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Decision Trees

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Machine Learning Datasets

What is Classification?

Contingency Tables

OLAP (Online Analytical Processing)

What is Data Mining?

Searching for High Information Gain

Learning an unpruned decision tree recursively

Training Set Error

Test Set Error

Overfitting

Avoiding Overfitting

Information Gain of a real valued input

Building Decision Trees with real Valued Inputs

Andrew's homebrewed hack: Binary Categorical Splits

Example Decision Trees

Here is a dataset

age	employe	education	edun	marital	...	job	relation	race	gender	hour	country	wealth
					...							
39	State_gov	Bachelors	13	Never_mar	...	Adm_cleric	Not_in_fan	White	Male	40	United_Stat	poor
51	Self_emp_	Bachelors	13	Married	...	Exec_man	Husband	White	Male	13	United_Stat	poor
39	Private	HS_grad	9	Divorced	...	Handlers_c	Not_in_fan	White	Male	40	United_Stat	poor
54	Private	11th	7	Married	...	Handlers_c	Husband	Black	Male	40	United_Stat	poor
28	Private	Bachelors	13	Married	...	Prof_speci	Wife	Black	Female	40	Cuba	poor
38	Private	Masters	14	Married	...	Exec_man	Wife	White	Female	40	United_Stat	poor
50	Private	9th	5	Married_sp	...	Other_serv	Not_in_fan	Black	Female	16	Jamaica	poor
52	Self_emp_	HS_grad	9	Married	...	Exec_man	Husband	White	Male	45	United_Stat	rich
31	Private	Masters	14	Never_mar	...	Prof_speci	Not_in_fan	White	Female	50	United_Stat	rich
42	Private	Bachelors	13	Married	...	Exec_man	Husband	White	Male	40	United_Stat	rich
37	Private	Some_coll	10	Married	...	Exec_man	Husband	Black	Male	80	United_Stat	rich
30	State_gov	Bachelors	13	Married	...	Prof_speci	Husband	Asian	Male	40	India	rich
24	Private	Bachelors	13	Never_mar	...	Adm_cleric	Own_child	White	Female	30	United_Stat	poor
33	Private	Assoc_acc	12	Never_mar	...	Sales	Not_in_fan	Black	Male	50	United_Stat	poor
41	Private	Assoc_voc	11	Married	...	Craft_repa	Husband	Asian	Male	40	*MissingV	rich
34	Private	7th_8th	4	Married	...	Transport	Husband	Amer_Indi	Male	45	Mexico	poor
26	Self_emp_	HS_grad	9	Never_mar	...	Farming_fi	Own_child	White	Male	35	United_Stat	poor
33	Private	HS_grad	9	Never_mar	...	Machine_c	Unmarried	White	Male	40	United_Stat	poor
38	Private	11th	7	Married	...	Sales	Husband	White	Male	50	United_Stat	poor
44	Self_emp_	Masters	14	Divorced	...	Exec_man	Unmarried	White	Female	45	United_Stat	rich
41	Private	Doctorate	16	Married	...	Prof_speci	Husband	White	Male	60	United_Stat	rich
:	:	:	:	:	:	:	:	:	:	:	:	:

48,000 records, 16 attributes [Kohavi 1995]

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Example Decision Trees

Classification

- A Major Data Mining Operation
- Give one attribute (e.g wealth), try to predict the value of new people's wealths by means of some of the other available attributes.
- Applies to categorical outputs
 - Categorical attribute: an attribute which takes on two or more discrete values. Also known as a symbolic attribute.
 - Real attribute: a column of real numbers

Today's lecture

- Information Gain for measuring association between inputs and outputs
- Learning a decision tree classifier from data

About this dataset

- It is a tiny subset of the 1990 US Census.
- It is publicly available online from the UCI Machine Learning Datasets repository

Used Attributes

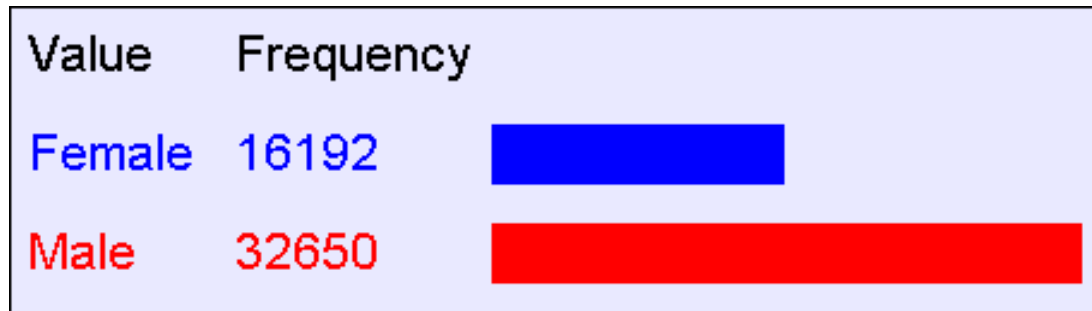
age	edunum	race	hours_worked
employment	marital	gender	country
taxweighting	job	capitalgain	wealth
education	relation	capitalloss	agegroup

This color = Real-valued This color = Symbol-valued

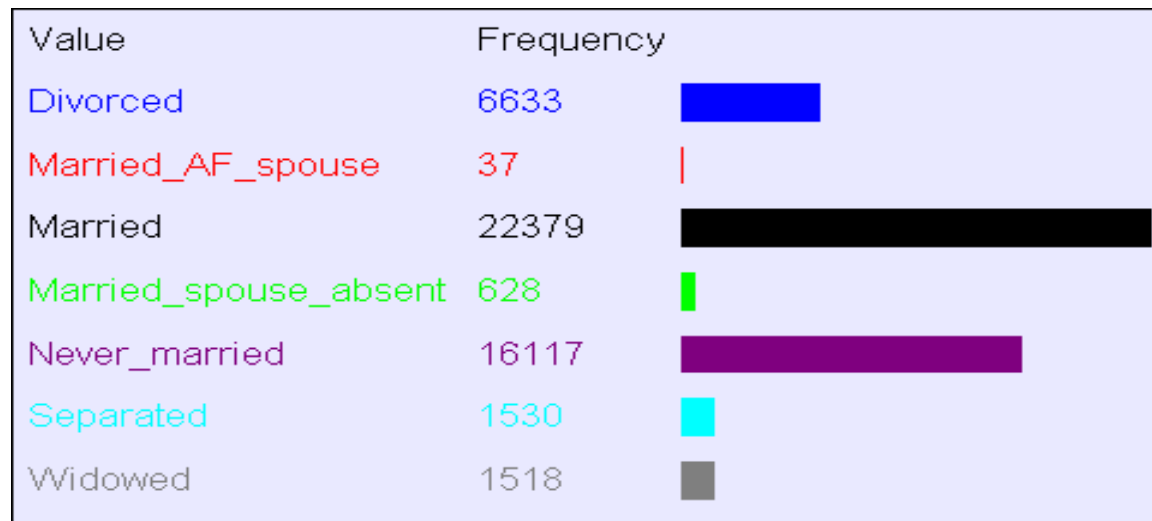
Successfully loaded a new dataset from the file \tadult.fds. It has 16 attributes and 48842 records.

What can you do with a dataset?

- Well, you can look at histograms...



Gender



Marital
Status

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Contingency Tables

- A better name for a histogram:
A One-dimensional Contingency Table
- Recipe for making a k-dimensional contingency table:
 1. Pick k attributes from your dataset. Call them a_1, a_2, \dots, a_k .
 2. For every possible combination of values, $a_1 = x_1, a_2 = x_2, \dots, a_k = x_k$, record how frequently that combination occurs

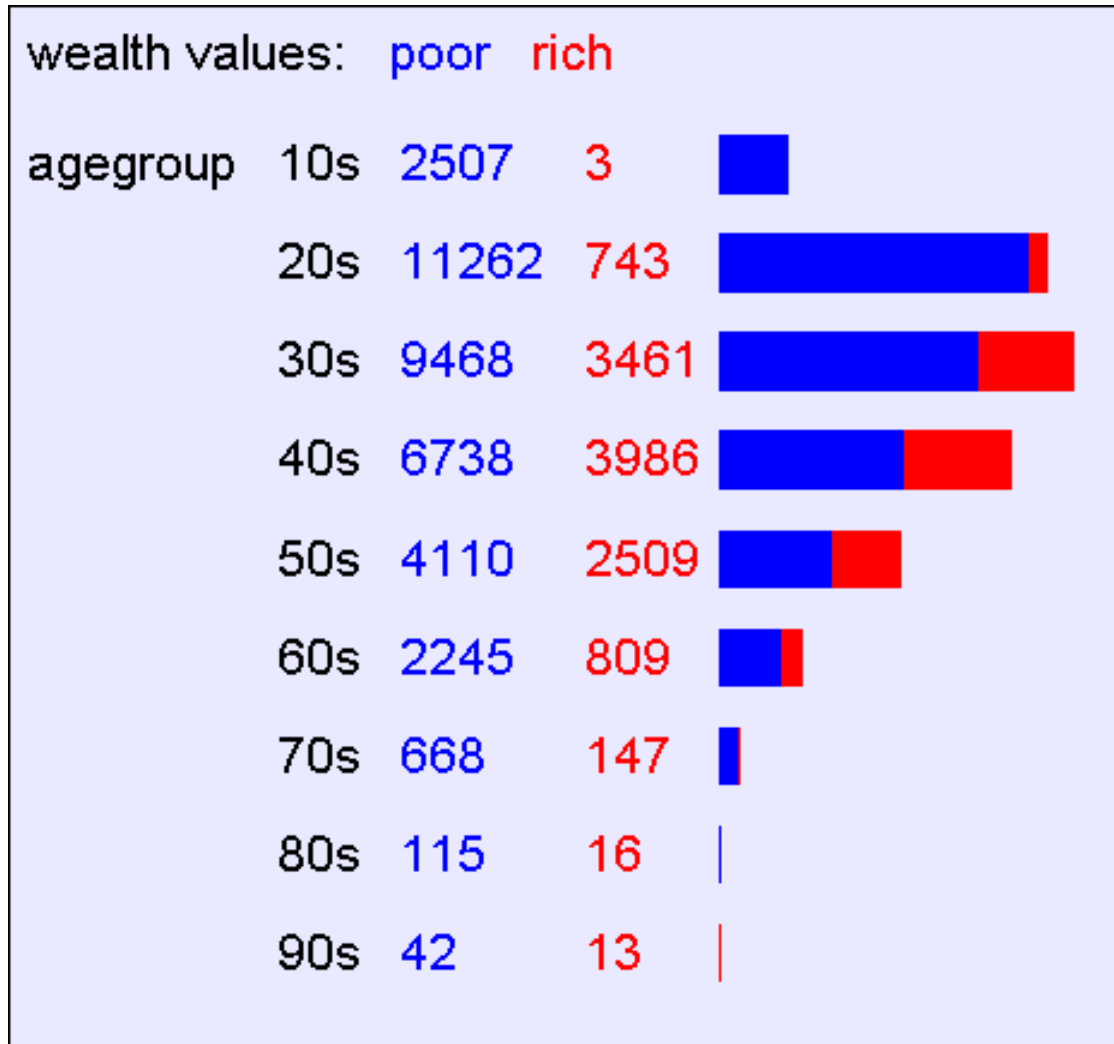
Fun fact: A database person would call this a “k-dimensional datacube”

A 2-d Contingency Table

wealth values:		poor	rich
agegroup	10s	2507	3
	20s	11262	743
	30s	9468	3461
	40s	6738	3986
	50s	4110	2509
	60s	2245	809
	70s	668	147
	80s	115	16
	90s	42	13

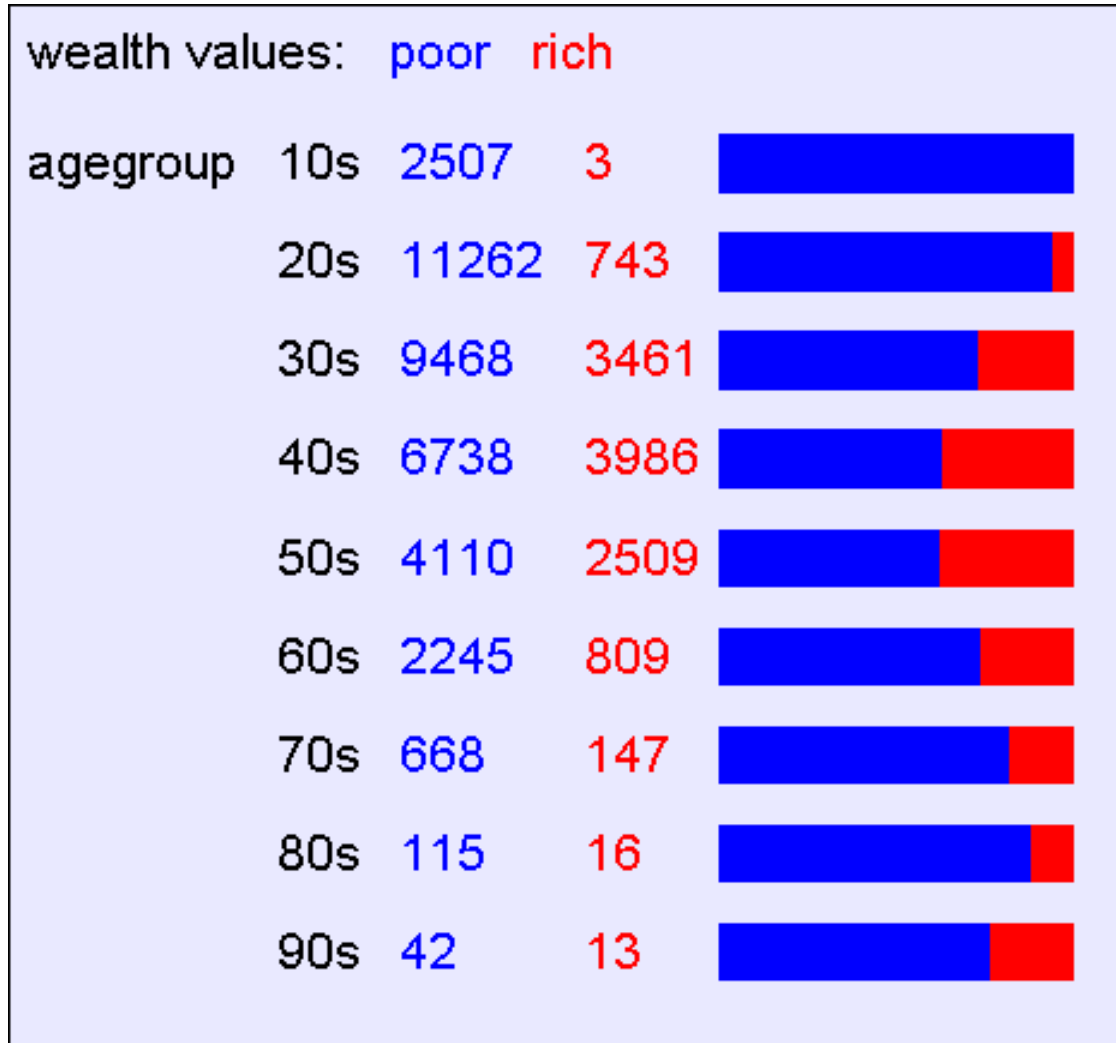
- For each pair of values for attributes (agegroup,wealth) we can see how many records match.

