Searching: Deterministic single-agent

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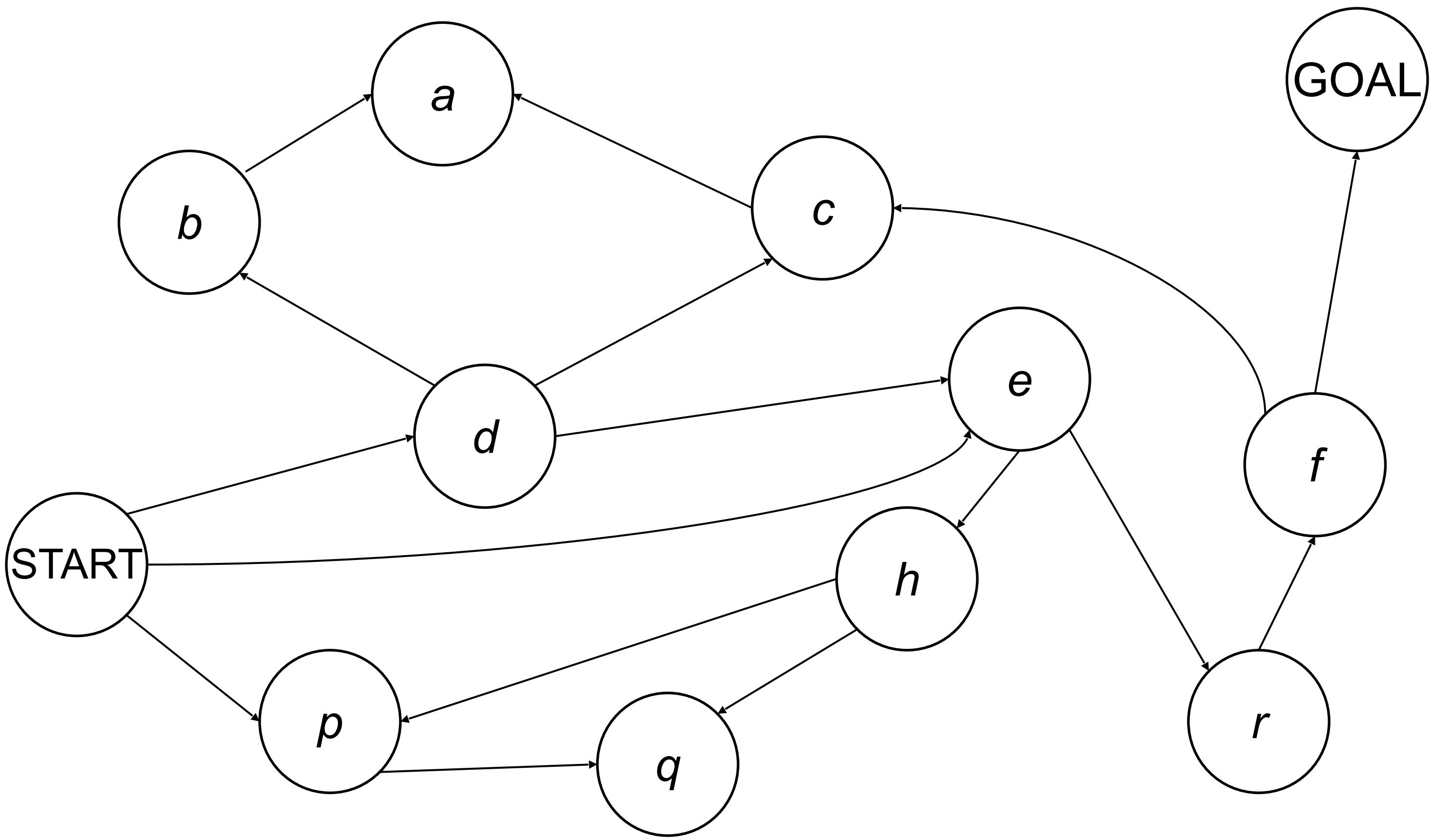
Overview

- Deterministic, single-agent, search problems
- Breadth First Search
- Optimality, Completeness, Time and Space complexity
- Search Trees
- Depth First Search
- Iterative Deepening
- Best First “Greedy” Search

A search problem



How do we get from S to G? And what's the smallest possible number of transitions?

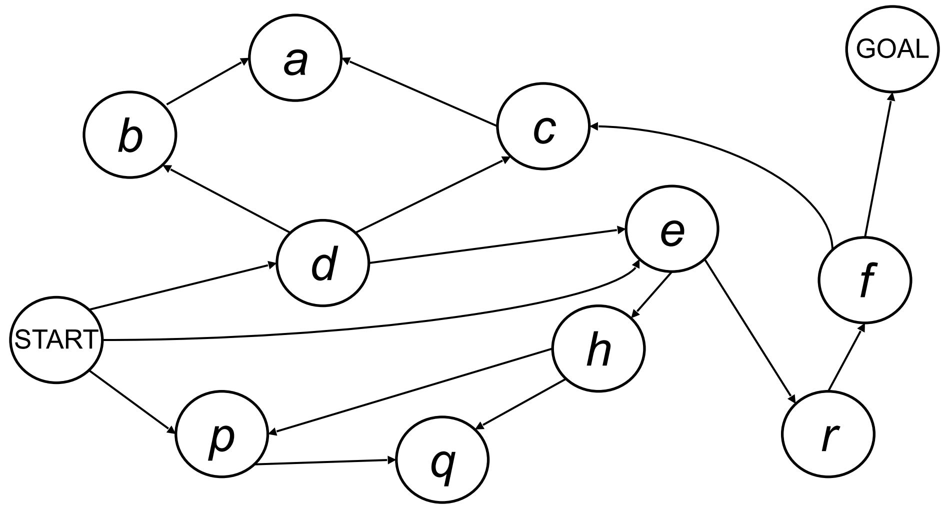
Formalizing a search problem

A search problem has five components:

Q , S , G , **succs** , **cost**

- Q is a finite set of states.
- $S \subseteq Q$ is a non-empty set of start states.
- $G \subseteq Q$ is a non-empty set of goal states.
- **succs** : $Q \rightarrow P(Q)$ is a function which takes a state as input and returns a set of states as output. **succs**(s) means "the set of states you can reach from s in one step".
- **cost** : Q , $Q \rightarrow$ *Positive Number* is a function which takes two states, s and s', as input. It returns the one-step cost of traveling from s to s'. The cost function is only defined when s' is a successor state of s.

Our Search Problem



Q = {START, *a* , *b* , *c* , *d* , *e* , *f* , *h* , *p* , *q* , *r* , GOAL}

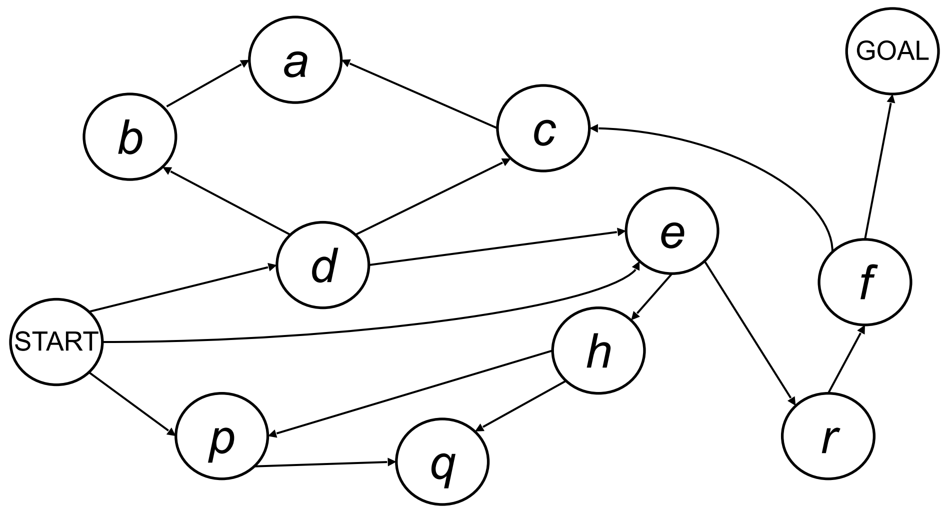
S = { START }

G = { GOAL }

succs(*b*) = { *a* }
succs(*e*) = { *h* , *r* }
succs(*a*) = NULL … etc.

cost(*s*,*s'*) = 1 for all transitions

Our Search Problem



Q = {START, *a* , *b* , *c* , *d* , *e* , *f* , *h* , *p* , *q* , *r* , GOAL}

S = { START }

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succs(*b*) = { *a* }
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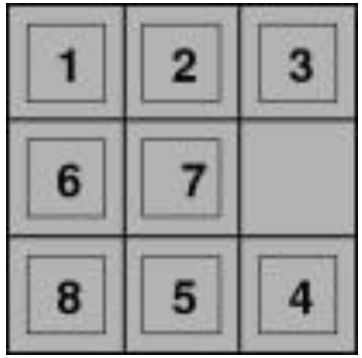
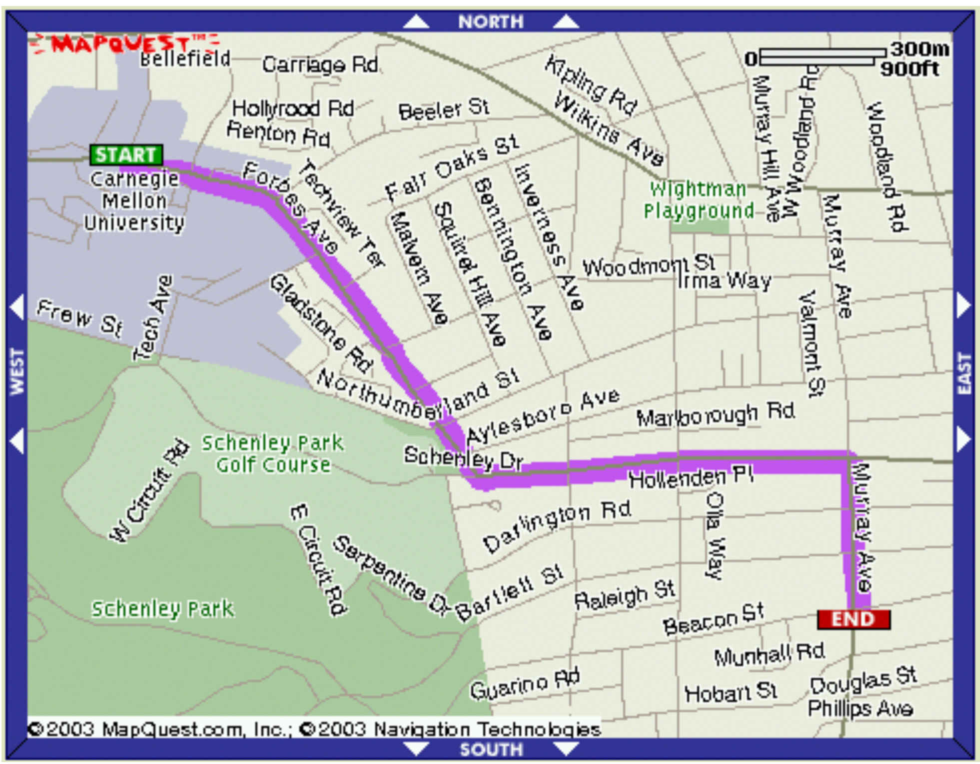
succs(*a*) = NULL … etc.

cost(*s*,*s'*) = 1 for all transitions

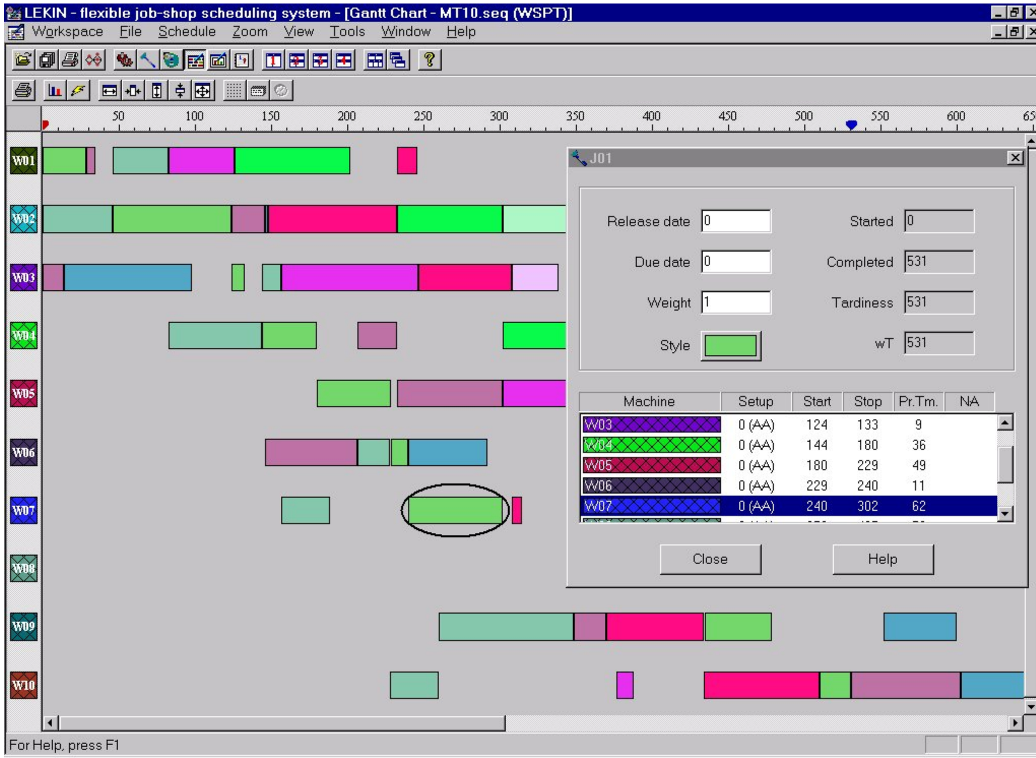
Why do we care? What problems are like this?

Search Problems



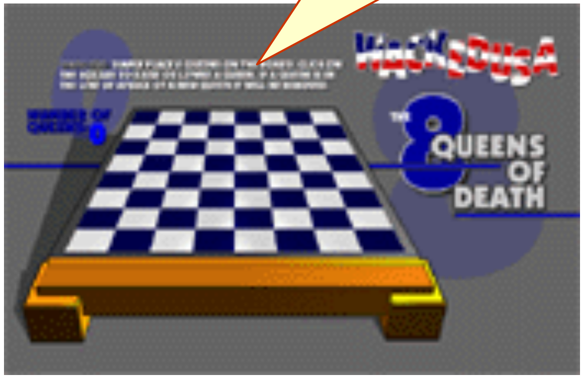


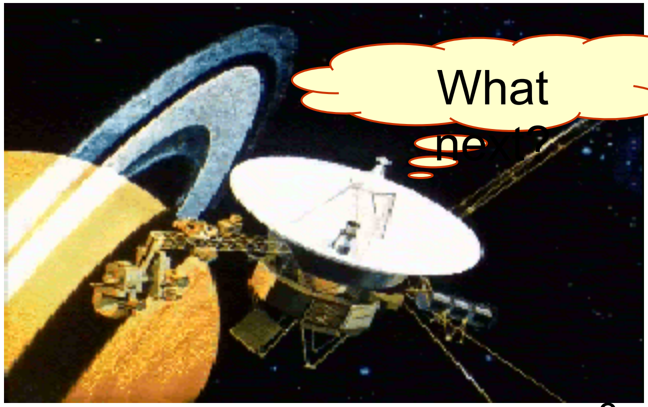
More Search Problems



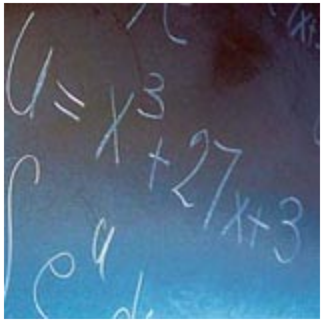
Scheduling

8-Queens





What



More Search Problems

But there are plenty of things which we'd normally call search problems that don't fit our rigid definition…

eduling

- A search problem has five components:
- *Q* , *S* , *G* , **succs** , **cost**
- *Q* is a finite set of states.
- $S \subseteq Q$ is a non-empty set of start states.
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- **succs** : $Q \rightarrow P(Q)$ is a function which takes a state as input and returns a set of states as output. **succs**(*s*) means "the set of states you can reach from *s* in one step".
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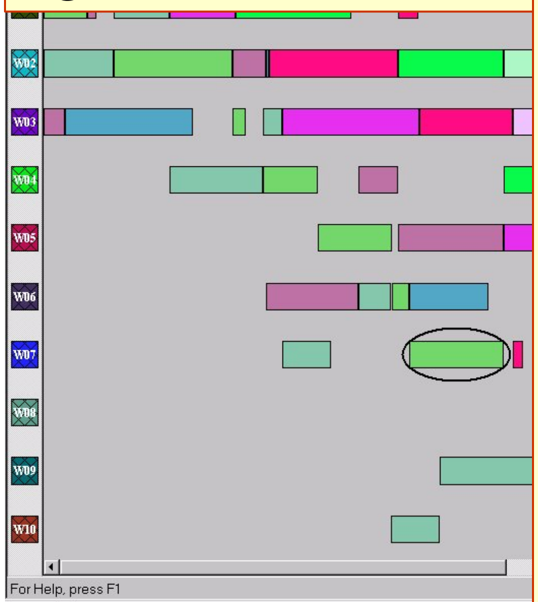
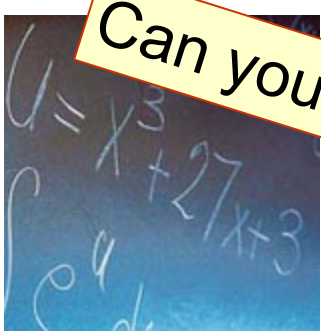
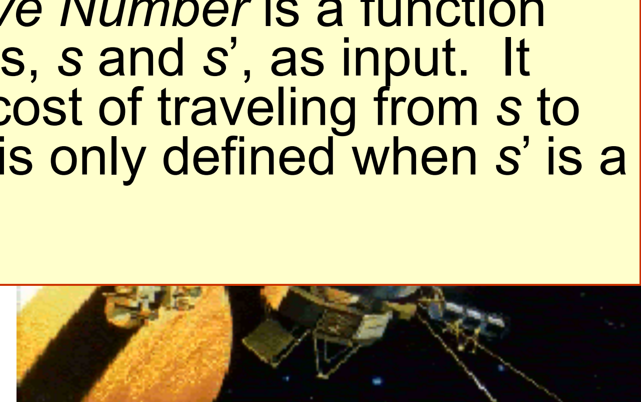
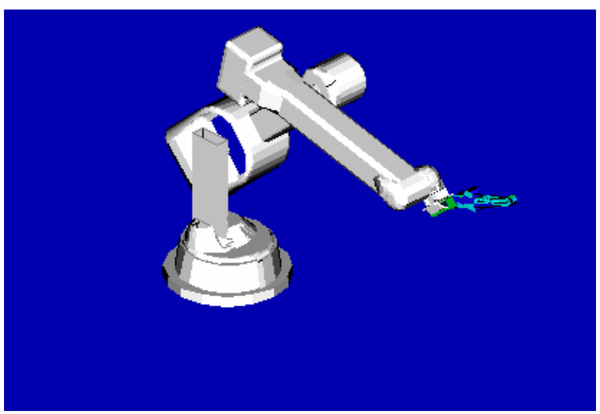






Can you think of examples?

Our definition excludes…



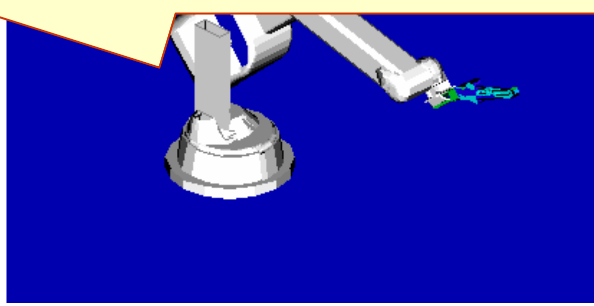
Our definition excludes

Game against adversary

Chance

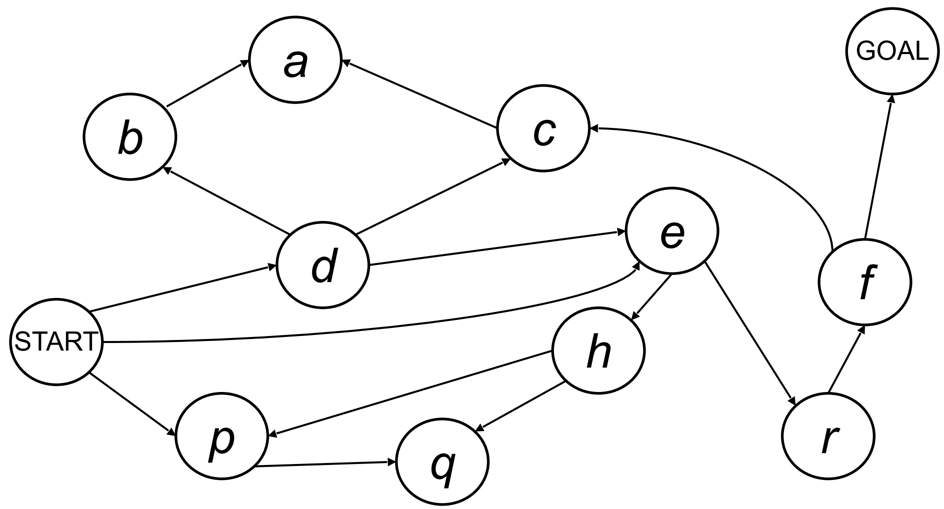
Hidden State

Continuum (infinite number) of states



All of the above, plus distributed team control

Breadth First Search



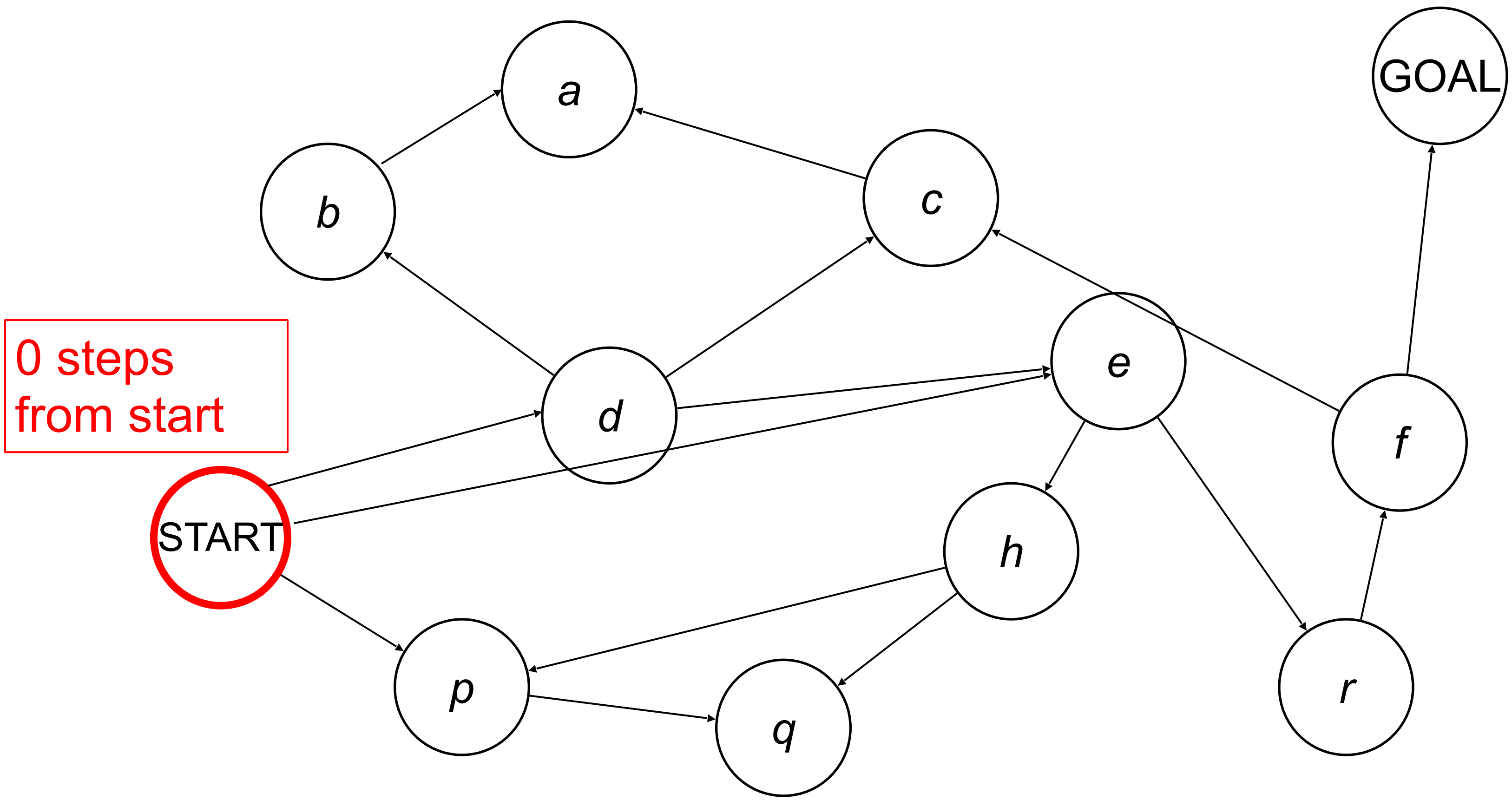
Label all states that are reachable from S in 1 step but aren't reachable in less than 1 step.

