1 Source Week 11 Syntax

Two functions/objects may have the same definition, but `===` returns **false** because each function is unique. This includes non-primitives, e.g. `[]===[]` returns true.
Note also that passing non-primitives as an argument to a function passes a pointer. (But javascript does not expose changing-pointer-pointing-to-which-memory-address mechanism). So, JS-to-C equivalent:

- `fn(arr)` ≡ `fn(&arr)`
- Inside `fn`: `arr[5]` ≡ `arr[5]=*(arr+5)`
- Hence, inside `fn`: `ret = []; arr = ret;` does not mutate the original arr as it is just a pointer, but `arr[2]=5;` does mutate the original array.

math_<name>, where `name` can be:

- **const**: `E, LN10, LN2, LOG10E, LOG2E, PI, SQRT1_2, SQRT2`
- **Functions**: `abs(x), cbrt(x), ceil(x), exp(x), floor(x), log(x), log10(x), log2(x), max(x1,...,xn), min(x1,...,xn), pow(base, exponent), random(), round(x), sqrt(x), trunc(x), acos(x), asin(x), atan(x), cos(x), sin(x), tan(x)`

General Functions

- `parseInt(string)`
- `display(value)`
- `alert(string)`
- `prompt(string)`

Primitive Data Type Checks

- `is_boolean`
- `is_string`
- `is_object`
- `is_number`
- `is_function`
- `is_array`

PS: `is_object(array)` returns true

List Library

- `pair(x,y)`: Makes a pair from x and y.
- `head(p)`: Returns the head (first component) of the pair x.
- `tail(p)`: Returns the tail (second component) of the pair x.
- `set_head(p,x)`: Sets the head (first component) of the pair p to be x; returns undefined.
- `set_tail(p,x)`: Sets the tail (second component) of the pair p to be x; returns undefined.
- `list(x1,x2,...,xn)`: Returns a list with n elements. The first element is x1, the second x2, etc. Iterative process; time: $O(n)$, space: $O(n)$, since the constructed list data structure consists of n pairs, each of which takes up a constant amount of space.
- `is_pair(p)`: Returns true if x is a pair and false otherwise.
- `is_empty_list(x)`: Returns true if xs is the empty list, and false otherwise.
- `is_list(x)`: Returns true if x is a list as defined in the lectures, and false otherwise. Iterative process; time: O(n), space: O(1), where n is the length of the chain of tail operations that can be applied to x.
- `length(xs)`: Returns the length of the list xs. Iterative process; time: O(n), space: O(1), where n is the length of xs.
- `map(f,xs)`: Returns the length of the list xs. Iterative process; time: O(n), space: O(1), where n is the length of xs.
- `build_list(n,f)`: Makes a list with n elements by applying the unary function f to the numbers 0 to n - 1. Recursive process; time: O(n), space: O(n).
- `for_each(f,xs)`: Applies f to every element of the list xs, and then returns true. Iterative process; time: O(n), space: O(1), where n is the length of xs
- `list_to_string(xs)`: Returns a string that represents list xs using the text-based box-and-pointer notation [...].
- `reverse(xs)`: Returns list xs in reverse order. Iterative process; time: O(n), space: O(n), where n is the length of xs. The process is iterative, but consumes space O(n) because of the result list.
- `append(xs,ys)`: Returns a list that results from appending the list ys to the list xs. Recursive process; time: O(n), space: O(n), where n is the length of xs.
- `member(x,xs)`: Returns first postfix sublist whose head is identical to x (===); returns [] if the element does not occur in the list. Iterative process; time: O(n), space: O(1), where n is the length of xs.
- `remove(x,xs)`: Returns a list that results from xs by removing the first item from xs that is identical (===) to x. Recursive process; time: O(n), space: O(n), where n is the length of xs.
- `remove_all(x)`:Returns a list that results from xs by removing all items from xs that are identical (===) to x. Recursive process; time: O(n), space: O(n), where n is the length of xs.
- `filter(pred,xs)`: Returns a list that contains only those elements for which the one-argument function pred returns true. Recursive process; time: O(n), space: O(n), where n is the length of xs.
- `enum_list(start,end)`: Returns a list that enumerates numbers starting from start using a step size of 1, until the number exceeds (>) end. Recursive process; time: O(n), space: O(n), where n is the length of xs.
- `list_ref(xs,n)`: Returns the element of list xs at position n, where the first element has index 0. Iterative process; time: O(n), space: O(1), where n is the length of xs.
- `accumulate(op,initial,xs)`: Applies binary function op to the elements of xs from right-to-left order, first applying op to the last element and the value initial, resulting in r1, then to the second-last element and r1, resulting in r2, etc, and finally to the first element and rn-1, where n is the length of the list. Thus, accumulate(op,zero,list(1,2,3)) results in op(1, op(2, op(3, zero))). Recursive process; time: O(n), space: O(n), where n is the length of xs, assuming op takes constant time.