A 2-d Contingency Table



- Easier to appreciate graphically

A 2-d Contingency Table



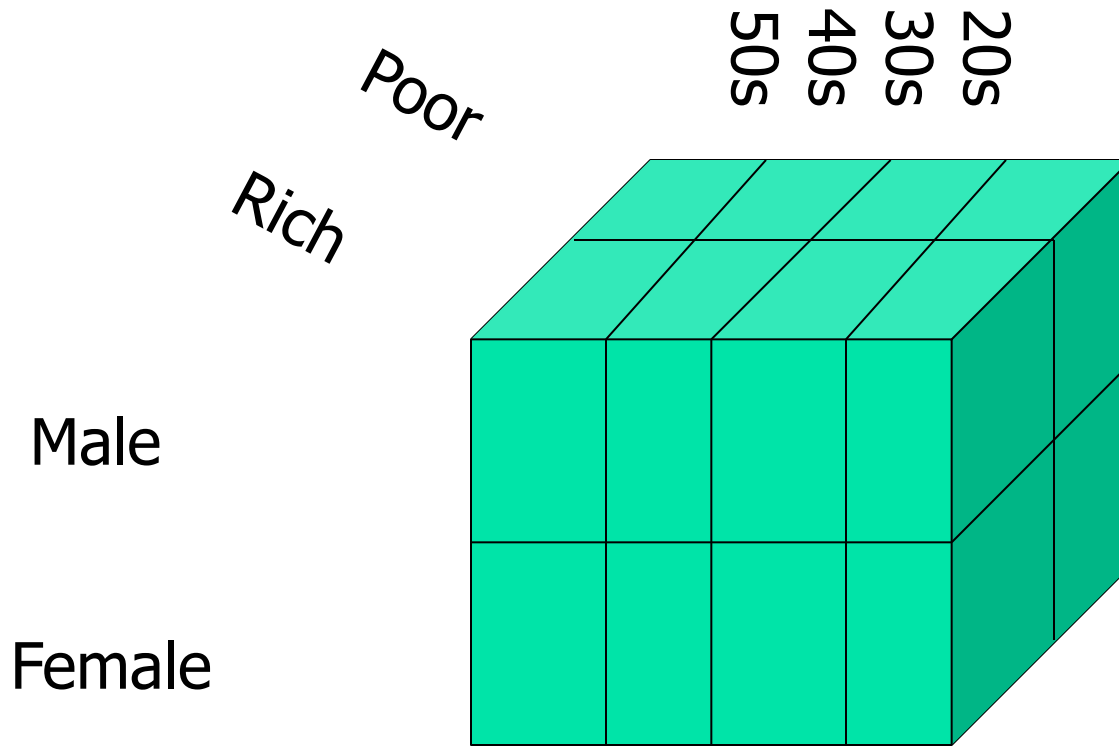
- Easier to see “interesting” things if we stretch out the histogram bars

A bigger 2-d contingency table

job values:		Adm_clerical	Craft_repair	Farming_fishing	Machine_op_inspct	Priv_house_serv	Protective_serv	Tech_support									
MissingValue		Armed_Forces	Exec_managerial	Handlers_cleaners	Other_service	Prof_specialty	Sales	Transport_moving									
marital	Divorced	270	1192	0	679	890	90	197	434	762	46	795	121	664	239	254	
	Married_AF_spouse	5	6	0	4	3	1	1	1	5	0	4	1	5	0	1	
	Married	928	1495	7	3818	3600	869	724	1469	1088	27	3182	583	2491	609	1489	
	Married_spouse_absent	45	84	0	77	52	35	32	37	92	9	64	7	55	9	30	
	Never_married	1242	2360	8	1301	1260	434	1029	872	2442	99	1849	237	1992	506	486	
	Separated	97	224	0	160	126	23	63	123	275	21	145	23	146	48	56	
	Widowed	222	250	0	73	155	38	26	86	259	40	133	11	151	35	39	

3-d contingency tables

- These are harder to look at!



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Example Decision Trees

On-Line Analytical Processing (OLAP)

- Software packages and database add-ons to do this are known as OLAP tools
- They usually include point and click navigation to view slices and aggregates of contingency tables
- They usually include nice histogram visualization

Time to stop and think

- Why would people want to look at contingency tables?

Let's continue to think

- With 16 attributes, how many 1-d contingency tables are there?
- How many 2-d contingency tables?
- How many 3-d tables?
- With 100 attributes how many 3-d tables are there?

Let's continue to think

- With 16 attributes, how many 1-d contingency tables are there? 16
- How many 2-d contingency tables? 16-choose-2 = $16 * 15 / 2 = 120$
- How many 3-d tables? 560
- With 100 attributes how many 3-d tables are there? 161,700

Manually looking at contingency tables

- **Looking at one contingency table:** can be as much fun as reading an interesting book
- **Looking at ten tables:** as much fun as watching CNN
- **Looking at 100 tables:** as much fun as watching an infomercial
- **Looking at 100,000 tables:** as much fun as a three-week November vacation in Duluth with a dying weasel.

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Example Decision Trees

Data Mining

- Data Mining is all about automating the process of searching for patterns in the data.

Which patterns are interesting?

Which might be mere illusions?

And how can they be exploited?

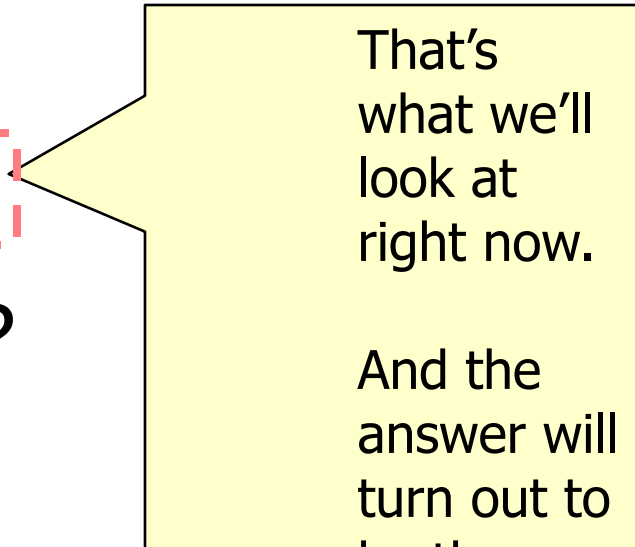
Data Mining

- Data Mining is all about automating the process of searching for patterns in the data.

Which patterns are interesting?

Which might be mere illusions?

And how can they be exploited?



That's what we'll look at right now.

And the answer will turn out to be the engine that drives decision tree

Deciding whether a pattern is interesting

- We will use **information theory**
- A very large topic, originally used for compressing signals
- But more recently used for data mining...

Deciding whether a pattern is interesting

- We will use **information theory**
- A very large topic, originally used for compressing signals
- But more recently used for data mining...

(The topic of Information Gain will now be discussed, but you will find it in a separate Andrew Handout)

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Example Decision Trees

Searching for High Info Gains

- Given something (e.g. wealth) you are trying to predict, it is easy to ask the computer to find which attribute has highest information gain for it.

wealth values: poor rich

relation	Husband	10870	8846		$H(\text{wealth} \mid \text{relation} = \text{Husband}) = 0.992385$
	Not_in_family	11307	1276		$H(\text{wealth} \mid \text{relation} = \text{Not_in_family}) = 0.473439$
	Other_relative	1454	52		$H(\text{wealth} \mid \text{relation} = \text{Other_relative}) = 0.216617$
	Own_child	7470	111		$H(\text{wealth} \mid \text{relation} = \text{Own_child}) = 0.110192$
	Unmarried	4816	309		$H(\text{wealth} \mid \text{relation} = \text{Unmarried}) = 0.328606$
	Wife	1238	1093		$H(\text{wealth} \mid \text{relation} = \text{Wife}) = 0.997207$

$H(\text{wealth}) = 0.793844$ $H(\text{wealth} \mid \text{relation}) = 0.628421$

$IG(\text{wealth} \mid \text{relation}) = 0.165423$

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Example Decision Trees

Learning Decision Trees

- A Decision Tree is a tree-structured plan of a set of attributes to test in order to predict the output.
- To decide which attribute should be tested first, simply find the one with the highest information gain.
- Then recurse...

A small dataset: Miles Per Gallon

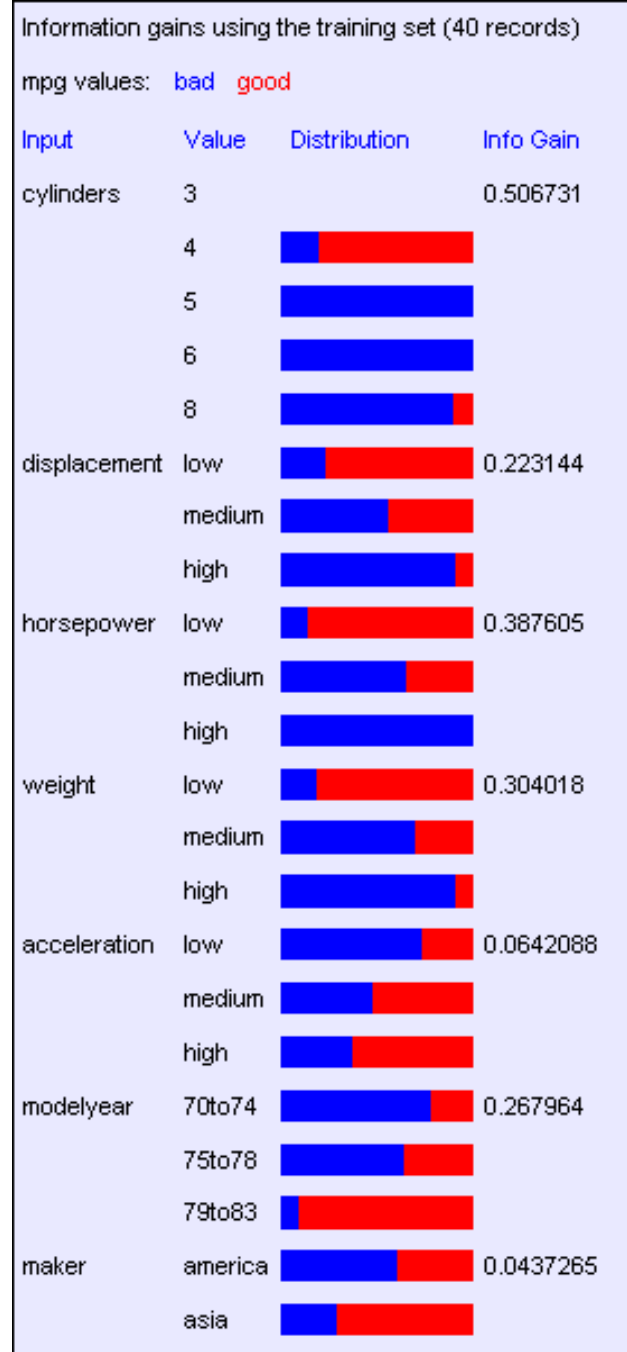
40
Records

mpg	cylinders	displacement	horsepower	weight	acceleration	modelyear	maker
good	4	low	low	low	high	75to78	asia
bad	6	medium	medium	medium	medium	70to74	america
bad	4	medium	medium	medium	low	75to78	europa
bad	8	high	high	high	low	70to74	america
bad	6	medium	medium	medium	medium	70to74	america
bad	4	low	medium	low	medium	70to74	asia
bad	4	low	medium	low	low	70to74	asia
bad	8	high	high	high	low	75to78	america
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
bad	8	high	high	high	low	70to74	america
good	8	high	medium	high	high	79to83	america
bad	8	high	high	high	low	75to78	america
good	4	low	low	low	low	79to83	america
bad	6	medium	medium	medium	high	75to78	america
good	4	medium	low	low	low	79to83	america
good	4	low	low	medium	high	79to83	america
bad	8	high	high	high	low	70to74	america
good	4	low	medium	low	medium	75to78	europa
bad	5	medium	medium	medium	medium	75to78	europa

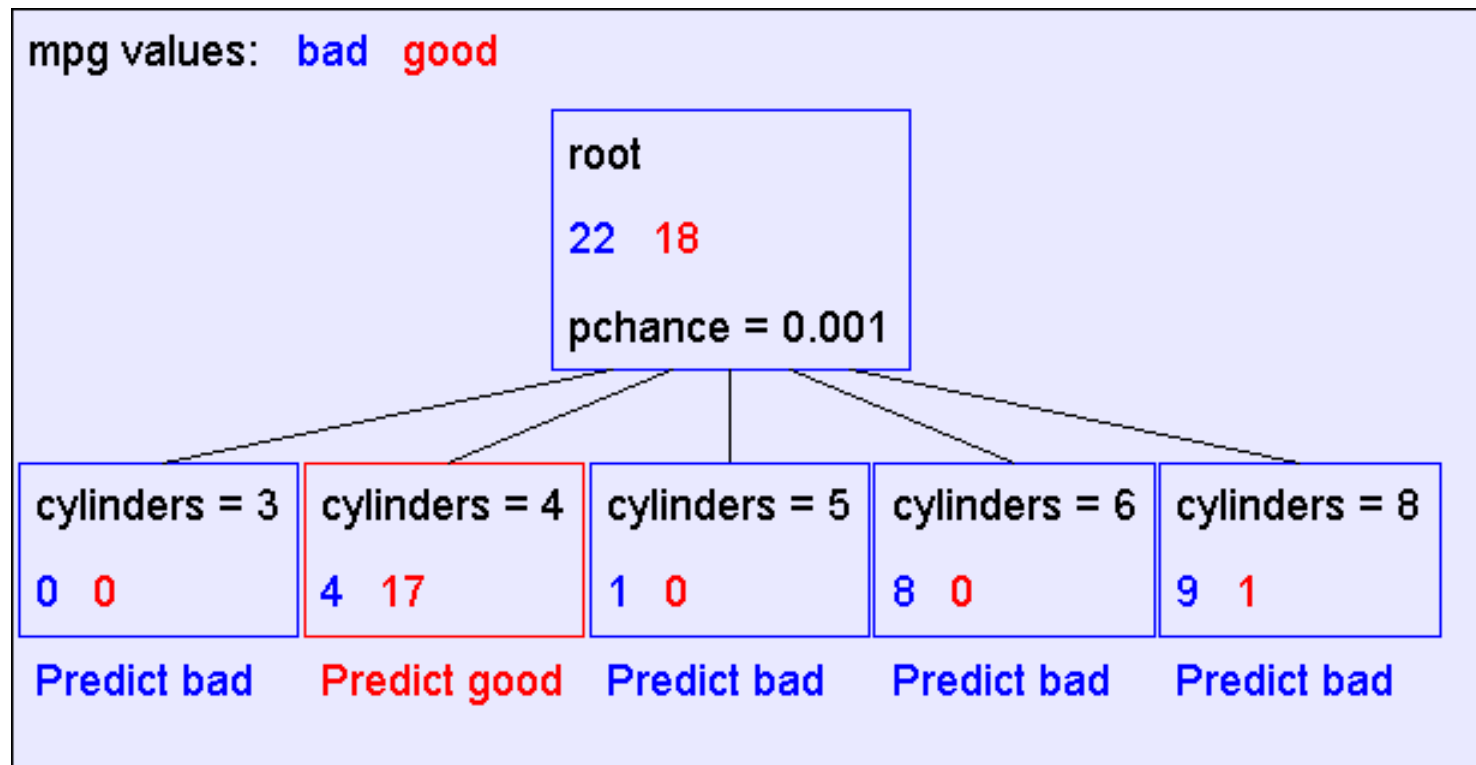
From the UCI repository (thanks to Ross Quinlan)

Suppose we want to
predict MPG.

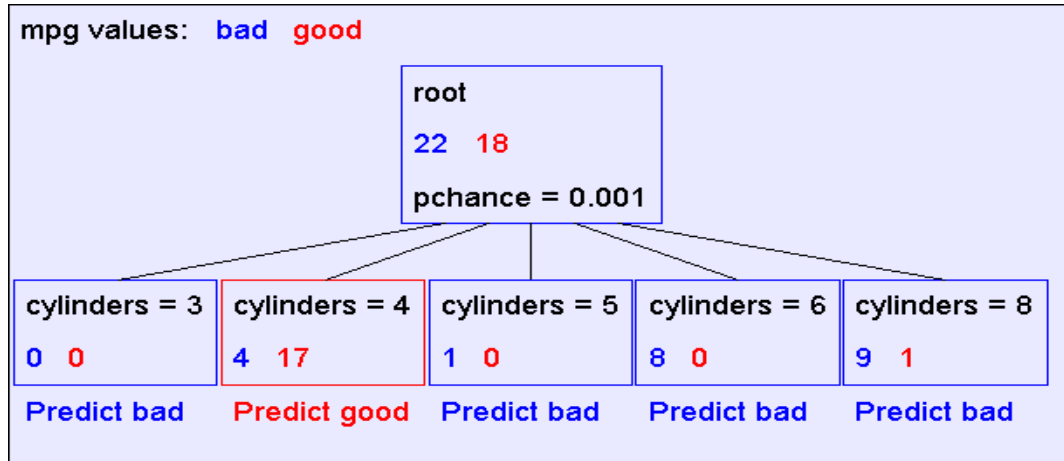
Look at all
the
information
gains...



