***What I Know***

### Pre-Assessment

**Multiple Choice.** Read the questions carefully. Write the letter of your answer in your notebook or answer sheet provided.

1. The moment of inertia of an object is greater when most of the mass is located in what area?
  - a. off-center
  - b. near the axis
  - c. on the rotational axis
  - d. away from the rotational axis
2. What quantity describes the ability of a force to rotate an object?
  - a. mass
  - b. moment of inertia
  - c. torque
  - d. velocity
3. Which of these statements best describes the torque produced when two children of different weights balance on a seesaw?
  - a. equal torques
  - b. unequal torques
  - c. equal torques in the same direction
  - d. equal torques in opposite direction
4. A force of 250 N is exerted on a cable wrapped around a drum that has a diameter of 120 mm. What is the torque produced about the center of the drum?
  - a. 15 N
  - b. 30 N
  - c. 45 N
  - d. 60 N
5. A 4.0 kg block travels around a 0.50 m radius circle with an angular velocity of 12 rad/s. What is its angular momentum at the center of the circle?
  - a. 12 kgm<sup>2</sup>/s
  - b. 24 kgm<sup>2</sup>/s
  - c. 6 kgm<sup>2</sup>/s
  - d. 3 kgm<sup>2</sup>/s
6. Which of the following tells if an object is in equilibrium?
  - a. the object is in motion
  - b. the forces acting are not equal
  - c. the object moving at a constant speed
  - d. the net force is equal to zero
7. What type of forces will cause an object to rotate?
  - a. concurrent forces
  - b. equilibrium forces
  - c. normal forces
  - d. non-concurrent forces
8. How many radians are there in 60 revolutions?
  - a. 0.49 rad
  - b. 0.98 rad
  - c. 2.5 rad
  - d. 18.8 rad
9. How long will the helicopter blade turn through 200 radians if its angular velocity is 60rad/s?
  - a. 2 seconds
  - b. 3.3 seconds
  - c. 4 seconds
  - d. 6 seconds

10. A bowling ball has a mass of 5.5 kg and a radius of 12.0 cm. It is released so that it rolls down the alley at a rate of 12 revs/s. What is the magnitude of its angular momentum?

- a.  $0.76 \text{ kgm}^2/\text{s}$       b.  $1.8 \text{ kgm}^2/\text{s}$       c.  $4.1 \text{ kgm}^2/\text{s}$       d.  $5.1 \text{ kgm}^2/\text{s}$

11. What is maximum torque on a 800 turn circular coil of radius 0.75 cm that carries a current of 1.9 mA and resides in a uniform magnetic field of 0.25T?

- a.  $6.41 \times 10^{-5} \text{ N.m}$       b.  $6.51 \times 10^{-5} \text{ N.m}$   
c.  $6.61 \times 10^{-5} \text{ N.m}$       d.  $6.71 \times 10^{-5} \text{ N.m}$

12. A child spins a top with a radius of 2 cm with a force of 0.5 N. How much torque is generated at the edge of the top?

- a.  $0.05 \text{ N.m}$       b.  $0.01 \text{ N.m}$       c.  $0.75 \text{ N.m}$       d.  $0.25 \text{ N.m}$

13. An object moves at a constant speed of 9.0 m/s in a circular path of radius of 1.5 m. What is the angular acceleration of an object?

- a.  $14 \text{ rad/s}^2$       b.  $36 \text{ rad/s}^2$       c.  $81 \text{ rad/s}^2$       d.  $54 \text{ rad/s}^2$

14. Which quantity below refers to any system where the sum of the force and torque on every particle of the system happens to be zero?

- a. rotational kinematics      b. Moment of inertia  
c. static equilibrium      d. torque

15. Which quantity below refers to moment of force, in physics, the tendency of a force to rotate the body to which it is applied?

- a. rotational kinematics      b. moment of inertia  
c. static equilibrium      d. torque



## Lesson

# 1

## Moment of Inertia

Learning Objectives:

1. Define and describe operationally moment of inertia; and
2. Calculate the magnitude of moment of inertia in a given system.



### *What's In*

#### **Activity 1. Ask Me!**

Direction: Read each situation below and identify what is it. Write your answer on a separate sheet of paper or on your activity notebook.

1. It refers to the location and considered that the mass of the whole system is concentrated at its center. It is the function of position and masses of particles that comprise the system. What is it?
2. It is the product of mass and velocity.
3. It is a term that quantifies the overall effect of a force acting over time.