Misc Functions

- `equal(x, y)` - true if pairs and corrs leaves are `===`
- `array_length(arr)` = 1+(highest index used)
- `apply_in_underlying_javascript(f, xs)`: calls the function f with arguments xs. For example:

```
function times(x, y) {
  return x * y;
}
apply_in_underlying_javascript(times, list(2, 3));
// returns 6
```

OOP Inheritance, method declaration, constructor example code

```
function icsbot(name){ Player.call(this, name); }
icsbot.Inherits(Player);
icsbot.prototype.__act = function(){
  Player.prototype.__act.call(this);
  ...
}
var anu = new Player("abc");
```

OOP Syntactic Sugar

- `Object.attribute = 5;` => `Object["attribute"] = 5;`
- `Object.method(arg1, arg2, ...)` => `var newid = object; newid["method"].call(newid, arg1, arg2, ...);`
- `ChildClass.Inherits(ParentClass)` => `ChildClass.prototype.__proto__=ParentClass.prototype;`
- `var anu = new Player("abc");` => `var anu = {}; anu.prototype.__proto__ = Player.prototype; Player.call(anu, "abc");`

2 Complexity

- **Big Oh** $O()$: Upper bound (worst case)
- **Big Omega** $\Omega()$: Lower bound (best case)
- **Big Theta** $\Theta()$: Both upper and lower bound
- **Order** : $\Theta(1) < \Theta(\log n) < \Theta(n) < \Theta(n \log n) < \Theta(n^2) < \Theta(n^3) < \Theta(2^n) < \Theta(3^n) < \Theta(n^n)$

3 Box-and-Pointer Diagrams

```
var w1 = list(1,2); var w2 = list(3,4);
var w3 = append(w1,w2);
   +---+  +---+
w1->|1| |-->|2||
   +---+  +---+
w2-----------------+
                   v
    +---+   +---+   +---+   +---+
w3->|1| |-->|2| |-->|3| |-->|4||
    +---+   +---+   +---+   +---+
var x = list(pair(1,2), pair(3,4));
set_tail(head(x), head(tail(x)));
   +---+   +---+
x->| | |-->| ||
   +---+   +---+
    V       V
   +---+   +---+
   |1| |-->|3|4|
   +---+   +---+
var input = list(1, pair(2,3), list(4));
var result = map(function(x){return x;}, input);
        +---+   +---+   +---+
input-->|1| |-->| | |-->| ||
        +---+   +---+   +---+
                 V       V
                +---+   +---+
                |2|3|   |4||
                +---+   +---+
                 ^       ^
        +---+   +---+   +---+
result->|1| |-->| | |-->| ||
        +---+   +---+   +---+
```

4 List-related

contains

```
function contains(x, xs) {
    return !is_empty_list(member(x, xs));
}
```

Map

```
function map(f, xs) {
  return is_empty_list(xs)
           ? []
           : pair(f(head(xs)), map(f, tail(xs)));
}
// iterative alternative
function new_map(f, xs) {
  function helper(xs, acc) {
    return is_empty_list(xs)
             ? acc
             : helper(tail(xs), pair(f(head(xs)), acc));
  }
  return reverse(helper(xs, []));
}
```

Filter

```
function filter(pred, xs) {
  if(is_empty_list(xs)) {
    return [];
  } else {
    return pred(head(x))
             ? pair(head(xs), filter(pred, tail(xs)))
             : filter(pred, tail(xs));
  }
}
// filter with accumulate
function new_filter(pred, xs) {
  return accumulate(function(e, acc){
                      return pred(e) ? pair(e, acc)
                                     : acc;
                    }, [], xs);
}
```

Reverse

```
function reverse(xs) {
  function rev(original, reversed) {
    return is_empty_list(original)
             ? reversed
             : rev(tail(original), pair(head(original),
               reversed));
  }
  return rev(xs,[]);
}
function reverse(lst) {
  return append(reverse(tail(lst)), list(head(lst)));
}
```

next_nth, every_n

```
// every_n returns every n-th element from lst
function next_nth(lst, n) {
  if (is_empty_list(lst)) {
    return [];
  } else if (n === 1) {
    return lst;
  } else {
    return next_nth(tail(lst), n - 1);
  }
}
function every_n(lst, n) {
  var next_element = next_nth(lst, n);
  if (is_empty_list(next_element)) {
    return [];
  } else {
    return pair(head(next_element),
                every_n(tail(next_element), n));
  }
}
function find(v, xs){
  return (is_empty_list(xs)) ? false
       : (v === head(xs)) ? true
                          : find(v, tail(xs));
}
```

Deep Copy

```
function deep_copy(xs) {
  return map(function(x) {
               return is_number(x) ? x : deep_copy(x);
             }, xs);
}
```