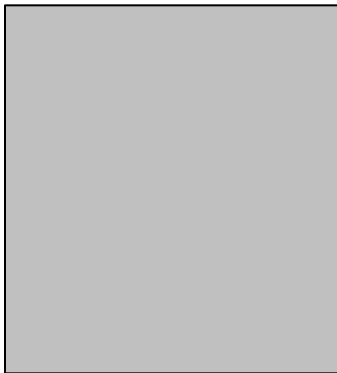
A Decision Stump



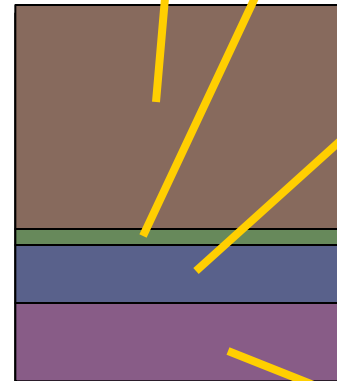
Recursion Step



Take the
Original
Dataset..



And partition it
according
to the value of
the attribute
we split on



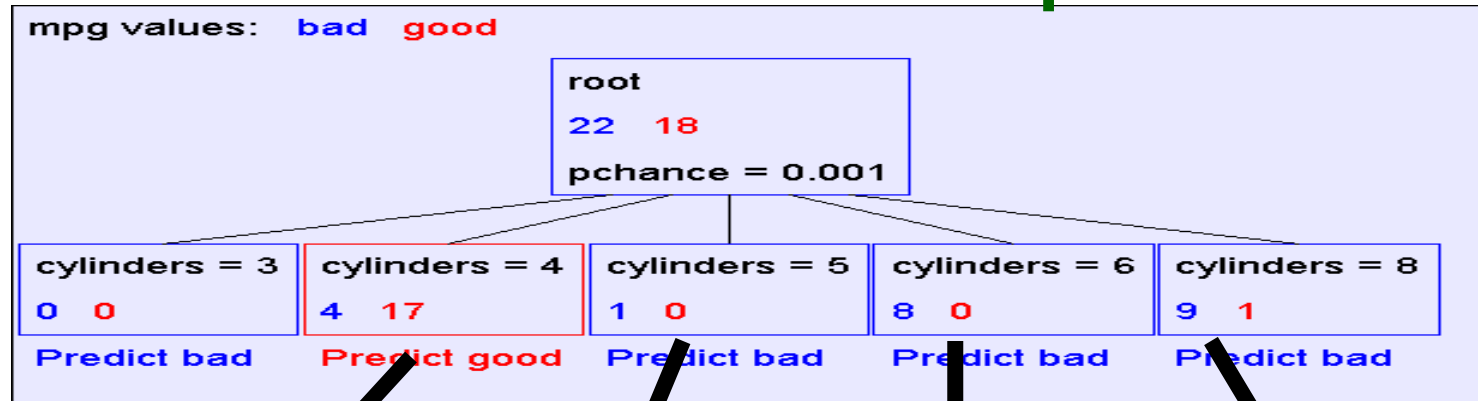
Records
in which
cylinders
= 4

Records
in which
cylinders
= 5

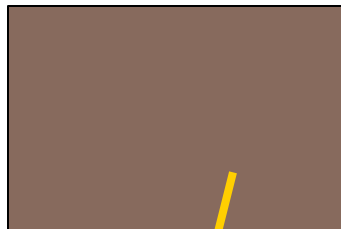
Records
in which
cylinders
= 6

Records
in which
cylinders
= 8

Recursion Step



Build tree from
These records..



Records in
which cylinders
= 4

Build tree from
These records..



Records in
which cylinders
= 5

Build tree from
These records..



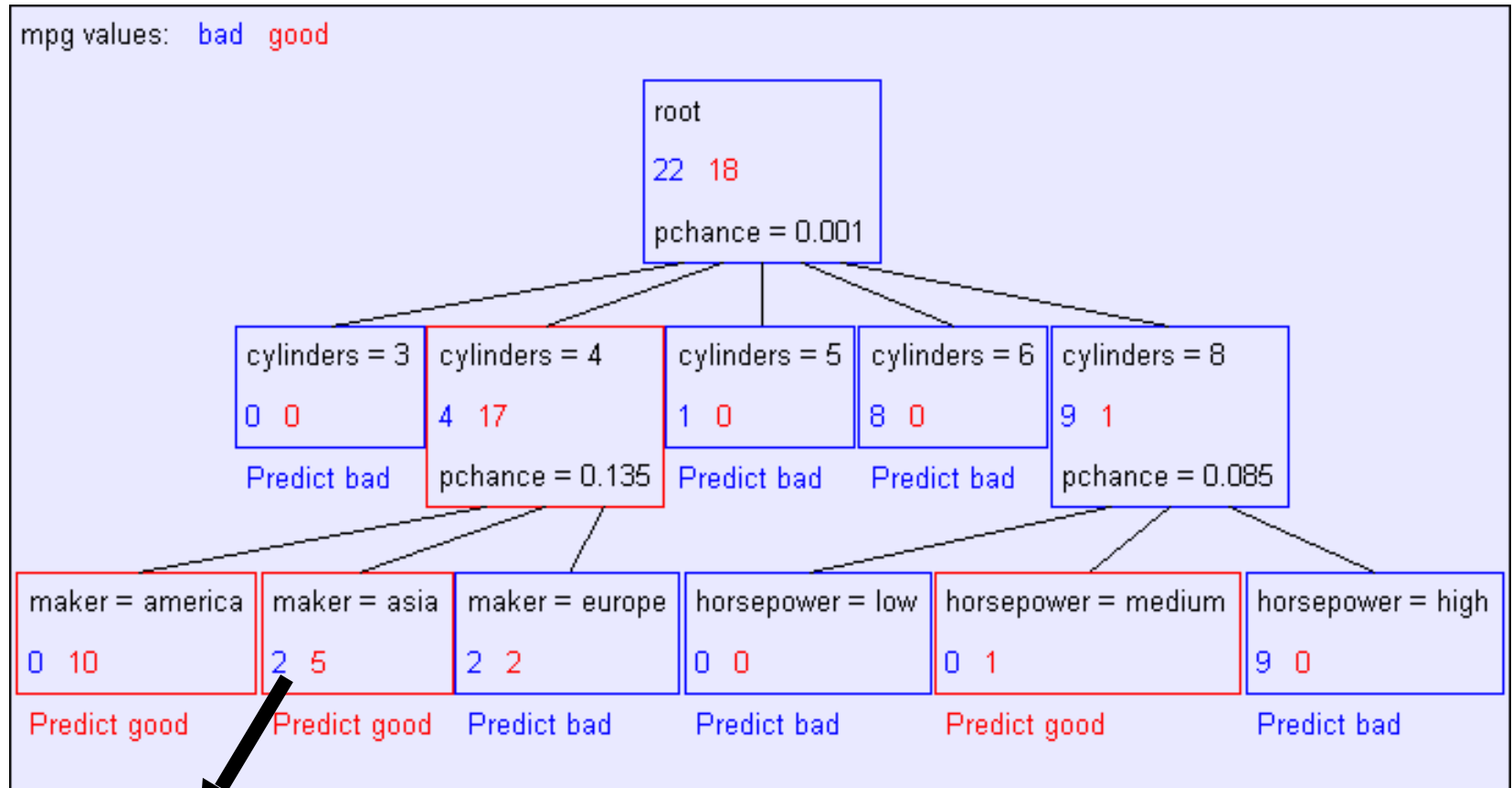
Records in
which cylinders
= 6

Build tree from
These records..



Records in
which cylinders
= 8

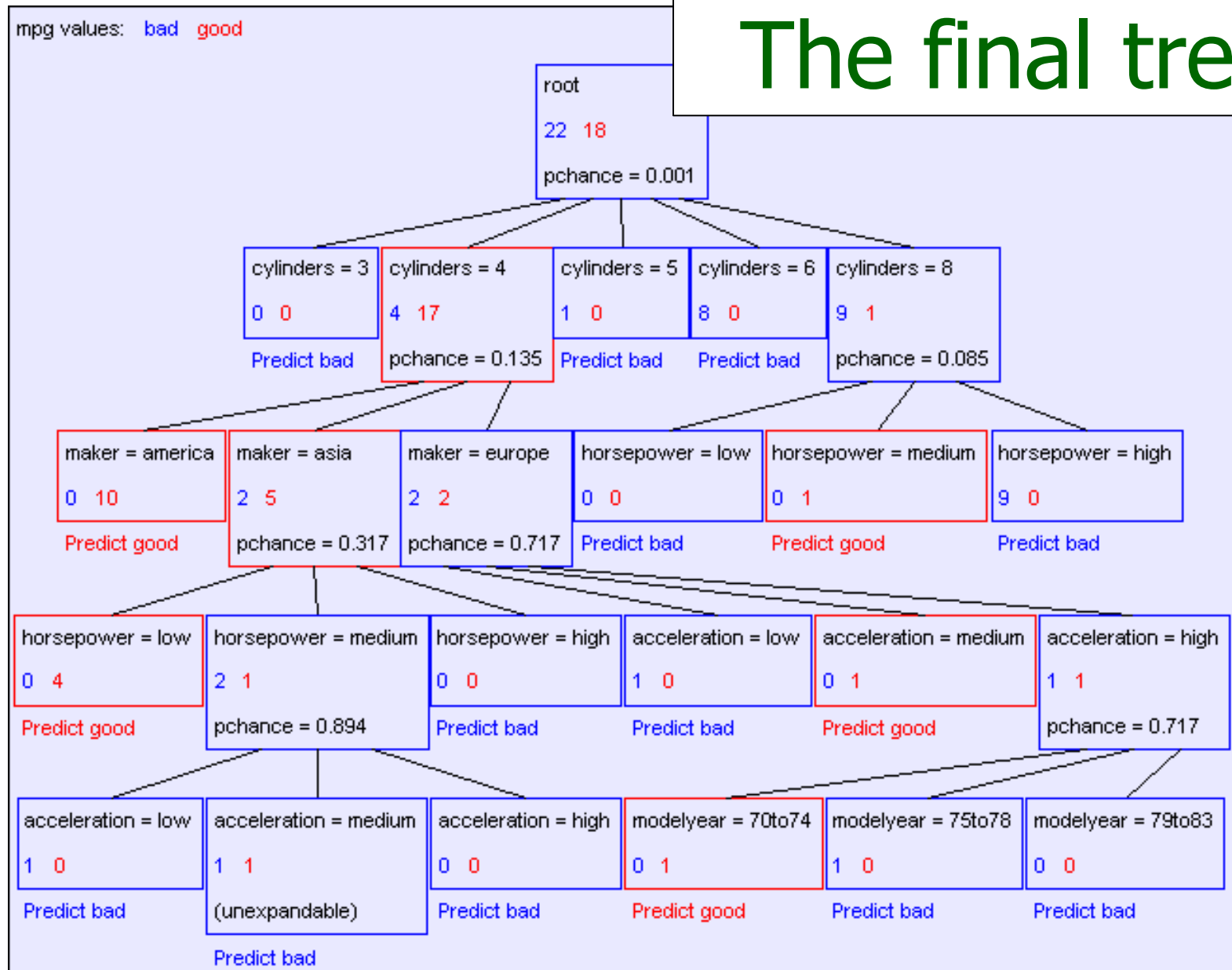
Second level of tree



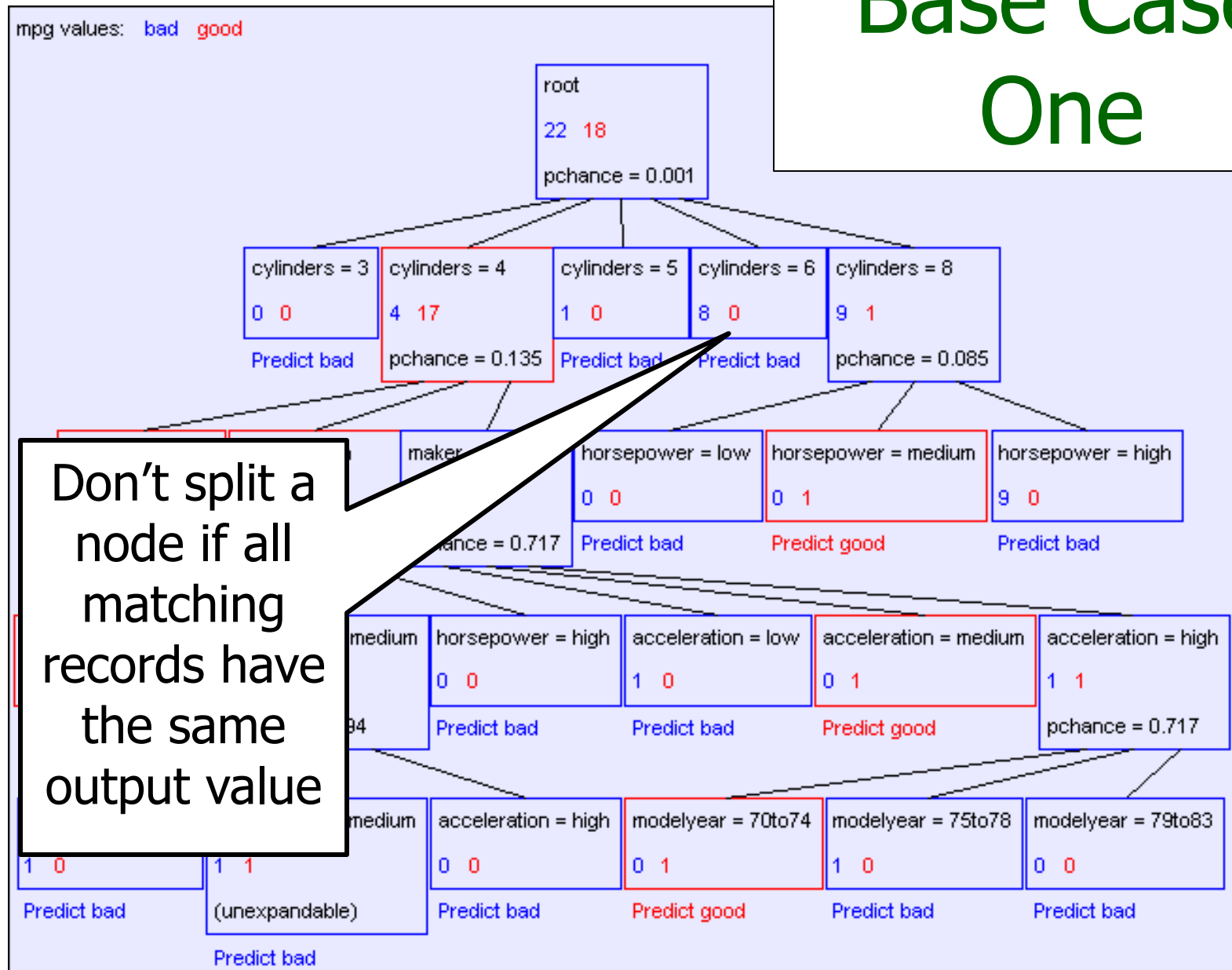
Recursively build a tree from the seven records in which there are four cylinders and the maker was based in Asia

(Similar recursion in the other cases)