Then label all states that are reachable from S in 2 steps but aren't reachable in less than 2 steps.

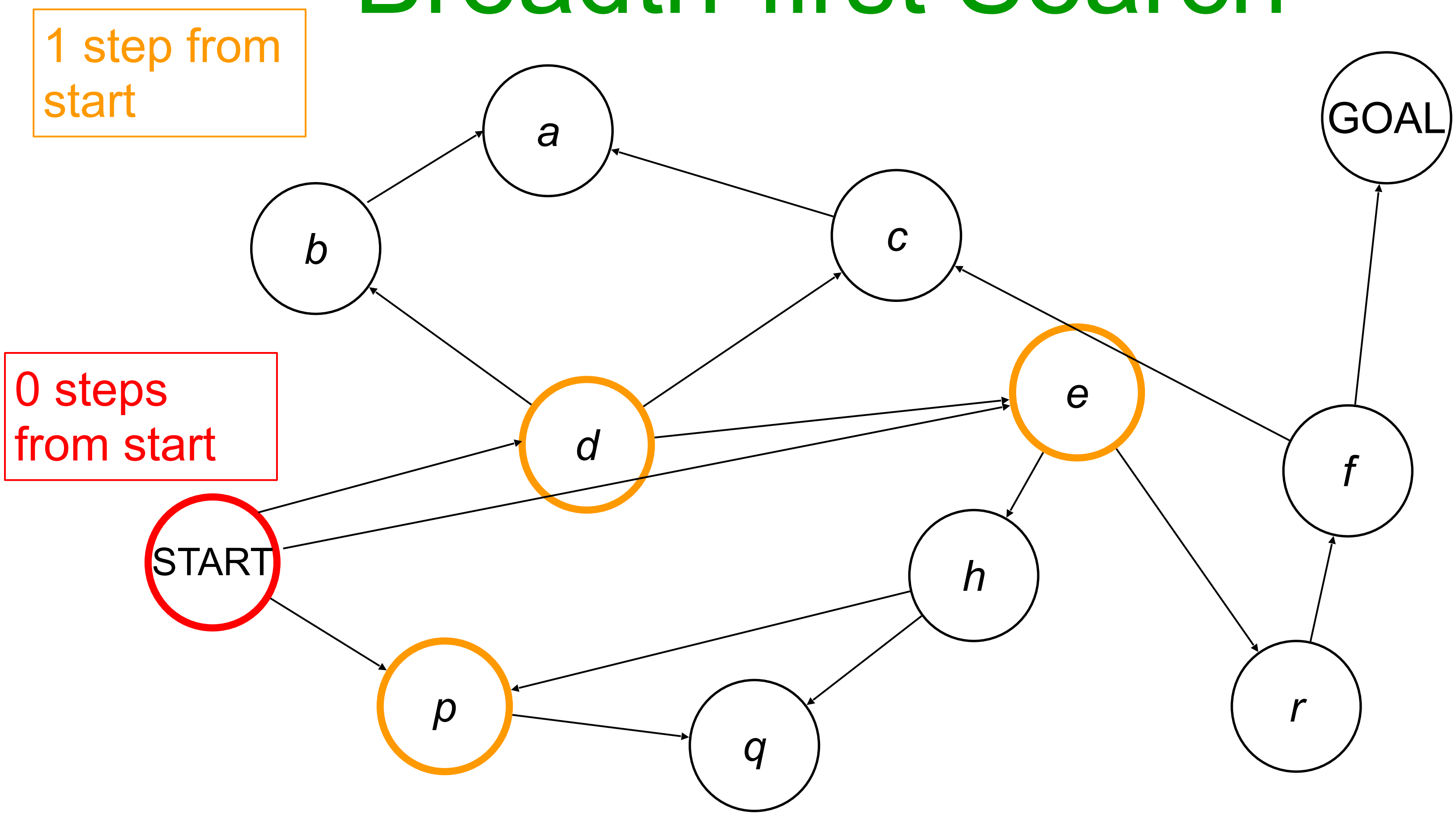
Then label all states that are reachable from S in 3 steps but aren't reachable in less than 3 steps.

Etc… until Goal state reached.

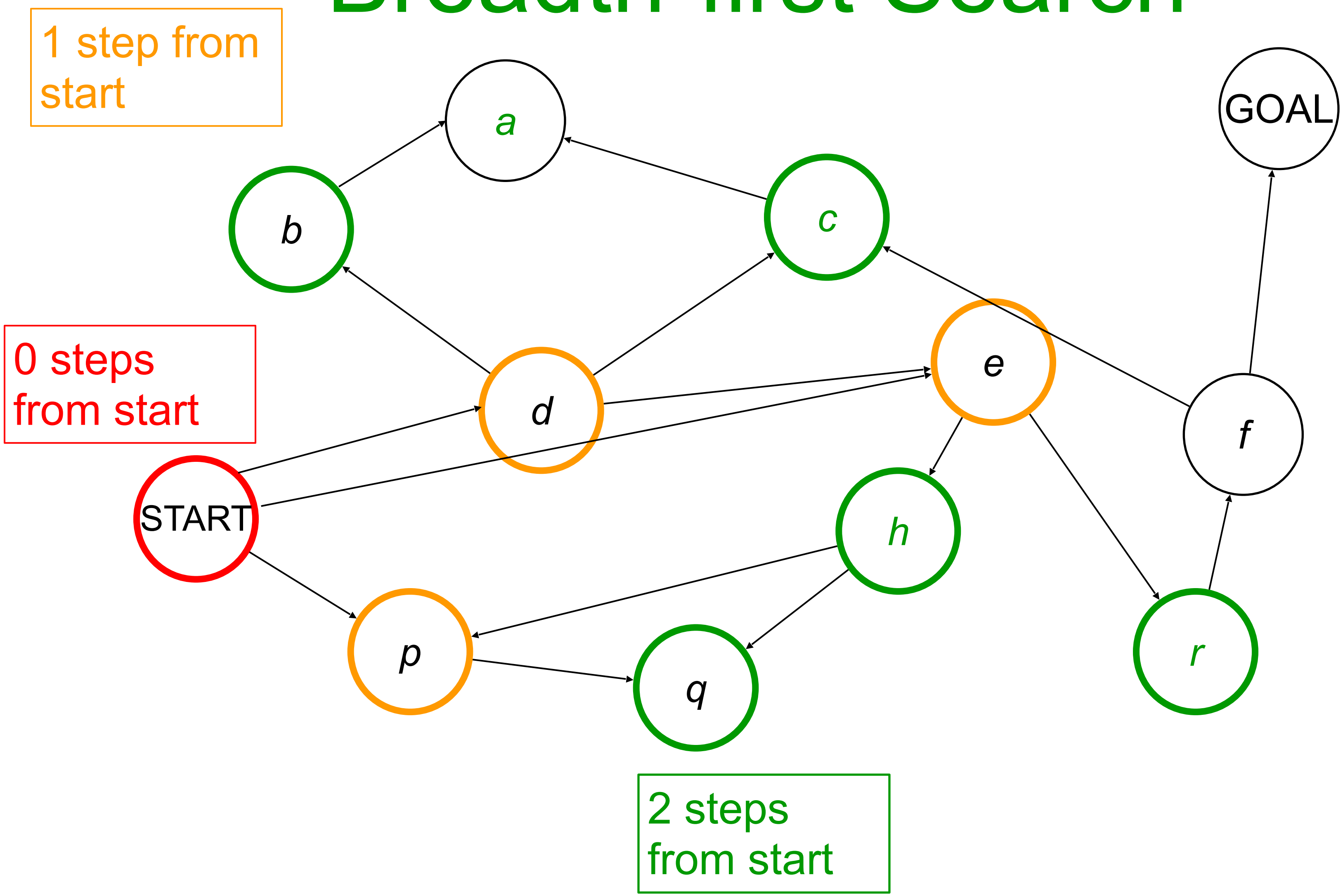
Breadth-first Search



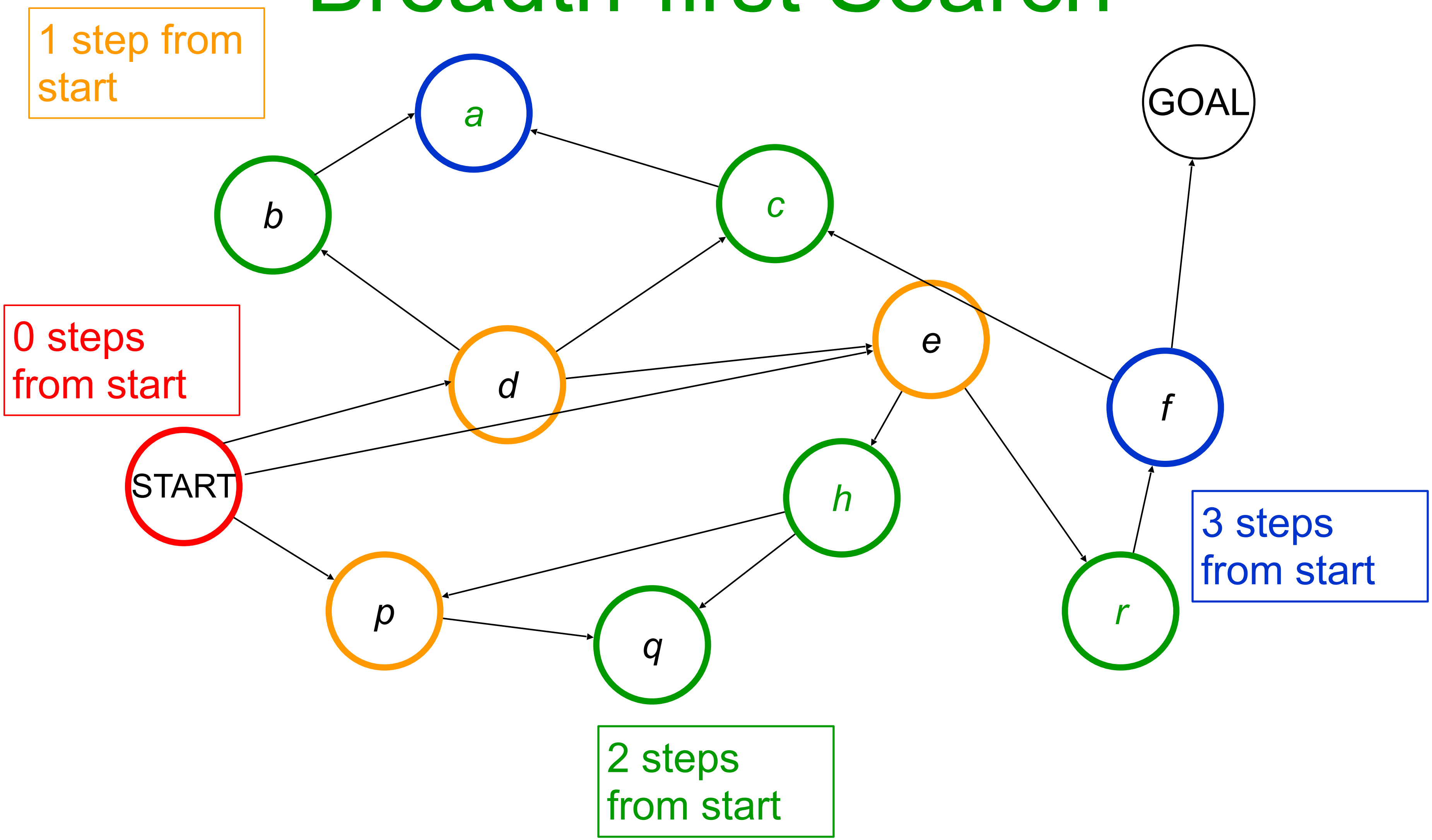
Breadth-first Search



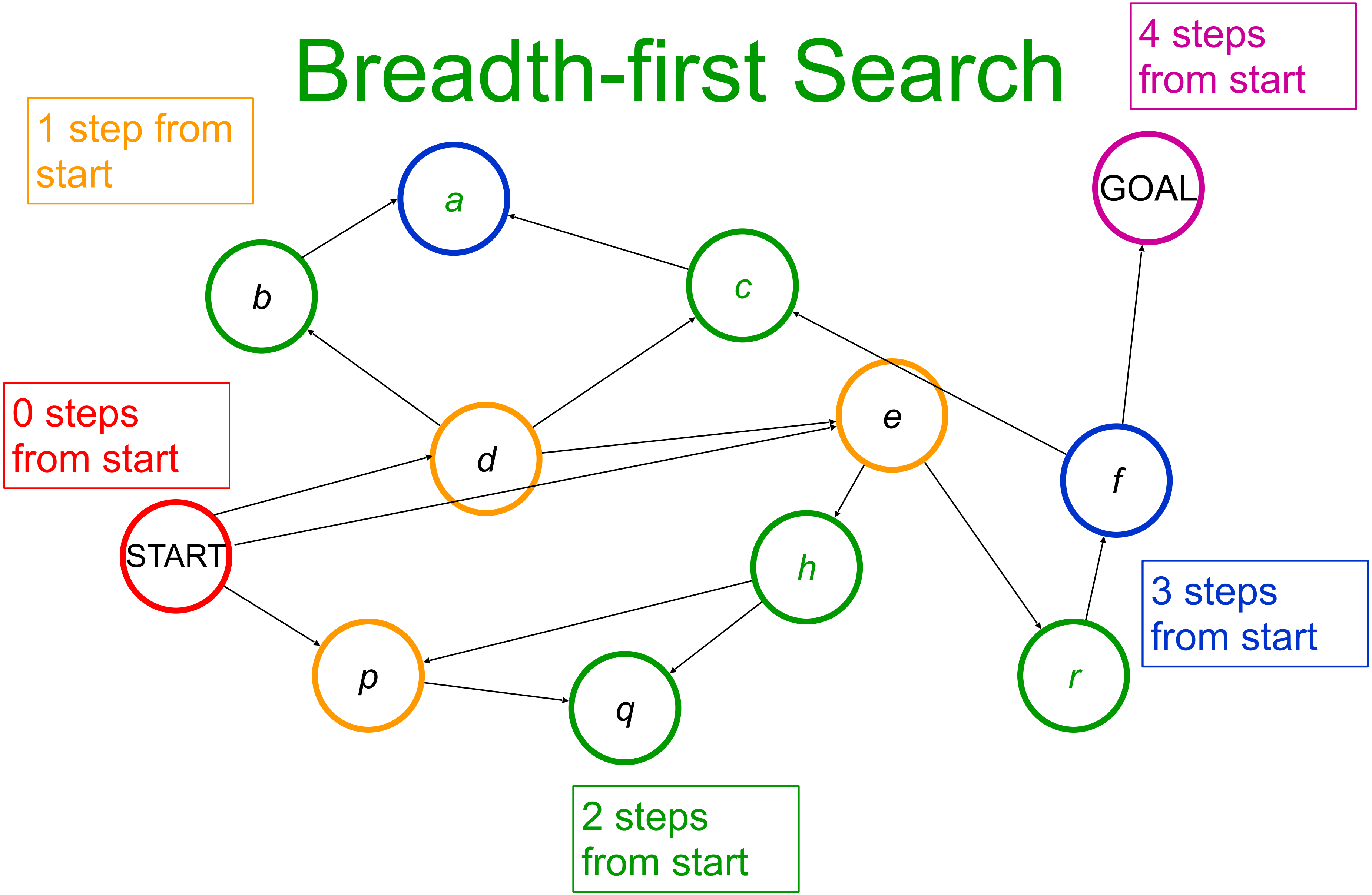
Breadth-first Search



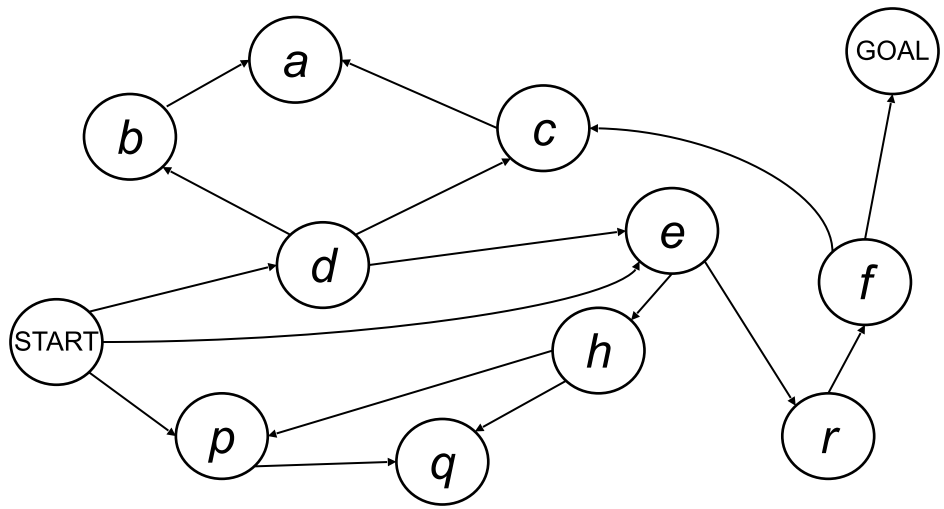
Breadth-first Search



Breadth-first Search



Remember the path!