Variations of accumulate

```
//accumulate_n(list(list(1,2,3),list(4,5,6),list(7,8,9)),
0, s) = list(12,15,18)
function accumulate_n(op, init, seqs) {
  if (is_empty_list(head(seqs))) {
    return [];
  } else {
    return pair(accumulate(op, init,
                  map(function(x){ return head(x); },
                      seqs)),
                accumulate_n(op, init,
                  map(function(x){ return tail(x); },
                      seqs)));
  }
}
//accumulate_tree(plus,0,list(1,2,list(3,4),
list(5,list(6,7)))) = 28
function accumulate_tree(op, initial, seq) {
  return accumulate(
           function(a, b){
             return a + (is_list(b) ?
               accumulate_tree(op, initial, b) :
               b);
           }, initial, seq);
}
var fold_right = accumulate;
function fold_left(op, initial, sequence) {
  function iter(result, rest) {
    return is_empty_list(rest)
             ? result
             : iter(op(result, head(rest)),
               tail(rest));
  }
  return iter(initial, sequence);
}
```

Count Pairs

```
function count_pairs(x) {
  var hist = [];
  function helper(x) {
    if(!is_pair(x)) {
      return 0;
    } else {
      if(is_empty_list(member(x, hist))) {
        hist = pair(x, hist);
        return 1 + helper(head(x)) + helper(tail(x));
      } else {
        return 0;
      }
    }
  }
  return helper(x);
}
```

5 Higher Order Function

Compose, Identity

```
function compose(f, g){
  return function(x){ return f(g(x)); };
}
function identity(x) {
  return x;
}
function repeated(f, n) {
  return (n === 0) ? identity : compose(f, repeated(f,
    n - 1));
}

function plus_one (x) {
  return x + 1;
}
function twice (f) {
  return function(x) { return f(f(x)); };
}
```

Evaluate: `(twice(twice(twice)))(twice)(plus_one)(4)`

$$t^{2^{((2^2)^2)}} = t^{2^{(4)^2}} = t^{2^{16}} = t^{65536}$$

`twice` does tetration (right-to-left).

Sum

```
//Sum: term - the operator, next - (de/in)crement
    counter, a - starting point, b - endpoint
// recursive version
function sum(term, a, next, b) {
  return a > b ? 0 : term(a) + sum(term, next(a), next,
    b);
}
// iterative version
function sum(term, a, next, b) {
  function sum_iter(x, total) {
    return x > b ? total : sum_iter(next(x), total +
      term(x));
  }
  return sum_iter(a, 0);
}
```

6 Data Structures

Queue (enqueue in $O(n)$, the rest in $O(1)$)

```
function empty_queue() { return []; }
function enqueue(x, q) {
  return is_empty_list(q)
           ? pair(x, empty_queue());
           : pair(head(q), enqueue(x, tail(q)));
  }
}
function dequeue(x, q){ return tail(q); }
function qhead(q){ return head(q); }
```

Trees

```
function count_data_items(tree) {
  return is_empty_list(tree)
           ? 0
           : (is_list(head(tree))
              ? count_data_items(head(tree))
              : 1)
             + count_data_items(tail(tree));
}
function map_tree(f, tree) {
  return map(function(sub_tree) {
               return !is_list(sub_tree) ? f(sub_tree) :
                 map_tree(f,sub_tree);
             }, tree);
}
function accumulate_tree(f, init, xs) {
  return accumulate(function(e, acc) {
                      return (is_list(e))
                        ? f(accumulate_tree(f, init, e),
                          acc)
                        : f(e, acc);
                    }, init, xs);
}
function is_number_tree(xs) {
  if (!is_list(xs)) { return false;
  } else {
    return accumulate(function(e, acc){
                        return acc && (is_list(e)
                                       ? is_number_tree(e)
                                       : is_number(e));
                      }, true, xs);
  }
}
```

Balanced BST

```
function make_node(left_tree, number, right_tree) {
  return list(left_tree, number, right_tree);
}
function middle(n) { return math_floor(n / 2); }
function take(xs, n) {
  return n === 0 ? [] : pair(head(xs), take(tail(xs), n
    - 1));
}
function drop(xs, n) {
  return n === 0 ? xs : drop(tail(xs), n - 1);
}
function make_balanced(xs) {
  var list_length = length(xs);
  if (list_length === 0) {
    return [];
  } else {
    var list_mid = middle(list_length);
    return make_node(make_balanced(take(xs, list_mid)),
                     list_ref(xs, list_mid),
                     make_balanced(drop(xs, list_mid +
                       1)));
  }
}
```