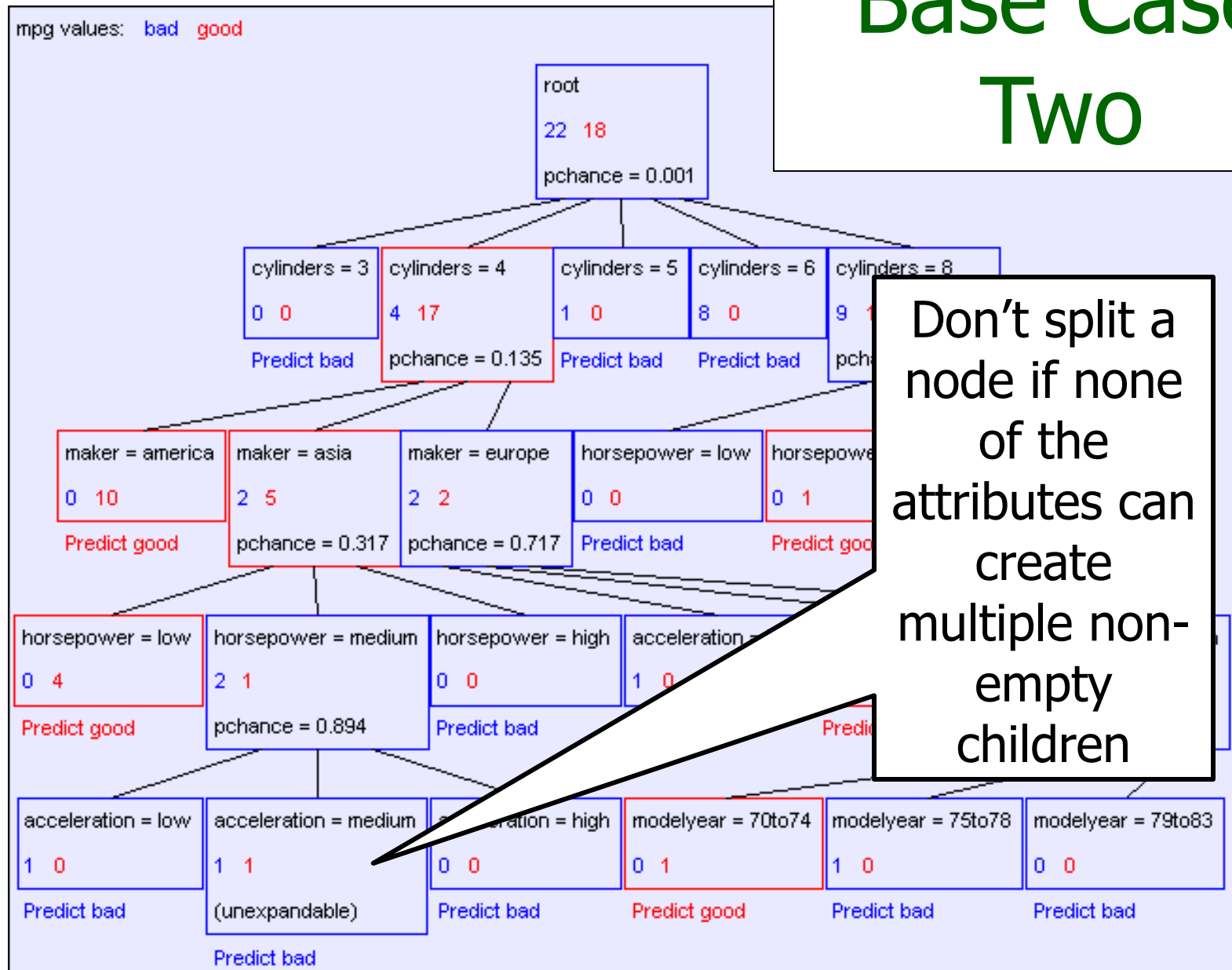
The final tree



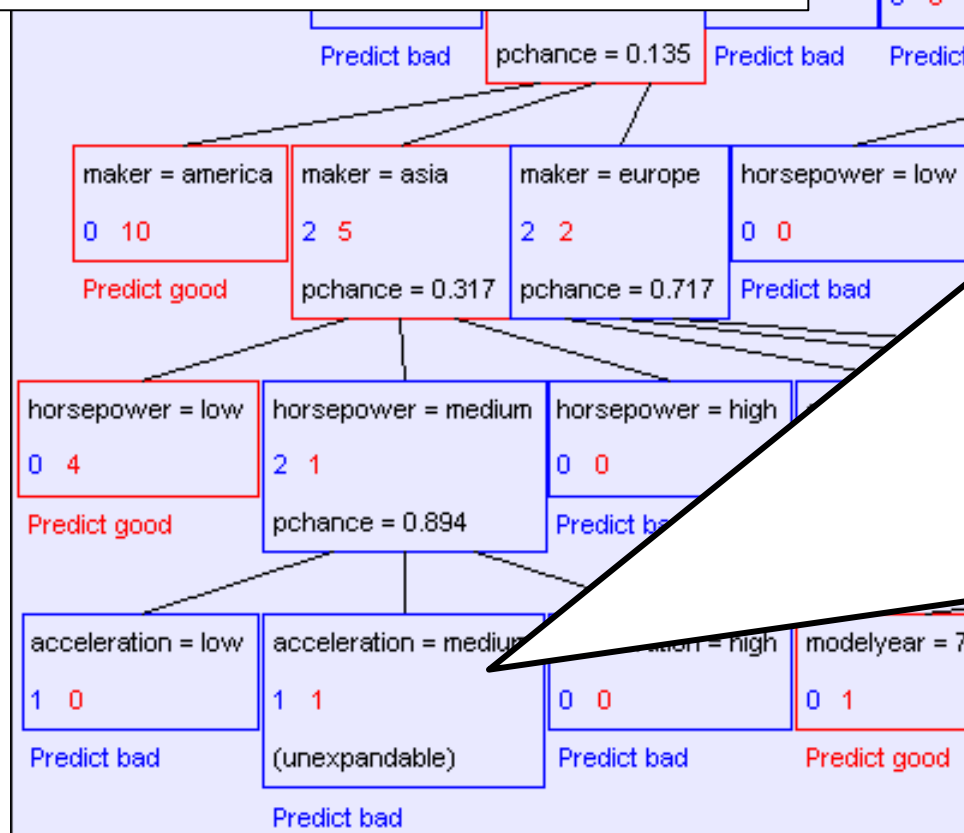
Base Case One



Base Case Two



Base Case Two: No attributes can distinguish



Information gains using the training set (2 records)

mpg values: bad good

Input	Value	Distribution	Info Gain
cylinders	3		0
	4		
	5		
	6		
	8		
displacement	low		0
	medium		
	high		
horsepower	low		0
	medium		
	high		
weight	low		0
	medium		
	high		
acceleration	low		0
	medium		
	high		
modelyear	70to74		0
	75to78		
	79to83		
maker	america		0
	asia		
	europe		

Base Cases

- Base Case One: If all records in current data subset have the same output then **don't recurse**
- Base Case Two: If all records have exactly the same set of input attributes then **don't recurse**

Base Cases: An idea

- Base Case One: If all records in current data subset have the same output then **don't recurse**
- Base Case Two: If all records have exactly the same set of input attributes then **don't recurse**



Proposed Base Case 3:

If all attributes have zero information gain then **don't recurse**

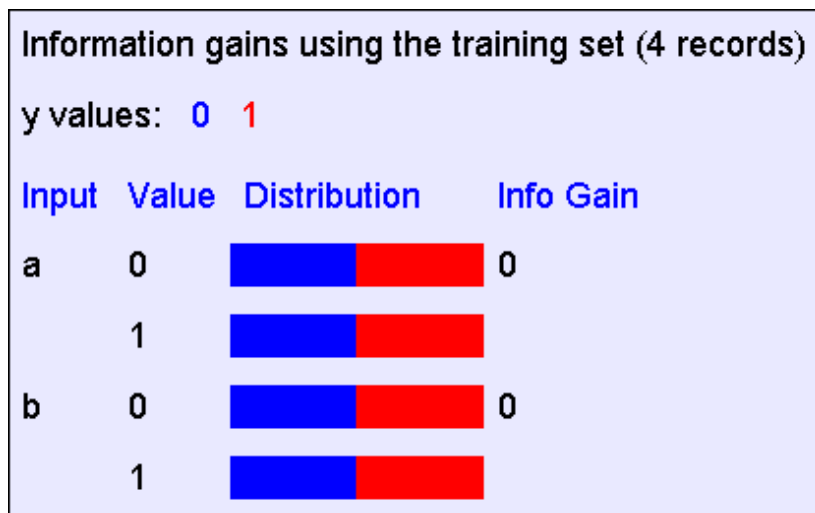
- Is this a good idea?

The problem with Base Case 3

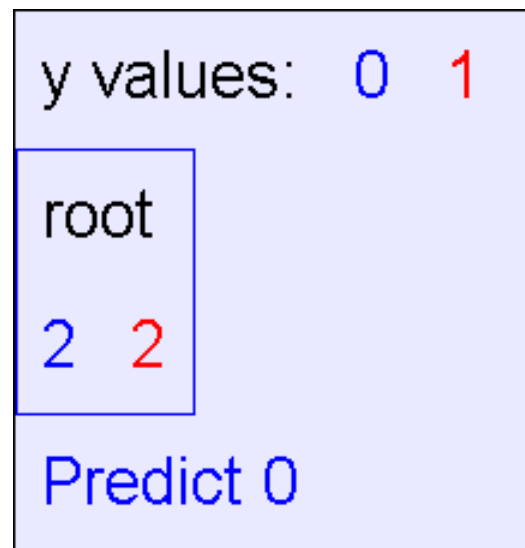
a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

$$y = a \text{ XOR } b$$

The information gains:



The resulting decision tree:

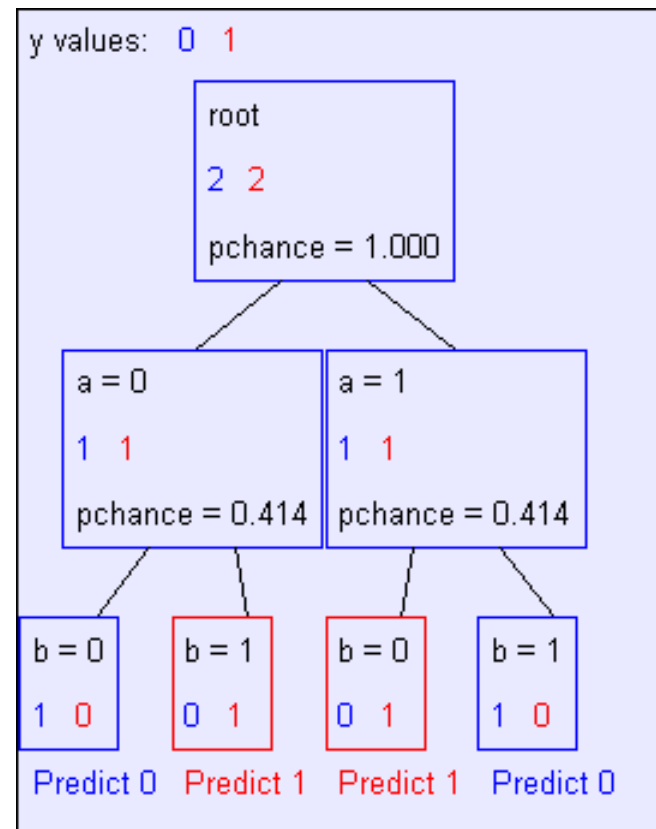


If we omit Base Case 3:

a	b	y
0	0	0
0	1	1
1	0	1
1	1	0

$$y = a \text{ XOR } b$$

The resulting decision tree:



Basic Decision Tree Building Summarized

BuildTree(DataSet,Output)

- If all output values are the same in DataSet, return a leaf node that says “predict this unique output”
- If all input values are the same, return a leaf node that says “predict the majority output”
- Else find attribute X with highest Info Gain
- Suppose X has n_x distinct values (i.e. X has arity n_x).
 - Create and return a non-leaf node with n_x children.
 - The i 'th child should be built by calling
BuildTree(DS _{i} ,Output)

Where DS _{i} built consists of all those records in DataSet for which X = i th distinct value of X.

Outline

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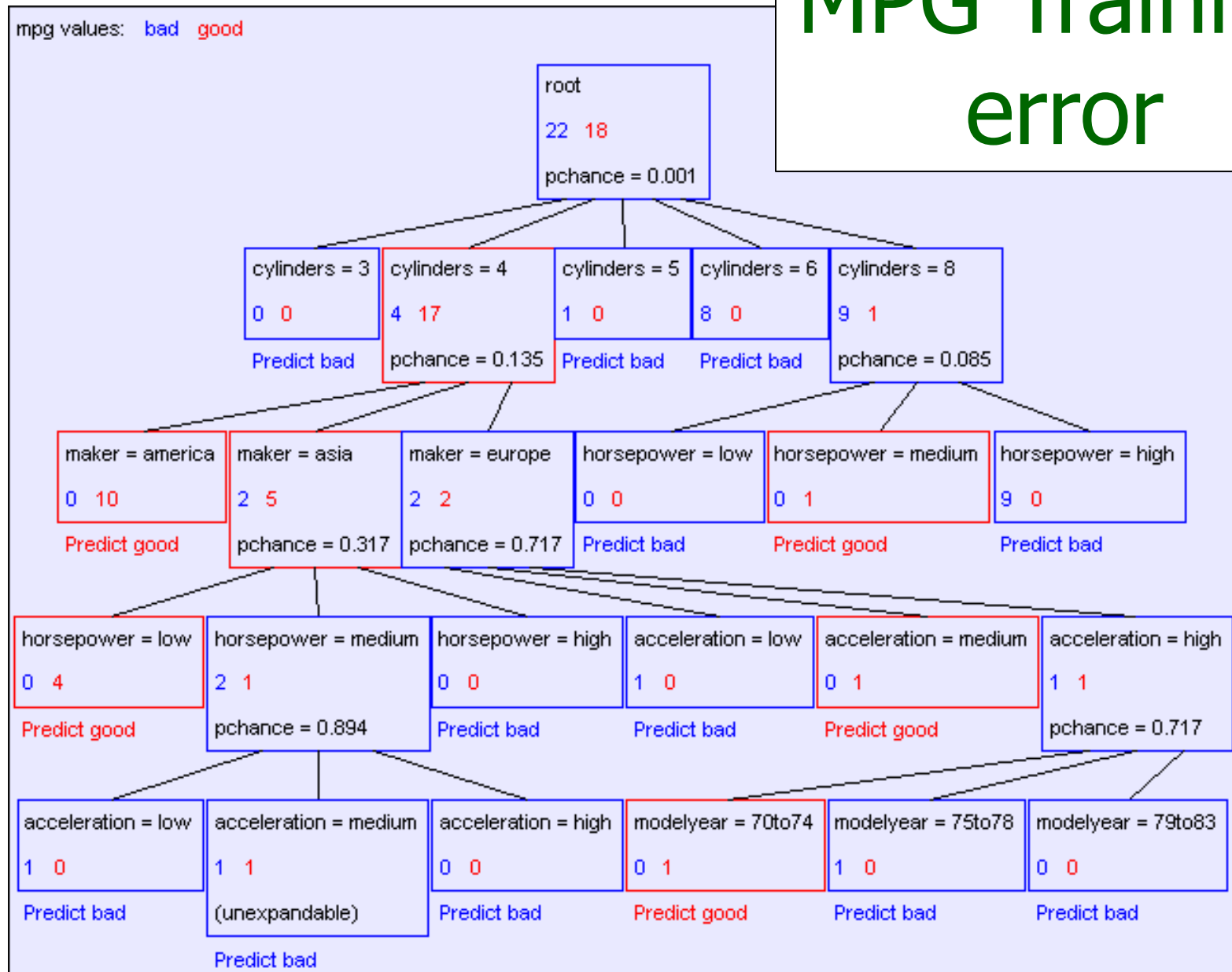
Andrew's homebrewed hack: Binary Categorical Splits

Example Decision Trees

Training Set Error

- For each record, follow the decision tree to see what it would predict
For what number of records does the decision tree's prediction disagree with the true value in the database?
- This quantity is called the **training set error**. The smaller the better.