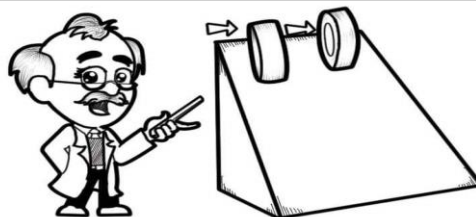


### *What's New*

#### **Activity 2: Roll and Drop Toilet Paper!**

Directions: Try to do the activity and analyze the situation. Compare the two systems. Answer the question below based on your observation. Write your answers on your activity notebook.

Toilet Paper Roll Drop – The Great Debate



Questions:

1. What is the spool's inner radius,  $R$ ? \_\_\_\_\_
2. At what time does the spool reach a rotation rate of 400 rpm? \_\_\_\_\_
3. If the spool holds only 40 m of paper and if the child maintains the same constant acceleration, at what time will the spool run out of paper? \_\_\_\_\_
4. At the moment that the spool runs out of paper, how many revolutions were executed? (Even though paper is unspooling from the roll, assume that  $R_1$  and  $M$  remains constant).



## What Is It

**Moment of Inertia** is defined as a measure of an object's resistance to changes to its rotation. It also tells the capacity of a cross-section to resist bending. It must be specified with respect to a chosen axis of rotation. It is usually quantified in  $\text{m}^4$  or  $\text{kgm}^2$ . It is also a name given to rotational inertia, the rotational analog of mass for linear motion. It appears in the relationships for the dynamics of rotational motion. The moment of inertia must be specified with respect to a chosen axis of rotation. For a point mass, the moment of inertia is just the mass times the square of perpendicular distance to the rotation axis,  $I = mr^2$ . That point mass relationship becomes the basis for all other moments of inertia since any object can be built up from a collection of point masses. The moment of inertia of a point mass with respect to an axis is defined as the product of the mass times the distance from the axis squared.

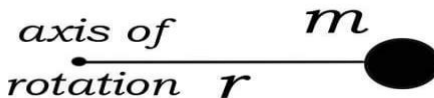
The total moment of inertia is due to the sum of masses at a distance from the axis of rotation. It shows below:

	$I = mr^2$	<p>For a point mass the moment of inertia is just the mass times the radius from the axis squared. For a collection of point masses (below) the moment of inertia is just the sum for the masses.</p>
	$I = kmr^2$	<p>For an object with an axis of symmetry, the moment of inertia is some fraction of that which it would have if all the mass were at the radius <math>r</math>.</p>
	$I = \sum_i m_i r_i^2 = m_1 r_1^2 + m_2 r_2^2 + m_3 r_3^2 + \dots$	<p>Sum of the point mass moments of inertia.</p>
	$I = \int_0^M r^2 dm$	<p>Continuous mass distributions require an infinite sum of all the point mass moments which make up the whole. This is accomplished by an integration over all the mass.</p>

A collection of two masses has a moment of inertia due to each separate mass.

$$I = mr^2 + mr^2 = 2mr^2$$

Since the moment of inertia of an ordinary object involves a continuous distribution of mass at a continually varying distance from any rotation axis, the calculation of moments of inertia generally involves calculus, the discipline of mathematics which can handle such continuous variables. Since the moment of inertia of a [point mass](#) is defined by

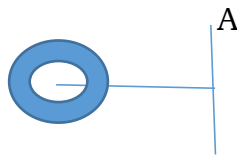
$$I = mr^2$$


The diagram illustrates the formula  $I = mr^2$ . It shows a horizontal line representing the distance  $r$  from a point labeled 'axis of rotation' to a solid black circle representing a point mass  $m$ .

Examples:

1. A 100 gram ball connected to one end of a cord with a length of 30 cm. What is the moment of inertia of a ball about the axis of rotation AB? Ignore cord's mass.

Illustration:



Given: The axis of rotation at AB  
 Mass ball ( $m$ ) = 100 gram/1000 = 0.1 kg  
 The distance between the ball and the axis rotation ( $r$ ) = 30 cm = 0.3 m

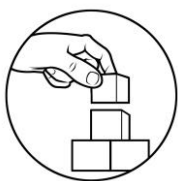
Unknown: Moment of inertia of a ball ( $I$ )

Solution:

$$I = mr^2 = (0.1 \text{ kg})(0.3 \text{ m})^2$$

$$I = (0.1 \text{ kg})(0.09 \text{ m}^2)$$

$$I = 0.009 \text{ kgm}^2$$

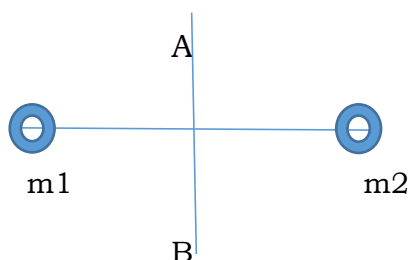


## ***What's More***

### **Activity 3: Calculate Me!**

1. A 100 gram ball  $m_1$ , and a 200 gram ball,  $m_2$ , connected by a rod with a length of 60 cm. The mass of the rod is ignored. The axis of rotation located at the center of the rod. What is the moment of inertia of the balls about the axis of rotation?

