

between diff types of flops

- quites, Numerical Aerody-
computer cluster), **Sim-**

- data and sync b/w processes
(data structure)
run-time product)

- single processor of same

- parallel computers, more
that reduces f
ndent on problem size n .

$$Sp^{(n)} = \frac{i}{1 + (p-1)f(n)} = p,$$

- ed to improve accuracy/-

- Advantage: no stale value,
only when cache block
may contain invalid

(Write Description)

- v_2) (Write Serialisation)

ference protocols)

- ack of the sharing status,
e line & take appropriate
block.

Most common in NUMA

- all before next mem op)

e to A is observed by all

- by all processors

5.3 Interconnection Networks

- 5.3.1 **Shuffle Sort**
1. Row sorting, odd rows sort asc, even rows sort desc
 2. Column sorting, all columns sort asc
 3. Repeat until sorted

For n numbers, $\log(n)$ phases, $O(\sqrt{n} \log n)$. Good for 2D mesh network, only comm w/ adj nodes.

5.4 Topology

5.4.1 Metric

- **Diameter**: max dist b/w any pair of nodes (small diameter, small dist for msg transmission)
- **Degree**: no of direct neighbour nodes (small node degree reduces node h/w overhead)
- **Bisection width**: min. no. of edges to be removed to divide the network into 2 equal halves (measure for capacity of network when transmitting messages simultaneously)
- **Node connectivity**: min. no of nodes failing to disconnect network (network robustness)
- **Edge connectivity**: min. no of edges failing to disconnect network (no of independent paths b/w any pair of nodes)

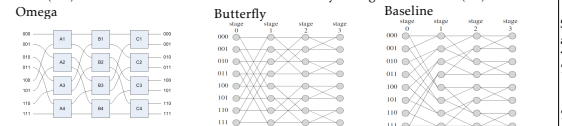
5.4.2 Direct Interconnection (Static/Point-To-Point)

Torus = 2 links for every dim. Mesh = nodes w/ no wraparound, CCC = k-dim hypercube but each node replaced with cycle of k nodes

network G with n nodes	degree $g(G)$	diameter $\delta(G)$	edge-connectivity $ec(G)$	bisection bandwidth $B(G)$
complete graph	$n-1$	1	$n-1$	$\left(\frac{n}{2}\right)^2$
linear array	2	$n-1$	1	1
ring	2	$\left\lfloor \frac{n}{2} \right\rfloor$	2	2
d -dimensional mesh ($n = r^d$)	$2d$	$d \left(\sqrt[d]{n} - 1 \right)$	d	$n^{\frac{d-1}{d}}$
d -dimensional torus ($n = r^d$)	$2d$	$d \left\lfloor \frac{\sqrt[d]{n}}{2} \right\rfloor$	$2d$	$2n^{\frac{d-1}{d}}$
k -dimensional hypercube ($n = 2^k$)	$\log n$	$\log n$	$\log n$	$\frac{n}{2}$
k -dimensional CCC-network ($n = k2^k$ for $k \geq 3$)	3	$2k-1 + \lfloor k/2 \rfloor$	3	$\frac{n}{2k}$
complete binary tree ($n = 2^k - 1$)	3	$2 \log \frac{n+1}{2}$	1	1
k -ary d -cube ($n = k^d$)	$2d$	$d \left\lfloor \frac{k}{2} \right\rfloor$	$2d$	$2k^{d-1}$

5.4.3 Indirect Interconnection (using switches) - examples for 8 to 8

- **Bus Network** – a set of wires, only one pair of devices can communicate at a time, bus arbiter for coordination
- **Crossbar Network** has $n \times m$ switches for n in, m out. Switch = either straight/change dir
- **Omega Network** – 1 unique path for every input to output, Uses $\log n$ stages, $\frac{n}{2}$ switches/stage. Edge from node (a,i) to two nodes $(\beta,i+1)$ where $\beta=a$ by a cyclic left shift (+ inversion of the LSBit)
- **Butterfly Network** – Node (a,i) connects to $(a,i+1)$ and $(a',i+1)$, a and a' differ in the $(i+1)$ th bit from the left (cross edge)
- **Baseline Network** – Node (a,i) connects to 2 nodes $(\beta,i+1)$ where $\beta=c$ cyclic right shift of last $(k-1)$ bits of a OR inversion of the LSBit of a + cyclic right shift of last $(k-1)$ bits.



