民科金融学

现代金融学股票估值的基本公式是错的，下面先简单介绍一下公式，然后说明其中的错误。

股票估值公式的基础是利率和时间价值。简单地说，因为货币可以由银行存款等方式实现自行增值，现在的 1 元钱比一年后的 1 元钱更有吸引力。也就是说，不同时点上的货币金额不能直接比较，需要经过折现换算。例如，银行年利率为 5%，则 2 年后的 1 元钱现在就值 $1/1.05^2=0.907$ 元。

在金融学中，股票的理论价格的基本求法是，假设股票被持有 1 年然后卖出，则理论价格为未来 1 年发放的股利与 1 年后股价分别折现然后求和。而 1 年后的股价又可以写成未来第 2 年发放的股利与第 2 年后股价分别折现然后求和……如果年数足够大，许多年后股价的折现数就可以忽略不计，从而得到股利贴现模型公式。如果替换股利，就得到自由现金流贴现模型公式，它在理论上更准确。

$$P_0=\frac{D_1}{1+k}+\frac{P_1}{1+k}$$

$$P_0=\frac{D_1}{1+k}+\frac{D_2}{(1+k)^2}+\frac{P_2}{(1+k)^2}$$

$$P_0=\frac{D_1}{1+k}+\frac{D_2}{(1+k)^2}+\frac{D_3}{(1+k)^3}+\frac{P_3}{(1+k)^3}$$

…

$$P_0=\sum_{t=1}^{T}\frac{D_t}{(1+k)^t}+\frac{P_T}{(1+k)^T}$$

$$P_0=\lim_{T\to\infty}\left[\sum_{t=1}^{T}\frac{D_t}{(1+k)^t}+\frac{P_T}{(1+k)^T}\right]$$

$$P_0=\lim_{T\to\infty}\sum_{t=1}^{T}\frac{D_t}{(1+k)^t}$$

$$P_0=\lim_{T\to\infty}\sum_{t=1}^{T}\frac{FCF_t}{(1+k)^t}$$

股利贴现公式最早是由威廉姆斯于 1938 年在《投资价值理论》中提出的，后于 1961 年由 MM 在论文《股息政策、增长和股票估值》中发展完善，并证明股利贴现、自由现金流贴现等公式实际等效。

先来看威廉姆斯的说法，他在书中没有详细数学推导直接给出了股利贴现公式。他认为，投资股票的目的就是获得股息，股价只是获得股息的手段，所以投资者应看重股息而不是追求股价变化，股票的理论价格就是股利贴现求和。这种论述，我认为错误，原因很简单，股息增值和股价增值都是股票的增值，股民没有必要把它们对立起来。在后面的章节中，为了应对质疑，威廉姆斯说，因为市场常常出错，所以不能用市场数据检验其理论的真伪。。。

compound interest for the benefit of the stockholder, as the critics imply, then these earnings should produce dividends later; if not, then they are money lost. Furthermore, if these reinvested earnings will produce dividends, then our formula will take account of them when it takes account of all future dividends; but if they will not, then our formula will rightly refrain from including them in any discounted annuity of benefits.

Earnings are only a means to an end, and the means should not be mistaken for the end. Therefore we must say that a stock derives its value from its dividends, not its earnings. In short, a stock is worth only *what you can get out of it.* Even so spoke the old farmer to his son:

t equals infinity, of π_t sub t, times v to the tth power" It should be noted that $\sum_{t=1}^{t=\infty}$ is not a factor to be multiplied by the other factors π_t and v^t, but is an operational sign applied to these two factors taken together

If the series runs from $t = 1$ to $t = n$, as in formula (1b) applying to bonds, the series is a finite series instead of an infinite series, because the number of terms is limited and is given in this case by the number of coupons payable during the life of the bond

收入只是达到目的的手段，手段不应该被误认为是目的。因此，我们必须说股票的价值来自其股息，而不是其收益。简而言之，股票的价值取决于您能从中获得的价值。

4. IS THE THEORY CORROBORATED BY EXPERIENCE?

Fourth, it may be asked, by those who are more hopeful than skeptical, if the true values for stocks as given by the new theory are usually verified later by the action of the market itself. The answer to this question is that such "verification" sometimes happens, sometimes not, merely according to chance. There is no theoretical reason, however, to expect corroboration or verification. After all, if the market makes a wrong estimate of true value today, why should it be expected to do better tomorrow? By tomorrow, today's facts may be more widely understood, of course; but by tomorrow new facts may be interjected into

the situation, with the result that a new estimate of true worth will be needed and a further correction of price will be called for. Thus the market may find itself as far away from a logical value tomorrow as today. But even if no new facts appear, the market may still refuse to move in the right direction between today and tomorrow. Since market price depends on popular opinion, and since the public is more emotional than logical, it is foolish to expect a relentless convergence of market price toward investment value.[2] Corroboration of estimates by subsequent market action, therefore, ought not to be expected. After all, investment value and market price are two quite different things.