Illustration:



## ***What I Have Learned***

### **Activity 4. Complete Me!**

Direction: Fill out the necessary word/s in the blank/s in every sentence below. Write your answer on your answer sheet.

1. The moment of inertia is defined with respect to a specific rotation \_\_\_\_\_.
2. The moment of inertia of a \_\_\_\_\_ with respect to an axis is defined as the product of the mass times the distance from the axis squared.
3. Moment of inertia is the name given to \_\_\_\_\_.
4. The point mass relationships become the \_\_\_\_\_ for all the other moments inertia.
5. The unit of moment of inertia of an area is \_\_\_\_\_.



## ***What I Can Do***

Draw or sketch a diagram that will describe the moment of inertia in a multi-system.

## Lesson

# 2

## Torque

Learning Objectives:

1. Define and describe torque; and
2. Calculate the magnitude and direction of torque using the definition of torque as a cross product.

When studying how objects rotate, it quickly becomes necessary to figure out how a given force results in a change in the rotational motion. The tendency of a force to cause or change rotational motion is called ***torque***, and it's one of the most important concepts to understand in resolving rotational motion situations.

***Torque*** (also called moment — mostly by engineers) is calculated by multiplying force and distance. The SI units of torque are newton-meters, or  $\text{N}\cdot\text{m}$  (even though these units are the same as Joules, torque isn't work or energy, so should just be newton-meters).

Torque is a measure of the force that can cause an object to rotate about an axis. Just as force is what causes an object to accelerate in linear kinematics, torque is what causes an object to acquire angular acceleration.

Torque is a vector quantity. The direction of the torque vector depends on the direction of the force on the axis.



### ***What's In***

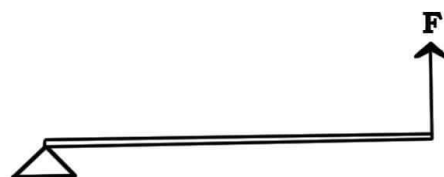
How was your performance in the test? As you go through this module, you will be able to deepen your understanding in our topic and do better in the next test.

In lesson 1, you were able to calculate the moment of inertia about a given axis of single-object and multiple-object systems. Here in lesson 2, you will be introduced to the concept on how to calculate magnitude and direction of torque using the definition of torque as a cross product.

Let us have a sample problem.

#### **Example Number 1:**

A force of  $5.0\text{ N}$  is applied at the end of a lever that has a length of  $2.0\text{ meters}$ . If the force is applied directly perpendicular to the lever, as shown in the diagram above, what will be the magnitude of the torque acting on the lever?



**Solution:**

This sample is a simple matter of plugging the values into the equation:

$$\text{Torque} = F * l$$

$$\text{Torque} = 5.0\text{N} * 2.0\text{m}$$

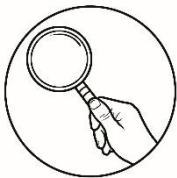
$$\text{Torque} = 10\text{N}\cdot\text{m}$$

**Activity Number 1. Problem on Torque**

1. If the same force as in example 1 is applied at an angle of 30 degrees at the end of the 2.0 meters lever, what will be the magnitude of the torque?
2. What force is necessary to generate a 20.0 N\*m torque at an angle of 50 degrees from along a 3.00 m rod?

***What' New***

In a previous lecture, we discussed one type of product of two vectors known as dot product, which gives us a scalar value. A very common example of such a product is work. Work is a scalar quantity that has only magnitude and no direction and is given by taking the product of the force and displacement vectors. Cross product (or vector product) is the product of two vectors that produces a third vector. This means that the result has both magnitude as well as direction. We can use different methods to calculate the cross product. One way to calculate it is to take the product of the magnitudes of the two vectors and multiply by the sine of the angle between them. Note that the direction of the vector is given by the right hand rule and is always perpendicular to the two vectors. If we are given two three-dimensional vectors and we are asked to calculate the cross product vector, we have to use the method as described in the lecture.

***What is It*****Torque as a vector quantity**

The vector or cross product is another way to combine two vectors; it creates a vector perpendicular to both the originals. In vector form, torque is the cross product of the radius vector (from axis of rotation to point of application of force) and the force vector.