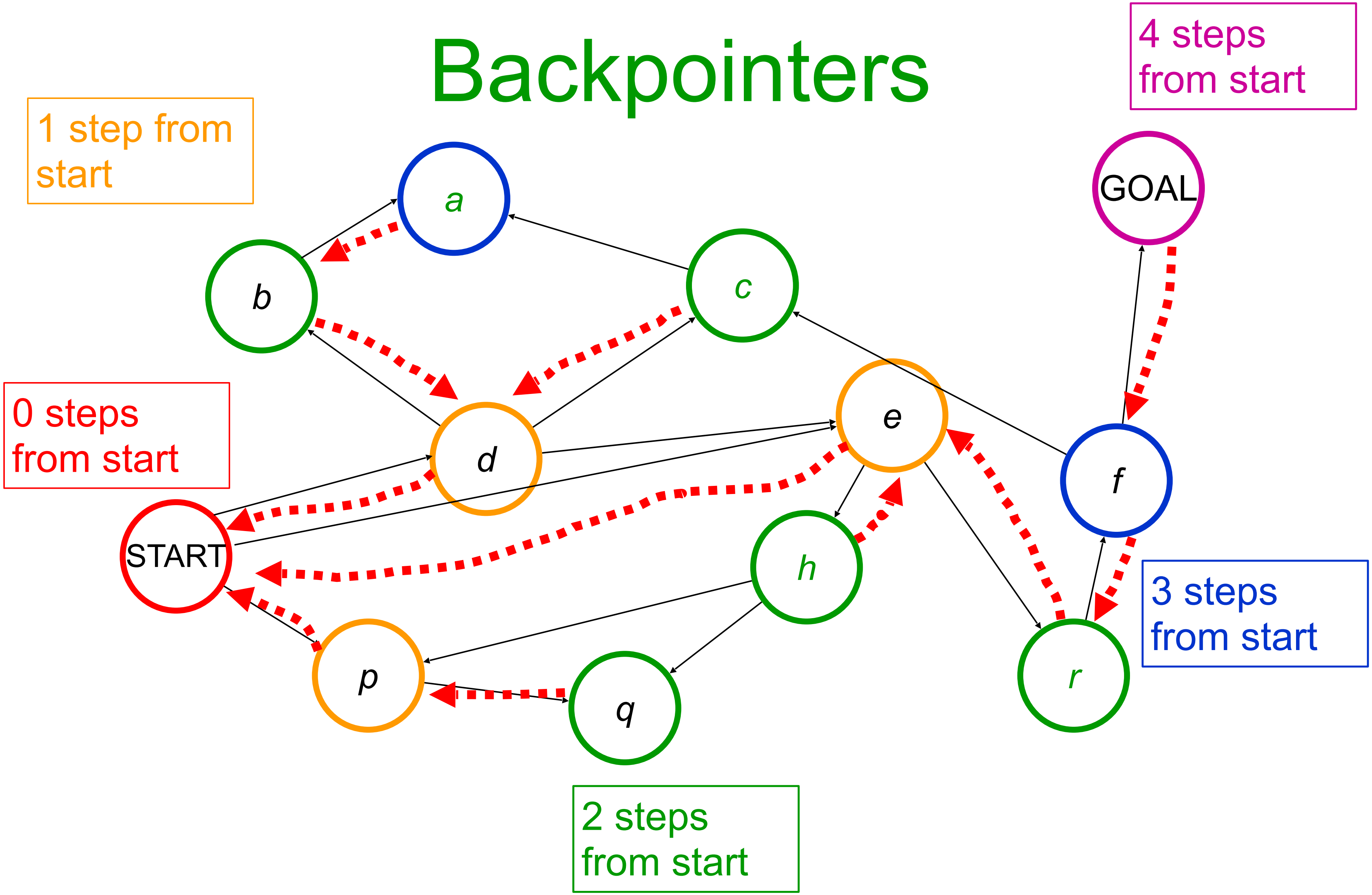


Also, when you label a state, record the predecessor state. This record is called a *backpointer*. The history of predecessors is used to generate the solution path, once you've found the goal:

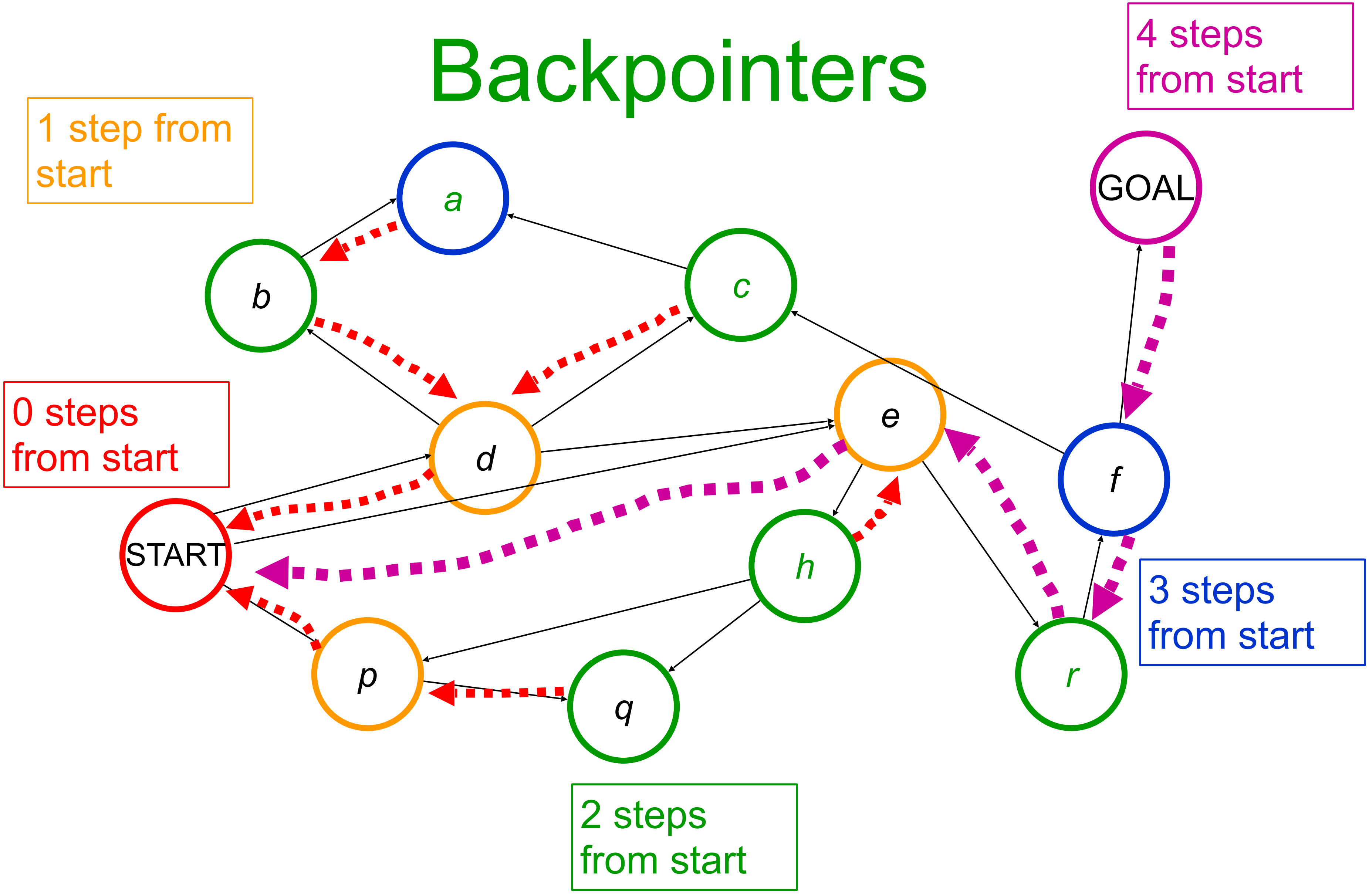
"I've got to the goal. I see I was at *f* before this. And I was at *r* before I was at *f*. And I was…

…. so solution path is S → *e* → *r* → *f* → G"

Backpointers



Backpointers



Starting Breadth First Search

For any state s that we've labeled, we'll remember:

•*previous*(s) as the previous state on a shortest path from START state to s.

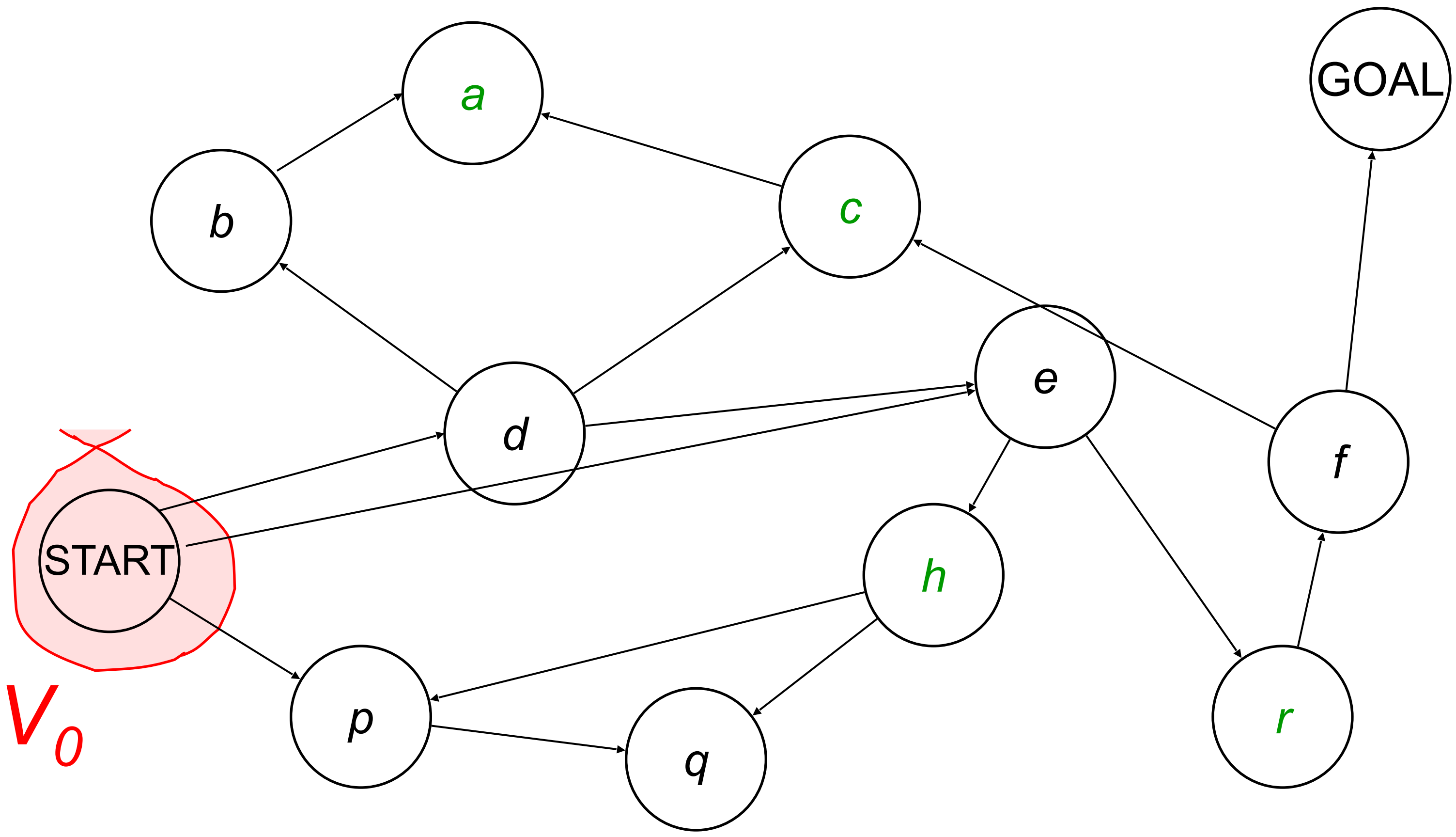
On the kth iteration of the algorithm we'll begin with V_k defined as the set of those states for which the shortest path from the start costs exactly k *steps*

Then, during that iteration, we'll compute V_{k+1}, defined as the set of those states for which the shortest path from the start costs exactly $k+1$ *steps*

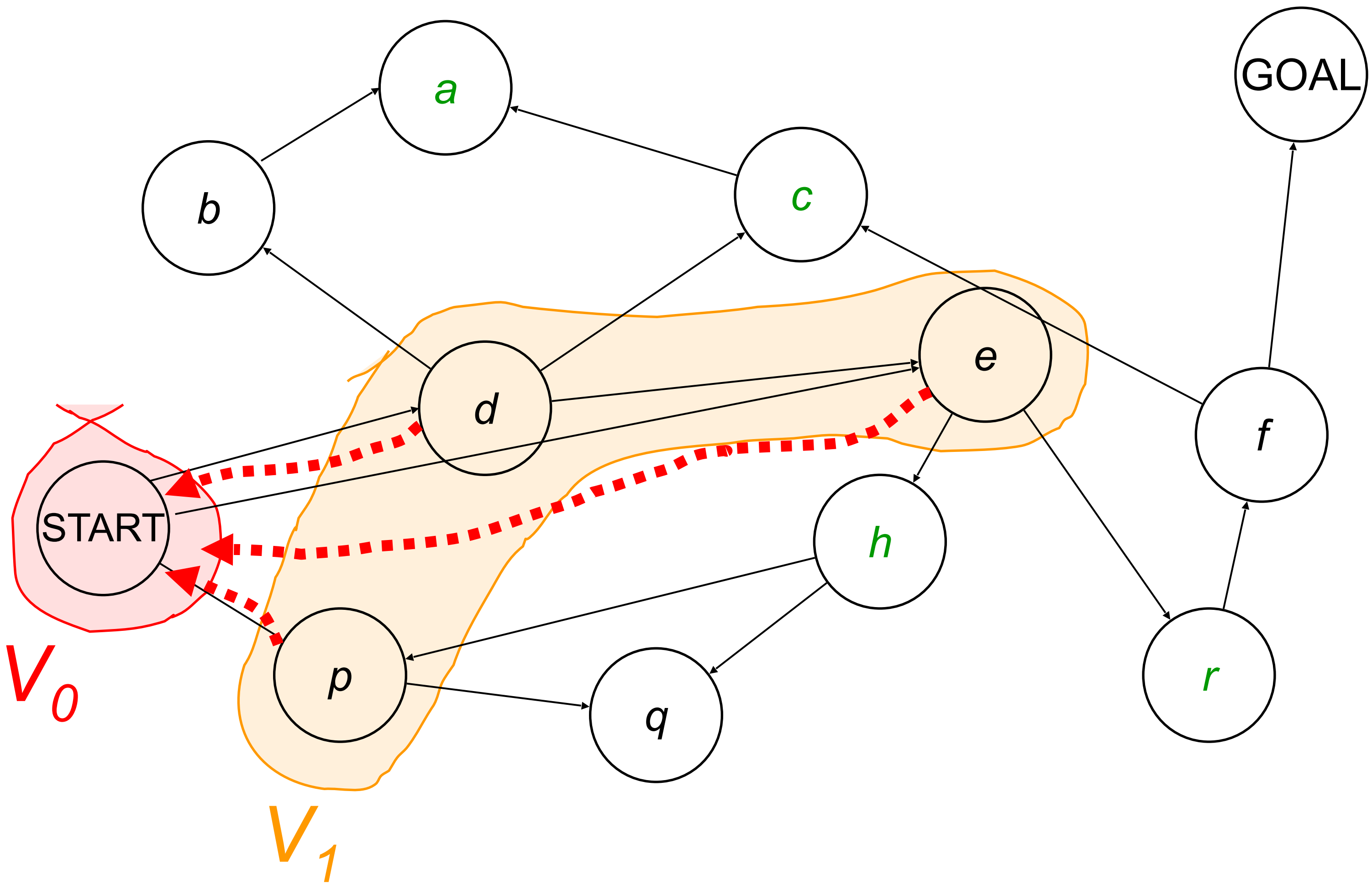
We begin with $k = 0$, V_0 = {START} and we'll define, *previous*(*START*) = *NULL*

Then we'll add in things one step from the START into V_1. And we'll keep going.

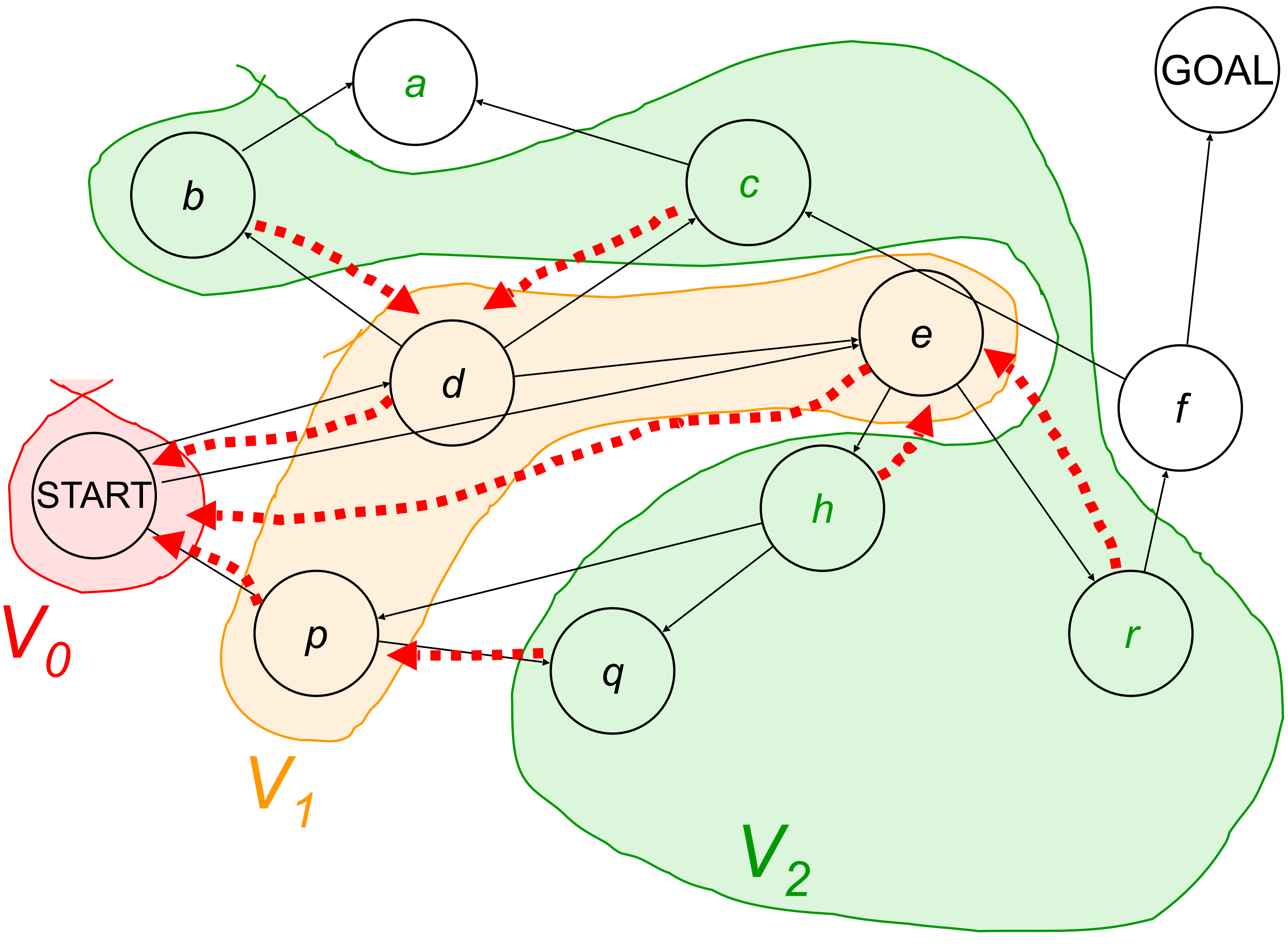
BFS



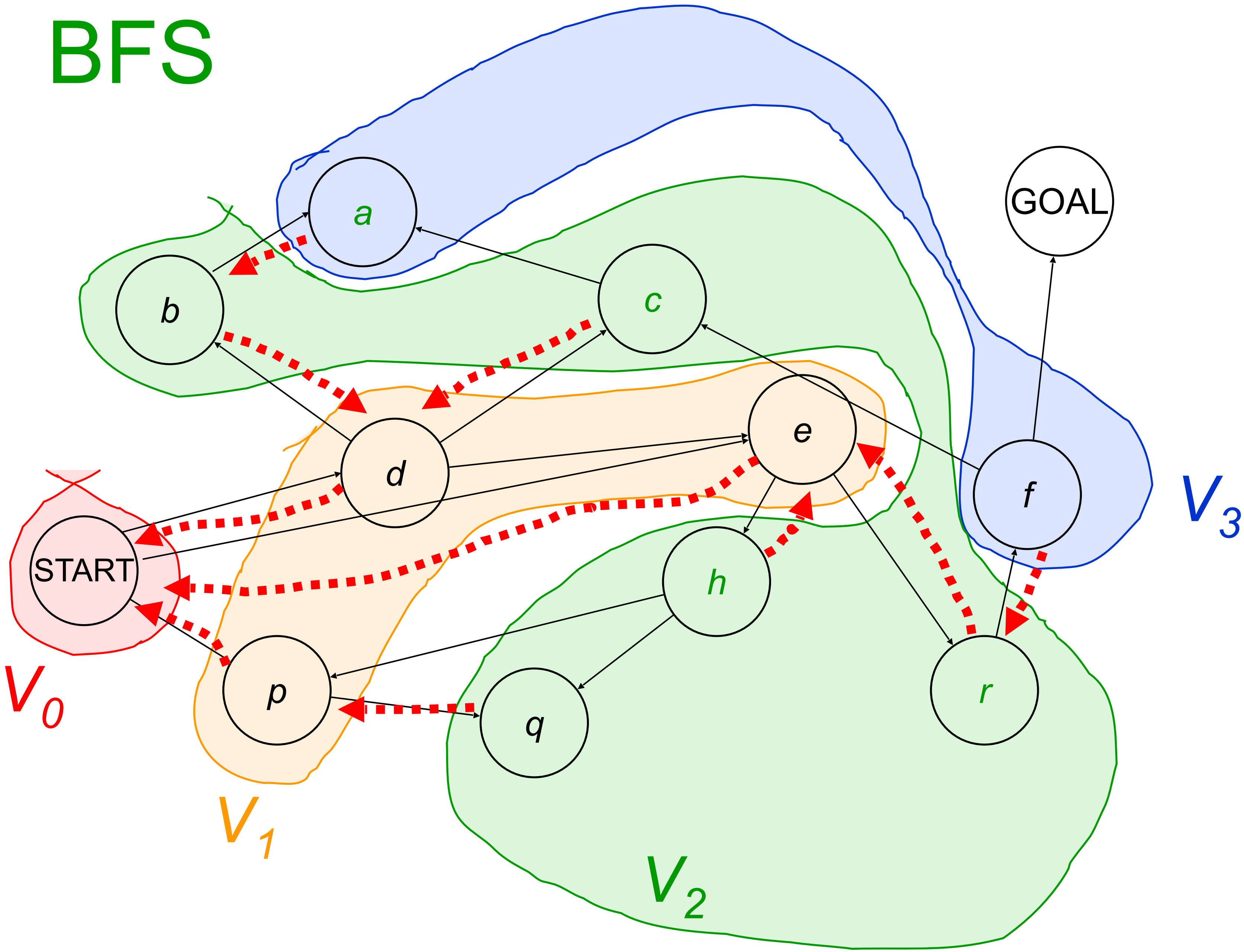
BFS



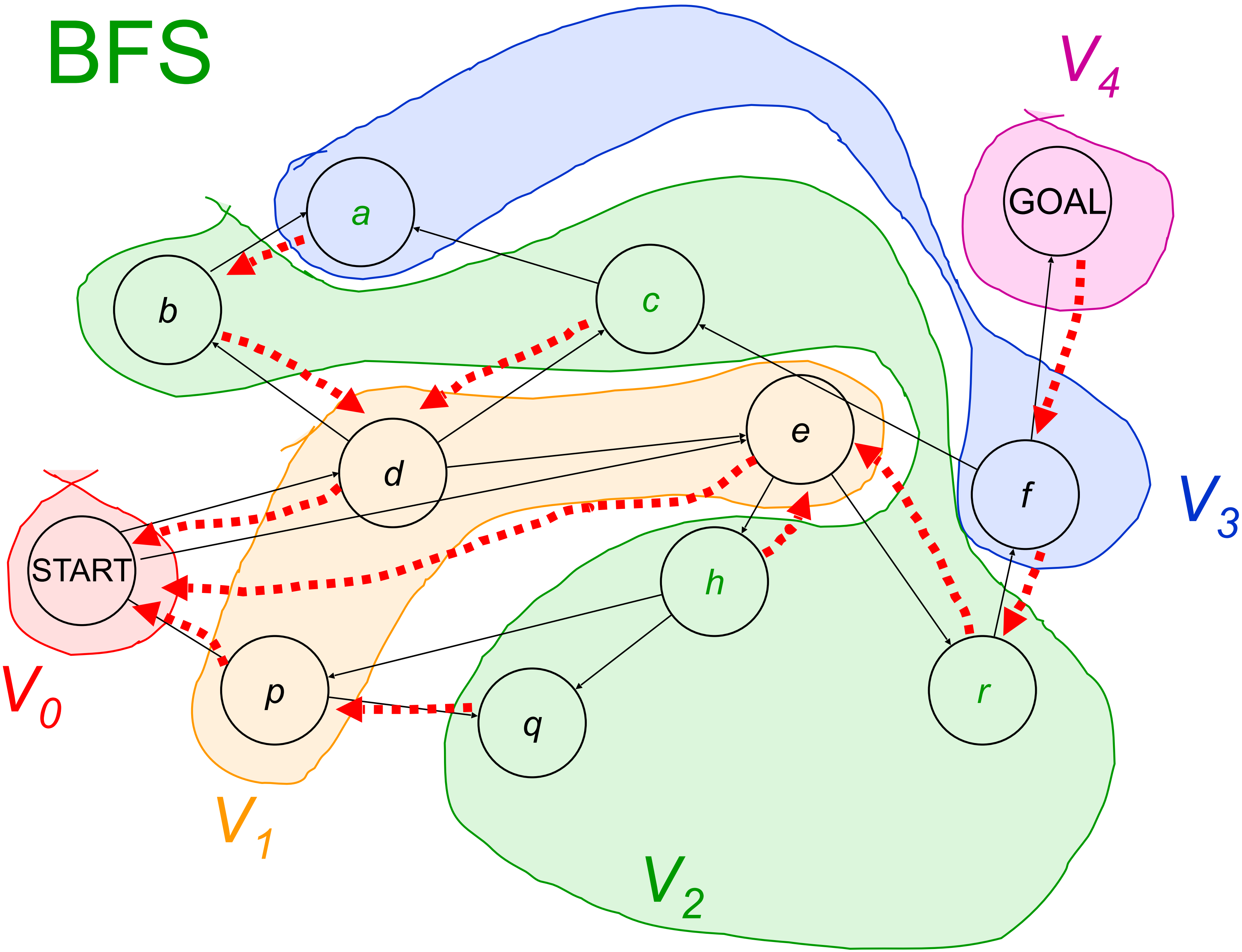
BFS



BFS



BFS



Breadth First Search

V_0 := S (the set of start states)

previous(*START*) := *NIL*

$k := 0$

while (no goal state is in V_k and V_k is not empty) **do**

    V_{k+1} := empty set

    For each state *s* in V_k

        For each state *s'* in **succs**(*s*)