MPG Training error

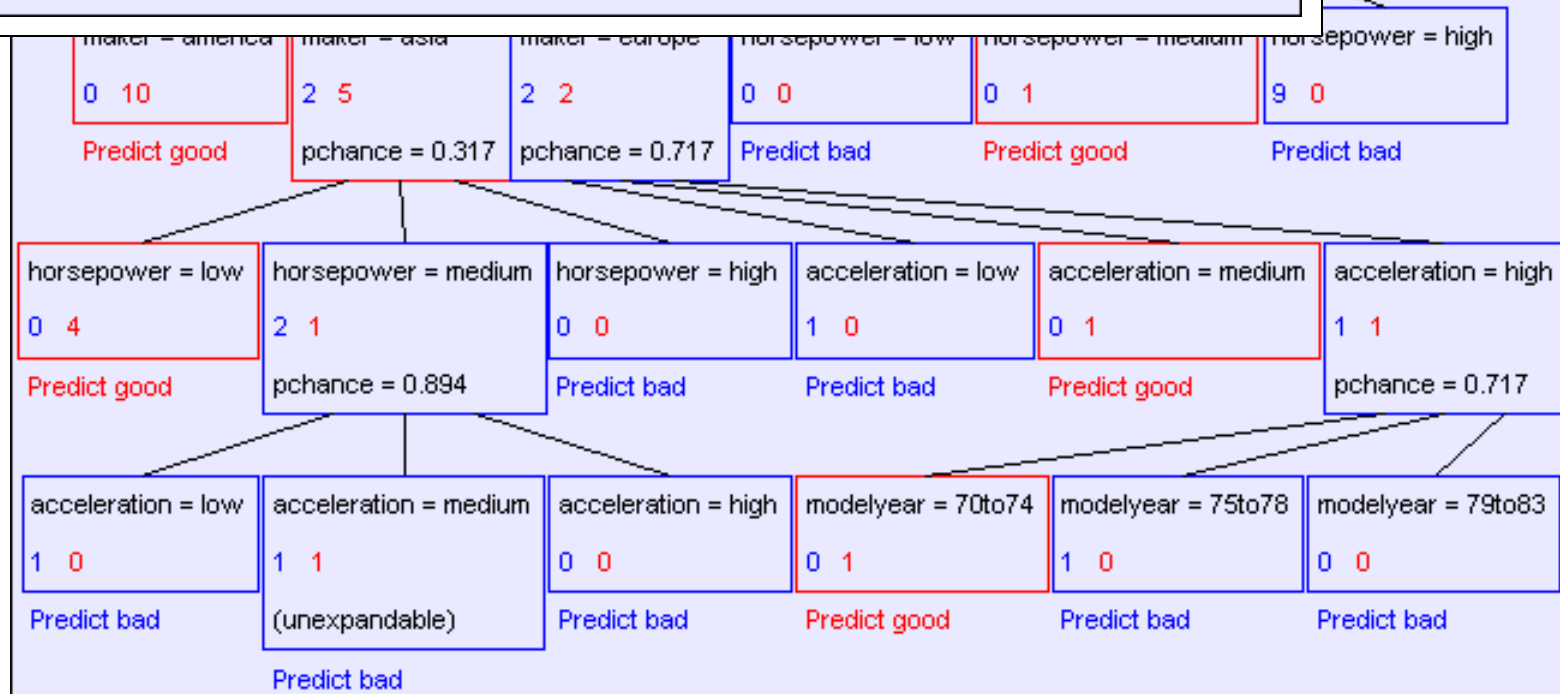


MPG Training error

mpg values: bad good

root
22 18
pchance = 0.001

	Num Errors	Set Size	Percent Wrong
Training Set 1	1	40	2.50

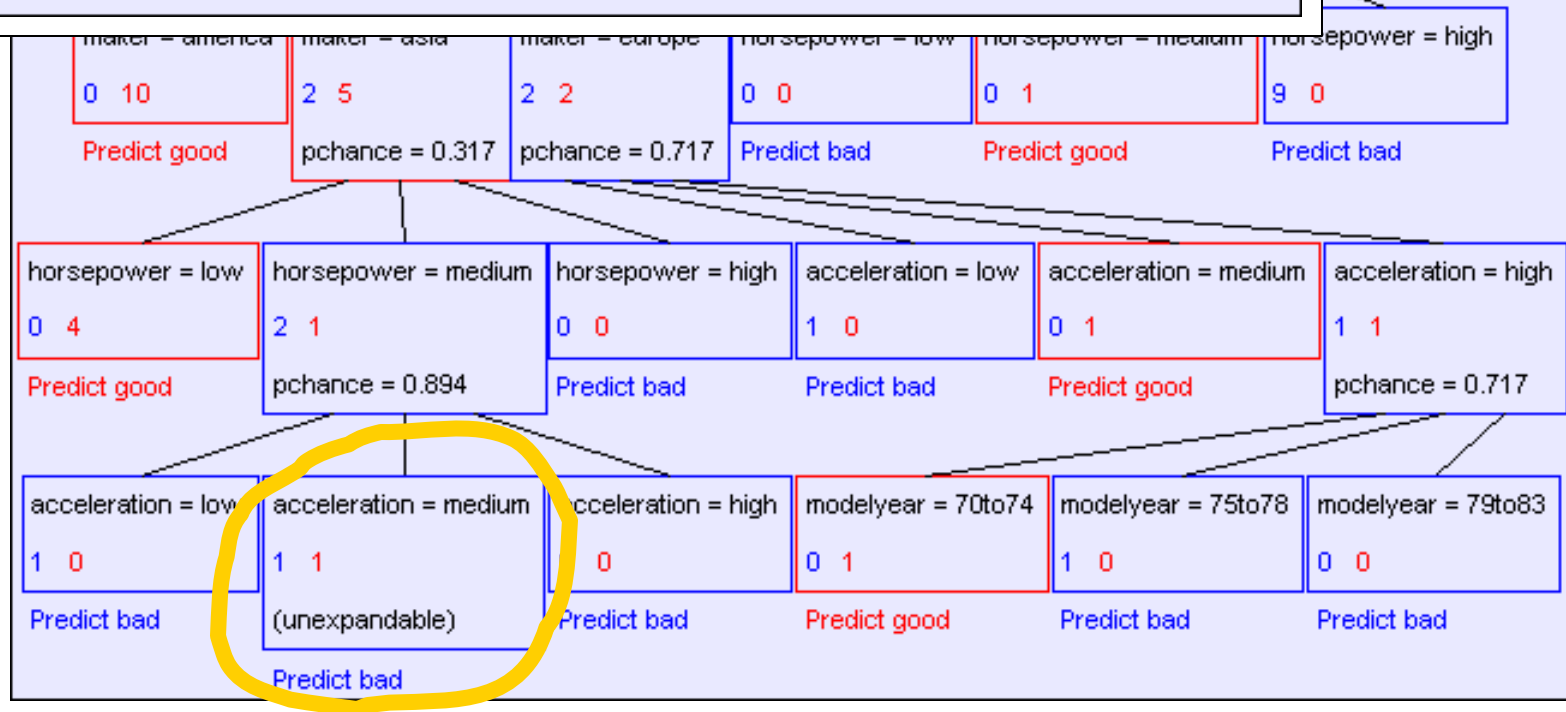


MPG Training error

mpg values: bad good

root
22 18
pchance = 0.001

	Num Errors	Set Size	Percent Wrong
Training Set 1	1	40	2.50



Stop and reflect: Why are we doing this learning anyway?

- It is not usually in order to predict the training data's output on data we have already seen.

Stop and reflect: Why are we doing this learning anyway?

- It is not usually in order to predict the training data's output on data we have already seen.
- It is more commonly in order to predict the output value for **future data** we have not yet seen.

Stop and reflect: Why are we doing this learning anyway?

- It is not usually in order to predict the training data's output on data we have already seen.
- It is more commonly in order to predict the output value for **future data** we have not yet seen.

Warning: A common data mining misperception is that the above two bullets are the only possible reasons for learning. There are at least a dozen others.

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Test Set Error

- Suppose we are forward thinking.
- We hide some data away when we learn the decision tree.
- But once learned, we see how well the tree predicts that data.
- This is a good simulation of what happens when we try to predict future data.
- And it is called **Test Set Error**.

MPG Test set error

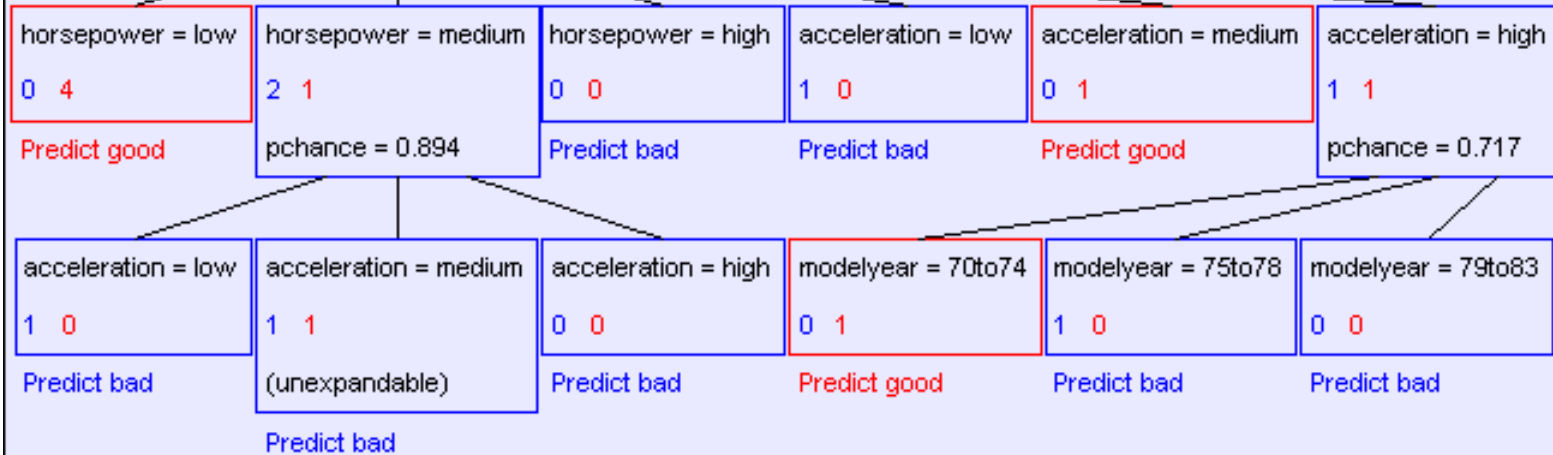
mpg values: bad good

root
22 18
pchance = 0.001

	Num Errors	Set Size	Percent Wrong
Training Set	1	40	2.50
Test Set	74	352	21.02

epower = high

ict bad



MPG Test set error

mpg values: bad good

root
22 18
pchance = 0.001

	Num Errors	Set Size	Percent Wrong
Training Set	1	40	2.50
Test Set	74	352	21.02

horsepower = high

Predict bad

horsepower = low

horsepower = medium

horsepower = high

acceleration = low

acceleration = medium

acceleration = high

0 1

Predict

ad

1

Predict bad

(unexpandable)

Predict bad

Predict good

Predict bad

Predict bad

= 0.717

= 79to83

The test set error is much worse than the training set error...

...why?

Outline

Machine Learning Datasets

What is Classification?

Contingency Tables

OLAP (Online Analytical Processing)

What is Data Mining?

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Overfitting

Avoiding Overfitting

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Example Decision Trees

An artificial example

- We'll create a training dataset

Five inputs, all bits, are generated in all 32 possible combinations

Output y = copy of e ,
Except a random 25% of the records have y set to the opposite of e

32 records

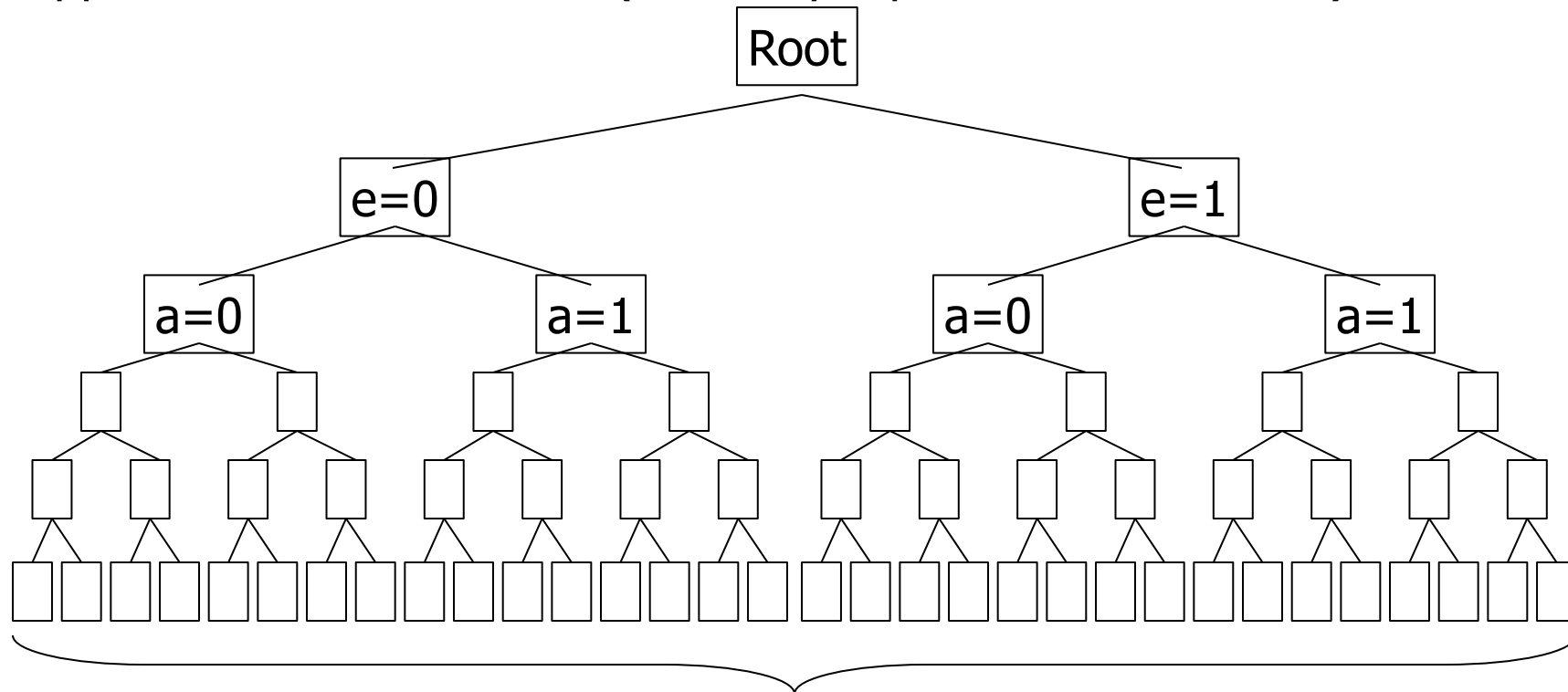
a	b	c	d	e	y
0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	1	0	0
0	0	0	1	1	1
0	0	1	0	0	1
:	:	:	:	:	:
1	1	1	1	1	1

In our artificial example

- Suppose someone generates a test set according to the same method.
- The test set is identical, except that some of the y 's will be different.
- Some y 's that were corrupted in the training set will be uncorrupted in the testing set.
- Some y 's that were uncorrupted in the training set will be corrupted in the test set.

Building a tree with the artificial training set

- Suppose we build a full tree (we always split until base case 2)



25% of these leaf node labels will be corrupted

Training set error for our artificial tree

All the leaf nodes contain exactly one record and so...

- We would have a training set error of zero

Testing the tree with the test set

	1/4 of the tree nodes are corrupted	3/4 are fine
1/4 of the test set records are corrupted	1/16 of the test set will be correctly predicted for the wrong reasons	3/16 of the test set will be wrongly predicted because the test record is corrupted
3/4 are fine	3/16 of the test predictions will be wrong because the tree node is corrupted	9/16 of the test predictions will be fine

In total, we expect to be wrong on 3/8 of the test set predictions

What's this example shown us?

- This explains the discrepancy between training and test set error
- But more importantly... ...it indicates there's something we should do about it if we want to predict well on future data.