Torque is the twisting **force** that tends to cause rotation. The **point** where the object rotates is known as the axis of rotation. Mathematically, torque can be written as  $T = F * r * \sin(\theta)$ , and it has units of Newton-meters.

Recall that torque is a vector and is obtained by taking the product of two vectors.

Cross-product  $\vec{A} \times \vec{B} = \vec{C}$  {//  
 $C = |\vec{A}| |\vec{B}| \sin\theta$  magnitude of the vector  $\vec{C}$  **Direction**  
**of  $\vec{C}$ :** Orient your fingers in direction of  $\vec{A}$  and curl in direction of  $\vec{B}$ , then your thumb will point in direction of  $\vec{C}$ . Note that the direction of  $\vec{C}$  is always perpendicular to  $\vec{A}$  and perpendicular to  $\vec{B}$ .

$$\text{If } \vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \text{ and } \vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

For the first element of the first row, the  $\hat{i}$ , take the product down and to the right,

$$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array}$$

( this yields  $\hat{i}(A_y B_z - A_z B_y)$  ) minus the product down and to the left

$$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{array}$$

( the product down-and-to-the-left is  $\hat{i}(A_z B_y - A_y B_z)$  )

For the first element in the first row, we thus have:  $\hat{i}(A_y B_z - A_z B_y) - \hat{i}(A_z B_y - A_y B_z)$  which can be written as:  $(A_y B_z - A_z B_y) \hat{i}$ . Repeating the process for the second and third elements in the first row (the  $\hat{j}$  and the  $\hat{k}$ ) we get  $(A_z B_x - A_x B_z) \hat{j}$  and  $(A_x B_y - A_y B_x) \hat{k}$  respectively. Adding the three results, to form the determinant of the matrix results in:

$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

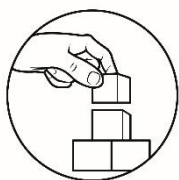
$$\vec{A} \times \vec{B} = (A_y B_z - A_z B_y) \hat{i} + (A_z B_x - A_x B_z) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

**Example Number 2:**

Suppose the lever arm vector  $\vec{r}$  is given by the equation  $\vec{r} = 5 \hat{i} + 6 \hat{j} + 1 \hat{k}$ . Calculate the torque if the force is  $F = 10 \hat{i}$  N

Solution:

$$\begin{aligned}
 \text{Torque} &= (5 \hat{i} + 6 \hat{j} + 1 \hat{k})(10 \hat{i} + 0 \hat{j} + 0 \hat{k}) \\
 &= (6 \times 0 - 1 \times 0) \hat{i} + (1 \times 10 - 5 \times 0) \hat{j} + (5 \times 0 - 6 \times 10) \hat{k} \\
 &= (0 - 0) \hat{i} + (10 - 0) \hat{j} + (0 - 60) \hat{k} \\
 &= 0 \hat{i} + 10 \hat{j} + -60 \hat{k} \\
 &= \sqrt{(10 \hat{j})^2 + (-60 \hat{k})^2} \\
 &= \sqrt{10^2 + 60^2} \\
 &= \sqrt{100 + 3600} \\
 &= \sqrt{3700} \\
 &= 60.83 \text{ N.m ( Magnitude of the Torque )}
 \end{aligned}$$

***What's More*****Activity Number 2. FIND MY  $\vec{A} \times \vec{B}$ !**

$$\vec{A} = 3 \hat{i} + 2 \hat{j} - 5 \hat{k}$$

$$\vec{B} = 2 \hat{i} - 6 \hat{j} + 4 \hat{k}$$

Solve for  $\vec{A} \times \vec{B}$ ?

***What I Have Learned***

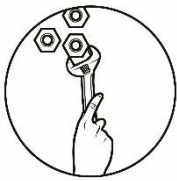
From the very start of our lesson, you have learned a lot about torque as a vector. In the succeeding activities, you will make generalizations of your learnings.



Now after we have done varied activities let us check how far you understood our lesson.

Answer the following questions as brief as you can.

1. Define operationally torque as a vector quantity.
2. Where does the direction of the torque vector depend?
3. What should always be the direction of  $\vec{C}$ ?



## ***What I Can Do***

### **Activity Number 3. FIND MY $\vec{A} \times \vec{B}$ and $\vec{C}$**

$$\vec{A} = 4 \hat{i} - 2 \hat{j} - 6 \hat{k}$$

$$\vec{B} = 5 \hat{i} + 4 \hat{j} + 4 \hat{k}$$

**Solve for:**

1.  $\vec{A} \times \vec{B}$

2.  $\vec{C}$

## Lesson

# 3

# Rotational Kinematics and Static Equilibrium

Learning Objectives:

1. Describe rotational quantities using vectors;
2. Determine whether a system is in static equilibrium or not; and
3. Apply the rotational kinematic relations for systems with constant angular acceleration.