            If *s'* has not already been labeled

                Set *previous*(*s'*) := *s*

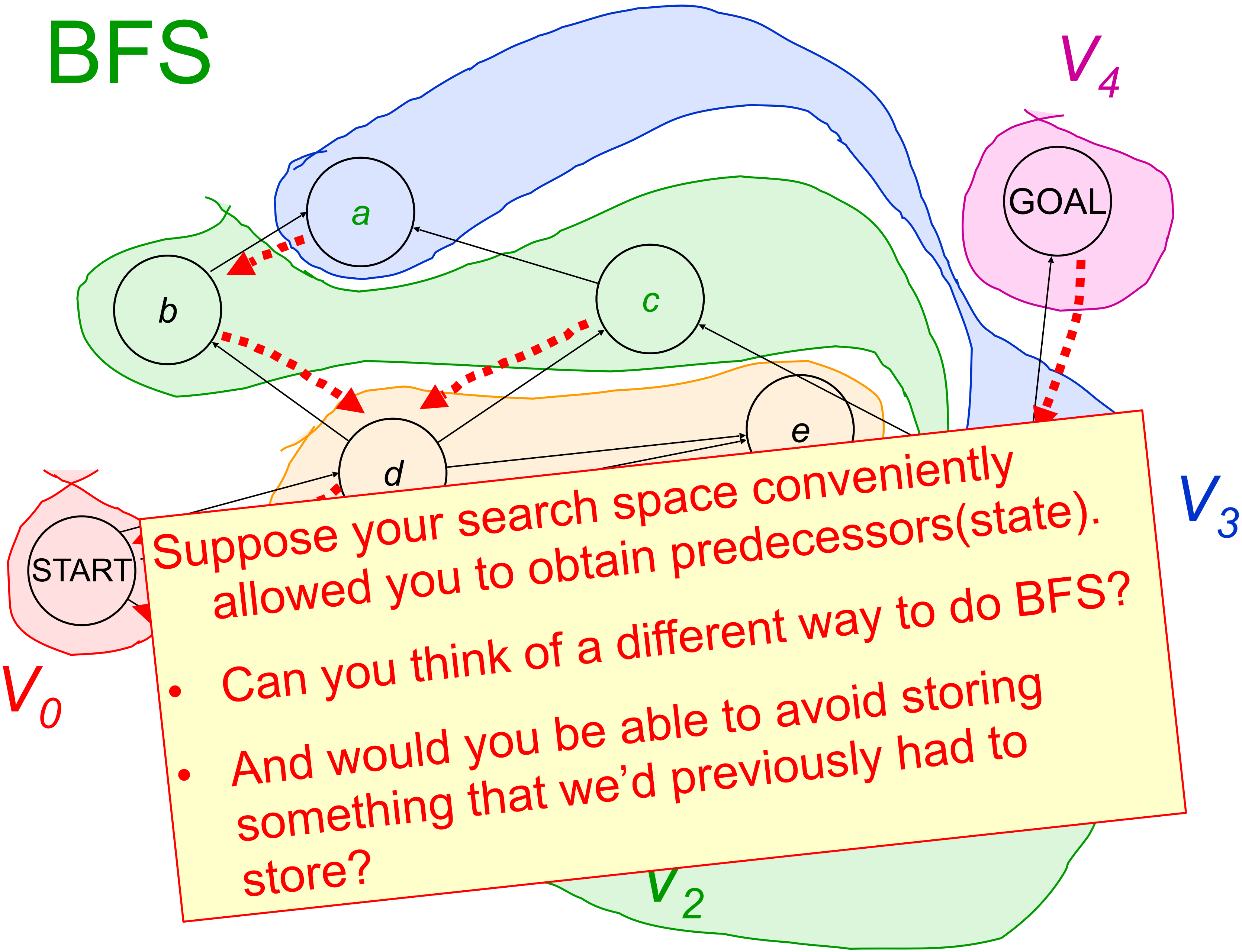
                Add *s'* into V_{k+1}

    $k := k+1$

If V_k is empty signal FAILURE

Else build the solution path thus: Let S_i be the *i*th state in the shortest path. Define S_k = GOAL, and forall $i <= k$, define S_{i-1} = *previous*(S_i).

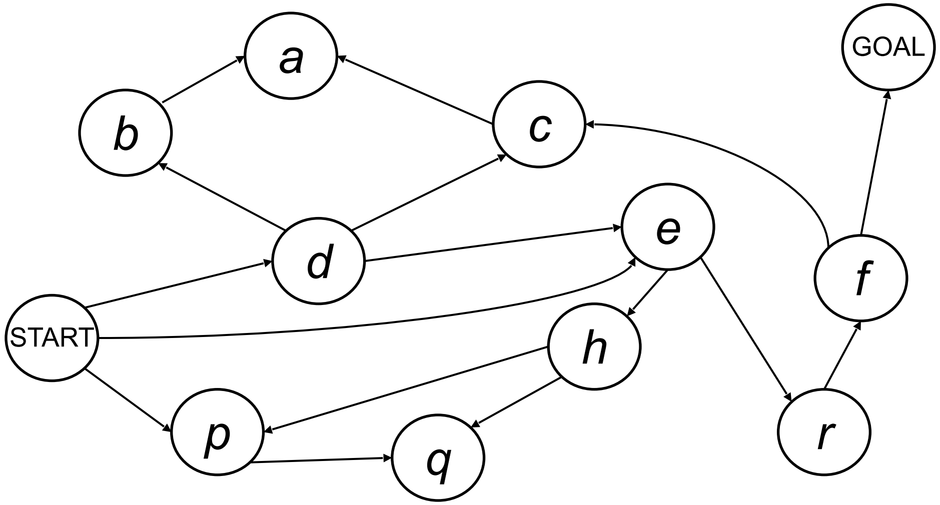
BFS



Suppose your search space conveniently allowed you to obtain predecessors(state).

- Can you think of a different way to do BFS?
- And would you be able to avoid storing something that we'd previously had to store?

Another way: Work back



Label all states that can reach G in 1 step but can't reach it in less than 1 step.

Label all states that can reach G in 2 steps but can't reach it in less than 2 steps.

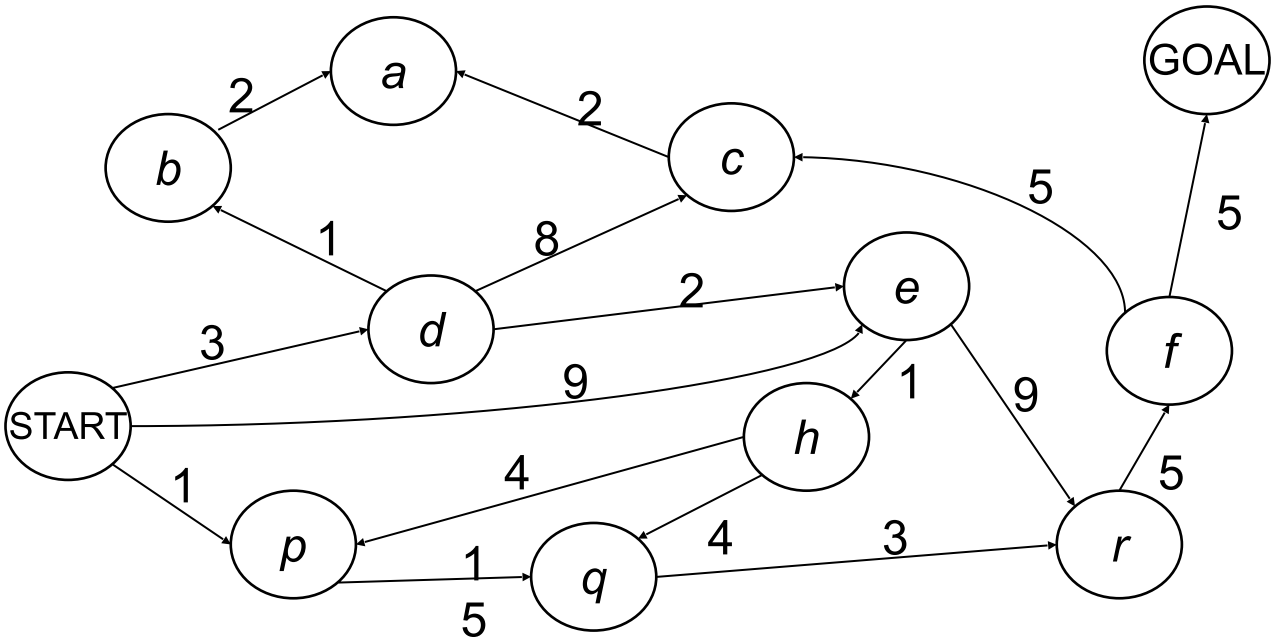
Etc. … until start is reached.

"number of steps to goal" labels determine the shortest path. Don't need extra bookkeeping info.

Breadth First Details

- It is fine for there to be more than one goal state.
- It is fine for there to be more than one start state.
- This algorithm works forwards from the start. Any algorithm which works forwards from the start is said to be *forward chaining*.
- You can also work backwards from the goal. This algorithm is very similar to Dijkstra's algorithm.
- Any algorithm which works backwards from the goal is said to be *backward chaining*.
- Backward versus forward. Which is better?

Costs on transitions



Notice that BFS finds the shortest path in terms of number of transitions. It does not find the least-cost path.

We will quickly review an algorithm which does find the least-cost path. On the *k*th iteration, for any state *S*, write *g*(*s*) as the least-cost path to *S* in *k* or fewer steps.

Least Cost Breadth First

V_k = the set of states which can be reached in exactly k steps, and for which the least-cost k-step path is less cost than any path of length less than k. In other words, V_k = the set of states whose values changed on the previous iteration.

V_0 := S (the set of start states)

previous(*START*) := *NIL*

g(*START*) = *0*

k := *0*

while (V_k is not empty) **do**

    V_{k+1} := empty set

    For each state *s* in V_k

        For each state *s'* in **succs**(*s*)

            If *s'* has not already been labeled

            OR if *g*(*s*) + *Cost*(*s*,*s'*) < *g*(*s'*)

                Set *previous*(*s'*) := *s*

                Set *g*(*s'*) := *g*(*s*) + *Cost*(*s*,*s'*)

                Add *s'* into V_{k+1}

    k := *k+1*

If GOAL not labeled, exit signaling FAILURE

Else build the solution path thus: Let S_k be the *k*th state in the shortest path. Define S_k = GOAL, and forall *i* <= *k*, define S_{i-1} = *previous*(S_i).

Uniform-Cost Search

- A conceptually simple BFS approach when there are costs on transitions
- It uses priority queues

Priority Queue Refresher

A priority queue is a data structure in which you can insert and retrieve (thing, value) pairs with the following operations:

Init-PriQueue(PQ)	initializes the PQ to be empty.
Insert-PriQueue(PQ, thing, value)	inserts *(thing, value)* into the queue.
Pop-least(PQ)	returns the *(thing, value)* pair with the lowest value, and removes it from the queue.

Priority Queue Refresher

A priority queue is a data structure in which you can insert and retrieve *(thing, value)* pairs with the following operations:

For more details, see Knuth or Sedgwick or basically any book with the word "algorithms" prominently appearing in the title.

Init-PriQueue(PQ)	initializes the PQ to be empty.
Insert-PriQueue(PQ, thing, value)	inserts *(thing, value)* into the queue.
Pop-least(PQ)	returns the *(thing, value)* pair with the lowest value, and removes it from the queue.

Priority Queues can be implemented in such a way that the cost of the insert and pop operations are

Very cheap (though not absolutely, incredibly cheap!)

O(log(number of things in priority queue))

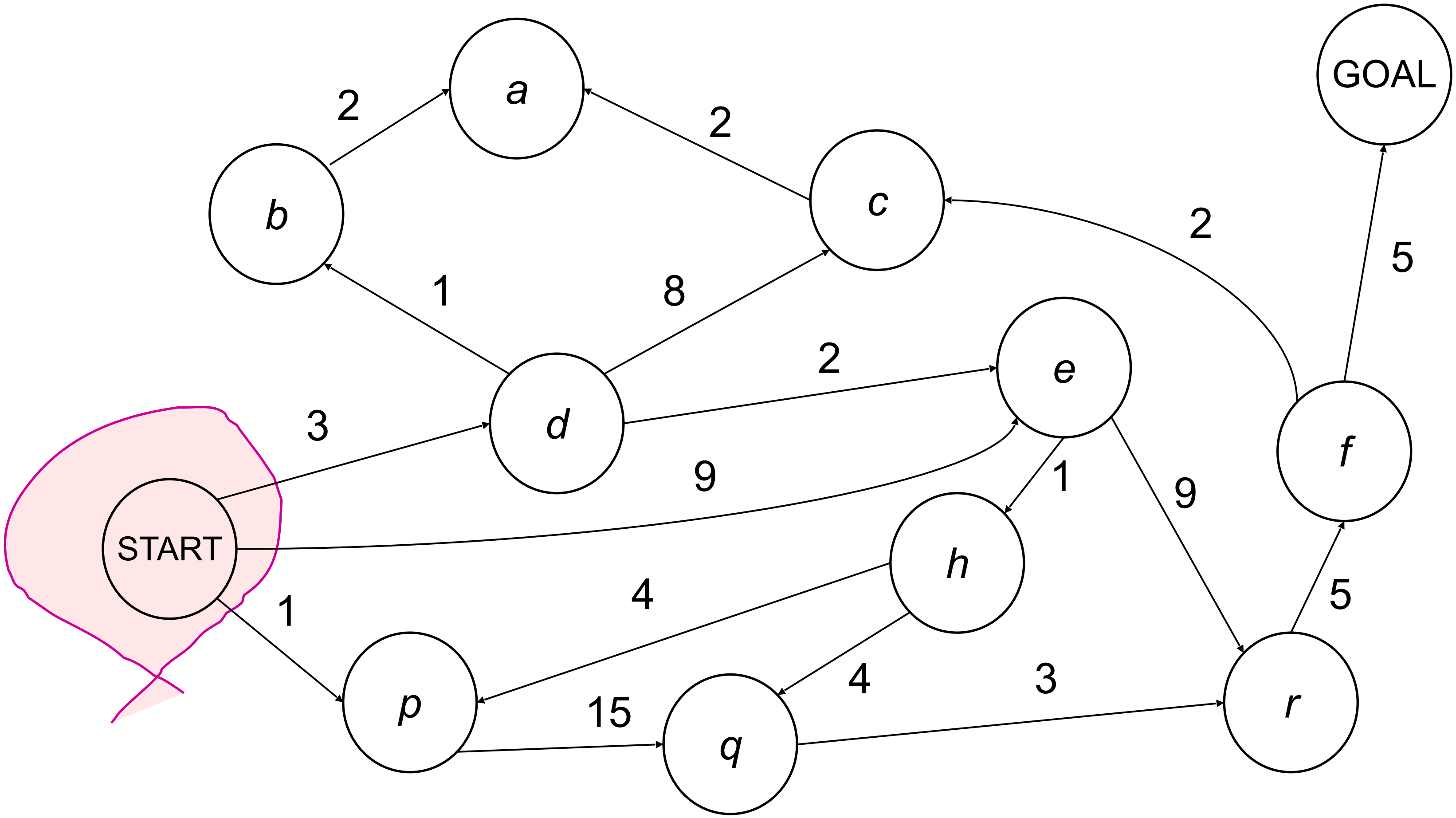
Uniform-Cost Search

- A conceptually simple BFS approach when there are costs on transitions
- It uses a priority queue

PQ = Set of states that have been expanded or are awaiting expansion

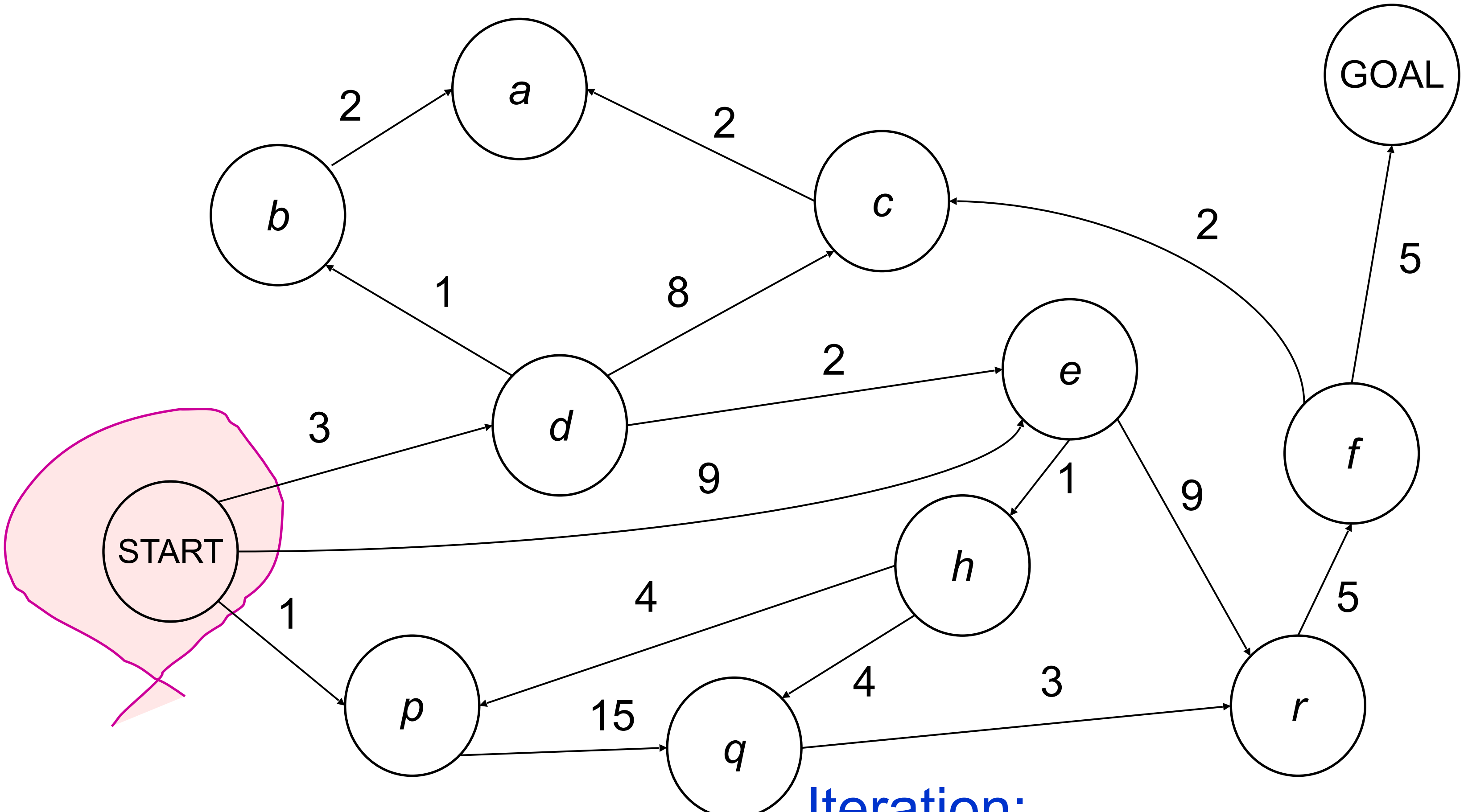
Priority of state s = $g(s)$ = cost of getting to s using path implied by backpointers.