7 Mutable Data Structure

Tortoise and Hare Algorithm (check if list has a loop)

```
function has_loop(lst) {
  function tortoise_and_hare(tortoise, hare) {
    if (is_empty_list(tortoise) || is_empty_list(hare)
      || is_empty_list(tail(hare))) {
      return false;
    } else {
      return tortoise === hare
               ? true
               : tortoise_and_hare(tail(tortoise),
                 tail(tail(hare)));
    }
  }
  return tortoise_and_hare(lst, tail(lst));
}
```

Circular List

```
function make_circular_copy(xs) {
  function inner(zs, ys) {
    if (is_empty_list(zs)) {
      return ys;
    } else {
      return pair(head(zs), inner(tail(zs), ys));
    }
  }
  if (is_empty_list(xs)) {
    return [];
  } else {
    var ys = pair(head(xs), []);
    set_tail(ys, inner(tail(xs), ys));
    return ys;
  }
}
function make_linear(xs) {
  function helper(lst) {
    if (tail(lst) === xs) {
      set_tail(lst, []);
    } else {
      helper(tail(lst));
    }x
  }
}
```

Queue in $O(1)$ using mutable data structure

```
// Queue data structure
function make_queue() { return pair([], []); }
function is_empty_queue(q) {
    return is_empty_list(head(q));
}
function enqueue(q, item) {
    if (is_empty_queue(q)) {
        set_head(q, pair(item, []));
        set_tail(q, head(q));
    } else {
        set_tail(tail(q), pair(item, []));
        set_tail(q, tail(tail(q)));
    }
}
function dequeue(q) {
    var front = head(head(q));
    set_head(q, tail(head(q)));
    return front;
}
function peek(q) { return head(head(q)); }
```

Stack

```
function make_stack() { return pair("stack", []); }
function is_empty_stack(stack) {
  return is_empty_list(tail(stack));
}
function peek(stack) {
  return is_empty_stack(stack) ? "stack underflow"
                               : head(tail(stack));
}
function push(stack, x) {
    set_tail(stack, pair(x, tail(stack)));
}
function pop(stack) {
    if (is_empty(stack)) {
        return "stack underflow";
    } else {
        var top = peek(stack);
        set_tail(stack, tail(tail(stack)));
        return top;
    }
}
```

Mutable version of list functions

```
function mutable_append(xs, ys) {
  if (is_empty_list(xs)) {
    return ys;
  } else {
    set_tail(xs, mutable_append(tail(xs), ys));
    return xs;
  }
}

function mutable_reverse(xs) {
  function help(ori, rev) {
    if (is_empty_list(ori)) {
      return rev;
    } else {
      var temp = tail(ori);
      set_tail(ori, rev);
      return help(temp, ori);
    }
  }
}
```

8 Coin Change

```
// cc(25, list(5, 10, 20, 50, 100));
function cc(amount, lst) {
  var coins = pair(0, lst);
  function helper(amount, coin_range) {
    if (amount === 0) { return 1;
    } else if (amount < 0 || coin_range === 0) {
      return 0;
    } else {
      return helper(amount, coin_range - 1)
             + helper(amount - list_ref(coins,
               coin_range), coin_range);
    }
  }
  return helper(amount, length(lst));
}
// Given coins in list l, return list of coin
   permutations that add up to x
function makeup_amount(x,l) {
  if (is_pair(l)) {
    return append(map(function(x){ return
      pair(head(l),x); },
                      makeup_amount(x - head(l),
                        tail(l))),
                    makeup_amount(x, tail(l)));
  } else if (x === 0) {
      return list([]);
  } else {
      return [];
  }
}
```

Doubly-linked List

```
function make_node(data, prev, next) {
  return list("node", data, prev, next);
}
function get_prev(node) {
  return head(tail(list(node)));
}
function get_next(node) {
  return head(tail(tail(tail(node))));
}
function get_data(node) { return head(tail(node)); }
function empty_node() { return make_node([], [], []); }
function is_empty_node(node) {
    return head(node) === "node" && equal(node,
      empty_node());
}
function set_data(n, v) {
    var temp = list(v, get_prev(n), get_next(n));
    set_tail(n, temp);
}
function set_previous(n, v) {
    var temp = list(get_data(n), v, get_next(n));
    set_tail(n, temp);
}
function set_next(n, v) {
    var temp = list(get_data(n), get_prev(n), v);
    set_tail(n, temp);
}
function insert_before(n1, n2) {
    set_prev(n2, get_prev(n1));
    set_next(get_prev(n1), n2);
    set_prev(n1, n2); set_next(n2, n1);
}
function insert_after(n1, n2) {
  set_next(n2, get_next(n1));
  set_prev(get_next(n1), n2);
  set_next(n1, n2); set_prev(n2, n1);
}
function remove(n) {
  var x = get_prev(n); var y = get_next(n);
  set_next(x, y); set_prev(y, x);
}
function dlist_to_list(dlst) {
  return is_empty_node(dlst)
           ? []
           : pair(get_data(dlst),
             dlist_to_list(get_next(dlst)));
}
function list_to_dlist(lst) {
  var first_node = make_node(head(lst), empty_node(),
    empty_node());
  var prev_node = first_node;
  for_each(function(x){
             var new_node = make_node(x, prev_node,
               empty_node());
             set_next(prev_node, new_node);
             prev_node = new_node;
           }, tail(lst));
  return first_node;
}
function reverse_dlist(dlst) {
  return list_to_dlist(reverse(dlist_to_list(dlst)));
}
```