By using vectors, we can define the angular quantities for rotation about an axis that can point in any direction. A **vector** is an object that has both a magnitude and a direction. Geometrically, we can picture a vector as a directed line segment, whose length is the magnitude of the vector and with an arrow indicating the direction.

**Rotation**—refers to the motion of a body turning about an axis, where each particle of the body moves along a circular path. Angular Acceleration can be thought of as the rotational equivalent of straight-line acceleration.

Physicists use angular quantities to describe rotation. The equations of kinematics and dynamics of rectilinear motion can be rewritten in terms of angular quantities. Each of the rectilinear motion of quantities has its rotational analog.



## What's In

How did you fare in the test? As you go through this module, you will be able to deepen your understanding in our topic and do better in the next test.

In lesson 2, you were able to calculate the magnitude and direction of torque using the definition of torque as a cross product. Here in lesson 3, you will be introduced to the concept on how describing rotational quantities using vectors, determining whether a system is in equilibrium or not and applying the rotational kinematic relation with constant angular acceleration.

Let's take example 1

A flywheel, in the form of uniform disk, is 4.0 m in diameter and weighs 637 N. What will be its angular acceleration if it is activated upon by net torque of 225J ?

Given:  $w = 637 \text{ N}$  or  $m = 65 \text{ kg}$

Find:  $\alpha = ?$

$r = 2.0 \text{ m}$

$\tau_{\text{net}} = 225 \text{ N.m}$

$I_{\text{uniform disk}} = \frac{1}{2}Mr^2$

Solution:  $\alpha = \tau / I = 225\text{-Nm} / 65\text{kg} (2\text{m})^2 / 2 = 1.73 \text{ rad/s}^2$



## What's New

Using Physics, you can calculate the angular acceleration of an object in circular motion.

Problem 1 : When you switch your room fan from medium to high speed, the blades accelerate at 1.2 radians per second squared for 1.5 seconds. If the initial angular speed of the fan blades is 3.0 radians per second, what is the final angular speed of the fan blades in radians per second ?

Angular is defined by  $\alpha = \frac{\Delta \hat{\omega}}{\Delta t}$

Where:  $\Delta \hat{\omega}$  = final angular speed minus initial angular speed

$\Delta t$  = time over which the angular speed changes, so you can write

$\Delta \hat{\omega} = \hat{\omega}_f - \hat{\omega}_i$  where  $\hat{\omega}_i = 3.0$  radians per second

Solve the equation for acceleration for the final angular speed and plug in the known quantities to get the answer. The result is

$$\alpha = \frac{\Delta \hat{\omega}}{\Delta t}$$

$$= \frac{\hat{\omega}_f - \hat{\omega}_i}{\Delta t}$$

$$\alpha \Delta t = \hat{\omega}_f - \hat{\omega}_i$$

$$\hat{\omega}_f = \alpha \Delta t + \hat{\omega}_i$$

$$= 1.2 \text{ rad/s}^2 \times 1.5 \text{ s} + 3.0 \text{ rad/s}$$

$$= 4.8 \text{ rad/s}$$



## What is It

In a previous lecture, you calculated the magnitude and direction of torque using the definition of torque as a cross product. Torque can be described as the rotational equivalent of straight-line force. Angular acceleration can be thought of as the rotational equivalent of straight-line acceleration. There is also a rotational equivalent of mass: rotational inertia, represented by the upper case italic letter  $I$ , in some text it is called moment of inertia. Thus, following Newton's second law of motion, torque is computed as  $\tau = I\alpha$  in N-m.

If the rotation about an axis is to be changed, torque must be applied. The rotational inertia of a particle about the chosen axis is the product of the mass of the particle and the square of the radius,

$$I = mr^2$$

With units of kilogram-meter squared ( $\text{kg}\cdot\text{m}^2$ )

In the simplest kind of rotation, points on a rigid object move on circular paths around an **axis of rotation**.