Starting UCS



PQ = { (S,0) }

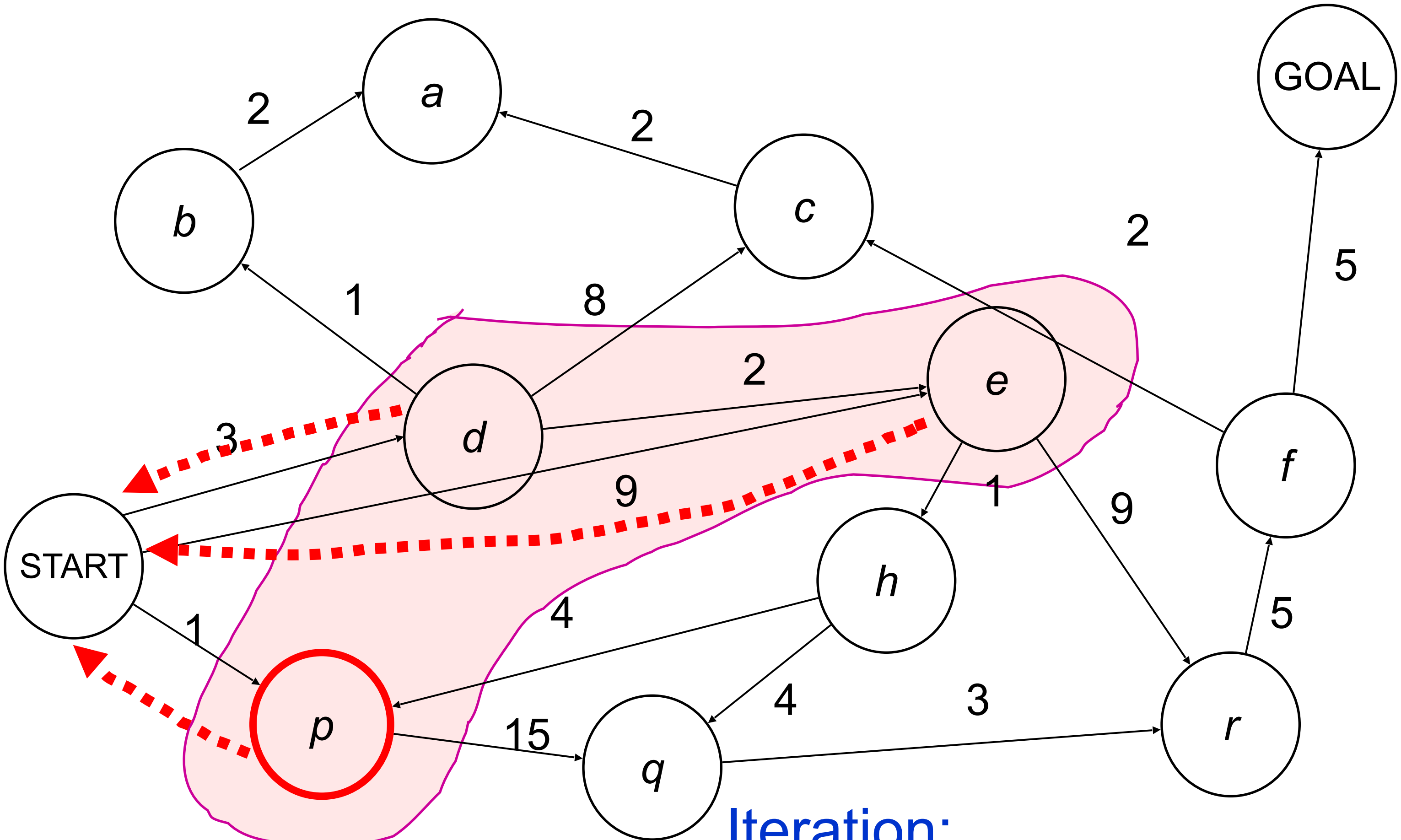
UCS Iterations



Iteration:
1. Pop least-cost state from PQ
2. Add successors

$PQ = \{ (S,0) \}$

UCS Iterations

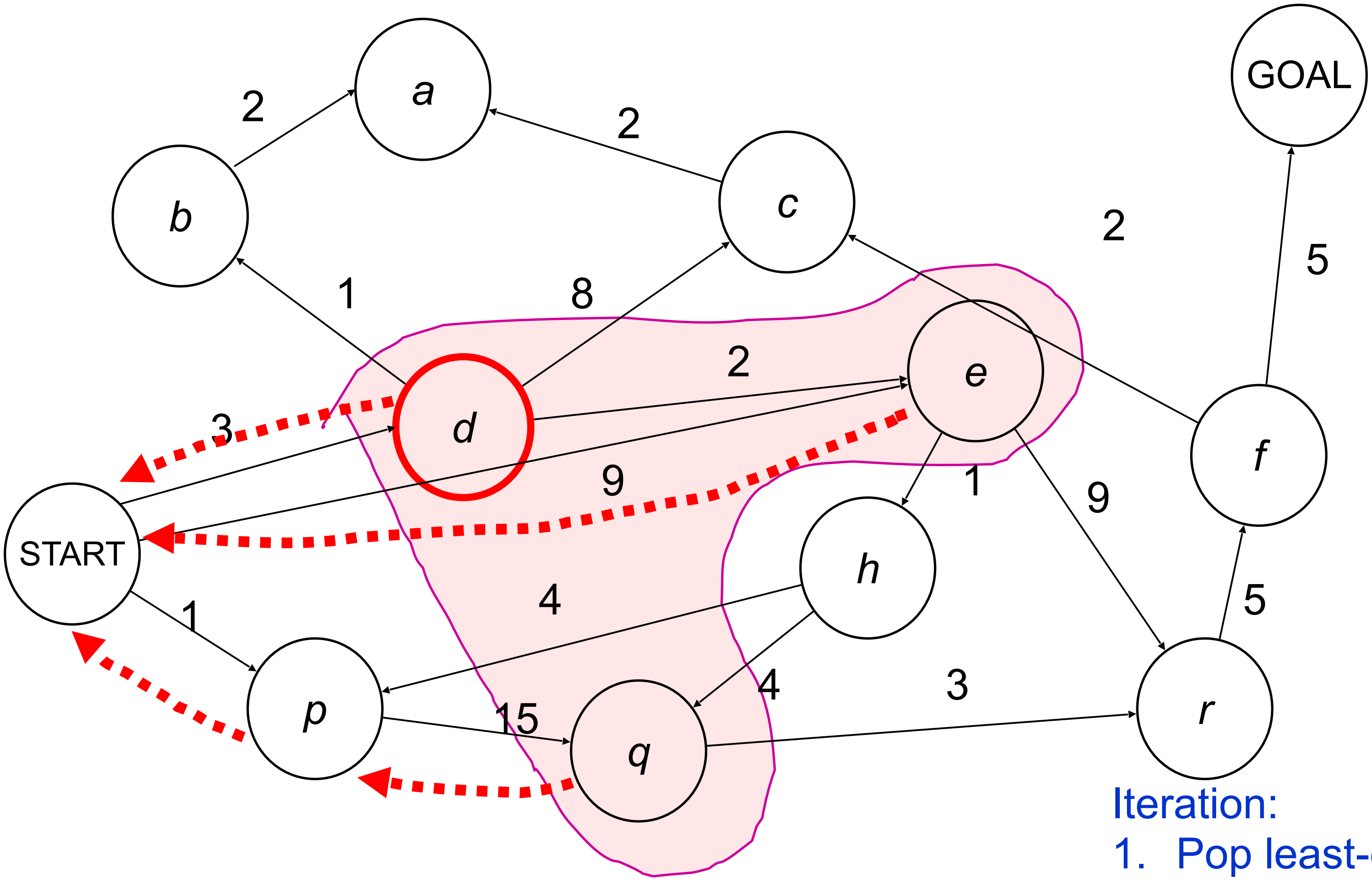


Iteration:

1. Pop least-cost state from PQ
2. Add successors

PQ = { (p,1), (d,3) , (e,9) }

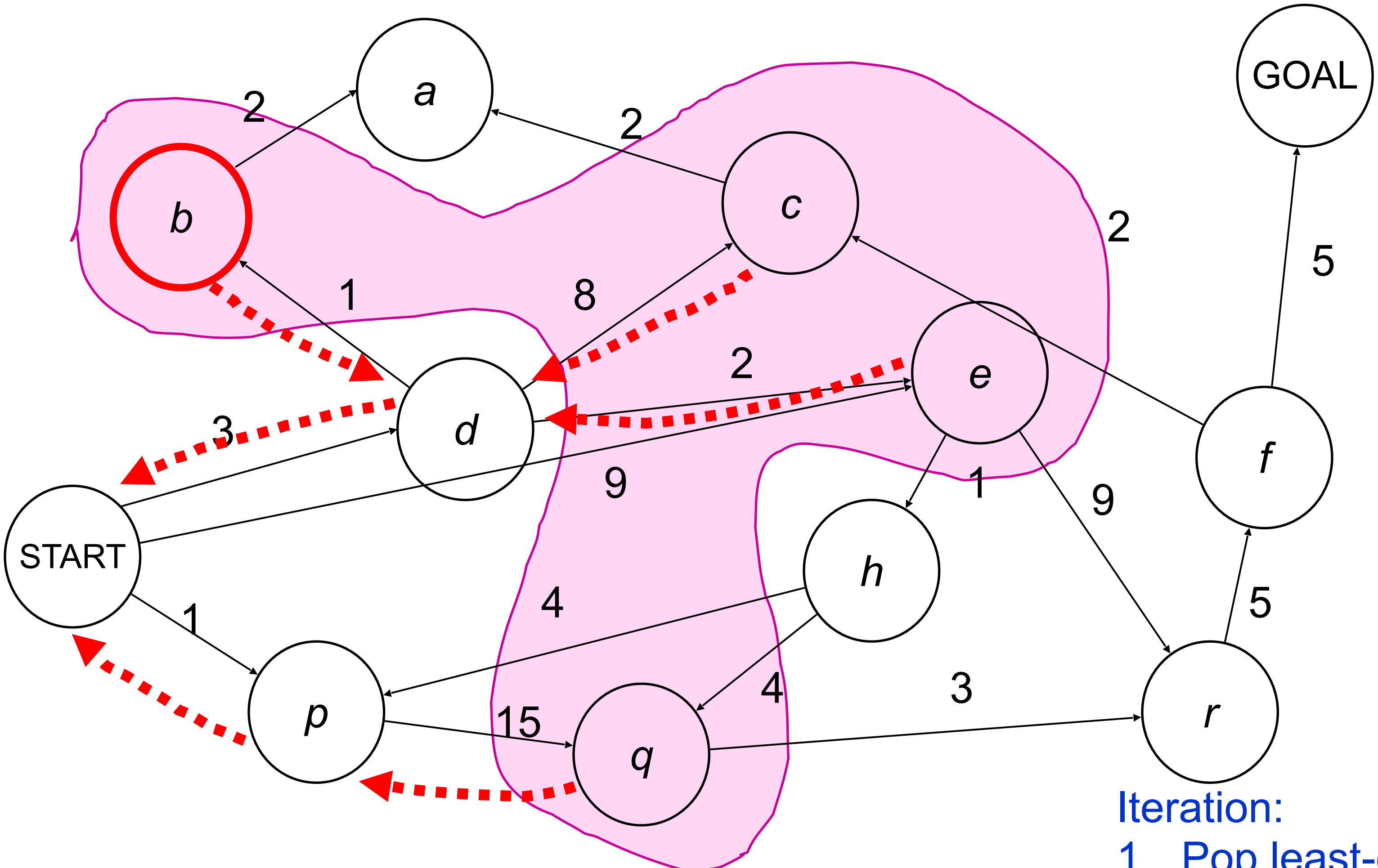
UCS Iterations



PQ = { (d,3) , (e,9) , (q,16) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

UCS Iterations

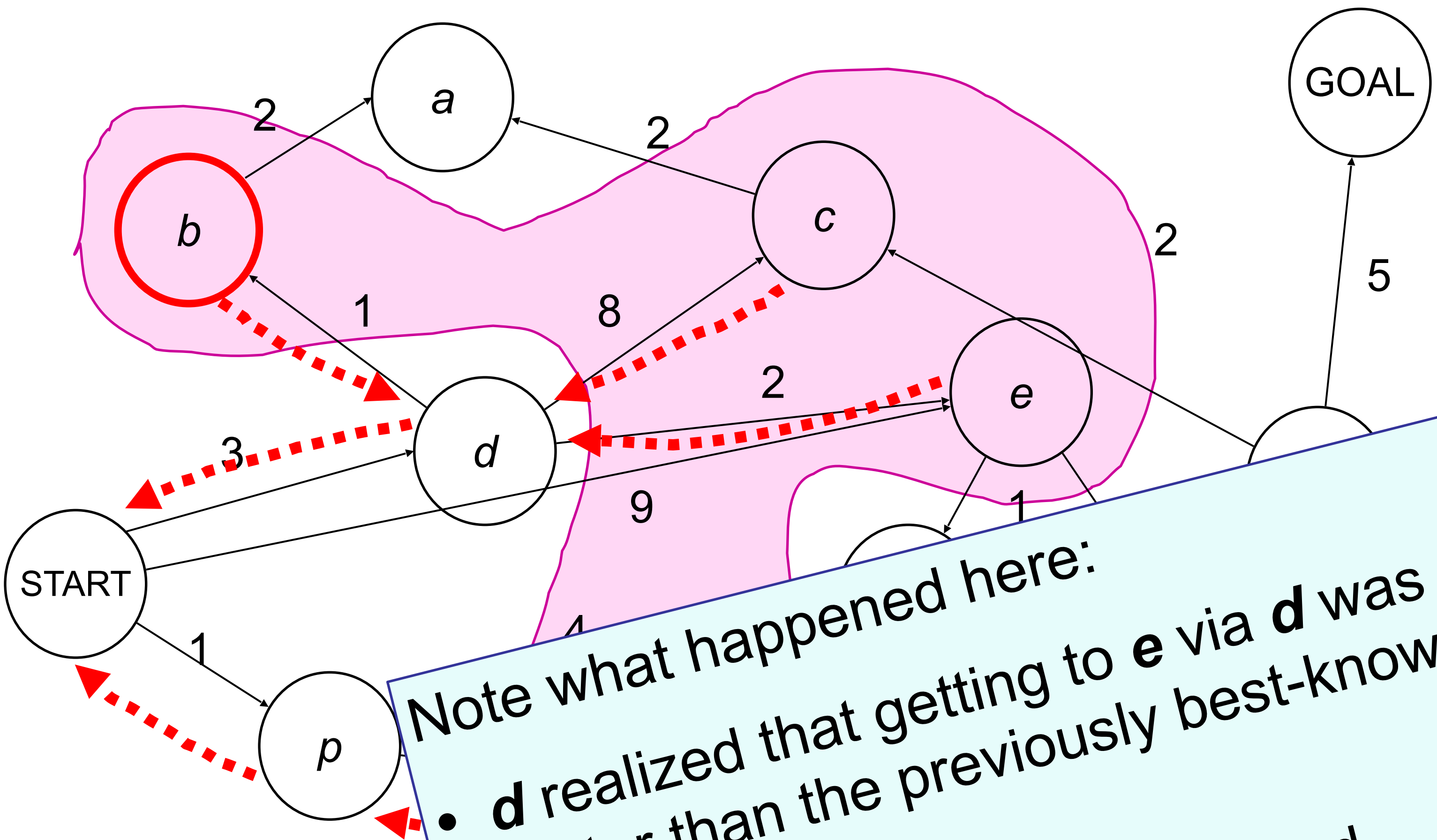


Iteration:

1. Pop least-cost state from PQ
2. Add successors

$PQ = \{\ (b,4)\ ,\ (e,5)\ ,\ (c,11)\ ,\ (q,16)\ \}$

UCS Iterations



Note what happened here:

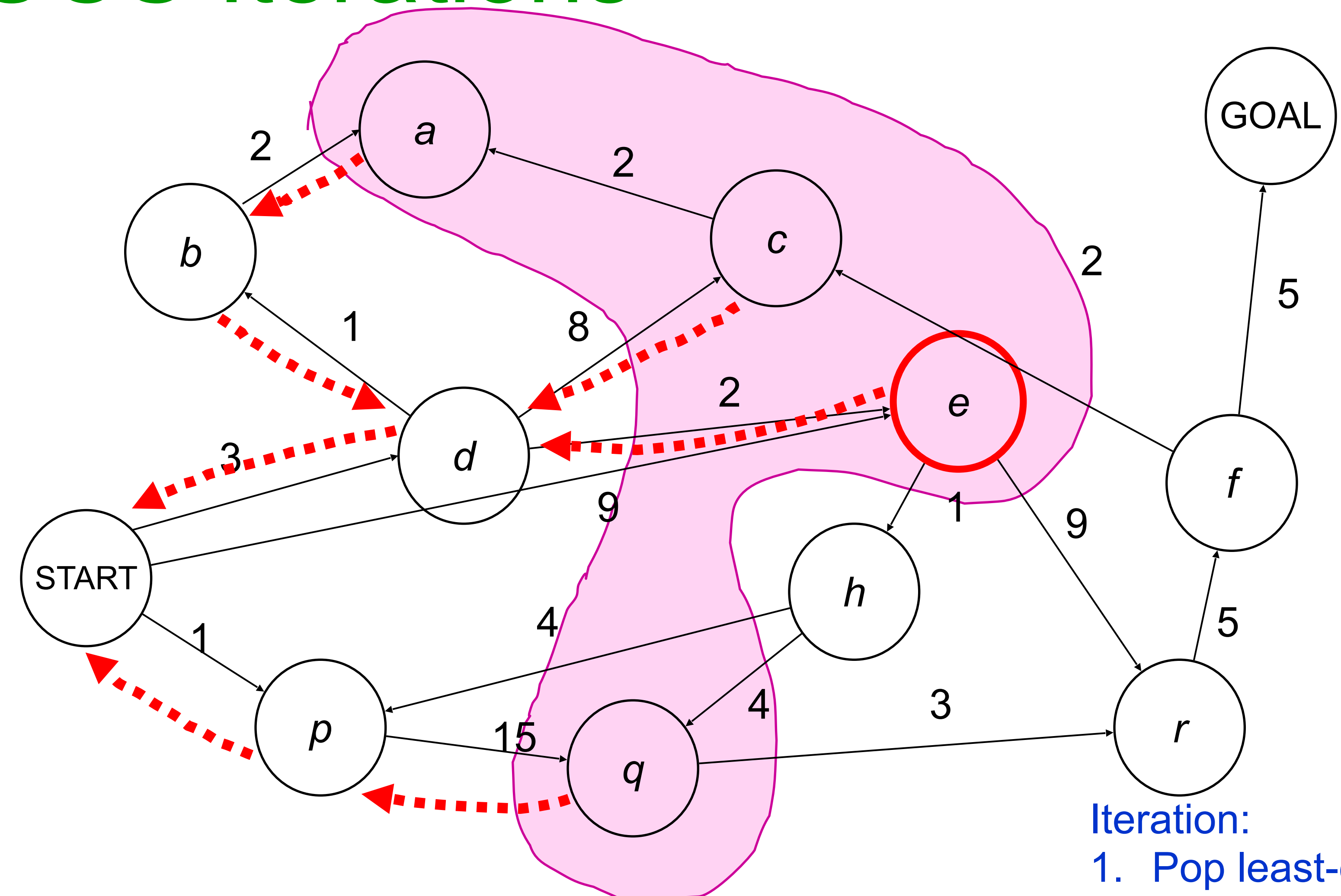
- **d** realized that getting to **e** via **d** was better than the previously best-known way to get to **e**
- and so **e**'s priority was changed

PQ = { (b,4) , (e,5) … *6) }*

… p least-cost state from PQ

2. Add successors

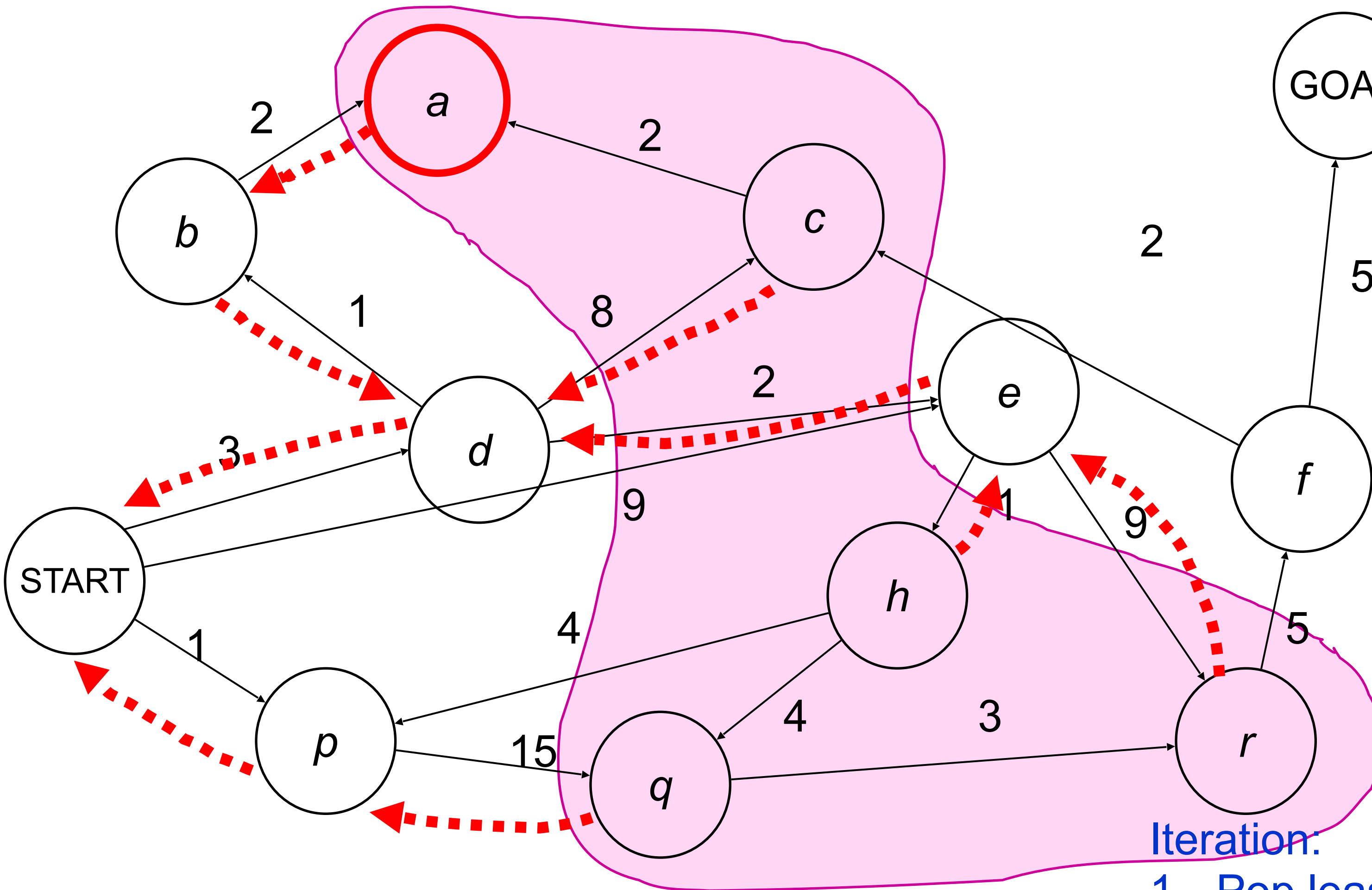
UCS Iterations



PQ = { (e,5) , (a,6) , (c,11) , (q,16) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