5.5 Routing

Based on:

- Path length (minimal/non-minimal) whether shortest path
- Adaptivity (deterministic/adaptive) whether always the same path for same pair of nodes/take into account network status

5.5.1 XY Routing for 2D Mesh (Deterministic)

(X_{src}, Y_{src}) to (X_{dst}, Y_{dst}) : move in x-direction until $X_{src}=X_{dst}$, then move in y-direction

5.5.2 E-Cube Routing for Hypercube (Deterministic)

No of bits difference b/w 2 pairs of nodes = number of hops (hamming distance). Start from MSB to LSB (vice versa), find first different bit, go to neighbouring node with the bit corrected.

5.5.3 XOR-Tag Routing for Omega Network (Deterministic)

T = Source Id \oplus (XOR) Destination ID

A stage- k : go straight if bit k of T is 0, crossover if bit k of T is 1

6 Parallel Programming Models

6.1 Data Distribution

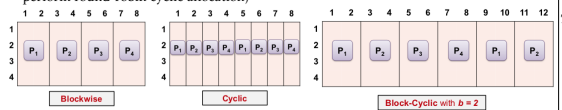
Assumptions: p identical processors, array starts from 1, Block size = $B = \left\lceil \frac{n}{p} \right\rceil$

6.1.1 1D Array

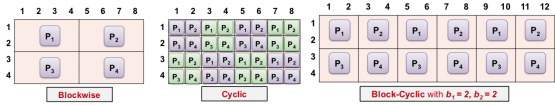
- **Blockwise**: P_i takes elements $\{(j-1) \times B + 1, \dots, j \times B\}$
- **Cyclic**: P_i takes elements $\{j, j+p, \dots, j+(B-1)p\}$ if $j \leq n \bmod p$, else $\{j, j+p, \dots, j+(B-2)p\}$

6.1.2 2D Array

- Combination of blockwise/cyclic in 1/both dimension(s)
- 1D distribution: column-wise blockwise, cyclic, or block-cyclic (Form blocks of size b , then perform round-robin cyclic allocation)



- Checkerboard distribution



6.2 Information Exchange

For controlling coordination of different parts of a parallel program execution

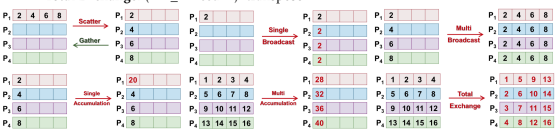
6.2.1 Shared Address Space (Shared variables)

Assumes a global memory accessible by all processors, need sync ops for safe concurrent access to prevent race condition using mutex to protect critical section, e.g. `#pragma omp critical` or `omp_init_lock(omp_lock_t+)`, `omp_set_lock`, `omp_unset_lock`

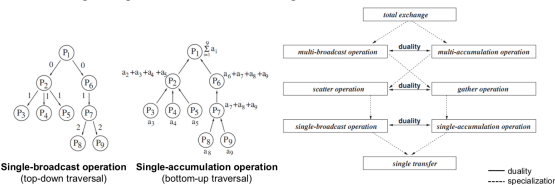
6.2.2 Distributed Address Space (Communication operations)

- Assumes disjoint memory space – data exchange through dedicated comm ops
- 2 main types of data exchange: point-to-point and global communication
- Examples

- **Single transfer**: point-to-point, send and receive
- **Scatter/Gather**: takes elements and distributes them in order of process rank
- **Single Broadcast**: takes a single data element at the root and copies it to all other
- **Multi Broadcast**: (MPI_Allgather) "Gather + Single Broadcast"
- **Single accumulation**: reduce (binary, associative, commutative) op applied to all elements, result stored in root (Gather + Reduce)
- **Multi accumulation**: "Accumulation + Scatter"
- **Total Exchange**: (MPI_Alltoall) "transpose"



- **Duality of comm ops**: interconnection can be represented by a spanning tree. Duality if the same spanning tree can be used for both ops



7 Message Passing Programming

Loosely synchronous paradigm, tasks sync to perform interactions, but otherwise exec is async: SPMD (Single Program Multiple Data) model