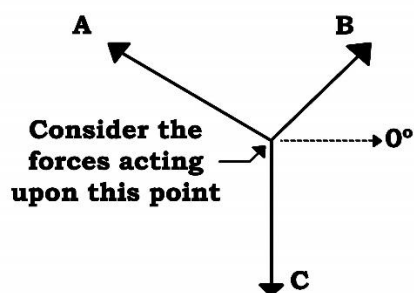
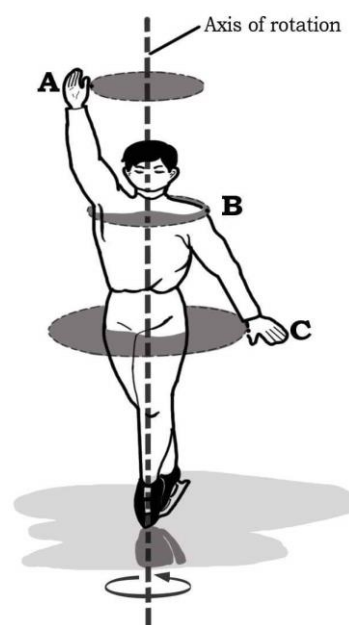
The term *equilibrium* has been used before to describe how an object experiences zero net force, and thus, its motion is unchanged. (If it rest, it will remain at rest; if in uniform motion, it will remain in uniform motion.) The Law of inertia actually describes the first condition of equilibrium, the case of linear equilibrium or zero linear acceleration.

The second condition of equilibrium is rotational equilibrium. As discussed in the previous lesson, when a number of forces are acting on an object, the total torque is the sum of the individual torques produced by each force acting on the object. In the event that the sum of these individual torques turns out to be zero, this means we have attained rotational equilibrium.

The second condition for equilibrium may be written as

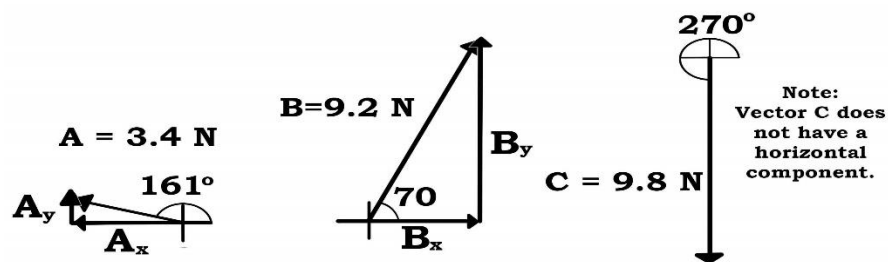
$$\sum \tau = 0$$

Thus, if all the forces are added together as vectors, then the resultant force (the vector sum) should be 0 Newton. (Recall that the net force is "the vector sum of all the forces" or the resultant of adding all the individual forces head-to-tail.) Thus, an accurately drawn vector addition diagram can be constructed to determine the resultant. Sample data for such a lab are shown below.



	Force A	Force B	Force C
<b>Magnitude</b>	3.4 N	9.2 N	9.8 N
<b>Direction</b>	161 deg.	70 deg.	270 deg

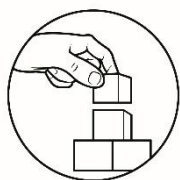
Another way of determining the net force (vector sum of all the forces) involves using the trigonometric functions to resolve each force into its horizontal and vertical components. Once the components are known, they can be compared to see if the vertical forces are balanced and if the horizontal forces are balanced. The diagram below shows vectors A, B, and C and their respective components. For vectors A and B, the vertical components can be determined using the sine of the angle and the horizontal components can be analyzed using the cosine of the angle. The magnitude and direction of each component for the sample data are shown in the table below the diagram.



Force	Horizontal Component (PSYW)	Vertical Component (PSYW)
A	$A_x = 3.4 \text{ N} \cdot \cos(161^\circ)$ $A_x = 3.2 \text{ N}$ , left	$A_y = 3.4 \text{ N} \cdot \sin(161^\circ)$ $A_y = 1.1 \text{ N}$ , up
B	$B_x = 9.2 \text{ N} \cdot \cos(70^\circ)$ $B_x = 3.1 \text{ N}$ , right	$B_y = 9.2 \text{ N} \cdot \sin(70^\circ)$ $B_y = 8.6 \text{ N}$ , up
C	$C_x = 0 \text{ N}$	$C_y = 9.8 \text{ N}$ , down

The data in the table above show that the forces *nearly* balance. An analysis of the horizontal components shows that the leftward component of A *nearly* balances the rightward component of B. An analysis of the vertical components show that the sum of the upward components of A + B *nearly* balance the downward component of C. The vector sum of all the forces is (*nearly*) equal to 0 Newton. But what about the 0.1 N difference between rightward and leftward forces and the 0.2 N difference between the upward and downward forces? Why do the components of force only *nearly* balance? The sample data used in this analysis are the result of measured data from an actual experimental setup. The difference between the actual results and the expected results is due to the error incurred when measuring force A and force B. We would have to conclude that this low margin of experimental error reflects an experiment with excellent results. We could say it's "close enough for government work."