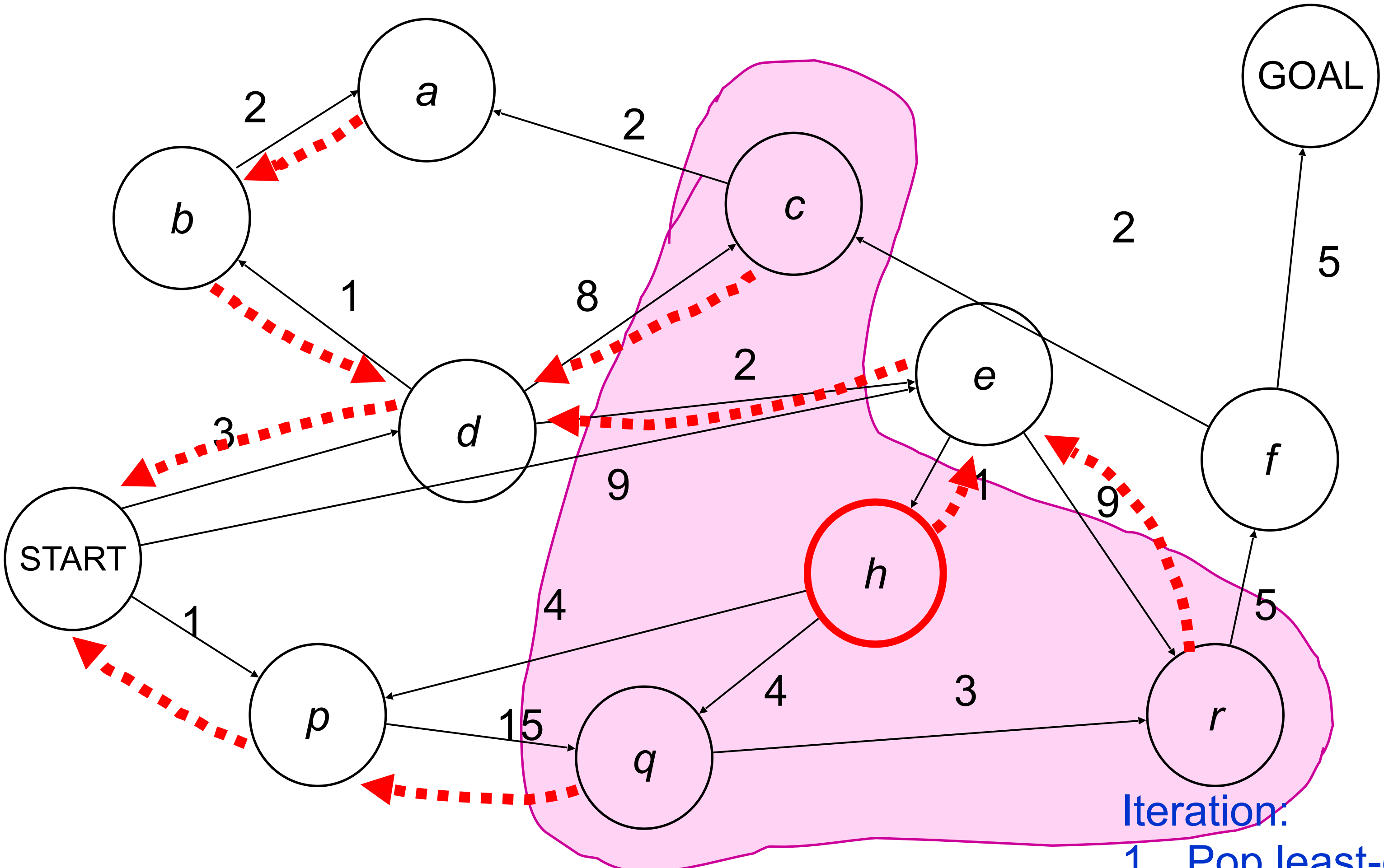
UCS Iterations



Iteration:
1. Pop least-cost state from PQ
2. Add successors

PQ = { (a,6),(h,6),(c,11),(r,14),(q,16) }

UCS Iterations

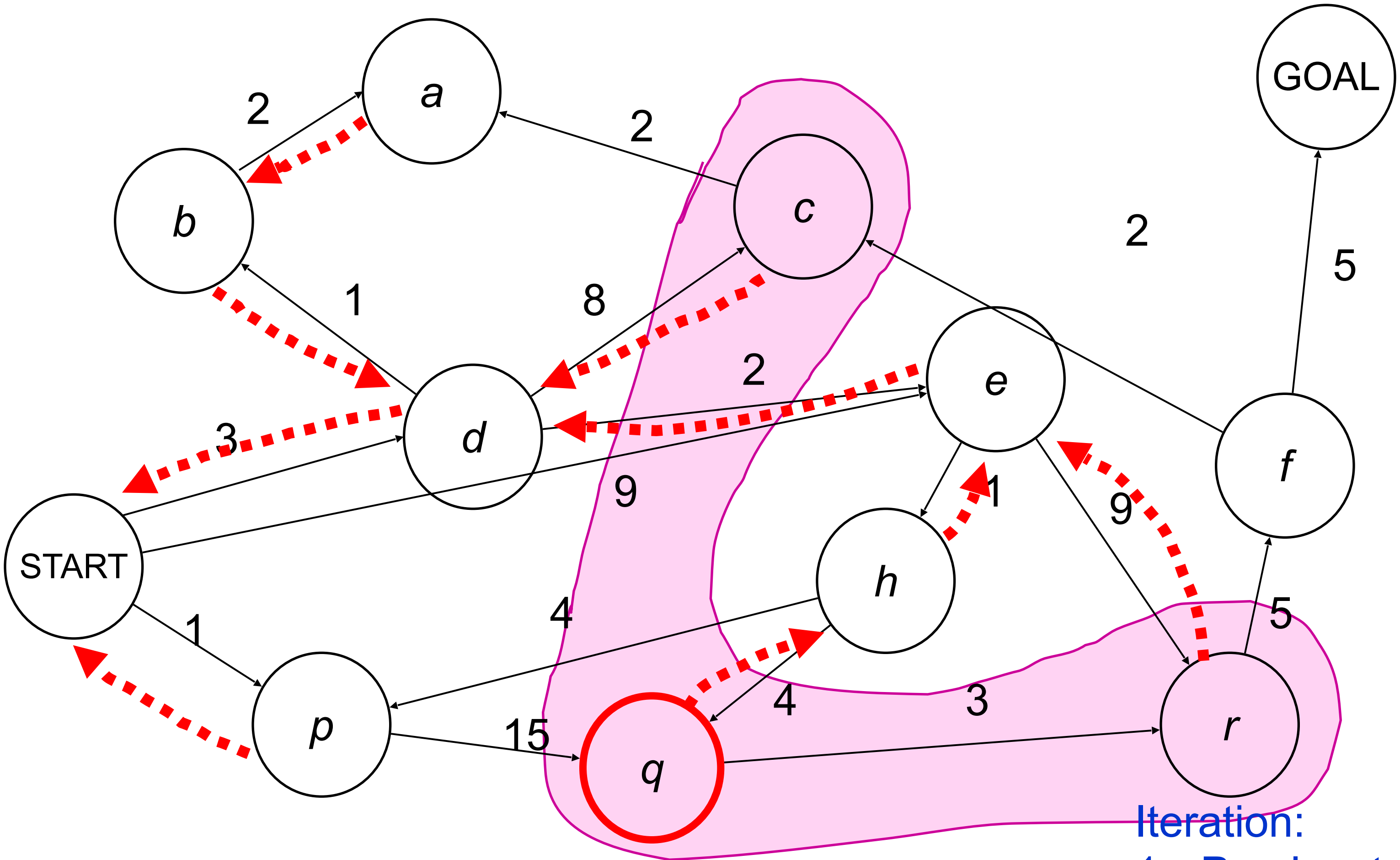


Iteration:

1. Pop least-cost state from PQ
2. Add successors

PQ = { (h,6),(c,11),(r,14),(q,16) }

UCS Iterations

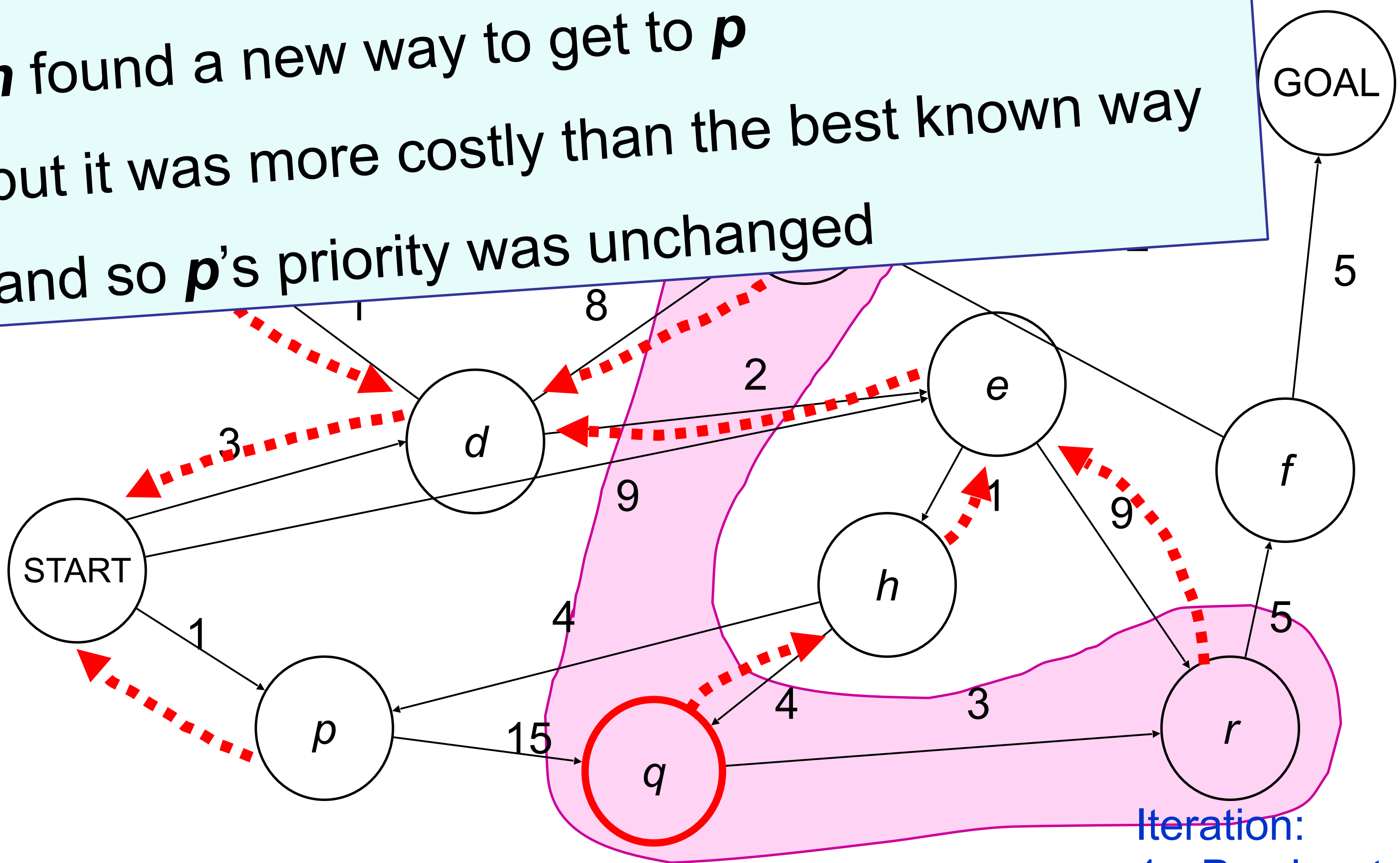


$PQ = \{ (q,10), (c,11), (r,14) \}$

Iteration:
1. Pop least-cost state from PQ
2. Add successors

Note what happened here:

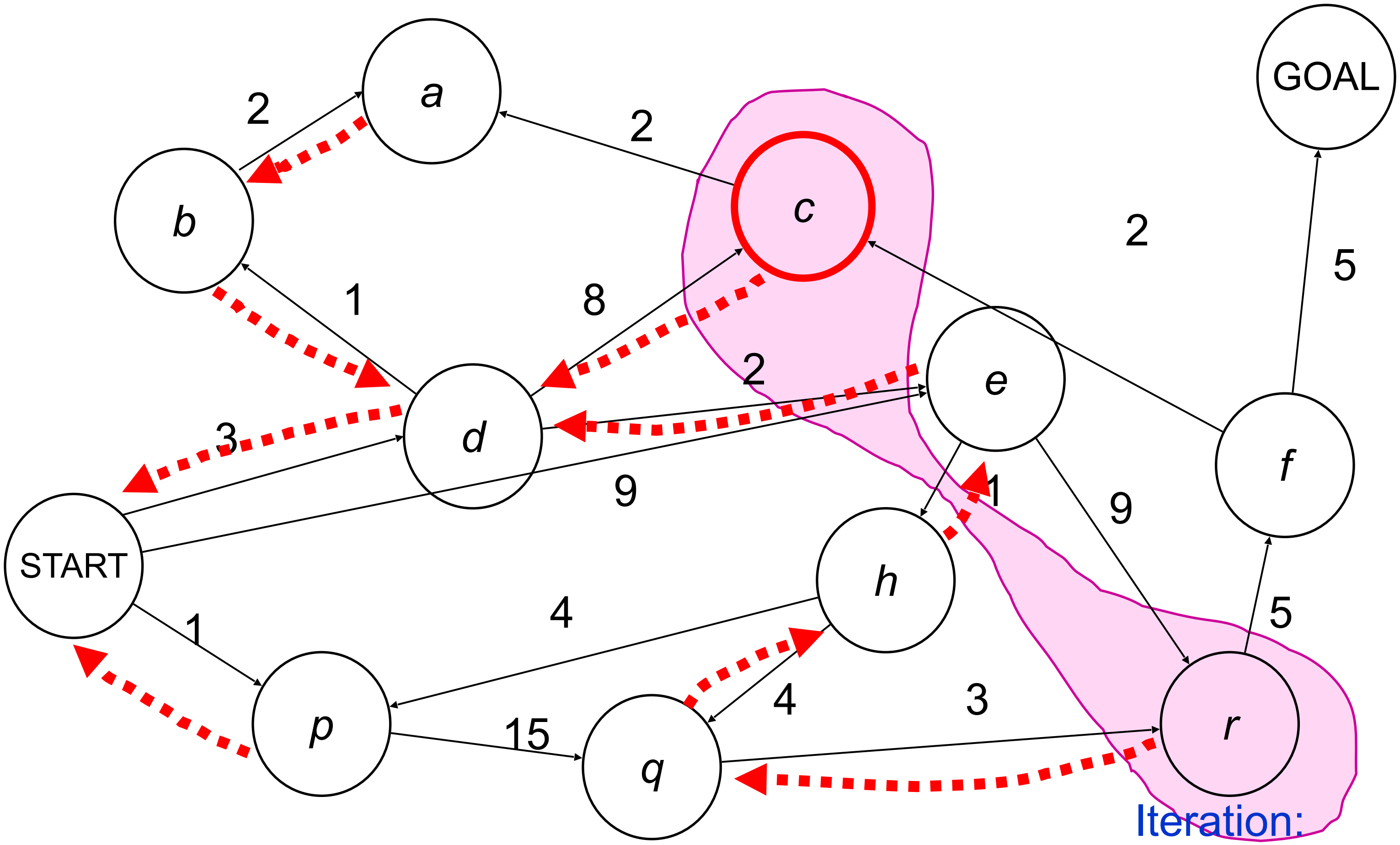
- ***h*** found a new way to get to ***p***
- but it was more costly than the best known way
- and so ***p***'s priority was unchanged



PQ = { (q,10), (c,11),(r,14) }

Iteration:
1. Pop least-cost state from PQ
2. Add successors

UCS Iterations

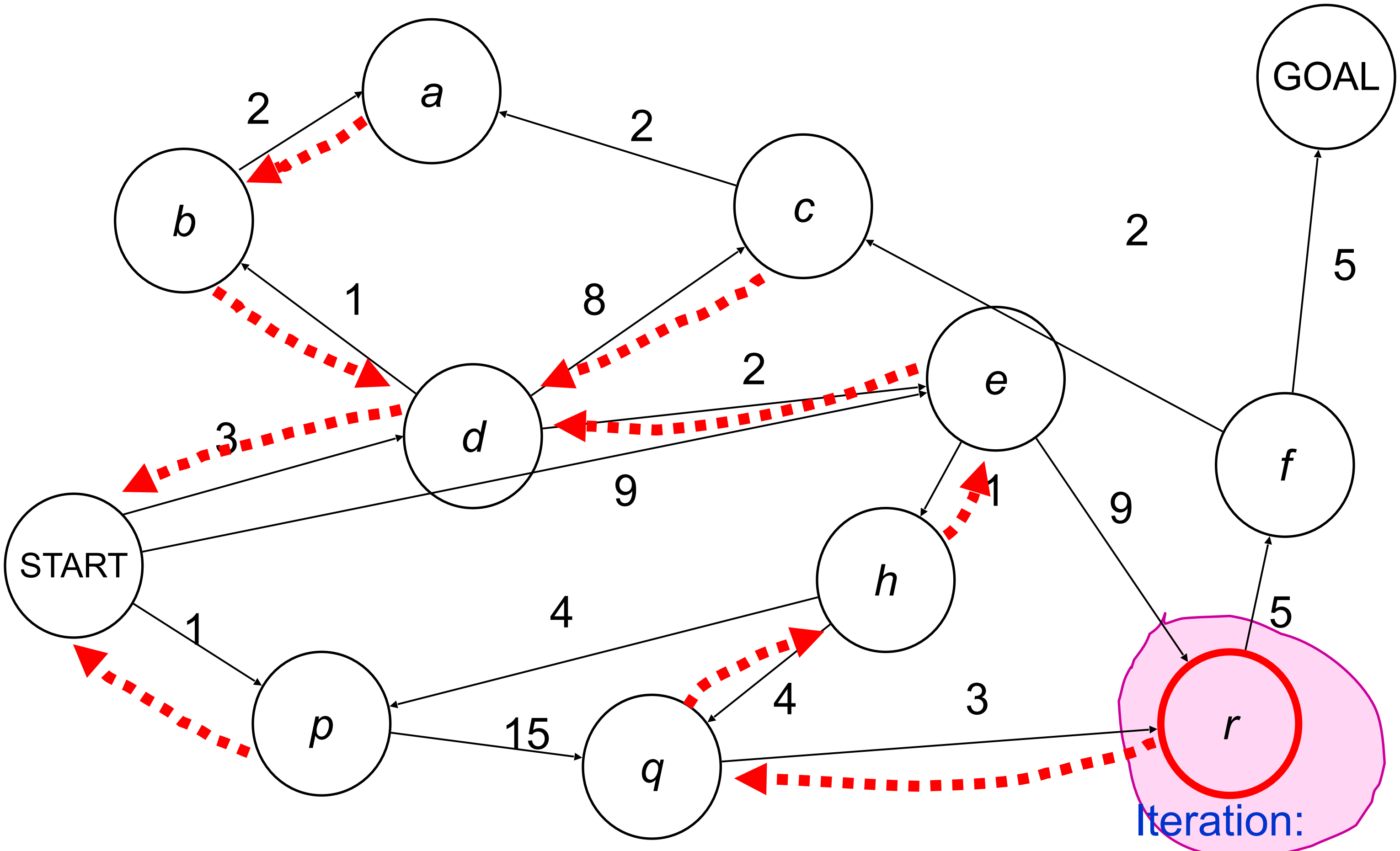


PQ = { (c,11),(r,13) }

Iteration:

1. Pop least-cost state from PQ
2. Add successors

UCS Iterations

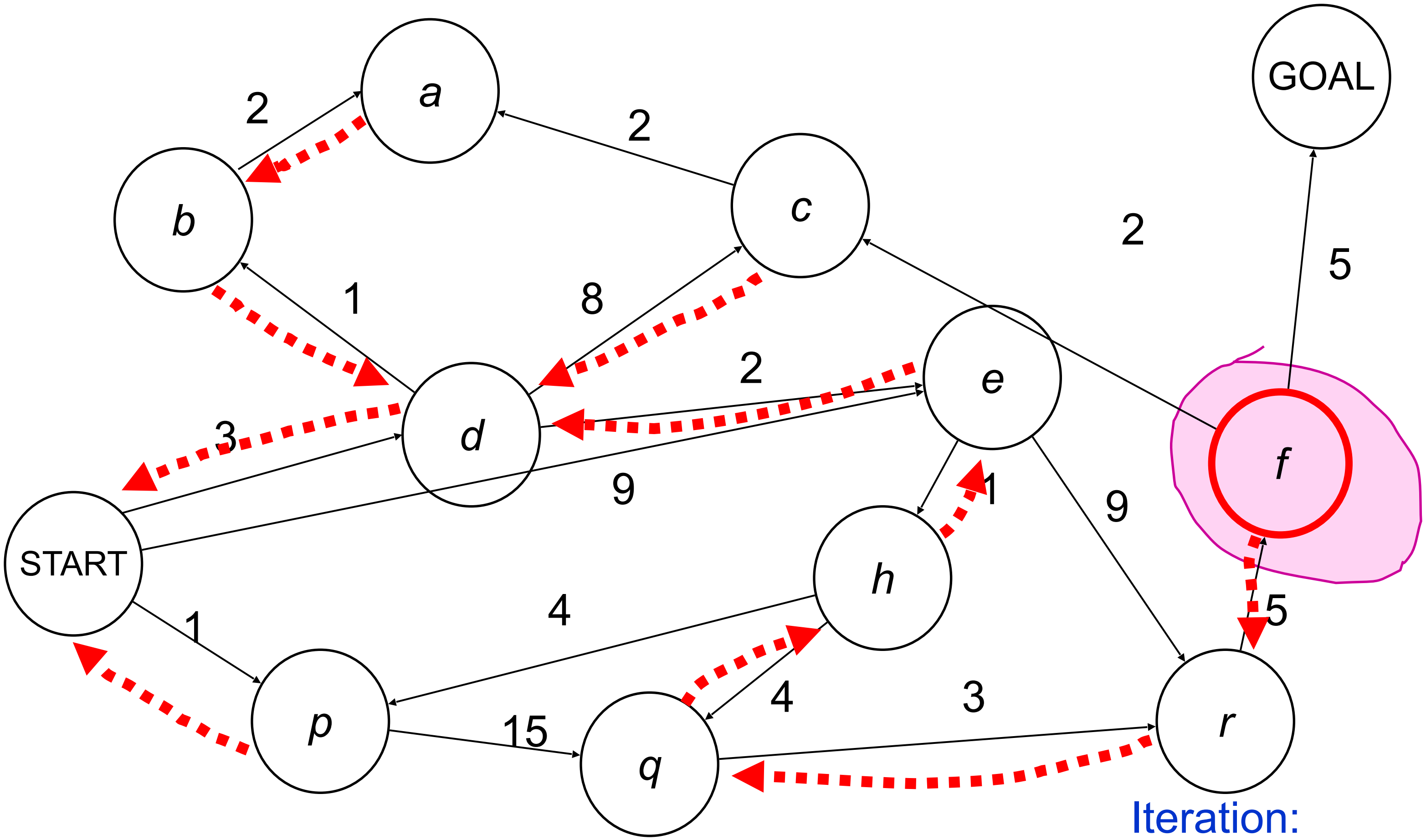


Iteration:

1. Pop least-cost state from PQ
2. Add successors

$PQ = \{ (r, 13) \}$

UCS Iterations

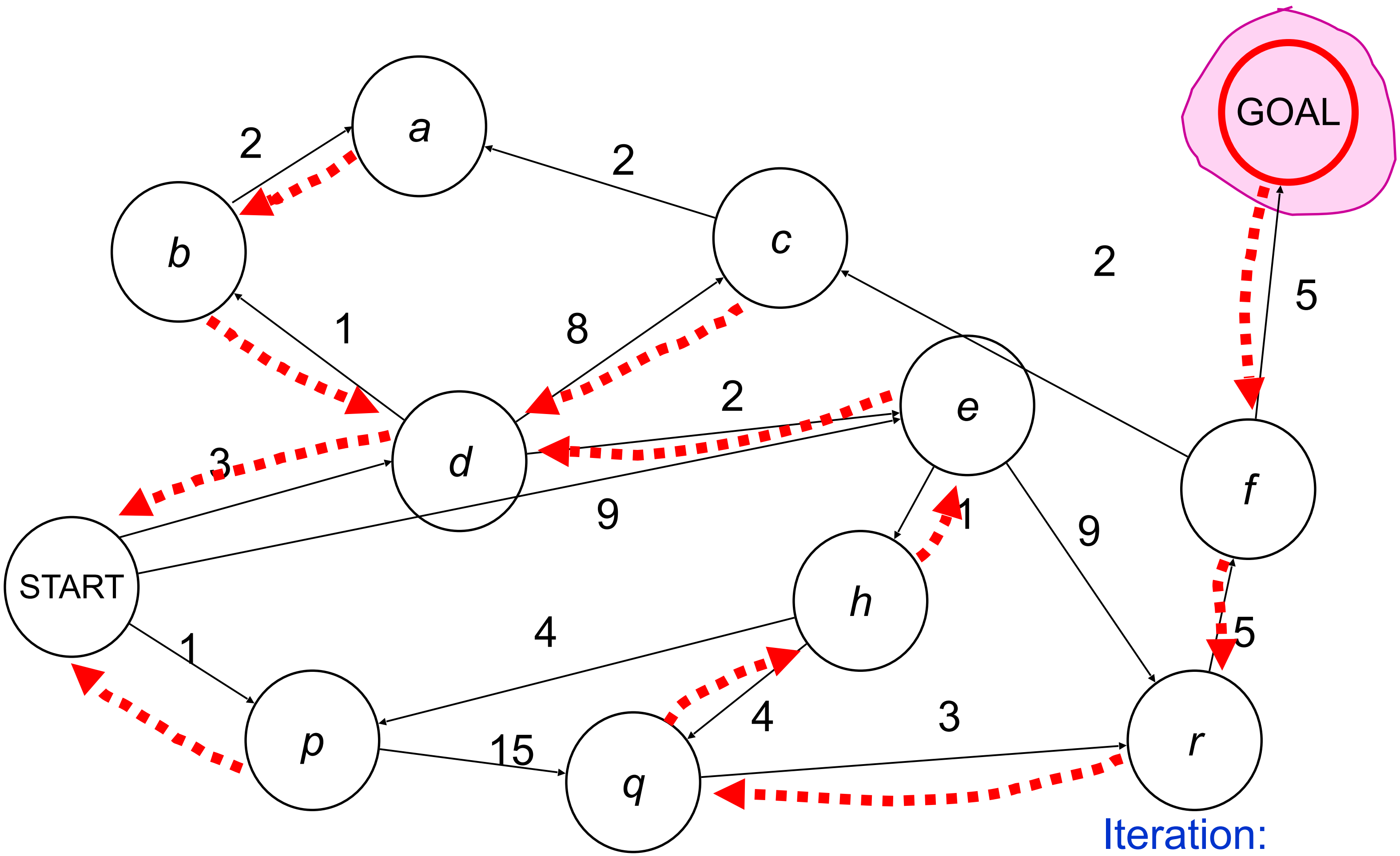


Iteration:

1. Pop least-cost state from PQ
2. Add successors

PQ = { (f,18) }

UCS Iterations



Iteration:

1. Pop least-cost state from PQ
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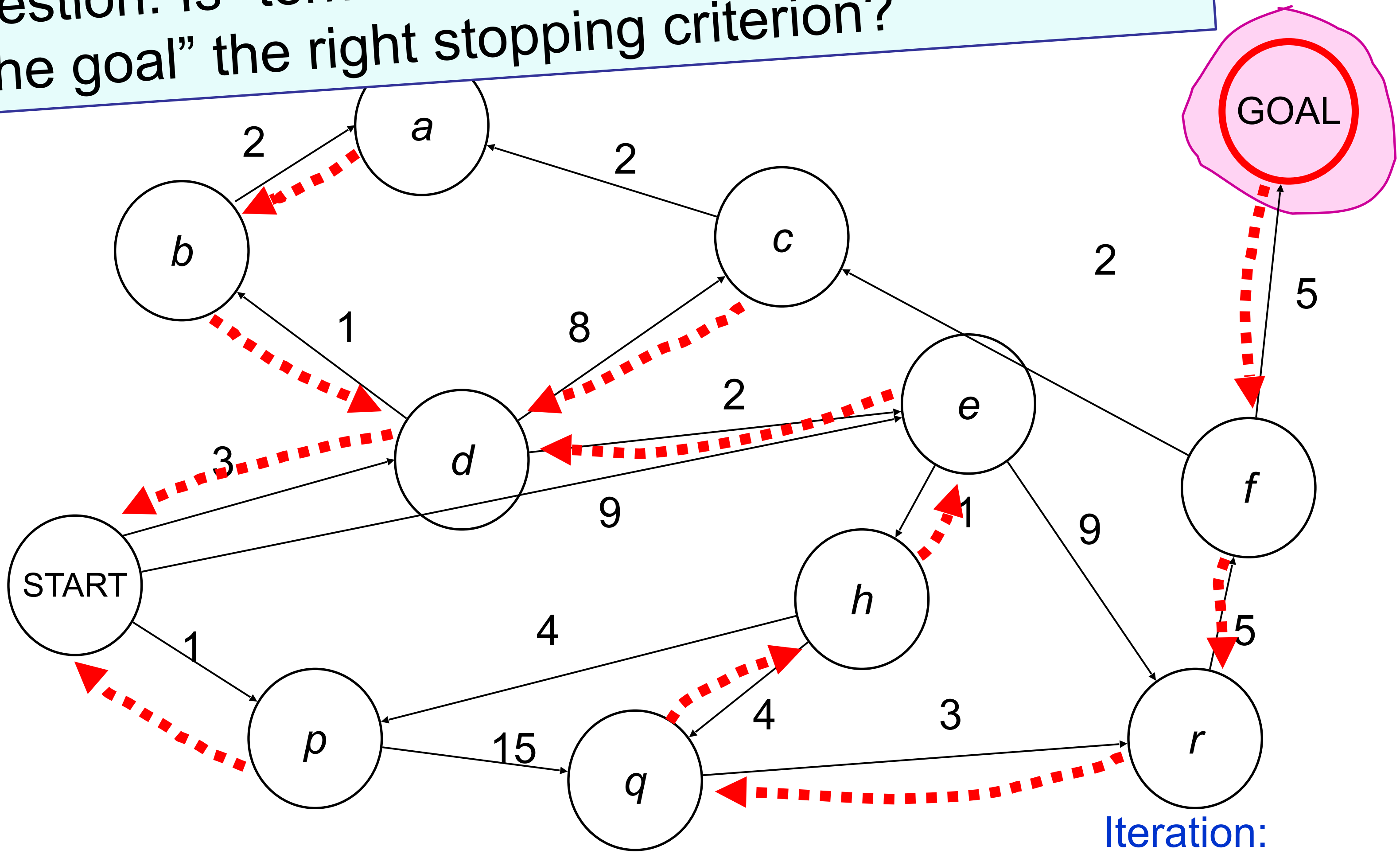
$PQ = \{ (G,23) \}$

Question: Is "terminate as soon as you discover the goal" the right stopping criterion?



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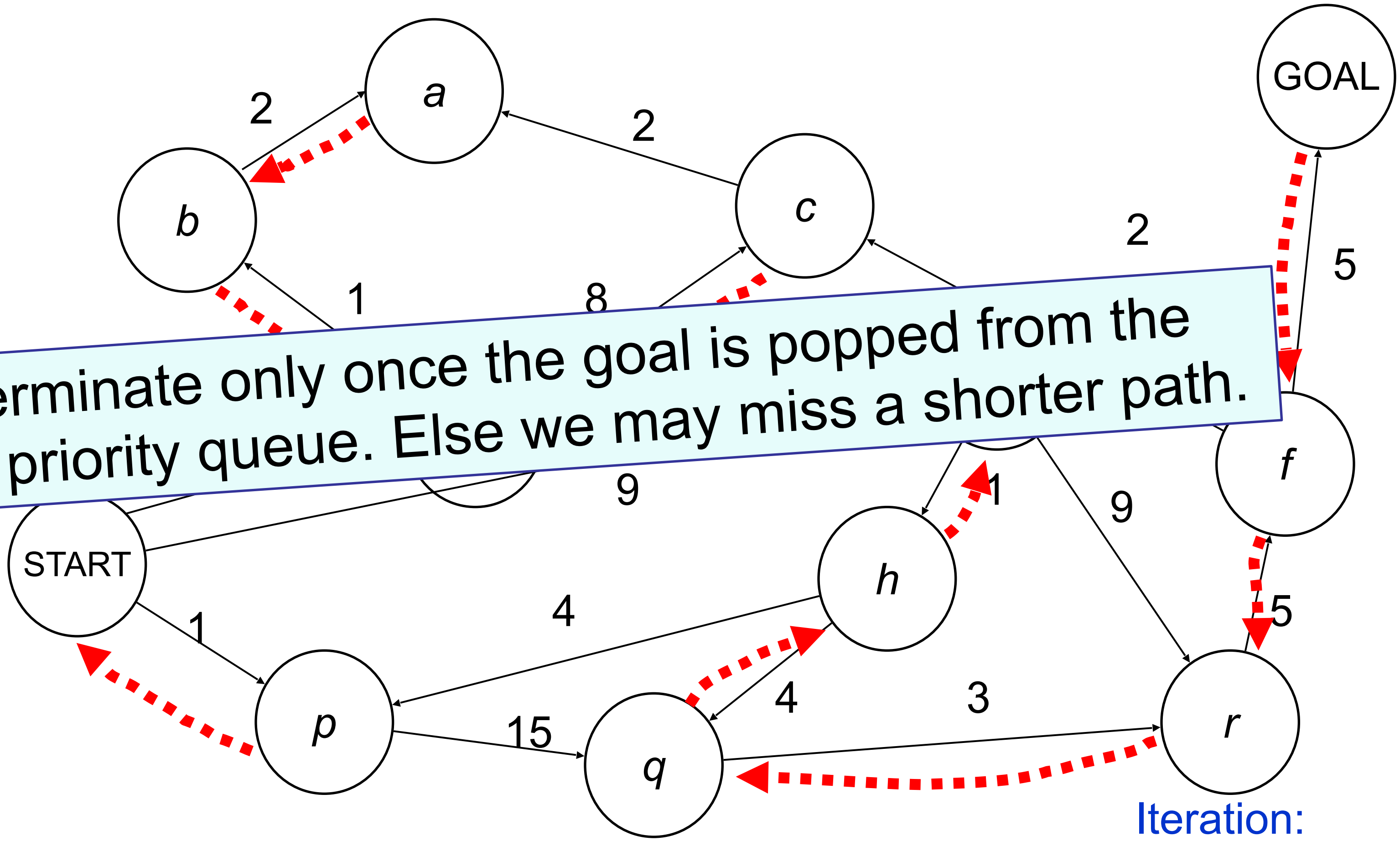
UCS terminates



Terminate only once the goal is popped from the priority queue. Else we may miss a shorter path.

PQ = { }

Iteration:

1. Pop least-cost state from PQ
2. Add successors

Judging a search algorithm

- Completeness: is the algorithm guaranteed to find a solution if a solution exists?
- Guaranteed to find optimal? (will it find the least cost path?)
- Algorithmic time complexity
- Space complexity (memory use)

Variables:

N	number of states in the problem
B	the average branching factor (the average number of successors) (B>1)
L	the length of the path from start to goal with the shortest number of steps

How would we judge our algorithms?

Judging a search algorithm

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Algorithm		Com plete	Optima l	Time	Space
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LCBFS	Least Cost BFS				
UCS	Uniform Cost Search				

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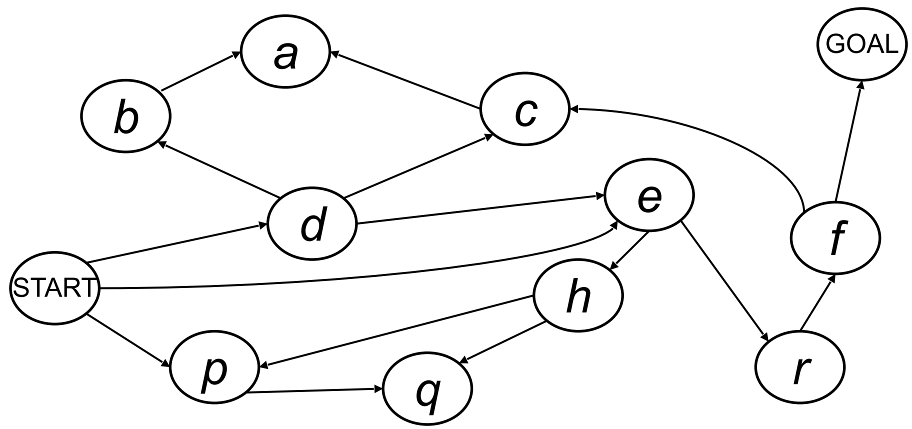
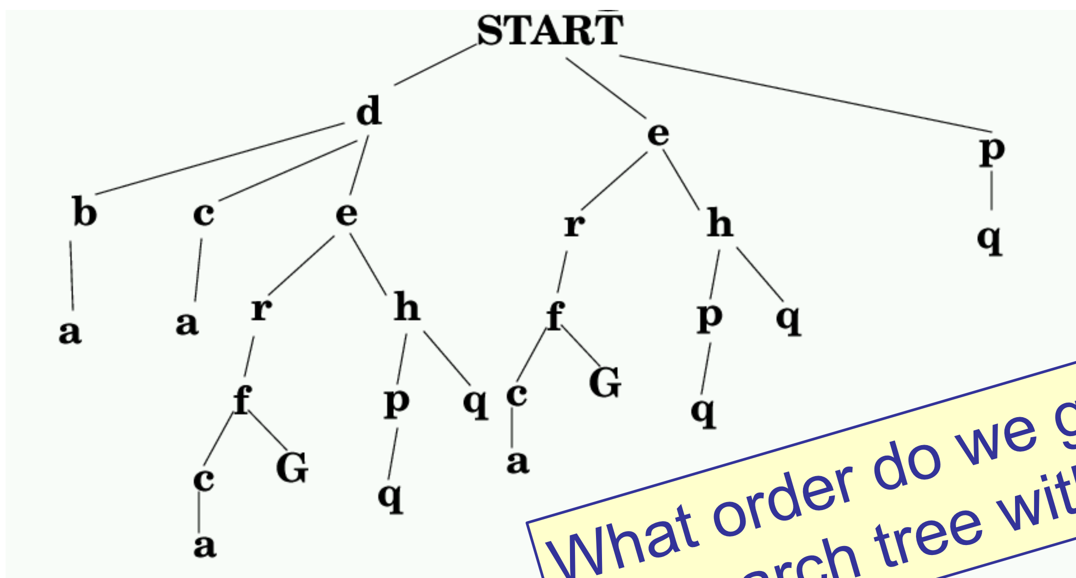
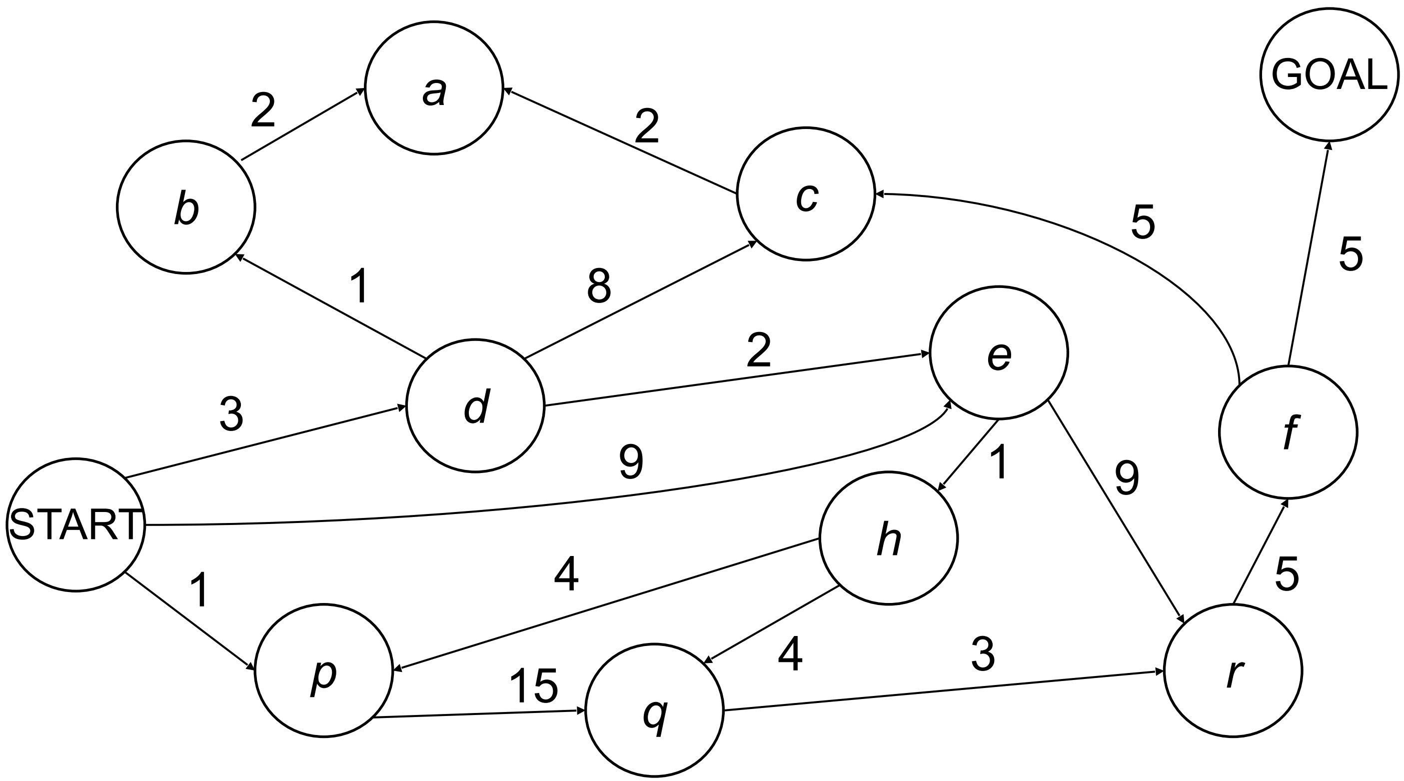
Search Tree Representation





What order do we go through the search tree with BFS?

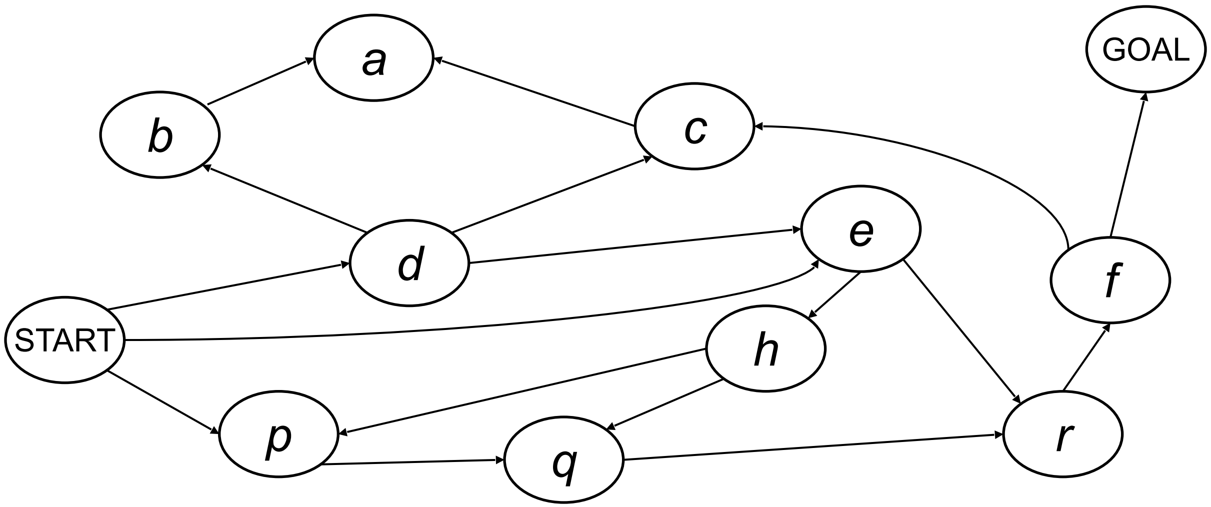
Depth First Search



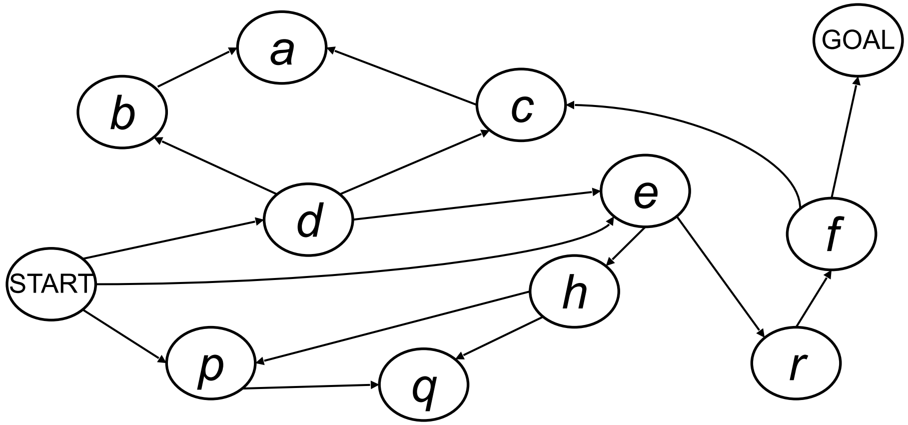
An alternative to BFS. Always expand from the most-recently-expanded node, if it has any untried successors. Else backup to the previous node on the current path.