9 Permutations & Combinations

Permutations ($O(n!)$) **and permutations_r** ($O\left(\frac{n!}{(n-r)!}\right)$)

```
function permutations(s) {
  if (is_empty_list(s)) {
      return list([]);
  } else {
      return accumulate(append, [], map(function(x) {
               return map(function(p) {
                            return pair(x, p);
                          }, permutations(remove(x,
                            s)));
             }, s));
  }
}
function permutations_r(s, r) {
  if (r === 0) { return list([]);
  } else if(is_empty_list(s)){ return list();
  } else {
    return accumulate(append, [], map(function(x) {
             return map(function(p) {
                          return pair(x, p);
                        }, permutations_r(remove(x, s),
                          r - 1));
           }, s));
  }
}
```

Combinations ($O\left(\frac{n!}{(n-k)!k!}\right) = O\left(n^{n \div 2}\right)$)

```
function combinations_k(xs, k) {
  if (k === 0) { return list([]);
  } else if (is_empty_list(xs)) { return [];
  } else {
    var s1 = combinations(tail(xs), k - 1);
    var s2 = combinations(tail(xs), k);
    var x = head(xs);
    var has_x = map(function(s) { return pair(x, s);},
      s1);
    return append(has_x, s2);
  }
}
```

Power Set

```
function power_set(xs) {
  if (is_empty_list(xs)) {
    return list([]);
  } else {
    // Either you pick the number, or you don't
    var without_head=power_set(tail(xs));
    var use_head = map(function(l) {
                         return pair(head(xs),l);
                       }, without_head);
    return append(use_head,without_head);
  }
}
function are_equal_sets(set1, set2) {
  if(length(set1) === length(set2)) {
    return false;
  } else {
    return accumulate(function(e1,acc){
              return accumulate(function(e2,acc) {
                        return e2 === e1 || acc;
              }, false, set2);
            }, true, set1);
  }
}
```

10 Tower of Hanoi

Tower of Hanoi (Running time: $O(2^n)$)

```
function make_move(from, to) { return list(from, to); }
function hanoi(n, A, B, C) {
  if (n === 1) { return list(make_move(A, B));
  } else {
    var moves_to_helper = hanoi(n - 1, A, C, B);
    var the_move = make_move(A, B);
    var moves_to_final = hanoi(n - 1, C, B, A);
    return append(moves_to_helper, pair(the_move,
    ↪  moves_to_final));
  }
}
```

11 Sorting functions & Binary Search

Selection Sort ($\Theta(n^2)$)

```
  function selection_sort(xs) {
    if (is_empty_list(xs)) { return xs;
    } else {
      var x = smallest(xs);
      return pair(x, selection_sort(remove(x, xs)));
    }
  }
  // find smallest element of a non-empty list xs
  function smallest(xs) {
    function sm(x, ys) {
      return is_empty_list(ys)
               ? x
               : x < head(ys) ? sm(x, tail(ys))
                              : sm(head(ys), tail(ys));
    }
    return sm(head(xs), tail(xs));
  }
```

Insertion Sort (Input-dependent, best:$\Omega(n)$, worst:$O(n^2)$)

```
function insert(x, xs) {
  return is_empty_list(xs)
           ? list(x)
           : x <= head(xs) ? pair(x,xs)
                           : pair(head(xs),
                             ↪  insert(x,tail(xs)));
}
function insertion_sort(xs) {
  return is_empty_list(xs) ? xs :
  ↪  insert(head(xs),insertion_sort(tail(xs)));
}
```

Merge Sort (Running time: $\Theta(n \log n)$)

```
function middle(n) { return math_floor(n / 2); }
function take(xs, n) {
    return n === 0 ? []
                   : pair(head(xs),take(tail(xs),n-1));
}
function drop(xs, n) {
    return n === 0 ? xs : drop(tail(xs), n - 1);
}
function merge(xs, ys) {
    if (is_empty_list(xs)) { return ys;
    } else if (is_empty_list(ys)) { return xs;
    } else {
        var x = head(xs); var y = head(ys);
        return (x < y) ? pair(x, merge(tail(xs), ys))
                       : pair(y, merge(xs, tail(ys)));
    }
}
function merge_sort(xs) {
    if (is_empty_list(xs) || is_empty_list(tail(xs))) {
        return xs;
    } else {
        var mid = middle(length(xs));
        return merge(merge_sort(take(xs, mid)),
                     merge_sort(drop(xs, mid)));
    }
}
```