In conclusion, equilibrium is the state of an object in which all the forces acting upon it are balanced. In such cases, the net force is 0 Newton. Knowing the forces acting upon an object, trigonometric functions can be utilized to determine the horizontal and vertical components of each force. If at equilibrium, then all the vertical components must balance and all the horizontal components must balance.



## What's More

### Activity Number 1. Let's Solve!

1. The radius of a car tire is about 0.35 meters. If the car accelerates in straight line from rest at 2.8 meters per second squared, what is the angular acceleration, both magnitude and direction, of the front passenger-side tire?

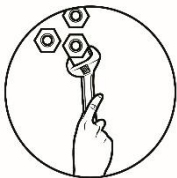
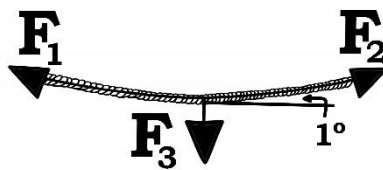


## ***What I Have Learned***

From the very start of our lesson, you have learned a lot about describing rotational quantities using vectors as well as determining whether a system is in static equilibrium or not and applying the rotational kinematic relation for a system with constant angular acceleration. In the succeeding activities, you will make generalizations of your learnings.

Now after we have done varied activities let us check how far you understood our lesson.

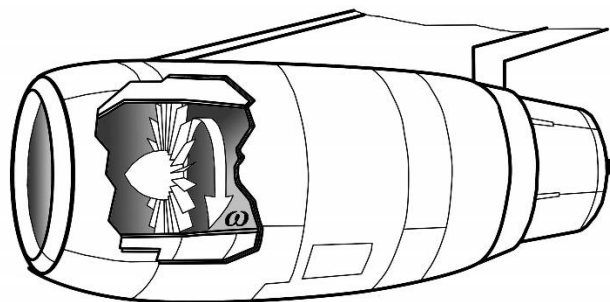
1. Suppose that a student pulls with two large forces ( $F_1$  and  $F_2$ ) in order to lift a 1-kg book by two cables. If the cables make a 1-degree angle with the horizontal, then what is the tension in the cable?



## ***What I Can Do***

### **A Jet Revving Its Engines**

As seen from the front of the engine, the fan blades are rotating with an angular speed of  $-110 \text{ rad/s}$ . As the plane takes off, the angular velocity of the blades reaches  $-330 \text{ rad/s}$  in a time of 14 s. Find the angular acceleration, assuming it to be constant.





## ASSESSMENT

Please read carefully and answer the questions below. Choose the letter of the best answer and write it on the separate sheet of paper.

1. A man spinning in free space changes the shape of his by e.g. by stretching his arms or curling up then, which of the following physical quantity will remain constant?  
a. moment of inertia      b. angular momentum  
c. angular velocity      d. rotational kinetic energy
2. What is the moment of inertia of a square of side  $b$  about an axis through its center of gravity?  
a.  $b^3/4$       b.  $b^4/12$       c.  $b^4/3$       d.  $b^4/8$
3. What is the unit of inertia of mass?  
a.  $\text{kg/m}$       b.  $\text{kg/m}^2$       c.  $\text{kg-m}^2$       d.  $\text{m}^3$
4. What will happen to the weight of a body if the gravitational acceleration at any place is doubled?  
a. reduced to  $\frac{1}{2}$       b. doubled      c. no change      d. none of these
5. What refers to the focus of the instantaneous center of a moving rigid body?  
a. straight line      b. involute      c. centroid      d. spiral
6. How many radians are there in 60 revolutions?  
a. 0.49 rad      b. 0.98 rad      c. 2.5 rad      d. 18.8 rad
7. How long will the helicopter blade turn through 200 radians if its angular velocity is 60 rad/s?  
a. 2 seconds      b. 3.3 seconds      c. 4 seconds      d. 6 seconds
8. Bowling ball has a mass of 5.5 kg and a radius of 12.0 cm. It is released so that it rolls down the alley at a rate of 12 revs/s. What is the magnitude of its angular momentum?  
a.  $0.76 \text{ kgm}^2/\text{s}$       b.  $1.8 \text{ kgm}^2/\text{s}$       c.  $4.1 \text{ kgm}^2/\text{s}$       d.  $5.1 \text{ kgm}^2/\text{s}$
9. The moment of inertia of an object is greater when most of the mass is located in what area?  
a. off-center      b. near the axis  
c. on the rotational axis      d. away from the rotational axis
10. What quantity describes the ability of a force to rotate an object?  
a. mass      b. moment of inertia      c. torque      d. velocity