DFS in action

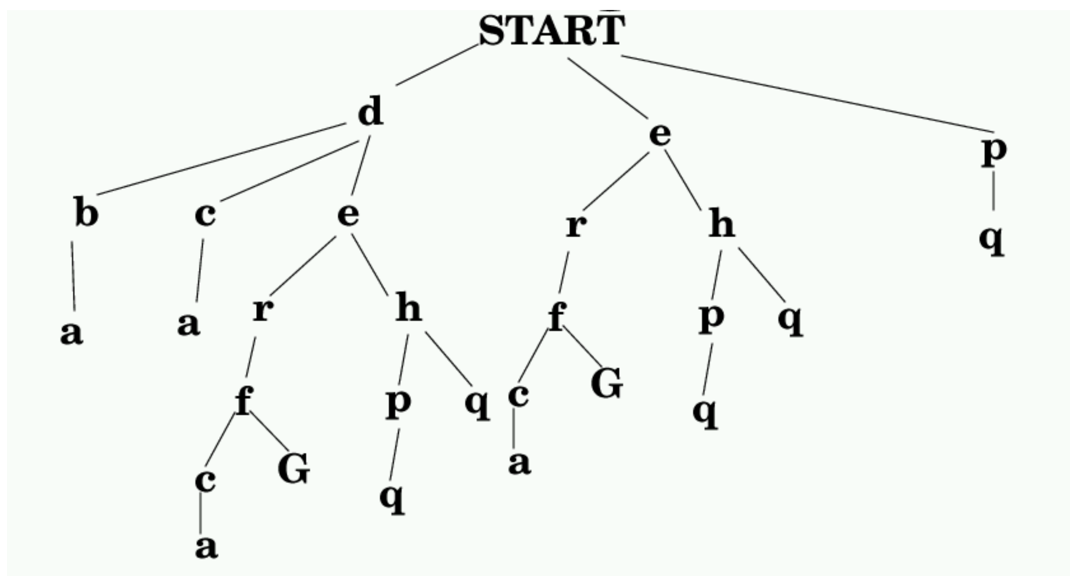
START
START *d*
START *d b*
START *d b a*
START *d c*
START *d c a*
START *d e*
START *d e r*
START *d e r f*
START *d e r f c*
START *d e r f c a*
START *d e r f* GOAL



DFS Search tree traversal



Can you draw in the order in which the search-tree nodes are visited?

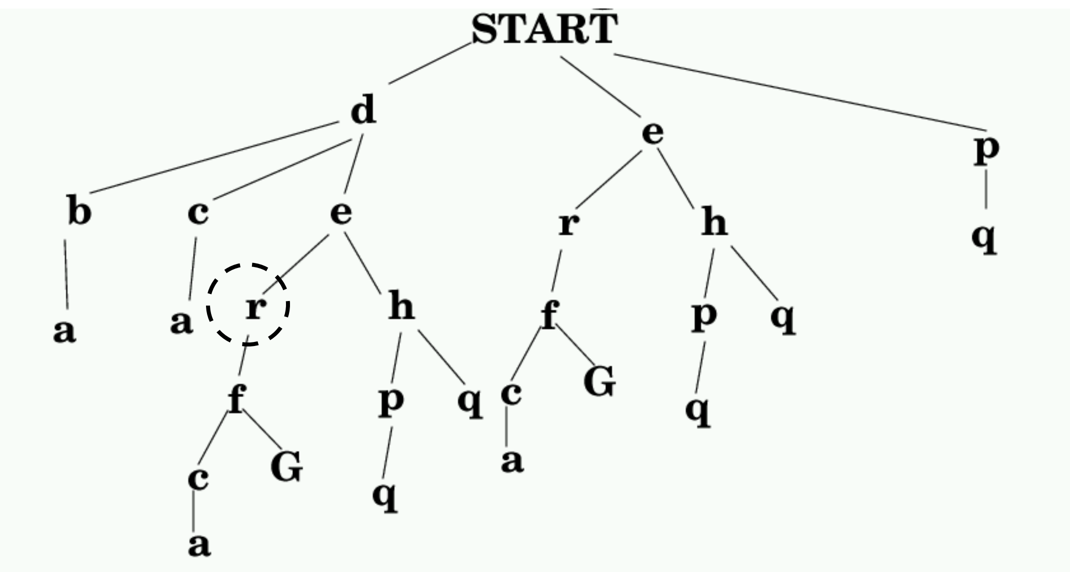


DFS Algorithm

We use a data structure we'll call a Path to represent the , er, path from the START to the current state.

E.G. Path P = <START, d, e, r >

Along with each node on the path, we must remember which successors we still have available to expand. E.G. at the following point, we'll have



P = <START (expand=e , p) ,

d (expand = NULL) ,

e (expand = h) ,

r (expand = f) >

DFS Algorithm

```
Let P = <START (expand = succs(START))>
While (P not empty and top(P) not a goal)
        if expand of top(P) is empty
        then
                remove top(P) (“pop the stack”)
        else
                let s be a member of expand of top(P)
                remove s from expand of top(P)
                make a new item on the top of path P:
                        s (expand = succs(s))
If P is empty
        return FAILURE
Else
        return the path consisting of states in P
```

This algorithm can be written neatly with recursion, i.e. using the program stack to implement P.

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BFS	Breadth First Search	Y	if all transitions same cost	$O(min(N,B^L))$	$O(min(N,B^L))$
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Assuming Acyclic Search Space

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DFS**	Depth First Search	Y	N	$O(B^{LMAX})$	$O(LMAX)$

Assuming Acyclic Search Space

Questions to ponder

- How would you prevent DFS from looping?

- How could you force it to give an optimal solution?

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Answer 1:

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Answer 2:

MEMDFS (Memoizing DFS):

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> Don’t recurse on a state if that state is already in the current path

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> Remember all states expanded so far. Never expand anything twice.

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PCDF S	Path Check DFS				
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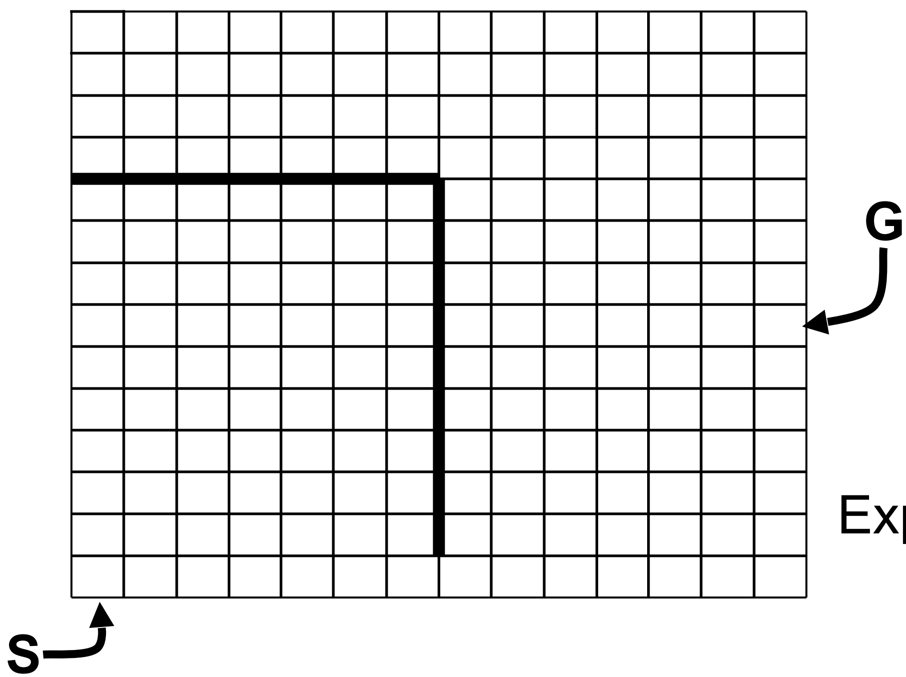
Maze example

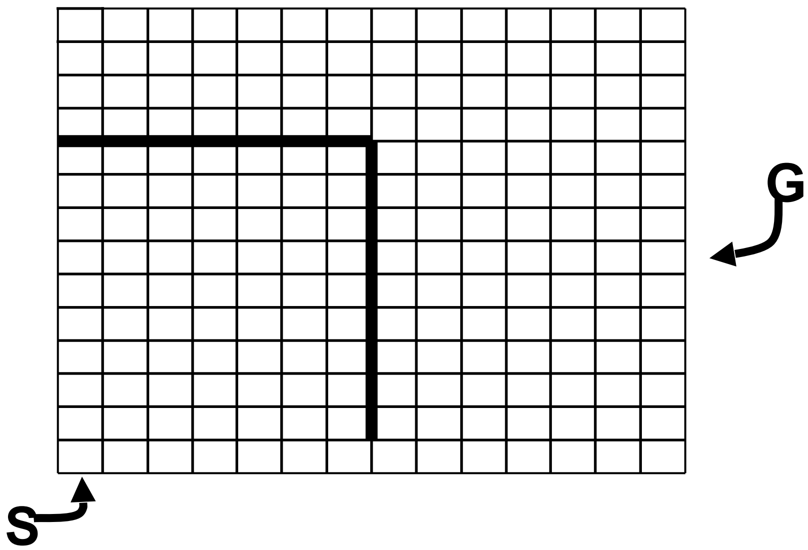
Imagine states are cells in a maze, you can move N, E, S, W. What would plain DFS do, assuming it always expanded the E successor first, then N, then W, then S?



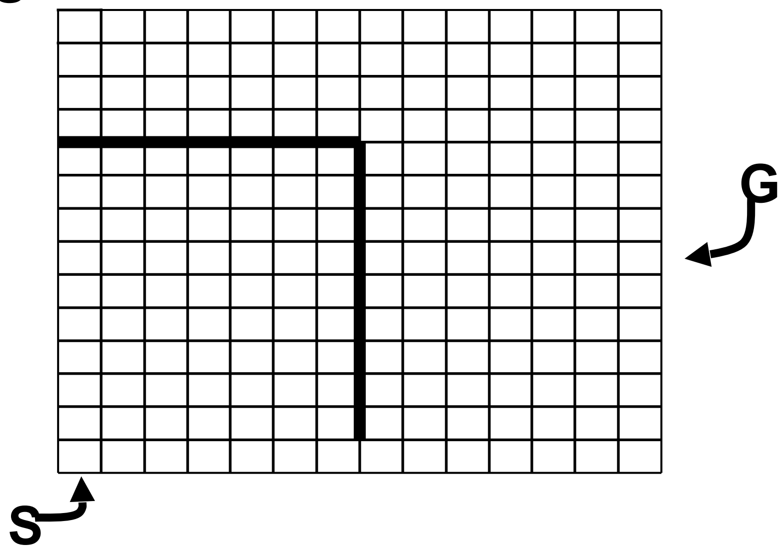
Expansion order E, N, W, S

Other questions:
- What would BFS do?
- What would PCDFS do?
- What would MEMDFS do?

Two other DFS examples



Order: N, E, S, W?



Order: N, E, S, W

with loops prevented

Forward DFSearch or Backward DFSearch

If you have a predecessors() function as well as a successors() function you can begin at the goal and depth-first-search backwards until you hit a start.

Why/When might this be a good idea?

Invent An Algorithm Time!

Here's a way to dramatically decrease costs sometimes. Bidirectional Search. Can you guess what this algorithm is, and why it can be a huge cost-saver?

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BIBFS	Bidirection BF Search	Y	All trans same cost	$O(min(N,2B^{L/2}))$	$O(min(N,2B^{L/2}))$

Iterative Deepening

Iterative deepening is a simple algorithm which uses DFS as a subroutine:

1. Do a DFS which only searches for paths of length 1 or less. (DFS gives up any path of length 2)
2. If "1" failed, do a DFS which only searches paths of length 2 or less.
3. If "2" failed, do a DFS which only searches paths of length 3 or less.

….and so on until success

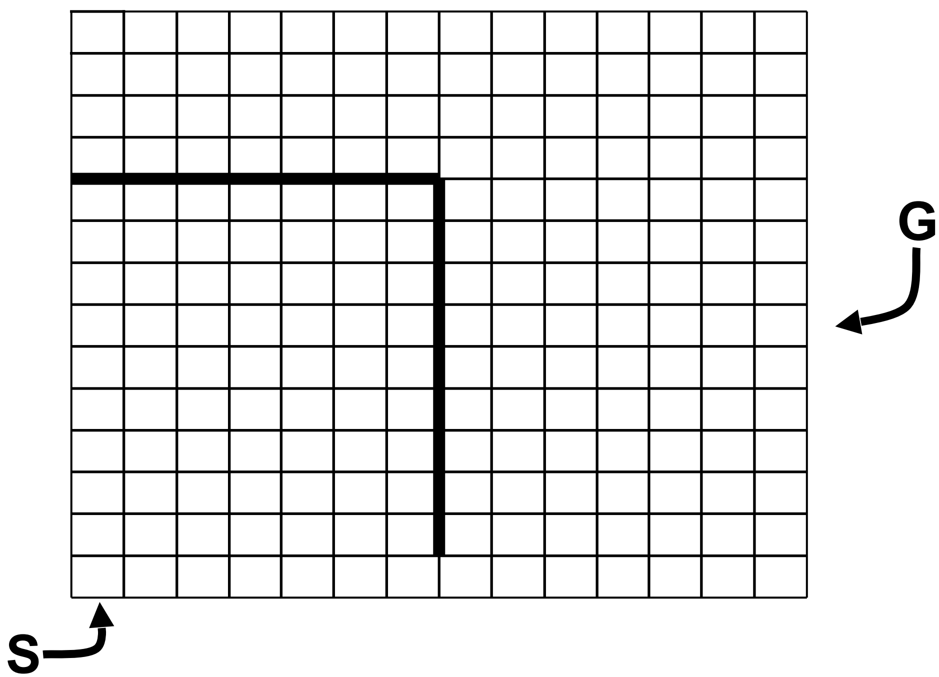
Cost is

$O(b^1 + b^2 + b^3 + b^4 \ldots + b^L) = O(b^L)$

Can be much better than regular DFS. But cost can be much greater than the number of states.

Maze example

Imagine states are cells in a maze, you can move N, E, S, W. What would **Iterative Deepening** do, assuming it always expanded the E successor first, then N, then W, then S?



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ID	Iterative Deepening	Y	if all transitions same cost	$O(B^L)$	$O(L)$

Best First “Greedy” Search

Needs a *heuristic*. A heuristic function maps a state onto an estimate of the cost to the goal from that state.

Can you think of examples of heuristics?

E.G. for the 8-puzzle?

E.G. for planning a path through a maze?

Denote the heuristic by a function $h(s)$ from states to a cost value.

Heuristic Search

Suppose in addition to the standard search specification we also have a *heuristic*.

A heuristic function maps a state onto an estimate of the cost to the goal from that state.

Can you think of examples of heuristics?

- E.G. for the 8-puzzle?
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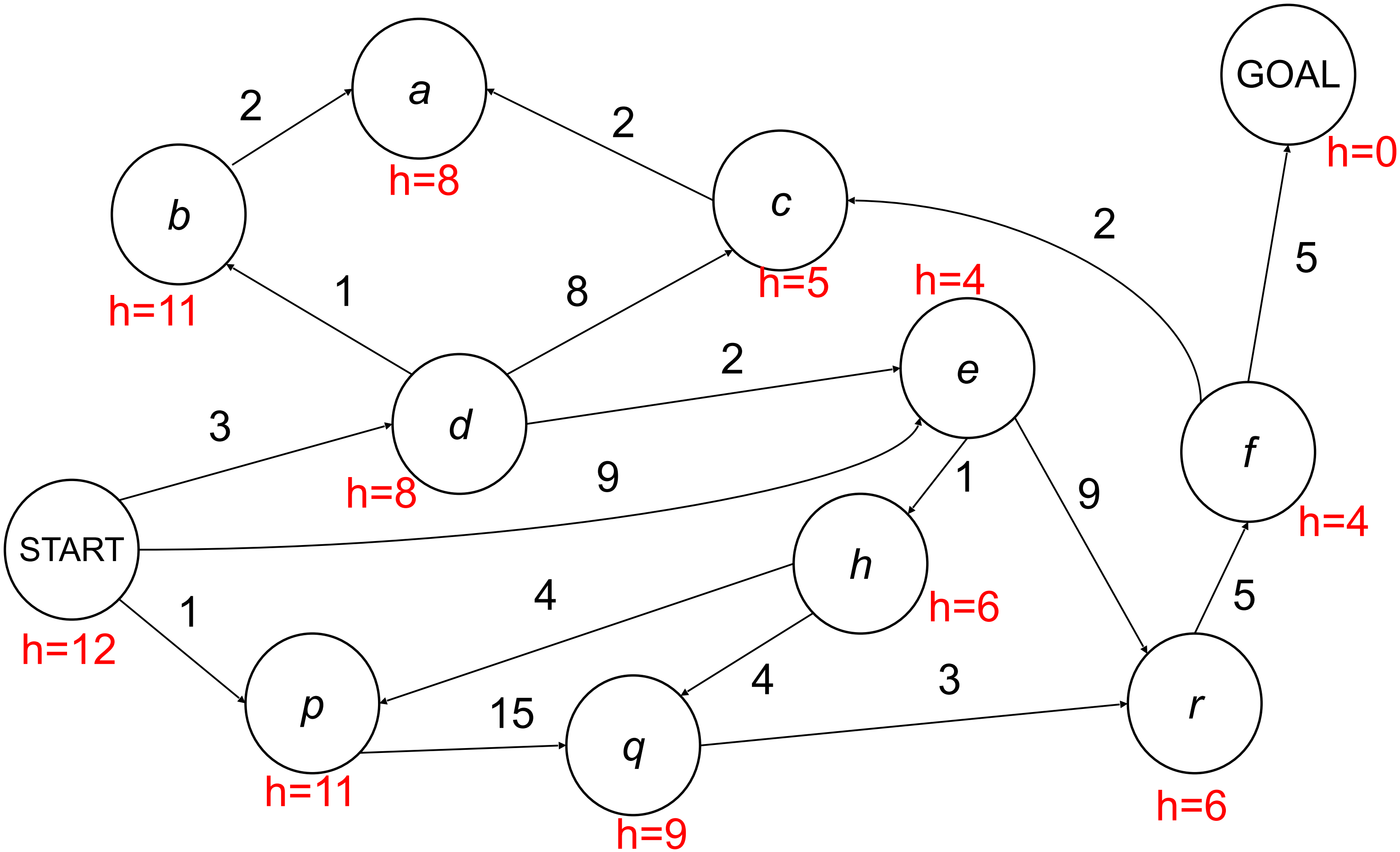
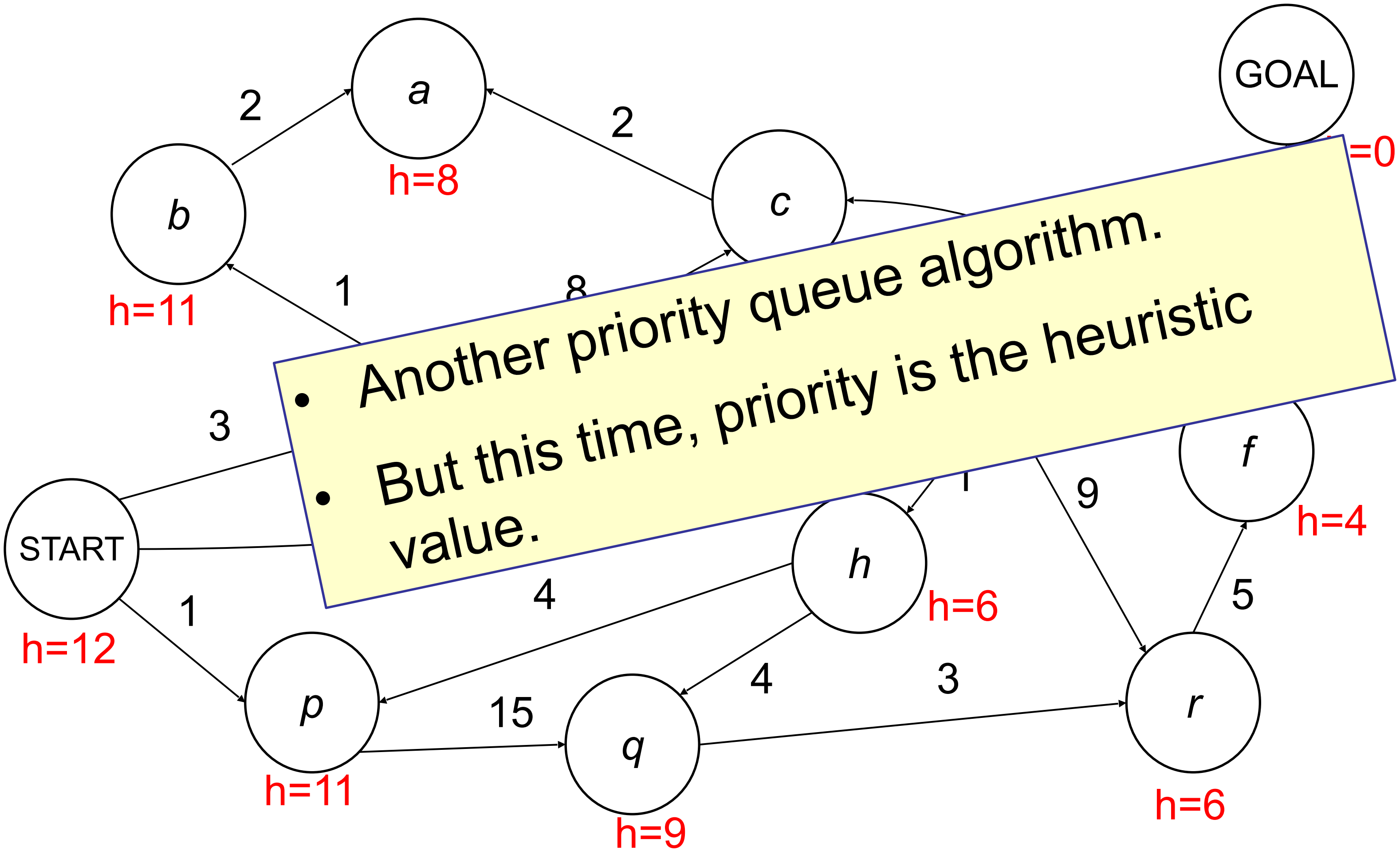
Denote the heuristic by a function $h(s)$ from states to a cost value.

Euclidian Heuristic



Euclidian Heuristic



- Another priority queue algorithm.
- But this time, priority is the heuristic value.

Best First "Greedy" Search

```
Init-PriQueue(PQ)
Insert-PriQueue(PQ,START,h(START))
while (PQ is not empty and PQ does not contain a goal state)
        (s , h ) := Pop-least(PQ)
        foreach s' in succs(s)
        if s' is not already in PQ and s' never previously been visited
                Insert-PriQueue(PQ,s',h(s'))
```

Algorithm		Complete	Optimal	Time	Space
BestFS	Best First Search	Y	**N**	$O(min(N,B^{LMAX}))$	$O(min(N,B^{LMAX}))$

vements to this algorithm can make things much better. It's a little thing we like to call: A*….

…to be continued!

What you should know

- Thorough understanding of BFS, LCBFS, UCS. PCDFS, MEMDFS
- Understand the concepts of whether a search is complete, optimal, its time and space complexity
- Understand the ideas behind iterative deepening and bidirectional search
- Be able to discuss at cocktail parties the pros and cons of the above searches