	Blocking	Non-Blocking
Buffered	Sending process returns after data has been copied into comm buffer	Sending process returns after initiating DMA transfer to buffer, may not be complete
Non-buffered	Sending process blocks until matching receive operation has been encountered	

- **Blocking** = send op blocks until it is safe. Issue: deadlock
- **Non-blocking** = send/recv returns before it is safe – generally accompanied by a check-status op. Used correctly, can overlap comm overheads with useful computations
- **Non-buffered**, issues: idling (timing mismatch sender <-> recvr)
- **Buffered**: buffers at both ends, trades idling overhead for buffer copying overhead

Summary = Overhead: idling (non-buffered) vs buffer management (buffered), **Side effect**: safe and easier programming (blocking) vs hiding comm overhead (non-blocking)

7.1 Message Passing Interface (MPI)

7.1.1 Semantic, Local View

- **Blocking**: return from lib call indicates user is allowed to reuse resources specified
- **Non-blocking**: may return before op completes, before user is allowed to reuse resources

7.1.2 Semantic, Global View

- **Synchronous**: comm op doesn't complete before both processes have started their comm op
- **Asynchronous**: sender can execute its comm op without any coordination with the receiver

	Synchronous	Asynchronous
Blocking	MPI_Ssend, MPI_SRecv	MPI_Send, MPI_Recv
Non-Blocking	MPI_Issend, MPI_IRecv	MPI_Isend, MPI_IRecv

7.1.3 Initialisation, Finalisation, Abort

- `int MPI_Init(int* argc, char** argv[])`, initialise MPI program, called only once
- `int MPI_Finalize(void)`, terminate all MPI processing, last MPI call
- `int MPI_Abort(MPI_Comm comm, int errorCode)`, force all processes to terminate, return `errorCode` to mpi run

Other functions: MPI_Comm_size, MPI_Comm_rank

7.1.4 MPI Messages Format

- **Message** = data + envelope (how to route the data)
- **Data** = start-buffer (pointer to data) + count + datatype
- **Envelope** = destination/source (using rank in a communicator) + tag (arbitrary number to distinguish messages) + communicator

7.1.5 Process Group

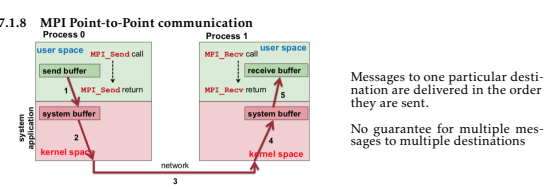
- An ordered set of processes where each process has a unique rank
- A process may be member of multiple groups (and may have diff ranks in each of these groups) – handled by MPI

7.1.6 Communicator

- Communication domain for a group of processes
- 2 types: **Intra-communicators** (supports the execution of arbitrary collective comm op on a single group, default: MPI_COMM_WORLD), **Inter-communicators** (supports the point-to-point comm op b/w 2 process groups)
- Allows to organise tasks based on func into task groups, Enable collective communication operations across a subset of related tasks, Provide basis for user-defined virtual topologies

7.1.7 Process Virtual Topologies

- A communicator with a Cartesian style of addressing the ranks of the processes
- MPI_Cartdim_get and MPI_Cart prefix: create, get, coords, rank, shift

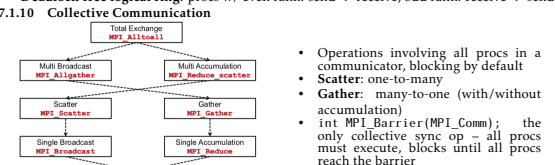


7.1.9 Deadlocks

- Assumes MPI_Recv(recvbuf, ...); MPI_Send(sendbuf, ...);
- For MPI_Recv, as above
- For MPI_Send, occurs when runtime doesn't use system buffers/system buffers are too small

- **Deadlock-free logical ring**: procs w/ even rank: send -> receive; odd rank: receive -> send

7.1.10 Collective Communication



- Operations involving all procs in a communicator, blocking by default
- **Scatter**: one-to-many
- **Gather**: many-to-one (with/without accumulation)
- `int MPI_Barrier(MPI_Comm)`: the only collective sync op – all procs must execute, blocks until all procs reach the barrier