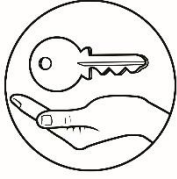
11. Which of these statements best describes the torque produced when two children of different weights balance on a seesaw?
  - a. equal torques
  - b. unequal torques
  - c. equal torques in the same direction
  - d. equal torques in opposite direction
12. A 4.0 kg block travels around a 0.50 m radius circle with an angular velocity of 12 rad/s. What is its angular momentum about the center of the circle?
  - a. 12 kgm<sup>2</sup>/s
  - b. 24 kgm<sup>2</sup>/s
  - c. 6 kgm<sup>2</sup>/s
  - d. 3 kgm<sup>2</sup>/s
13. A force of 250 N is exerted on a cable wrapped around a drum that has a diameter of 120 mm. What is the torque produced about the center of the drum?
  - a. 15 N
  - 30 N
  - c. 45 N
  - d. 60 N
14. Which of the following tells if an object is in equilibrium?
  - a. the object is in motion
  - b. the forces acting are not equal
  - c. the object moving at a constant speed
  - d. the net force is equal to zero
15. At what point will takes place when determining whether a rigid body is in equilibrium if the vector sum of the gravitational forces acting on the individual particles of the body can always be replaced by a single force?
  - a. a point of the boundary
  - b. the geometrical center
  - c. the center of gravity
  - d. any of the above



## ***Additional Activities***

Angular acceleration is a vector, having both magnitude and direction. How do we denote its magnitude and direction? Illustrate with an example.





Answer Key

<p><b>What's More ( Lesson 1)</b></p> <p>Solution: <math>I = m_1 + r_2 + m_2r_2</math> <math>I = (0.1\text{kg})(0.3\text{m})^2 + (0.2\text{kg})(0.3\text{m})^2</math> <math>I = (0.1\text{kg})(0.09\text{m}^2) + (0.2\text{kg})(0.09\text{m}^2)</math> <math>I = 0.009\text{kgm}^2 + 0.018\text{kgm}^2</math> <math>I = 0.027\text{kgm}^2</math></p>	<p><b>What's In ( Lesson 1)</b></p> <p>1. Center of mass (0.2kg) 2. Momentum 3. impulse</p>	<p><b>What I Know</b></p> <p>1. B 2. C 3. D 4. A 5. A 6. C 7. D 8. D 9. B 10. A 11. D 12. B 13. B 14. C 15. D</p>
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<p><b>What I Have Learned</b></p> <p><b>Lesson 1:</b> 1. Axis 2. Point mass 3. Rotational inertia 4. Basis 5. <math>M^4</math></p> <p><b>For lesson 3</b></p> <p>The tension <b>281 Newtons!</b></p> <p>Since the mass is 1 kg, the weight is 9.8 N. Each cable must pull upwards with 4.9 N of force. Thus,</p> <p><math>\sin(1 \text{ degree}) = (4.9 \text{ N}) / (F_{\text{tens}})</math>.</p> <p>Proper use of algebra leads to the equation</p> <p><math>F_{\text{tens}} = (4.9 \text{ N}) / [\sin(1 \text{ degree})] = 281 \text{ N}.</math></p>	<p><b>Assessment</b></p> <p><b>POST TEST</b></p> <p>1. B 2. B 3. D 4. B 5. C 6. D 7. B 8. A 9. C 10. C 11. D 12. A 13. A 14. C 15. D</p>	<p><b>What I Can Do</b></p> <p>Lesson 3</p> <p><math>\vec{a} = -16 \text{ rad/s}^2</math></p>
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### **EDITOR'S NOTE**

This Self-learning Module (SLM) was developed by DepEd SOCCSKSARGEN with the primary objective of preparing for and addressing the new normal. Contents of this module were based on DepEd's Most Essential Learning Competencies (MELC). This is a supplementary material to be used by all learners of SOCCSKSARGEN Region in all public schools beginning SY 2020-2021. The process of LR development was observed in the production of this module. This is version 1.0. We highly encourage feedback, comments, and recommendations

**For inquiries or feedback, please write or call:**

Department of Education – SOCCSKSARGEN  
Learning Resource Management System (LRMS)

Regional Center, Brgy. Carpenter Hill, City of Koronadal

Telefax No.: (083) 2288825/ (083) 2281893

Email Address: [region12@deped.gov.ph](mailto:region12@deped.gov.ph)