该理论是否得到经验的证实？

没有任何理论上的理由可以期待证实或验证。

市场可能仍然拒绝在今天和明天之间朝着正确的方向发展。由于市场价格取决于大众意见，并且由于公众的情感多于逻辑，因此期望市场价格不断向投资价值趋同是愚蠢的。因此，不应期望随后的市场行为会证实估计。毕竟，投资价值和市场价格是两个完全不同的东西。

profits, and the rate of growth of dividends per share. Once these fundamentals have been established, we shall proceed in Section IV to drop the assumption of certainty and to see the extent to which the earlier conclusions about dividend policy must be modified. Finally, in Section V, we shall briefly examine the implications for the dividend policy problem of certain kinds of market imperfections.

I. EFFECT OF DIVIDEND POLICY WITH PERFECT MARKETS, RATIONAL BEHAVIOR, AND PERFECT CERTAINTY

The meaning of the basic assumptions.—Although the terms "perfect markets," "rational behavior," and "perfect certainty" are widely used throughout economic theory, it may be helpful to start by spelling out the precise meaning of these assumptions in the present context.

1. In "perfect capital markets," no buyer or seller (or issuer) of securities is large enough for his transactions to have an appreciable impact on the then ruling price. All traders have equal and costless access to information about the ruling price and about all other relevant characteristics of shares (to be detailed specifically later). No brokerage fees, transfer taxes, or other transaction costs are incurred when securities are bought, sold, or issued, and there are no tax differentials either between distributed and undistributed profits or between dividends and capital gains.

2. "Rational behavior" means that investors always prefer more wealth to less and are indifferent as to whether a given increment to their wealth takes the form of cash payments or an increase in the market value of their holdings of shares.

3. "Perfect certainty" implies complete assurance on the part of every investor as to the future investment program and the future profits of every corporation. Because of this assurance, there is, among other things, no need to distinguish between stocks and bonds as sources of funds at this stage of the analysis. We can, therefore, proceed as if there were only a single type of financial instrument which, for convenience, we shall refer to as shares of stock.

The fundamental principle of valuation.—Under these assumptions the valuation of all shares would be governed by the following fundamental principle: the price of each share must be such that the rate of return (dividends plus capital gains per dollar invested) on every share will be the same throughout the market over any given interval of time. That is, if we let

$d_j(t)$ = dividends per share paid by firm j during period t

$p_j(t)$ = the price (ex any dividend in $t-1$) of a share in firm j at the start of period t,

we must have

$$\frac{d_j(t)+p_j(t+1)-p_j(t)}{p_j(t)} = \rho(t) \text{ independent of } j\,; \tag{1}$$

or, equivalently,

$$p_j(t)=\frac{1}{1+\rho(t)}\left[d_j(t)+p_j(t+1)\right] \tag{2}$$

for each j and for all t. Otherwise, holders of low-return (high-priced) shares could increase their terminal wealth by selling these shares and investing the proceeds in shares offering a higher rate of return. This process would tend to drive down the prices of the low-return shares and drive up the prices of high-return shares until the differential in rates of return had been eliminated.

The effect of dividend policy.—The im-

plications of this principle for our problem of dividend policy can be seen somewhat more easily if equation (2) is restated in terms of the value of the enterprise as a whole rather than in terms of the value of an individual share. Dropping the firm subscript j since this will lead to no ambiguity in the present context and letting

$n(t)$ = the number of shares of record at the start of t

$m(t+1)$ = the number of new shares (if any) sold during t at the ex dividend closing price $p(t+1)$, so that