7.1.11 Measuring Program Timing

- `double MPI_Wtime(void)` Time in seconds since an arbitrary time in the past.
- `double MPI_Wtick(void)` Time in seconds of resolution of MPI_Wtime

8 CUDA Programming

8.1 GPU Architecture

- Multiple **Streaming Multiprocessors (SMs)** – memory, cache, connecting interface
- SM consists of multiple compute cores, memories (registers, L1 cache, texture memory, shared memory), logic for thread & instruction management

8.2 CUDA Kernel and Threads

- **Device** = GPU, **Host** = CPU, **Kernel** = function that runs on device
- Parallel portions execute on device as threads, one kernel at a time, 1 kernel = many threads
- CUDA threads are extremely lightweight, very little creation overhead, instant switching
- All threads run the same code – SPMD (Single Program Multiple Data)

8.3 Thread Cooperation

- Share results to save computation, share memory accesses to reduce bandwidth
- Divide monolithic thread array into multiple blocks
- In a block: **shared memory**, **atomic ops**, **barrier sync**. Threads in diff blocks can't cooperate
- HW is free to schedule thread blocks to any processor at any time

8.4 Thread Hierarchy

- A kernel is executed by a grid of thread blocks. A block executes on 1 SM, does not migrate.
- Several blocks can reside concurrently on 1 SM, limited by control limitations (32 block/s/SM, 1024 threads/block) and resources (register file and shared memory are partitioned among all resident threads)

8.5 Thread Execution Mapping to Architecture

- SIMT (Single Instruction Multiple Thread) execution model
- SM creates, manages, schedules, executes threads in SIMT warps (group of 32 parallel threads). Threads in a warp start together at the same program address. A block is always split into warps the same way
- Warp executes 1 common instruction at a time. Scheduler optimises by grouping threads with the same exec paths in the same SIMT unit

8.6 CUDA Memory Model

Memory	Location	Cached	Access	Scope	Lifetime
Register	on chip	n/a	R/W	1 thread	thread
Local	off chip	No	R/W	1 thread	thread
Shared	on chip	n/a	R/W	All threads in block	block
Global	off chip	No	R/W	All threads + host	host alloc
Constant	off chip	Yes	R	All threads + host	host alloc
Texture	off chip	Yes	R	All threads + host	host alloc

- **Shared** = higher bandwidth and lower latency than local/global. Divided into equal-sized mem mods called **banks**. Access to diff banks can be simultaneous. **Bank conflict**: mem request in the same bank, has to be serialised
- **Local**: automatic array vars alloc by compiler, **Texture**: for spatially coherent random-access I/O data (provides filtering, address clamping, wrapping)

8.7 Programming in CUDA

- **Func qual**: `__host__ __device__ __global__`, **Launching kernel**: `kernel <<= 80, 64>;`
- **Var qual**: `__device__ __constant__ __shared__ __unqualified`; scalar, built-in vector stored in registers, arrays of more than 4 elements or runtime indices stored in local mem

- **Thread synchronisation**: `void __syncthreads()`, generates barrier sync instruction

8.8 Memory Optimisations

Minimise data transfer between host and device mem, use page-locked/pinned mem transfer. Overlapping async transfers with GPU computations

8.8.1 Coalesced access to Global Mem

Simultaneous accesses to global mem by threads in a half-warp can be coalesced into as few as single mem txn of 32, 64, or 128 bytes

8.8.2 Overall Memory Optimisations

Minimize data transfer between host and device. Ensure global memory accesses are coalesced whenever possible. Minimize global memory accesses by using shared memory. Minimize bank conflicts in shared memory accesses

8.9 Overall Optimisation

- **Maximise parallel exec**: restructure algo to expose as much data parallelism as possible, map to HW carefully choosing exec config of each kernel invocation
- **Optimizing memory usage** to achieve maximum memory bandwidth: diff mem spaces and access patterns
- **Optimizing instruction usage** to achieve maximum instruction throughput: Use high throughput arithmetic instructions, avoid diff exec paths within the same warp

9 Parallel Algorithm Design

9.1 Overheads of Parallelism

- **Tradeoff**: task granularity – small to reduce overhead, large to still have enough parallel work