Suppose we had less data

- Let's not look at the irrelevant bits

These bits are hidden

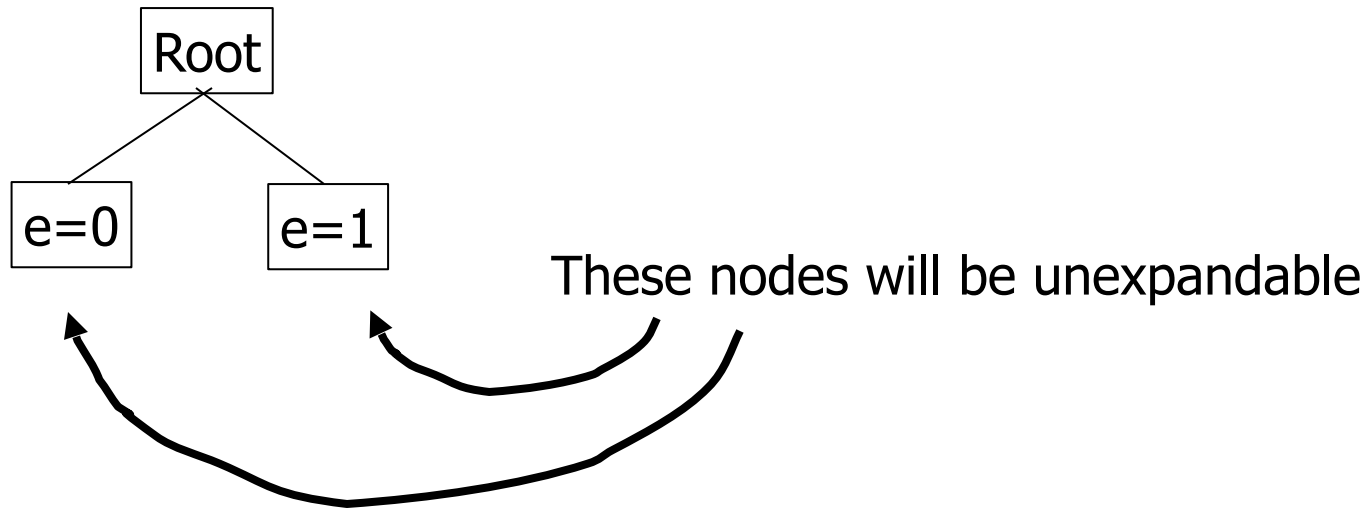
Output y = copy of e , except a random 25% of the records have y set to the opposite of e

32 records

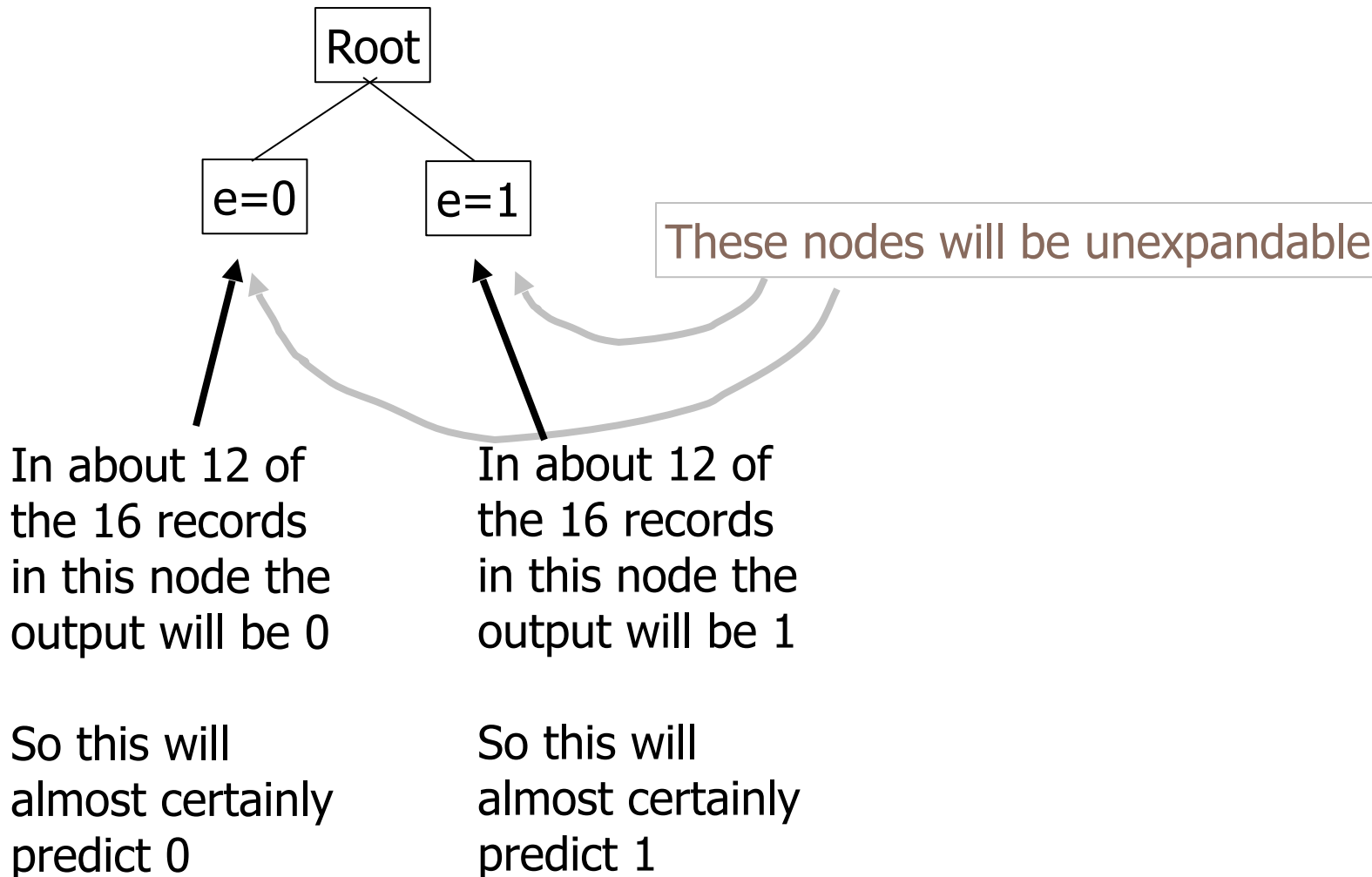
a	b	c	d	e	y
0	0	0	0	0	0
0	0	0	0	1	0
0	0	0	1	0	0
0	0	0	1	1	1
0	0	1	0	0	1
:	:	:	:	:	:
1	1	1	1	1	1

What decision tree would we learn now?

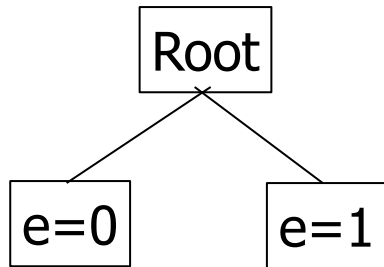
Without access to the irrelevant bits...



Without access to the irrelevant bits...



Without access to the irrelevant bits...



	almost certainly none of the tree nodes are corrupted	almost certainly all are fine
1/4 of the test set records are corrupted	n/a	1/4 of the test set will be wrongly predicted because the test record is corrupted
3/4 are fine	n/a	3/4 of the test predictions will be fine

In total, we expect to be wrong on only 1/4 of the test set predictions

Overfitting

- Definition: If your machine learning algorithm fits noise (i.e. pays attention to parts of the data that are irrelevant) it is **overfitting**.
- Fact (theoretical and empirical): If your machine learning algorithm is overfitting then it may perform less well on test set data.

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Test Set Error

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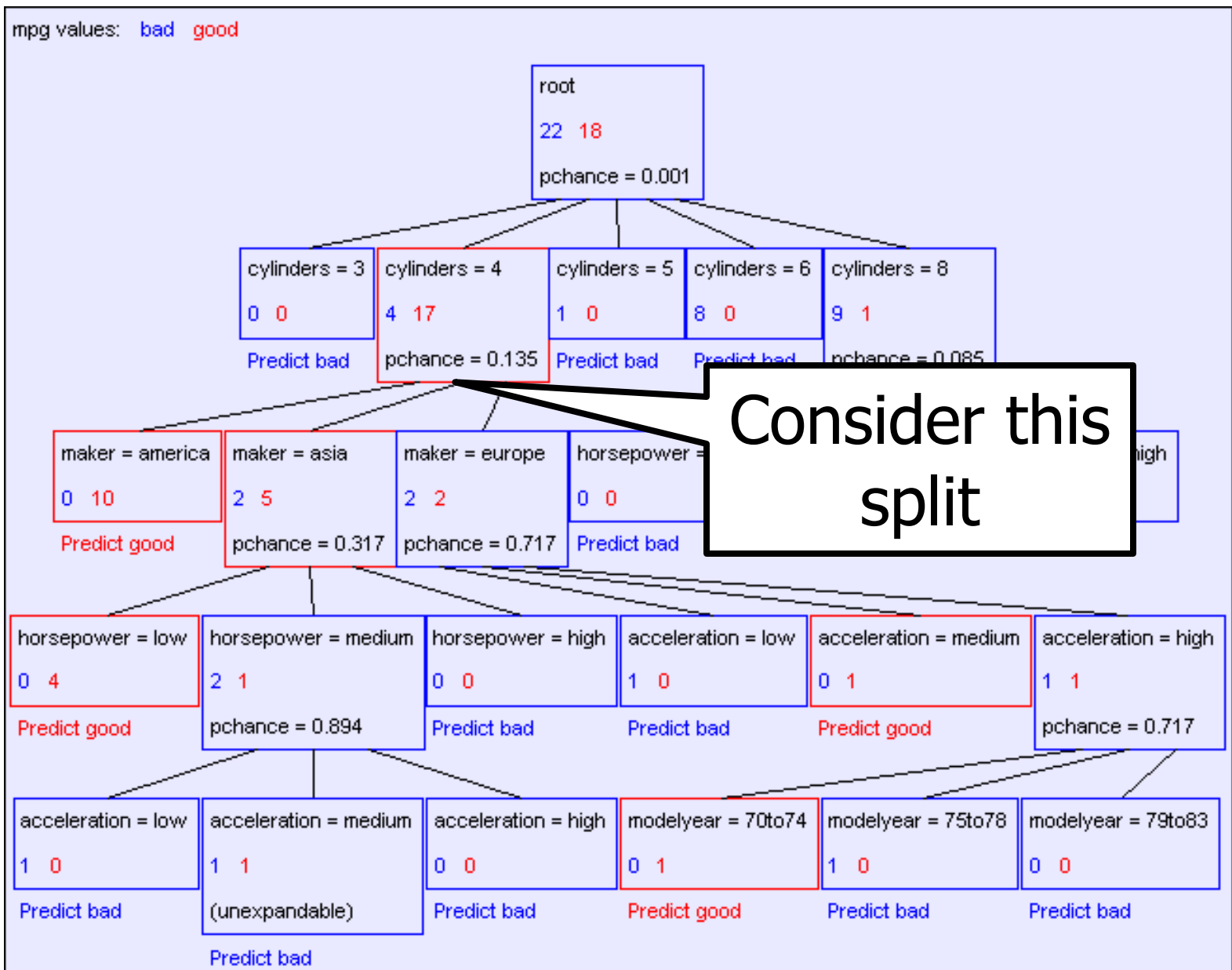
Andrew's homebrewed hack: Binary Categorical Splits

Example Decision Trees






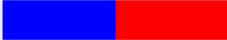
Avoiding overfitting

- Usually we do not know in advance which are the irrelevant variables
- ...and it may depend on the context
For example, if $y = a \text{ AND } b$ then b is an irrelevant variable only in the portion of the tree in which $a=0$

But we can use simple statistics to warn us that we might be overfitting.






A chi-squared test

mpg values:		bad	good		
maker	america	0	10		 $H(\text{mpg} \mid \text{maker} = \text{america}) = 0$
	asia	2	5		 $H(\text{mpg} \mid \text{maker} = \text{asia}) = 0.863121$
	europa	2	2		 $H(\text{mpg} \mid \text{maker} = \text{europa}) = 1$
$H(\text{mpg}) = 0.702467$ $H(\text{mpg} \mid \text{maker}) = 0.478183$					
$IG(\text{mpg} \mid \text{maker}) = 0.224284$					

- Suppose that mpg was completely uncorrelated with maker.
- What is the chance we'd have seen data of at least this apparent level of association anyway?

A chi-squared test

mpg values: bad good				
maker	america	0 10		$H(\text{mpg} \mid \text{maker} = \text{america}) = 0$
	asia	2 5		$H(\text{mpg} \mid \text{maker} = \text{asia}) = 0.863121$
	europa	2 2		$H(\text{mpg} \mid \text{maker} = \text{europa}) = 1$
$H(\text{mpg}) = 0.702467 \quad H(\text{mpg} \mid \text{maker}) = 0.478183$				
$IG(\text{mpg} \mid \text{maker}) = 0.224284$				

- Suppose that mpg was completely uncorrelated with maker.
- What is the chance we'd have seen data of at least this apparent level of association anyway?

By using a particular kind of chi-squared test, the answer is 13.5%.

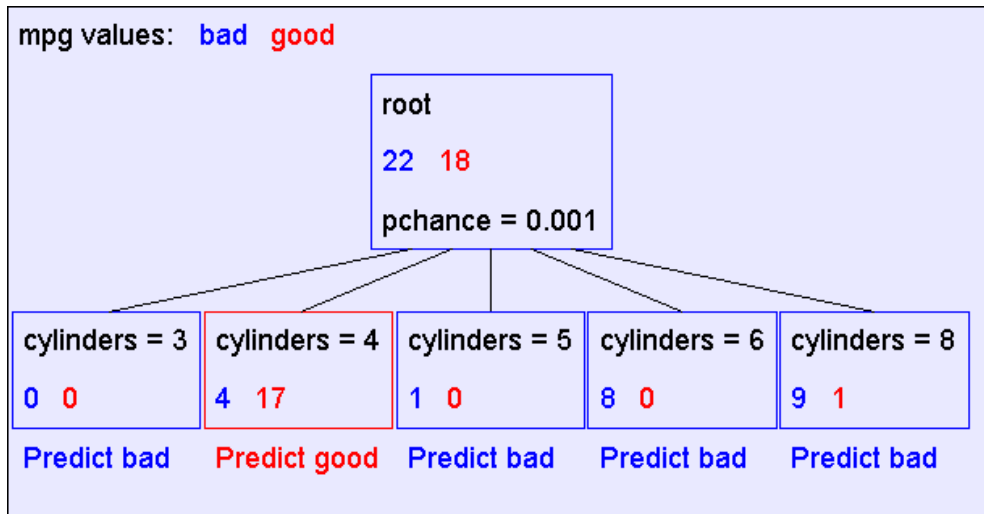
Using Chi-squared to avoid overfitting

- Build the full decision tree as before.
- But when you can grow it no more, start to prune:
 - Beginning at the bottom of the tree, delete splits in which $p_{\text{chance}} > \text{MaxPchance}$.
 - Continue working your way up until there are no more prunable nodes.

MaxPchance is a magic parameter you must specify to the decision tree, indicating your willingness to risk fitting noise.

Pruning example

- With $\text{MaxPchance} = 0.1$, you will see the following MPG decision tree:



Note the improved test set accuracy compared with the unpruned tree

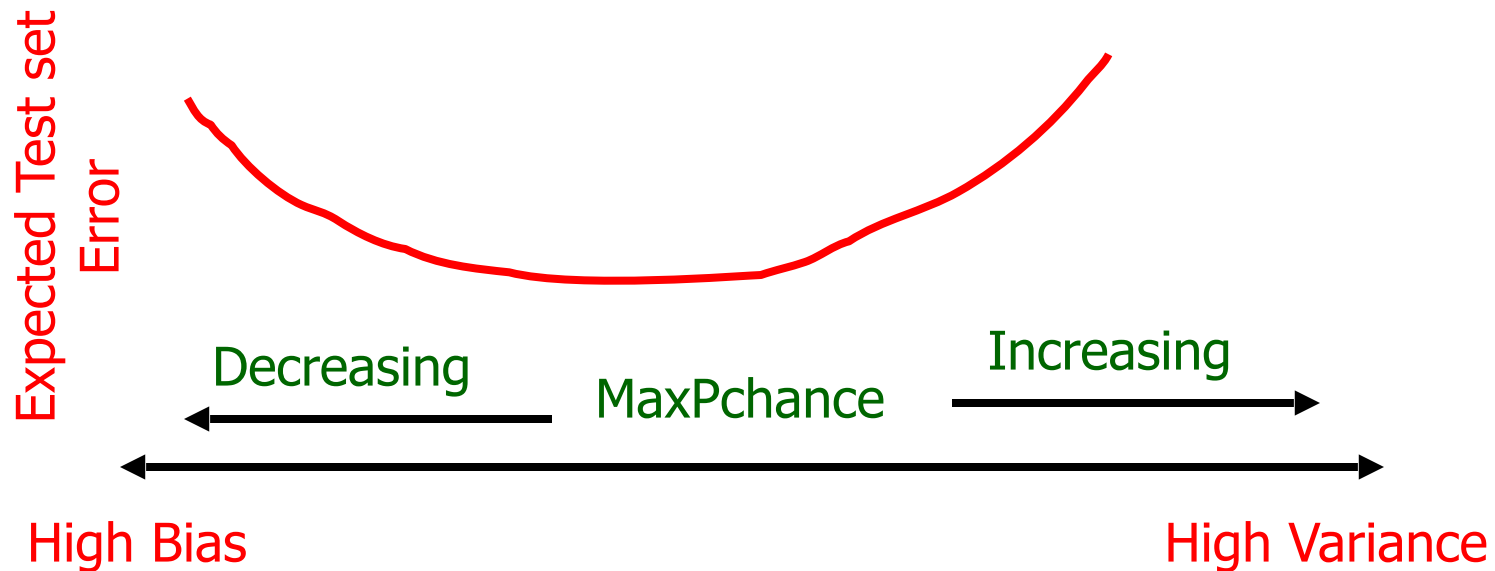
	Num Errors	Set Size	Percent Wrong
Training Set	5	40	12.50
Test Set	56	352	15.91

MaxPchance

- **Good news:** The decision tree can automatically adjust its pruning decisions according to the amount of apparent noise and data.
- **Bad news:** The user must come up with a good value of MaxPchance. (Note, Andrew usually uses 0.05, which is his favorite value for any magic parameter).
- **Good news:** But with extra work, the best MaxPchance value can be estimated automatically by a technique called cross-validation.