$n(t+1) = n(t) + m(t+1)$

$V(t) = n(t)\,p(t)$ = the total value of the enterprise and

$D(t) = n(t)\,d(t)$ = the total dividends paid during t to holders of record at the start of t,

we can rewrite (2)

$$\begin{aligned} V(t) &= \frac{1}{1+\rho(t)}\left[D(t)+n(t)p(t+1)\right] \\ &= \frac{1}{1+\rho(t)}\left[D(t)+V(t+1)\right. \\ &\qquad \left.-\, m(t+1)\,p(t+1)\right]. \end{aligned} \tag{3}$$

The advantage of restating the fundamental rule in this form is that it brings into sharper focus the three possible routes by which current dividends might affect the current market value of the firm $V(t)$, or equivalently the price of its individual shares, $p(t)$. Current dividends will clearly affect $V(t)$ via the first term in the bracket, $D(t)$. In principle, current dividends might also affect $V(t)$ indirectly via the second term, $V(t+1)$, the new ex dividend market value. Since $V(t+1)$ must depend only on future and not on past events, such could be the case, however, only if both (*a*) $V(t+1)$ were a function of future dividend policy and (*b*) the current distribution $D(t)$ served to convey some otherwise unavailable information as to what that future dividend policy would be. The first possibility being the relevant one from the standpoint of assessing the effects of dividend policy, it will clarify matters to assume, provisionally, that the future dividend policy of the firm is known and given for $t+1$ and all subsequent periods and is independent of the actual dividend decision in t. Then $V(t+1)$ will also be independent of the current dividend decision, though it may very well be affected by $D(t+1)$ and all subsequent distributions. Finally, current dividends can influence $V(t)$ through the third term, $-m(t+1)\,p(t+1)$, the value of new shares sold to outsiders during the period. For the higher the dividend payout in any period the more the new capital that must be raised from external sources to maintain any desired level of investment.

The fact that the dividend decision effects price not in one but in these two conflicting ways—directly via $D(t)$ and inversely via $-m(t)\,p(t+1)$—is, of course, precisely why one speaks of there being a dividend policy *problem*. If the firm raises its dividend in t, given its investment decision, will the increase in the cash payments to the current holders be more or less than enough to offset their lower share of the terminal value? Which is the better strategy for the firm in financing the investment: to reduce dividends and rely on retained earnings or to raise dividends but float more new shares?

In our ideal world at least these and related questions can be simply and immediately answered: the two dividend effects must always exactly cancel out so that the payout policy to be followed in t will have *no* effect on the price at t.

We need only express $m(t+1)\cdot p(t+1)$ in terms of $D(t)$ to show that such must

indeed be the case. Specifically, if $I(t)$ is the given level of the firm's investment or increase in its holding of physical assets in t and if $X(t)$ is the firm's total net profit for the period, we know that the amount of outside capital required will be

$$\begin{aligned} m(t+1)\,p(t+1) &= I(t) \\ &\quad -\left[X(t)-D(t)\right]. \end{aligned} \tag{4}$$

Substituting expression (4) into (3), the $D(t)$ cancel and we obtain for the value of the firm as of the start of t

$$\begin{aligned} V(t) &\equiv n(t)\,p(t) \\ &= \frac{1}{1+\rho(t)}\left[X(t)-I(t)+V(t+1)\right]. \end{aligned} \tag{5}$$

Since $D(t)$ does not appear directly among the arguments and since $X(t)$, $I(t)$, $V(t+1)$ and $\rho(t)$ are all independent of $D(t)$ (either by their nature or by assumption) it follows that the current value of the firm must be independent of the current dividend decision.

Having established that $V(t)$ is unaffected by the current dividend decision it is easy to go on to show that $V(t)$ must also be unaffected by any future dividend decisions as well. Such future decisions can influence $V(t)$ only via their effect on $V(t+1)$. But we can repeat the reasoning above and show that $V(t+1)$—and hence $V(t)$—is unaffected by dividend policy in $t+1$; that $V(t+2)$—and hence $V(t+1)$ and $V(t)$—is unaffected by dividend policy in $t+2$; and so on for as far into the future as we care to look. Thus, we may conclude that given a firm's investment policy, the dividend payout policy it chooses to follow will affect neither the current price of its shares nor the total return to its shareholders.

Like many other propositions in economics, the irrelevance of dividend policy, given investment policy, is "obvious, once you think of it." It is, after all, merely one more instance of the general principle that there are no "financial illusions" in a rational and perfect economic environment. Values there are determined solely by "real" considerations—in this case the earning power of the firm's assets and its investment policy—and not by how the fruits of the earning power are "packaged" for distribution.