9.2 General Design Approach

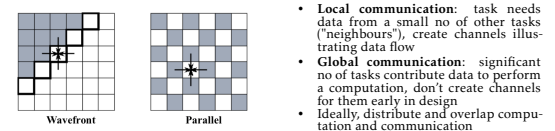
- Consider machine independent issues before machine specific aspects of design
- Task/Channel model: **Tasks** interact by sending messages through **channels**

9.3 Foster's Design Methodology

9.3.1 Partitioning

- Divide **computation & data** into independent pieces to discover max parallelism.
- Data centric – **Domain decomposition**: divide data into pieces of approx. equal size, determine how to associate computations with the data
- Computation centric – **Functional decomposition**: divide computation into pieces, determine how to associate data with the computations.
- **Rules of thumb**: at least 10x more primitive tasks than processors, minimise redundant computations and data storage, primitive tasks roughly same size, no of tasks an increasing function of problem size

9.3.2 Communication



- **Local communication**: task needs data from a small no of other tasks ("neighbours"), create channels illustrating data flow
- **Global communication**: significant no of tasks contribute data to perform a computation, don't create channels for them early in design
- Ideally, distribute and overlap computation and communication

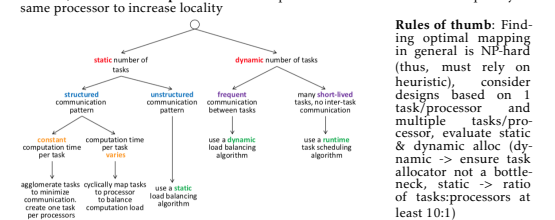
- **Local comm**: 2 finite diff update strategies. Parallel: time t unshaded, time (t+1) shaded
- **Global comm**: optimisation through pipeline/divide-and-conquer

9.3.3 Agglomeration

- Combine tasks into larger tasks to improve perf (reduce task creation and comm overhead), maintain scalability, simplify programming
- In MPI, goal: create 1 agglomerated task/processor
- E.g. reduce dim of decomposition from 3 to 2, **3D decomposition**: combine adjacent tasks, **divide-and-conquer**: coalesce sub-trees, **tree algo** – nodes are combined
- **Rules of thumb**: increase locality of parallel algo, no of tasks increases with problem size, no of tasks suitable for target, reasonable trade-off b/w agglomeration and code modification

9.3.4 Mapping

- Assigning tasks to processors, performed by OS (centralised multiprocessor), User (distributed mem systems)
- **Conflicting goals**: **Maximise processor util** – place tasks on diff processors to increase parallelism, **Minimise inter-processor comm** – place tasks that communicate frequently on the same processor to increase locality



10 Energy-efficient Computing and Data Centers

10.0.1 ARM Cortex-A family Vs general purpose Intel/AMD x86 servers

Similarities	Differences
• Processors cores + RAM + I/O interfaces + misc. peripherals	• RISC Instruction-Set Architecture
• Cores use similar exec model (pipelines)	• Heterogeneous cores (big.LITTLE) – Fast cores / slow cores
• Memory hierarchy (L1 + L2 + RAM)	• Lower instruction-level parallelism exploitation
• Uses Virtual Memory	• Smaller L1/L2 caches
• Uses commodity Linux – Supports all programming models available on Linux, most server SW is available, Porting is trivial	• Less RAM (0.5 GB – 4GB), typically non-upgradable, depends on config
• Hardware virtualization, hardware-level security	• Lower main-memory bandwidth
	• Simpler I/O interfaces (USB2.0/USB 3.0/SATA2), next gen: PCI-Express, SATA 3

10.1 Cloud Computing

- Abstraction of underlying applications, information, content and resources, allows resources to be provided and consumed in a more elastic manner and on demand.
- Models: Software/Platform/Infrastructure as a Service (SaaS e.g. Google Apps; PaaS e.g. Google App Engine; IaaS e.g. Amazon EC2)
- Deployment models: Private, Community, Public, Hybrid Cloud
- Characteristics: On-demand self-service, Broad network access, Resource pooling, Rapid elasticity, Measured service

10.2 Virtualisation

- Creation of a virtual version of something. Access resource without being concerned with where and how the resource is physically located/managed.
- Types: **Services**, **Server**, **Storage**, **Network**