MaxPchance

- Technical note (dealt with in other lectures): MaxPchance is a regularization parameter.



The simplest tree

- Note that this pruning is heuristically trying to find
The simplest tree structure for which all within-leaf-node disagreements can be explained by chance
- This is not the same as saying “the simplest classification scheme for which...”
- Decision trees are biased to prefer classifiers that can be expressed as trees.

Expressiveness of Decision Trees

- Assume all inputs are Boolean and all outputs are Boolean.
- What is the class of Boolean functions that are possible to represent by decision trees?
- Answer: All Boolean functions.

Simple proof:

1. Take any Boolean function
2. Convert it into a truth table
3. Construct a decision tree in which each row of the truth table corresponds to one path through the decision tree.

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Example Decision Trees

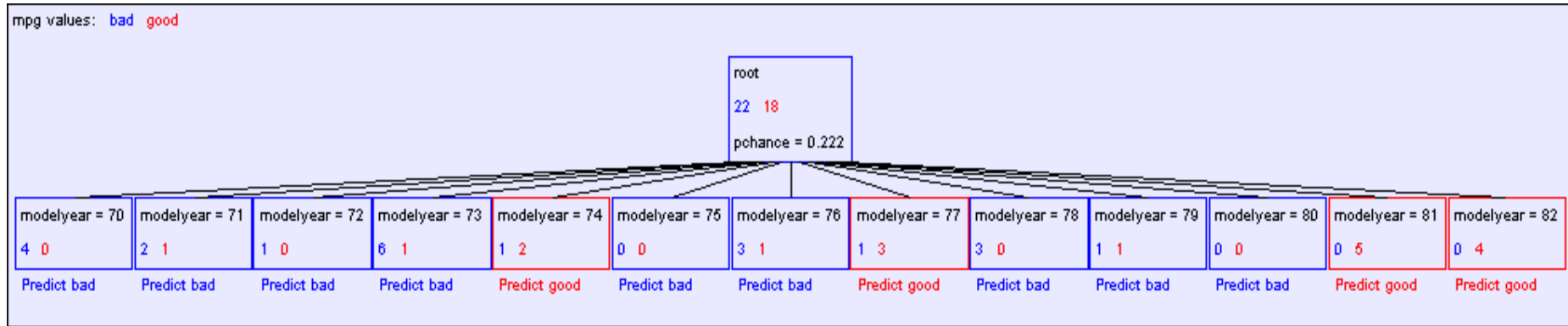
Real-Valued inputs

- What should we do if some of the inputs are real-valued?

mpg	cylinders	displacemen	horsepower	weight	acceleration	modelyear	maker
good	4	97	75	2265	18.2	77	asia
bad	6	199	90	2648	15	70	america
bad	4	121	110	2600	12.8	77	europa
bad	8	350	175	4100	13	73	america
bad	6	198	95	3102	16.5	74	america
bad	4	108	94	2379	16.5	73	asia
bad	4	113	95	2228	14	71	asia
bad	8	302	139	3570	12.8	78	america
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
:	:	:	:	:	:	:	:
good	4	120	79	2625	18.6	82	america
bad	8	455	225	4425	10	70	america
good	4	107	86	2464	15.5	76	europa
bad	5	131	103	2830	15.9	78	europa

Idea One: Branch on each possible real value

“One branch for each numeric value” idea:



Hopeless: with such high branching factor will shatter the dataset and over fit

Note pchance is 0.222 in the above...if MaxPchance was 0.05 that would end up pruning away to a single root node.

A better idea: thresholded splits

- Suppose X is real valued.
- Define $IG(Y|X:t)$ as $H(Y) - H(Y|X:t)$
- Define $H(Y|X:t) =$
$$H(Y|X < t) P(X < t) + H(Y|X \geq t) P(X \geq t)$$
 - $IG(Y|X:t)$ is the information gain for predicting Y if all you know is whether X is greater than or less than t
- Then define $IG^*(Y|X) = \max_t IG(Y|X:t)$
- For each real-valued attribute, use $IG^*(Y|X)$ for assessing its suitability as a split

Computational Issues

- You can compute $IG^*(Y|X)$ in time

$$R \log R + 2 R n_y$$

- Where
















R is the number of records in the node under consideration
 n_y is the arity (number of distinct values of) Y

How?

Sort records according to increasing values of X . Then create a $2 \times n_y$ contingency table corresponding to computation of $IG(Y|X:x_{\min})$. Then iterate through the records, testing for each threshold between adjacent values of X , incrementally updating the contingency table as you go. For a minor additional speedup, only test between values of Y that differ.

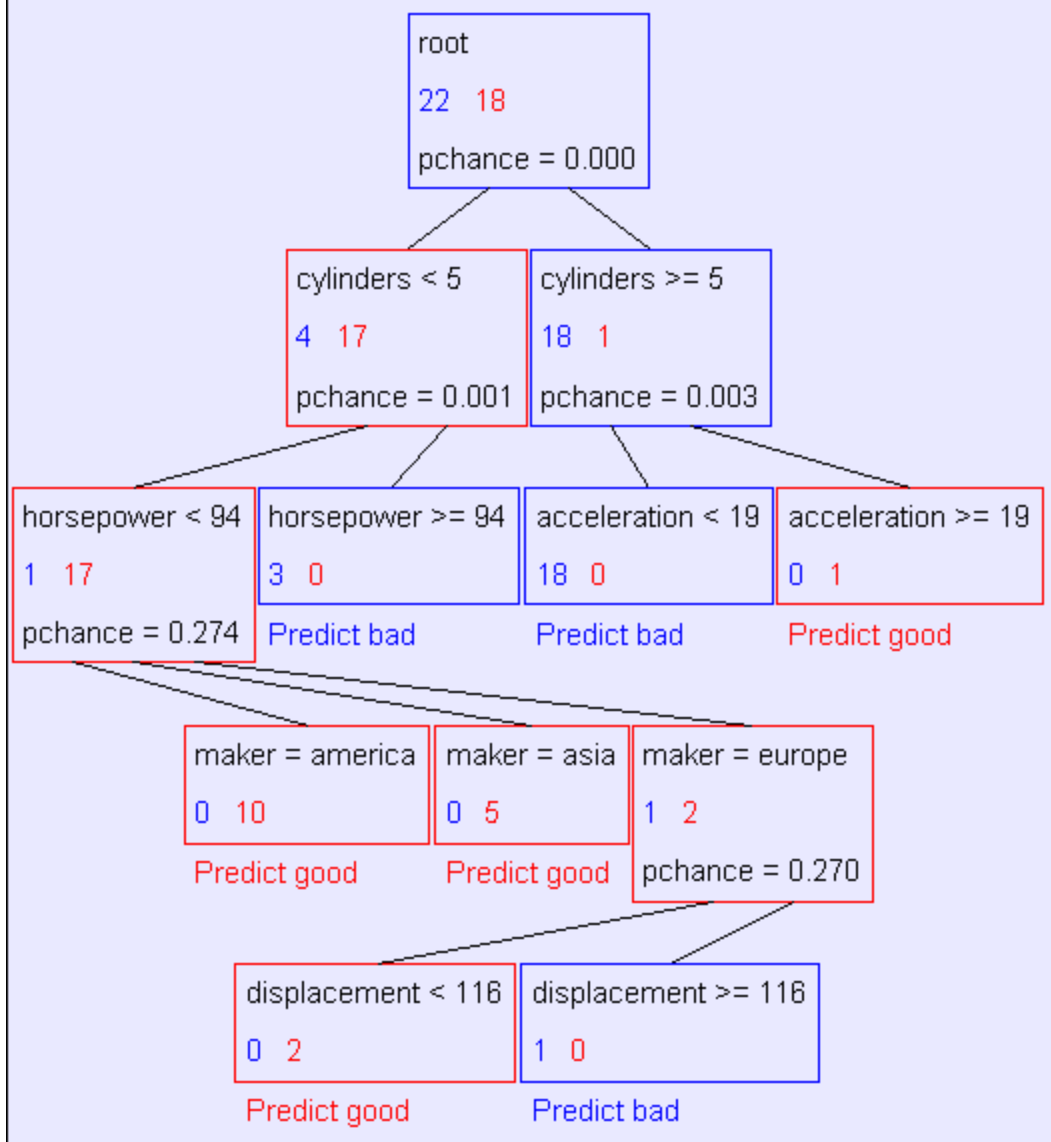
Information gains using the training set (40 records)

mpg values: bad good

Input	Value	Distribution	Info Gain
cylinders	< 5		0.48268
	>= 5		
displacement	< 198		0.428205
	>= 198		
horsepower	< 94		0.48268
	>= 94		
weight	< 2789		0.379471
	>= 2789		
acceleration	< 18.2		0.159982
	>= 18.2		
modelyear	< 81		0.319193
	>= 81		
maker	america		0.0437265
	asia		
	europe		

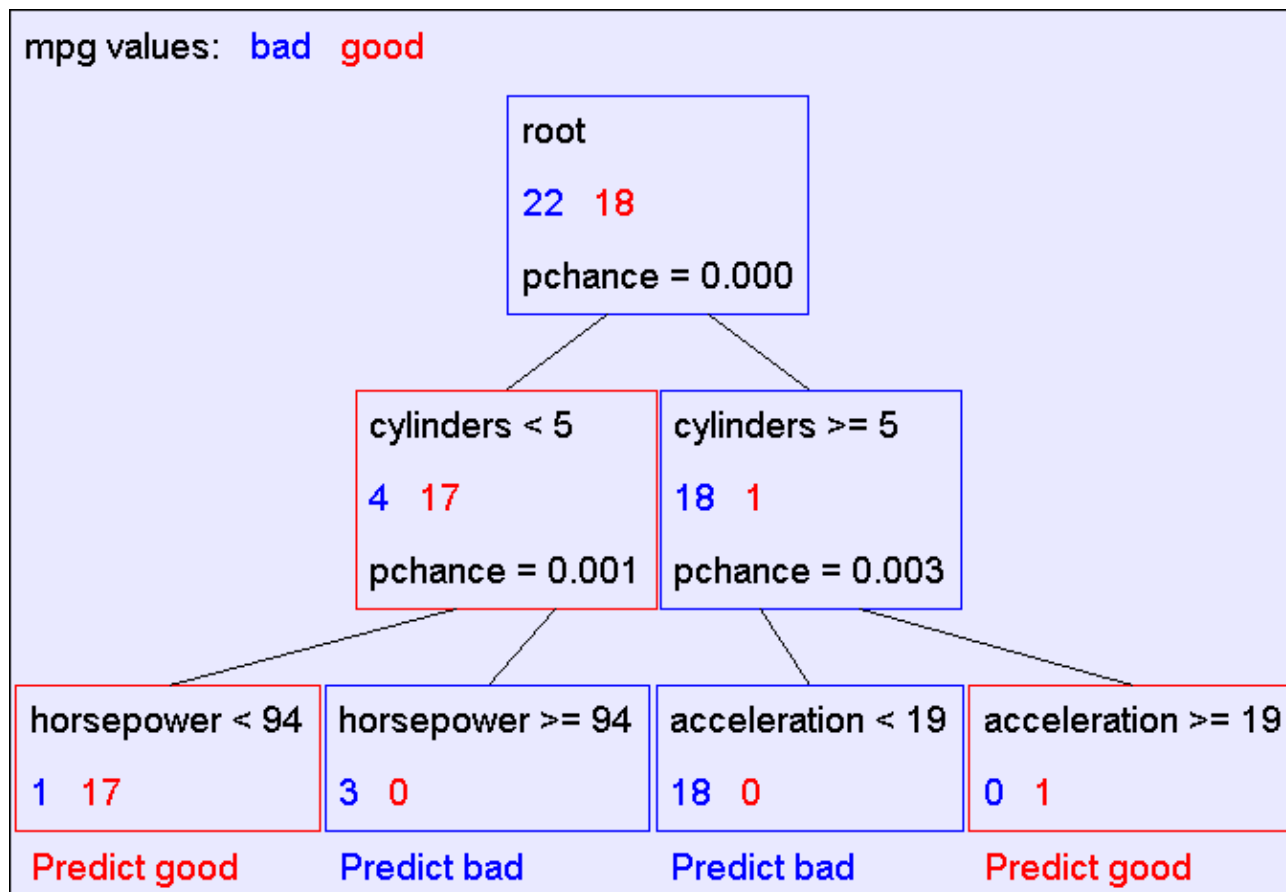
Example with MPG

mpg values: bad good



Unpruned tree using reals

Pruned tree using reals



	Num Errors	Set Size	Percent Wrong
Training Set	1	40	2.50
Test Set	53	352	15.06

LearnUnprunedTree(X,Y)

Input: X a matrix of R rows and M columns where X_{ij} = the value of the j'th attribute in the i'th input datapoint. Each column consists of either all real values or all categorical values.

Input: Y a vector of R elements, where Y_i = the output class of the i'th datapoint. The Y_i values are categorical.

Output: An Unpruned decision tree

If all records in X have identical values in all their attributes (this includes the case where $R < 2$), return a Leaf Node predicting the majority output, breaking ties randomly. This case also includes

If all values in Y are the same, return a Leaf Node predicting this value as the output

Else

For $j = 1 \dots M$

If j'th attribute is categorical

$IG_j = IG(Y|X_j)$

Else (j'th attribute is real-valued)

$IG_j = IG^*(Y|X_j)$ from about four slides back

Let $j^* = \operatorname{argmax}_j IG_j$ (this is the splitting attribute we'll use)

If j^* is categorical then

For each value v of the j'th attribute

Let X^v = subset of rows of X in which $X_{ij} = v$. Let Y^v = corresponding subset of Y

Let $Child^v = \text{LearnUnprunedTree}(X^v, Y^v)$

Return a decision tree node, splitting on j'th attribute. The number of children equals the number of values of the j'th attribute, and the v'th child is $Child^v$

Else j^* is real-valued and let t be the best split threshold

Let X^{LO} = subset of rows of X in which $X_{ij} \leq t$. Let Y^{LO} = corresponding subset of Y

Let $Child^{LO} = \text{LearnUnprunedTree}(X^{LO}, Y^{LO})$

Let X^{HI} = subset of rows of X in which $X_{ij} > t$. Let Y^{HI} = corresponding subset of Y

Let $Child^{HI} = \text{LearnUnprunedTree}(X^{HI}, Y^{HI})$

Return a decision tree node, splitting on j'th attribute. It has two children corresponding to whether the j'th attribute is above or below the given threshold.

LearnUnprunedTree(X,Y)

Input: X a matrix of R rows and M columns where X_{ij} = the value of the j'th attribute in the i'th input datapoint. Each column consists of either all real-valued values or all categorical values.

Input: Y a vector of R elements where Y_i = the class of the i'th input datapoint. The Y values are categorical.

Output: An Unpruned decision tree

If all records in X have identical values for all attributes, return a Leaf Node predicting the majority class.

If all values in Y are the same, return a Leaf Node.

Else

For j = 1 .. M

If j'th attribute is categorical

$IG_j = I(Y|X_j)$

Else (j'th attribute is real-valued)

$IG_j = I(Y|X_j)$

Let $j^* = \text{argmax}_j IG_j$

If j^* is categorical then

For each value v of X_{j^*}

Let X^v = subset of X with $X_{j^*} = v$

Let C^v = subset of Y corresponding to X^v

Return a decision tree with children C^v

values of Y

Else j^* is real-valued

Let $X^{LO} = \{x \in X \mid x_{j^*} \leq \text{median}(X_{j^*})\}$

Let $Child^{LO}$ = subset of Y corresponding to X^{LO}

Let $X^{HI} = \{x \in X \mid x_{j^*} > \text{median}(X_{j^*})\}$

Let $Child^{HI}$ = subset of Y corresponding to X^{HI}

Return a decision tree with children $Child^{LO}$ and $Child^{HI}$

j'th attribute

Things to note:

Below the root node, there is no point testing categorical attributes that have already been split upon further up the tree. This is because all the values of that attribute will be the same and IG must therefore be zero.

But it's worth retesting real-valued attributes, since they may have different values below the binary split, and may benefit from splitting further.

To achieve the above optimization, you should pass down through the recursion a current active set of attributes.

Pedantic detail: a third termination condition should occur if the best split attribute puts all its records in exactly one child (note that this means it and all other attributes have $IG=0$).