Quicksort (Input-dependent, best: $\Omega(n \log n)$, worst: $O(n^2)$)

```
function partition(xs, p) {
  function helper(lst, smaller, larger) {
    if (is_empty_list(lst)) {
      return pair(smaller, larger);
    } else {
      var cur = head(lst);
      return cur <= p
               ? helper(tail(lst), pair(cur, smaller),
               ↪  larger)
               : helper(tail(lst), smaller, pair(cur,
               ↪  larger));
    }
  }
  return helper(xs, [], []);
}
function quicksort(xs) {
  if (is_empty_list(xs)) { return xs;
  } else if (is_empty_list(tail(xs))) { return xs;
  } else {
    var cur = head(xs);
    var partitioned = partition(tail(xs), cur);
    var sorted_smaller = quicksort(head(partitioned));
    var sorted_larger = quicksort(tail(partitioned));
    return append(sorted_smaller, pair(cur,
    ↪  sorted_larger));
  }
}
```

Binary Search ($O(\log n)$), returns true/false

```
function binary_search(a, v) {
  function search(low, high) {
    if (low > high) {
      return false;
    } else {
      var mid = math_floor((low + high) / 2);
      return (v === a[mid])
               || (v < a[mid]) ? search(low, mid - 1)
                               : search(mid + 1, high);
    }
  }
  return search(0, array_length(a) - 1);
}
```

Binary Search on sorted rotated array, ($O(\log n)$), returns the index if found, -1 if not found

```
function find(target, arr) {
  function findPivot(low, high) {
    if (low > high) {
      return -1;
    } else {
      var mid = math_floor((low + high) / 2);
      if (mid < high && arr[mid] > arr[mid + 1]) {
        return mid;
      } else if (mid>low && arr[mid] < arr[mid - 1]) {
        return mid - 1;
      } else if (arr[mid] > arr[low]) {
        return findPivot(mid + 1, high);
      } else {
        return findPivot(low, mid - 1);
      }
    }
  }
  function search(low, high) {
    if (low > high) {
      return -1;
    } else {
      var mid = math_floor((low + high) / 2);
      if (target === arr[mid]) {
        return mid;
      } else {
        return target < arr[mid]
                 ? search(low, mid - 1)
                 : search(mid + 1, high);
      }
    }
  }
  var len = array_length(arr);
  var pivot = findPivot(0, len - 1);
  if (pivot === -1) {
    return search(0, len - 1);
  } else if (arr[0] > target) {
    return search(pivot+1, len - 1);
  } else {
    return search(0, pivot);
  }
}
```

12 Functions of questionable usefulness

DG Week 3

```
function sum_squares_two_larger(a, b, c){
  return a > b ? a * a + ((b > c) ? b * b : c * c)
               : b * b + ((a > c) ? a * a : b * b)
}
function is_leap_year(year){
    return (year % 4 === 0 && year % 100 !== 0) || year
    ↪  % 400 === 0;
}
```

DG Week 4

```
// Pascal Triangle
function pascal(row, column){
  if(column === 1 || row === column){ return 1;
  } else {
      return pascal(row - 1, column - 1) + pascal(row -
      ↪  1, column);
  }
}
// Additional
function addition(a,n){return repeated(add_one,n)(a);}
function multiplication(a,n){
  var addition_a = function(x){return addition(a,x);};
  return repeated(addition_a,n)(0);
}
function exponentiation(a,n){
  var mul_a = function(x){return multiplication(a,x);};
  return repeated(mul_a,n)(1);
}
function tetration(a,n){
  var exp_a = function(x){return exponentiation(a,x);};
  return repeated(exp_a,n)(1);
}
```

13 Functions from Past Year Papers

```
function is_tree_of_numbers(x) {
  // Solution 1 [5 marks]
  return is_list(x)
         && accumulate(function(a,b){
                         return (is_number(a) ||
                         ↪  is_tree_of_numbers(a)) &&
                         ↪  b;
                       }, true, x);
  // Solution 2 [3 marks]
  if (is_empty_list(x)) { return true;
  } else if (is_pair(x)) {
    return (is_number(head(x)) ||
    ↪  is_tree_of_numbers(head(x))) &&
    ↪  is_tree_of_numbers(tail(x));
  } else { return false;
  }
}
function my_filter(pred, xs) {
  return accumulate(function(a, b) {
                      return pred(a) ? pair(a, b) : b;
                    }, [], xs);
}
```

