Mandatory hand-in 5 ITDMAT

It can be made by a maximum of two students. It must be accepted (along with the other hand-ins) in order to enrol for the exam. It must be digitally handed in, in a pdf format.

Solve the following sequence exercises:

S1: Let $q_n = 2q_{n-1} + 2n + 5$, and $q_0 = 4$. Compute q_1, q_2, q_3 and q_4. (Almost problem 6.2)

S2: Define a sequence $\{x_n\}$ by $x_0 = 1$, and $x_n = 2x_{n-1}\ if\ n \geq 1$. Find a closed-form for the n^{th} term of this sequence. Prove that your solution is correct. (Problem 6.5)

Solve the following induction exercises:

R1: Use mathematical induction to prove that

$$1 + 5 + 9 + \ldots + (4n - 3) = 2n^2 - n$$

for every positive integer n.

R2: Prove that $2^n > n^3$ for every integer $n \geq 10$. Hint: you will need to really work with inequalities.

R3: Using mathematical induction, prove that $n! > n^3$ for $n \geq 6$.

R4: Suppose that we have two non-empty piles of cards each containing *n* cards. Two players play a game as follows. Each player, in turn, chooses one pile and then removes any number of cards, but at least one, from the chosen pile. The player who removes the last card on the table wins the game. Show (using strong induction) that the second player can always win the game.

R5: Use the Strong Principle of Mathematical Induction to prove the following:

If d and n are integers, d > 0, there exists integers Q (quotient) and r (remainder) satisfying

$$n = dq + r \qquad 0 \leq r < d.$$

Furthermore, q and r are unique; that is, if

$$n = dq_1 + r_1 \qquad 0 \leq r_1 < d.$$

and

$$n = dq_2 + r_2 \qquad 0 \leq r_2 < d.$$

then $q_1 = q_2$ and $r_1 = r_2$.