Obvious as the proposition may be, however, one finds few references to it in the extensive literature on the problem.[1] It is true that the literature abounds with statements that in some "theoretical" sense, dividend policy ought not to count; but either that sense is not clearly specified or, more frequently and especially among economists, it is (wrongly) identified with a situation in which the firm's internal rate of return is the same as the external or market rate of return.[2]

A major source of these and related misunderstandings of the role of the dividend policy has been the fruitless concern and controversy over what investors "really" capitalize when they buy shares. We say fruitless because as we shall now proceed to show, it is actually possible to derive from the basic principle of valuation (1) not merely one, but several valuation formulas each starting from one of the "classical" views of what is being capitalized by investors. Though differing somewhat in outward appearance, the various formulas can be shown to be equivalent in all essential respects including, of course, their implication that dividend policy is irrelevant. While the

[1] Apart from the references to it in our earlier papers, especially [16], the closest approximation seems to be that in Bodenhorn [1, p. 492], but even his treatment of the role of dividend policy is not completely explicit. (The numbers in brackets refer to references listed below, pp. 432–33).

[2] See below p. 424.

controversy itself thus turns out to be an empty one, the different expressions do have some intrinsic interest since, by highlighting different combinations of variables they provide additional insights into the process of valuation and they open alternative lines of attack on some of the problems of empirical testing.

II. WHAT DOES THE MARKET "REALLY" CAPITALIZE?

In the literature on valuation one can find at least the following four more or less distinct approaches to the valuation of shares: (1) the discounted cash flow approach; (2) the current earnings plus future investment opportunities approach; (3) the stream of dividends approach; and (4) the stream of earnings approach. To demonstrate that these approaches are, in fact, equivalent it will be helpful to begin by first going back to equation (5) and developing from it a valuation formula to serve as a point of reference and comparison. Specifically, if we assume, for simplicity, that the market rate of yield $\rho(t) = \rho$ for all t,[3] then, setting $t = 0$, we can rewrite (5) as

$$\begin{aligned} V(0) &= \frac{1}{1+\rho}\left[X(0)-I(0)\right] \\ &\quad +\frac{1}{1+\rho}\,V(1). \end{aligned} \tag{6}$$

Since (5) holds for all t, setting $t = 1$ permits us to express $V(1)$ in terms of $V(2)$ which in turn can be expressed in terms of $V(3)$ and so on up to any arbitrary terminal period T. Carrying out these substitutions, we obtain

$$\begin{aligned} V(0) &= \sum_{t=0}^{T-1}\frac{1}{(1+\rho)^{t+1}}\left[X(t)-I(t)\right] \\ &\quad +\frac{1}{(1+\rho)^{T}}\,V(T). \end{aligned} \tag{7}$$

In general, the remainder term $(1+\rho)^{-T}\cdot V(T)$ can be expected to approach zero as T approaches infinity[4] so that (7) can be expressed as

$$\begin{aligned} V(0) &= \lim_{T\to\infty}\sum_{t=0}^{T-1}\frac{1}{(1+\rho)^{t+1}} \\ &\quad \times\left[X(t)-I(t)\right], \end{aligned} \tag{8}$$

which we shall further abbreviate to

$$V(0)=\sum_{t=0}^{\infty}\frac{1}{(1+\rho)^{t+1}}\left[X(t)-I(t)\right]. \tag{9}$$

The discounted cash flow approach.—Consider now the so-called discounted cash flow approach familiar in discussions of capital budgeting. There, in valuing any specific machine we discount at the market rate of interest the stream of cash receipts generated by the machine; plus any scrap or terminal value of the machine; and minus the stream of cash outlays for direct labor, materials, repairs, and capital additions. The same approach, of course, can also be applied to the firm as a whole which may be thought of in this context as simply a large, composite machine.[5] This ap-

[3] More general formulas in which $\rho(t)$ is allowed to vary with time can always be derived from those presented here merely by substituting the cumbersome product

$$\prod_{\tau=0}^{t}\left[1+\rho(\tau)\right] \quad \text{for} \quad (1+\rho)^{t+1}.$$

[4] The assumption that the remainder vanishes is introduced for the sake of simplicity of exposition only and is in no way essential to the argument. What is essential, of course, is that $V(0)$, i.e., the sum of the two terms in (7), be finite, but this can always be safely assumed in economic analysis. See below, n. 14.