If $R < 2$, return a Leaf Node

subset of Y

children equals the number of

subset of Y

subset of Y

corresponding to whether the

Outline

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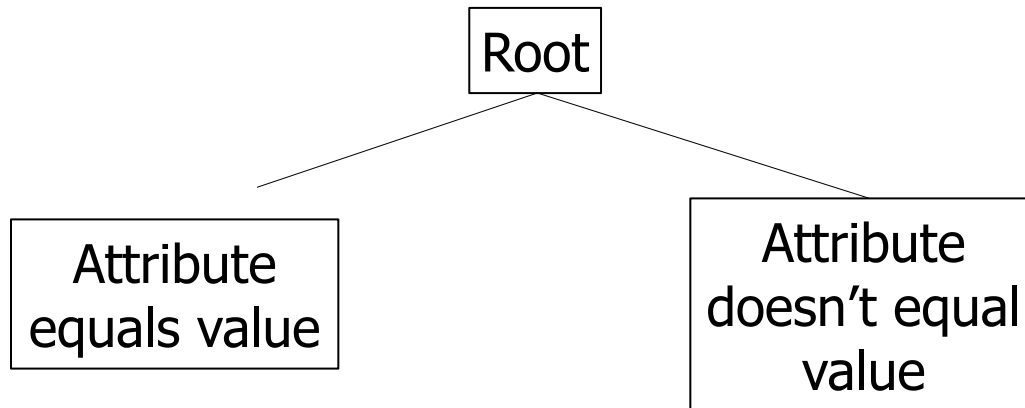
▶ Andrew's homebrewed hack: Binary Categorical Splits

Example Decision Trees

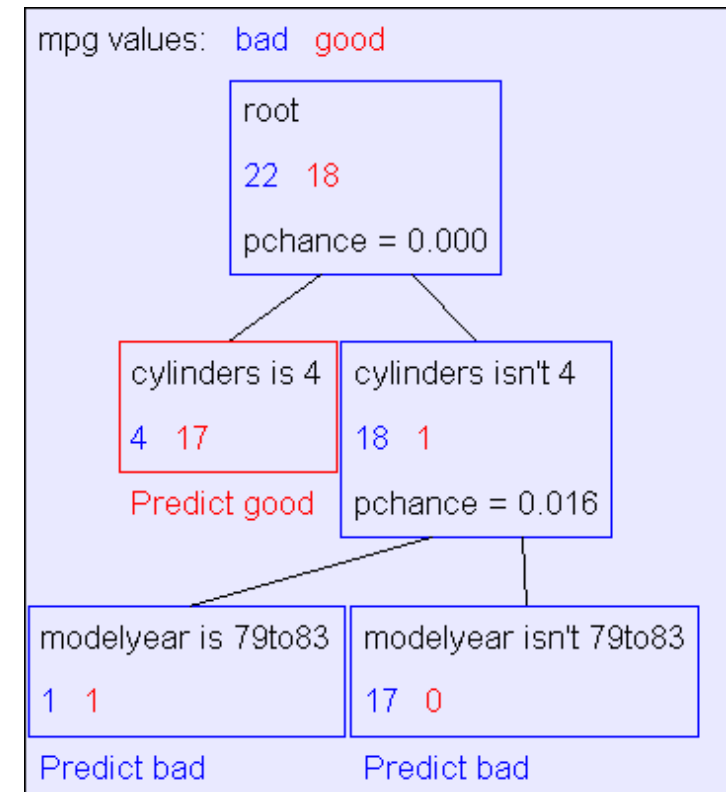
Warning: unlike what went before, this is an editorial "trick of the trade": not part of the official Decision Tree algorithm.

Binary categorical splits

- One of Andrew's favorite tricks
- Allow splits of the following form



Example:



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Test Set Error

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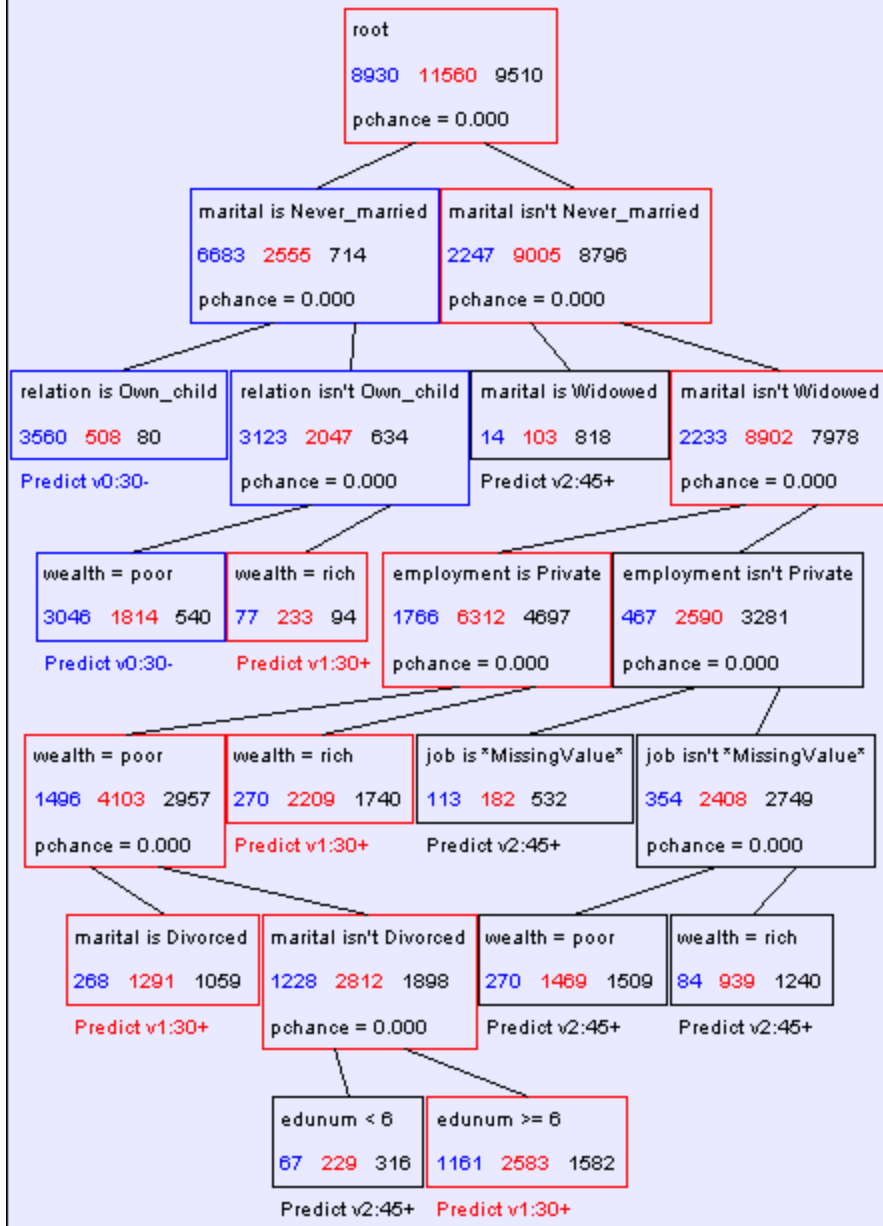
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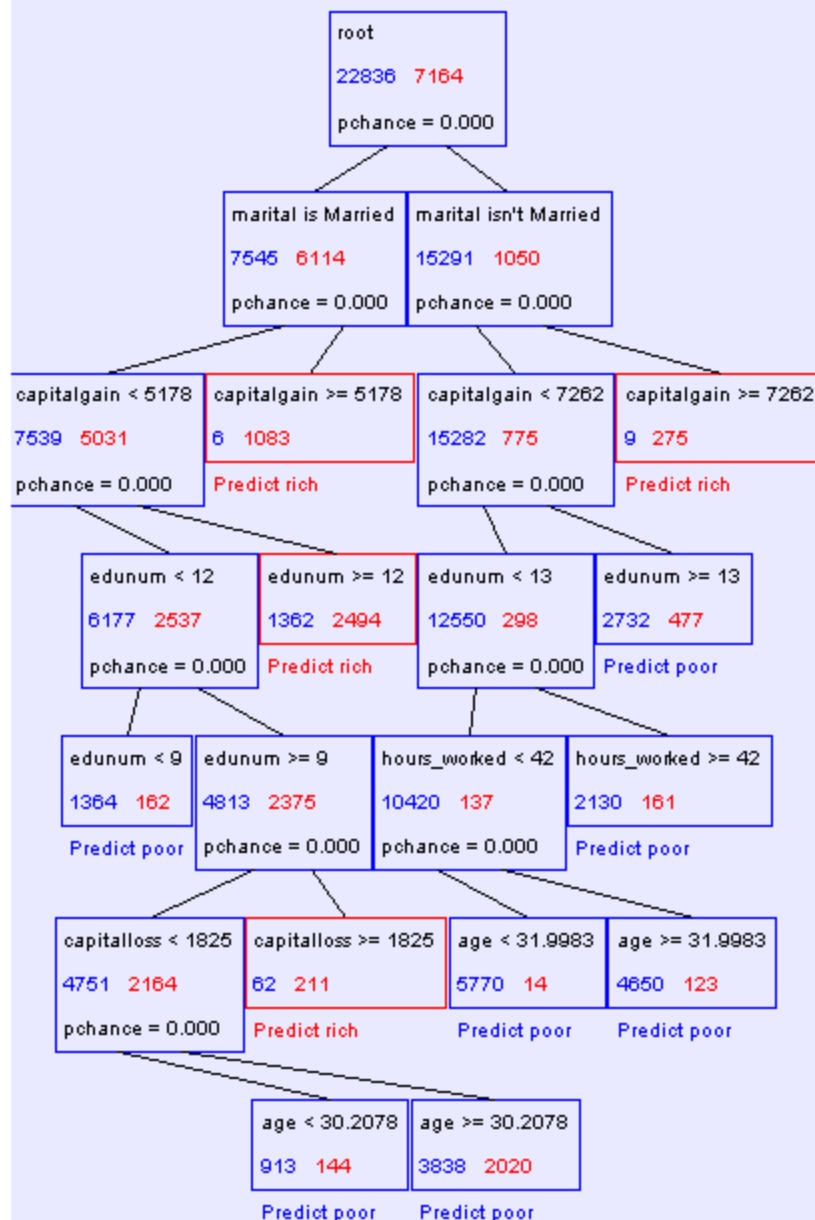
Example Decision Trees

age values: v0:30- v1:30+ v2:45+



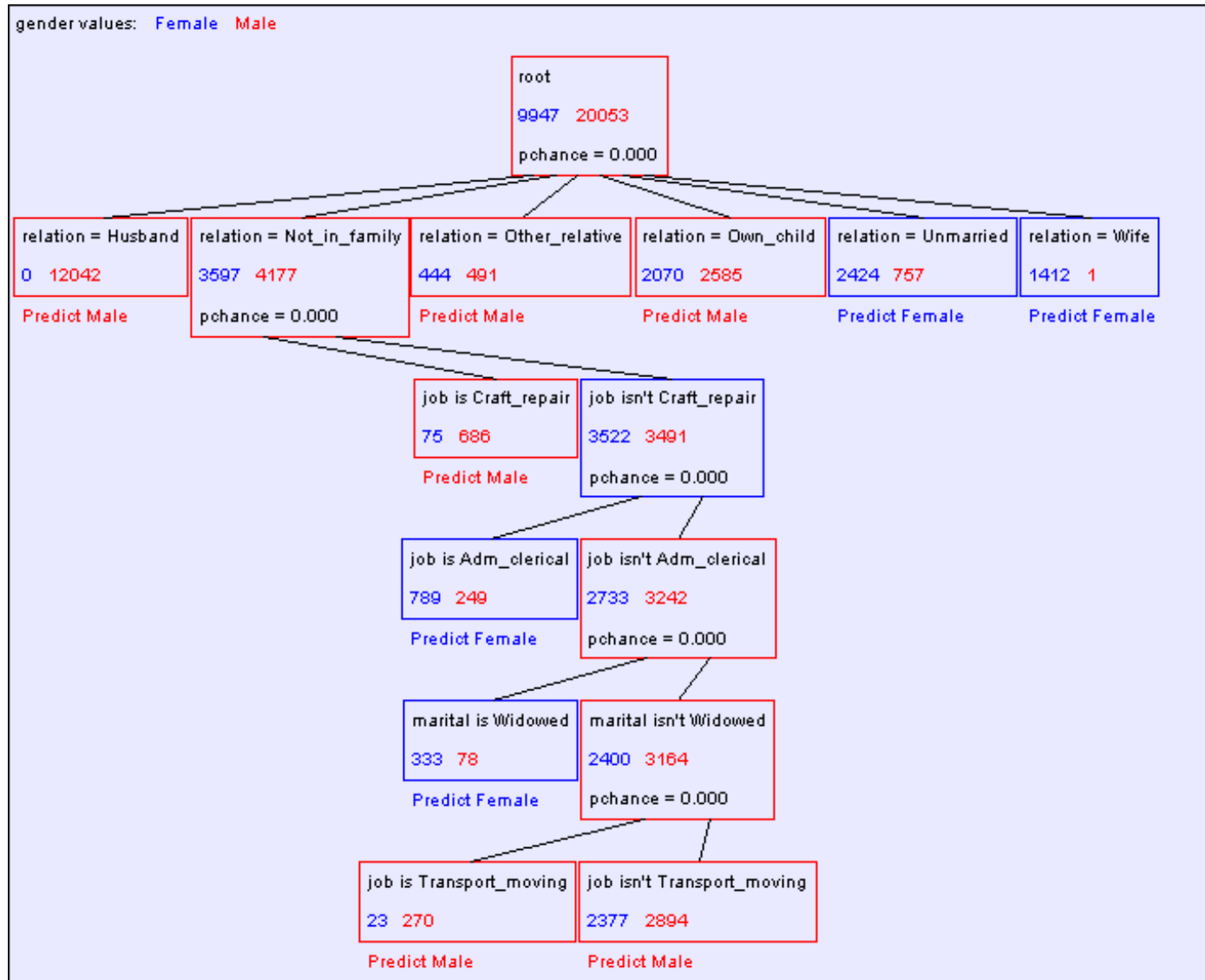
Predicting age from census

wealth values: poor rich



Predicting wealth from census

Predicting gender from census



Conclusions

- Decision trees are the single most popular data mining tool
 - Easy to understand
 - Easy to implement
 - Easy to use
 - Computationally cheap
- It's possible to get in trouble with overfitting
- They do classification: predict a categorical output from categorical and/or real inputs

What you should know

- What's a contingency table?
- What's information gain, and why we use it
- The recursive algorithm for building an unpruned decision tree
- What are training and test set errors
- Why test set errors can be bigger than training set
- Why pruning can reduce test set error
- How to exploit real-valued inputs

What we haven't discussed

- It's easy to have real-valued outputs too---these are called Regression Trees*
- Bayesian Decision Trees can take a different approach to preventing overfitting
- Computational complexity (straightforward and cheap) *
- Alternatives to Information Gain for splitting nodes
- How to choose MaxPchance automatically *
- The details of Chi-Squared testing *
- Boosting---a simple way to improve accuracy *

lectures

* = discussed in other Andrew

For more information

- Two nice books
 - L. Breiman, J. H. Friedman, R. A. Olshen, and C. J. Stone. Classification and Regression Trees. Wadsworth, Belmont, CA, 1984.
 - C4.5 : Programs for Machine Learning (Morgan Kaufmann Series in Machine Learning) by J. Ross Quinlan
- Dozens of nice papers, including
 - Learning Classification Trees, Wray Buntine, Statistics and Computation (1992), Vol 2, pages 63-73
 - Kearns and Mansour, On the Boosting Ability of Top-Down Decision Tree Learning Algorithms, STOC: ACM Symposium on Theory of Computing, 1996"
- Dozens of software implementations available on the web for free and commercially for prices ranging between \$50 - \$300,000

Discussion

- Instead of using information gain, why not choose the splitting attribute to be the one with the highest prediction accuracy?
- Instead of greedily, heuristically, building the tree, why not do a combinatorial search for the optimal tree?
- If you build a decision tree to predict wealth, and marital status, age and gender are chosen as attributes near the top of the tree, is it reasonable to conclude that those three inputs are the major causes of wealth?
- ..would it be reasonable to assume that attributes not mentioned in the tree are not causes of wealth?
- ..would it be reasonable to assume that attributes not mentioned in the tree are not correlated with wealth?