2048

```
function slideRowToTheLeft(arr) {
  for (var i = 0; i < 4; i = i + 1) {
    for (var j = i + 1; j < 4 ; j = j + 1) {
      if (arr[i]===arr[j] && arr[i]!==0 && arr[j] !==
      ↪  0) {
        arr[i] = 2 * arr[i];
        arr[j]=0;
        break;
      } else { }
    }
  }
  var n = -1;
  for(i = 0; i < 4; i = i + 1) {
    if (arr[i]===0 && n === -1) {
      n = i;
    } else if (arr[i]!== 0 && n > -1) {
      arr[n]=arr[i];
      arr[i]=0;
      i=n;
      n=-1;
    } else { }
  }
  return arr;
}
function slideLeft(grid) {
  for(var i = 0; i < 4; i = i + 1) {
    slideRowToTheLeft(grid[i]);
  }
  return grid;
}

function rotateLeft(grid) {
  var ret = [];
  for(var i = 0; i < 4; i = i + 1) {
    ret[i]=[];
  }
  for(var i=0; i < 4; i = i + 1) {
    for(var j=0; j < 4; j = j + 1) {
      ret[3-j][i]=grid[i][j];
    }
  }
  return ret;
}
function performSlide(grid, direction) {
  if (direction === "left") {
    return slideLeft(grid);
  } else if (direction === "up") {
    return rotateLeft(rotateLeft(rotateLeft(
             slideLeft(rotateLeft(grid)))));
  } else if (direction === "right") {
    return rotateLeft(rotateLeft(slideLeft(rotateLeft(
             rotateLeft(grid)))));
  } else  {
    return rotateLeft(slideLeft(rotateLeft(rotateLeft(
             rotateLeft(grid)))));
  }
}
```

14 Memoization

1D Memoization Abstraction

```
function memoize(f) {
  var mem = [];
  return function(x) {
    if (mem[x] !== undefined) { return mem[x];
    } else {
      var result = f(x);
      mem[x] = result;
      return result;
    }
  };
}
// example
var fib=memoize(function(n){
  return n <= 1 ? n : fib(n-1) + fib(n-2);
});
```

Memoized Coin Change

```
var mem = [];
function read(amount, coin_range) {
  return (mem[amount] === undefined) ? undefined :
  ↪  mem[amount][coin_range];
}
function write(amount, coin_range, value) {
  if (mem[amount] === undefined) {
    mem[amount] = [];
  } else {}
  mem[amount][coin_range] = value;
}
function cc(amount, coin_range) {
  if (read(amount, coin_range) !== undefined) {
    return read(amount, coin_range);
  } else {
    var result = amount === 0 ? 1
                 : (amount < 0 || range === 0) ? 0
                 : cc(amount, range - 1) + cc(amount
                 ↪  - highest(range), coin_range);
    write(amount, coin_range, result);
    return result;
  }
}
```

Multi-dimensional Memoization (e.g. Tower of Hanoi)

```
var mem = [];
function read(w, x, y, z) {
  return (mem[w] === undefined) ? undefined
         : (mem[w][x] === undefined) ? undefined
         : (mem[w][x][y] === undefined) ? undefined
         ↪  : mem[w][x][y][z];
}
function write(w, x, y, z, value) {
  if (mem[w] === undefined) { mem[w] = [];
  } else {}
  if (mem[w][x] === undefined) { mem[w][x] = [];
  } else {}
  if (mem[w][x][y] === undefined) { mem[w][x][y] = [];
  } else {}
  mem[w][x][y][z] = value;
}
function make_move(From, To) { return list(From, To); }
function retriever(n, A, B, C) {
  var result = read(n, A, B, C);
  if (result === undefined) {
    result = mhanoi(n, A, B, C);
    write(n, A, B, C, result);
  } else { }
  return result;
}
function mhanoi(n, A, B, C) {
  var result = retriever(n, A, B, C);
  if (result !== undefined) { return result;
  } else {
    if (n === 1) { return list(make_move(A, B));
    } else {
      var moves_to_helper = retriever(n - 1, A, C, B);
      var the_move = make_move(A, B);
      var moves_to_final = retriever(n - 1, C, B, A);
      return append(moves_to_helper,
                    pair(the_move, moves_to_final));
    }
  }
}
// mhanoi(8, 1, 2, 3);
```

Variation of Knapsack Problem

A thief has a getaway car that can hold at most m kg (m is an integer). Write a program that takes in a list of the gold bar masses and determines the total mass of gold bars he can steal. Time complexity:

Non-Memoized: $O(2^{\text{length(bar\_masses)}})$,

Memoized: $O(\text{length(bar\_masses)} \cdot m)$

```
// Non-memoized version
function max_loot_mass(bar_masses, m) {
  if(is_empty_list(bar_masses)||head(bar_masses) > m) {
    return 0;
  } else {
    var cur = head(bar_masses);
    // Either take the current bar or not
    return math_max(cur +
    ↪  max_loot_mass(tail(bar_masses), m - cur),
                    max_loot_mass(tail(bar_masses),
                    ↪  m));
  }
}
// Memoized version
function max_loot_mass(bar_masses, m) {
  var mem = [];
  function write(len, m, value) {
    if (mem[len] === undefined) {
      mem[len] = [];
    } else { }
    countw = countw + 1;
    mem[len][m] = value;
  }
  function read(len, m) {
    if (mem[len] === undefined) { mem[len] = [];
    } else { countr = countr + 1;
    }
    return mem[len][m];
  }
  function helper(bar_masses, m) {
    if (is_empty_list(bar_masses) || head(bar_masses) >
    ↪  m) {
      return 0;
    } else {
      var cur = head(bar_masses);
      var len = length(bar_masses);
      var res = read(len, m);
      if (res === undefined) {
          res = math_max(cur + helper(tail(bar_masses),
          ↪  m - cur), helper(tail(bar_masses), m));
          write(len, m, res);
      } else { }
      return res;
    }
  }
  return helper(bar_masses, m);
}
```

Knapsack Problem: Herbert the Clown

Most expensive combination with a given budget from a list of ingredients.

`b` = Budget

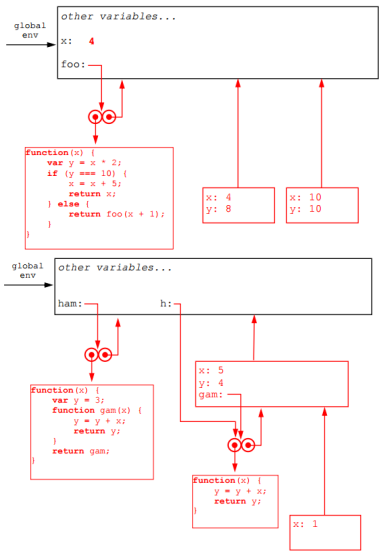
`i_l` = List of Ingredients Price List

```
// Assume all sublists are sorted in ascending order
function getMaxSpend(b, i_l) {
  function is_sufficient_fund(b, i_l) {
    return is_empty_list(i_l)
             ? b >= 0
             :  is_sufficient_fund(b - head(head(i_l)),
             ↪  tail(i_l));
  }
  if (is_sufficient_fund(b, i_l)) {
      if (is_empty_list(i_l) ||
      ↪  is_empty_list(head(i_l)) || head(head(i_l)) >
      ↪  b) {
        return 0;
      } else {
        var cur = head(head(i_l));
        var x = cur + getMaxSpend(b - cur, tail(i_l));
        var y = getMaxSpend(b, pair(tail(head(i_l)),
        ↪  tail(i_l)));
        return math_max(x, y);
      }
  } else {
    return -1;
  }
}
var i_l = list(list(4,6,8), list(5,10), list(1,3,5,5));
getMaxSpend(11, i_l); // returns 10 (4+5+1)
getMaxSpend(20, i_l); // returns 19 (8+10+1)
```

15 Environment Model

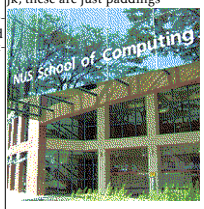
```
var x = 4;
function foo(x) {
  var y = x * 2;
  if(y===10) {
    x = x + 5;
    return x;
  } else {
    return foo(x+1);
  }
}
```

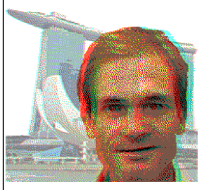
```
function ham(x) {
  var y = 3;
  function gam(x) {
    y = y + x;
    return y;
  }
  return gam;
}
var h = ham(5);
h(1);
```

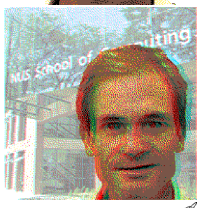


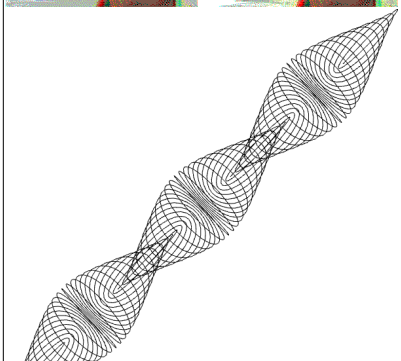
16 Contest Entries for "inspiration"

jk, these are just paddings

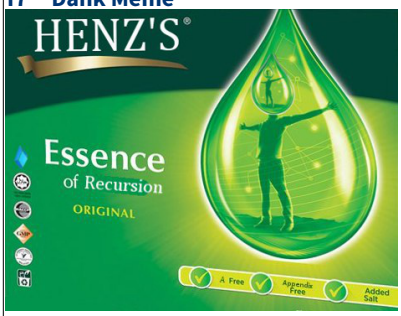








17 Dank Meme



18 20th Anniversary, from IC1101S to CS1101S, from 1997 to 2017

Woah the module is as old as me!