[5] This is, in fact, the approach to valuation normally taken in economic theory when discussing the value of the *assets* of an enterprise, but much more rarely applied, unfortunately, to the value of the liability side. One of the few to apply the approach to the shares as well as the assets is Bodenhorn in [1], who uses it to derive a formula closely similar to (9) above.

公司	ρ	$d_j(-1)$	d_j(0以后)	$p_j(-1)$	$p_j(0)$	$p_j(1)$	$p_j(2)$
A	0.05	2.5	1	50	50	51.5	53.075
B	0.05	2.5	3	50	50	49.5	48.975

公式 7 末项$\lim_{T\to\infty}\frac{1}{(1+\rho)^T}V(T)=\lim_{T\to\infty}n(T)\times\lim_{T\to\infty}\frac{p(T)}{(1+\rho)^T}$

由公式 2，$p_j(t+1)=p_j(t)(1+\rho)-d_j(t)$

$$p(0)=50$$

$$p(1)=p(0)\times 1.05-1$$

$$p(2)=p(1)\times 1.05-1=p(0)\times 1.05^2-1.05-1$$

$$p(3)=p(2)\times 1.05-1=p(0)\times 1.05^3-1.05^2-1.05-1$$

$$\cdots$$

$$p(T)=p(0)\times 1.05^T-1.05^{T-1}-\cdots-1.05^2-1.05-1$$

$$=p(0)\times 1.05^T-\frac{1.05^T-1}{1.05-1}$$

$$\lim_{T\to\infty}\frac{p(T)}{1.05^T}=30$$

$$\lim_{T\to\infty}\frac{1}{(1+\rho)^T}V(T)=30n(T)$$

MM 在其论文的开始部分，就假设了投资者的理性行为，即投资者只在意更多的财富，而不在意它是以股利还是股价形式实现。再结合其它假设，MM 得出公式 1 和公式 2，即在任一时期都存在与上市公司无关的收益率 ρ(t)使得投资者只能按其获取平均收益。其中，$d_j(t)$是 j 公司在 t 期间支付的每股股息，$p_j(t)$是 t 期初时 j 公司股票的价格。当仅考虑一个公司时，下标 j 可以省略。令 n(t)为 t 开始时的股票数量，m(t+1)为 t 期间以除息收盘价 p(t+1)出售的新股数量，有 n(t+1)=n(t)+m(t+1)，企业的总价值 V(t)=n(t)p(t)，t 期初支付的股息总额 D(t)=n(t)d(t)，于是将公式 2 变化整理得到公式 3。再令 I(t)为公司在 t 期间的投资额，X(t)为公司在 t 期间的净利润总额。公司的净利润要么成为投资，要么成为股息，故得到所需的外部资本数额即公式 4，将 4 代入 3，得到公式 5。再假设 ρ(t)为定值推出公式 6，递归公式 6 的末项得到公式 7。在公式 7 中，最后一项的极限是 0，故推出公式 8，整理为公式 9。当投资为零且不增发股票时，公式 9 两边同除以股票总量可以得到股利贴现模型。但是，问题并没有解决好。在对公式 1 和 2 的解释中，MM 写到市场会压低低回报（即高价格）股票价格并推高高回报股票价格，直到消除回报率差异，但这与两个公式并不符合。如表所示，假定市场的平均收益率恒为 5%，AB 两公司根据公式 1 和 2 计算的股价。可以看出，低回报的 A 股价越来越高，而高回报的 B 股价越来越低。如果想要回报率与价格匹配，各时期的 p_j 就必须相等，但如此就意味着，$d_j(t)$一期的股利即可决定股价，与公式 9 矛盾，也失去了公式存在的意义。如果去掉这个表述，虽然问题看似可以解决，但公式 7 中末项的极限就不为 0，从而无法推出公式 9。仍以上表为例，由公式 2 和 7，求得 A 公司的该极限为 30n(T)。由此可见 MM 的证明方法不可行。强调一下，这里反对 MM 的证明方法，不是全部反对论文的结论。

不仅如此，MM 在论文中递归公式 6 的末项得到公式 7 是错误的，因为在卖出后股票和公司并不为股民带来利益，自然也不会被股民考虑。也就是说，从自身利益出发，股民没有必要认同公式 7 并按其交易，所以市场不一定会按其后的公式运行。

由前面的分析可知，股票估值公式错误。