GENERAL ASSEMBLY

INTRO TO DATA SCIENCE

LECTURE 6: REGRESSION & REGULARIZATION

LAST TIME:

- INTRO TO MACHINE LEARNING
- SUPERVISED LEARNING

QUESTIONS?

I. REVIEW SUPERVISED LEARNING

II. LINEAR REGRESSION

III. REGULARIZATION

I. SUPERVISED LEARNING

Q: How does a classification problem work?

A: Data in, predicted labels out.

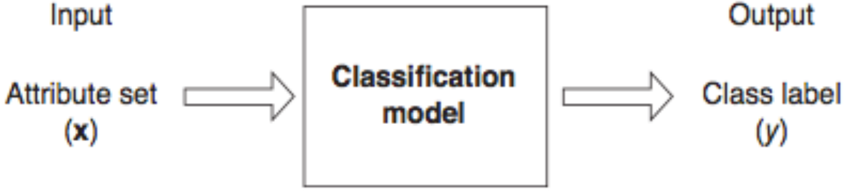
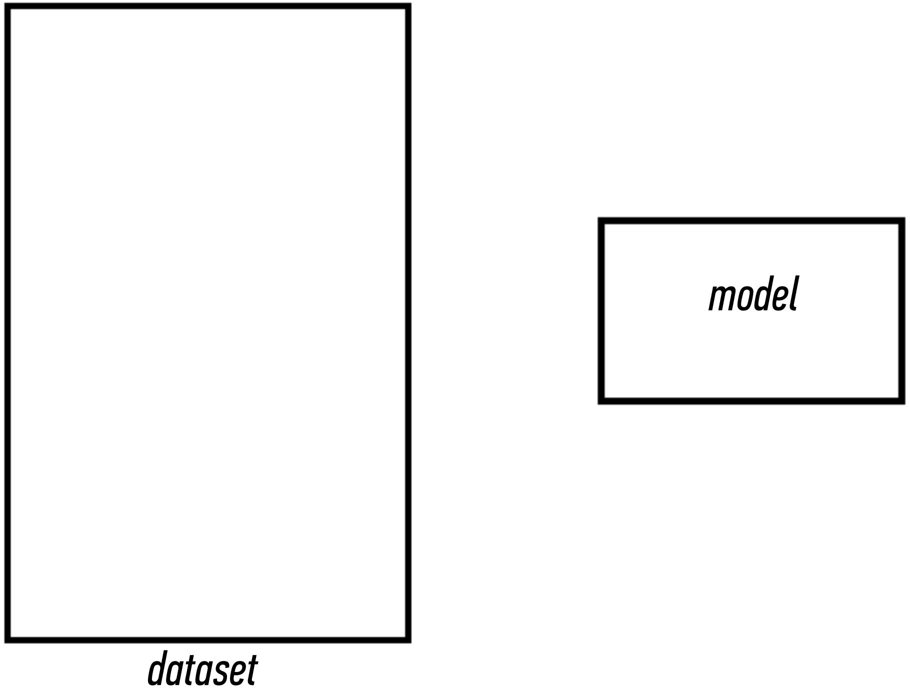


Figure 4.2. Classification as the task of mapping an input attribute set $\mathbf{x}$ into its class label y.

source: http://www-users.cs.umn.edu/~kumar/dmbook/ch4.pdf

Q: What steps does a classification problem require?



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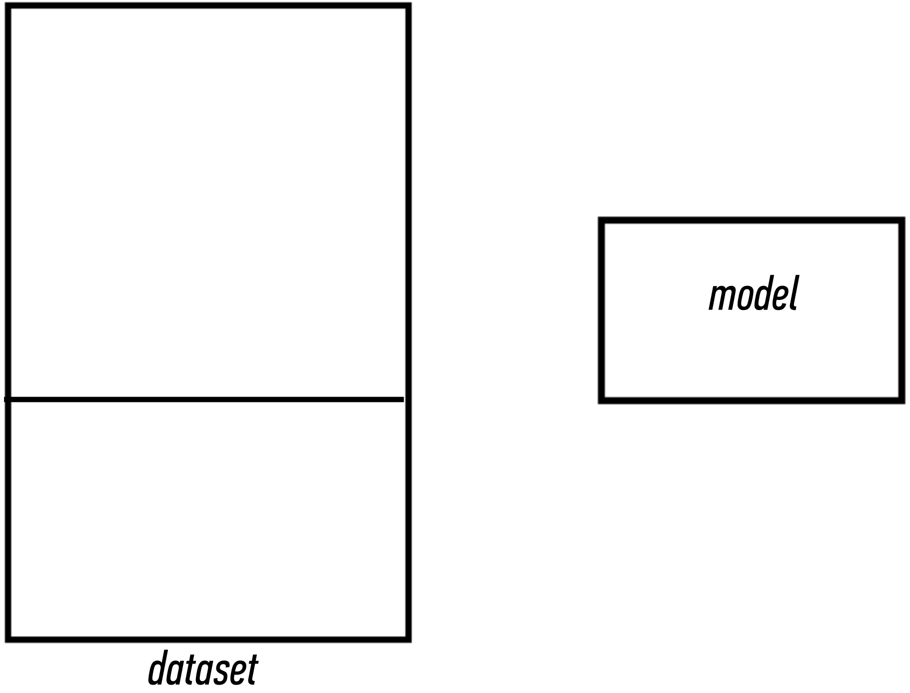
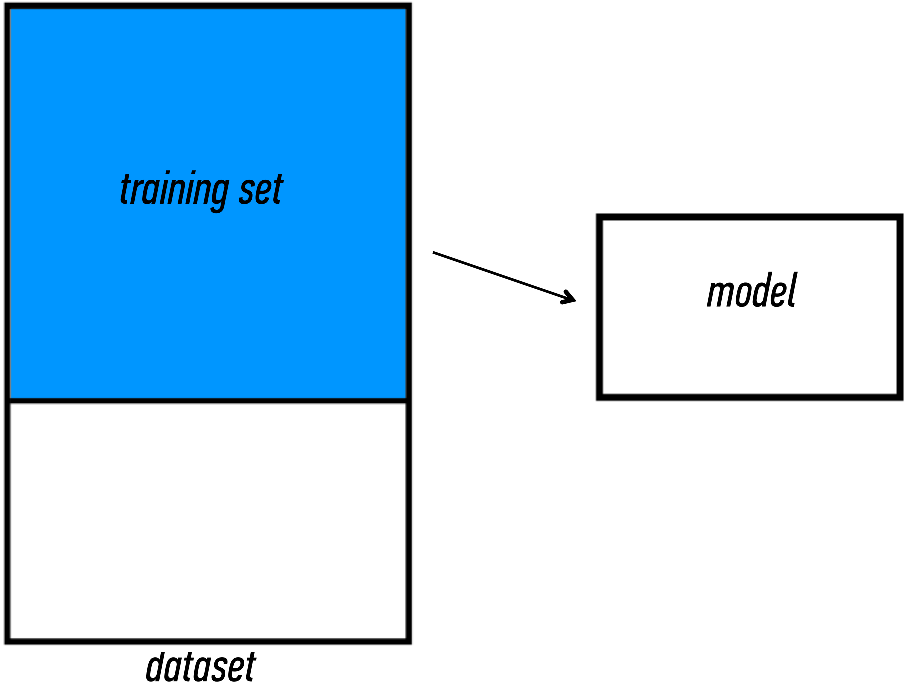


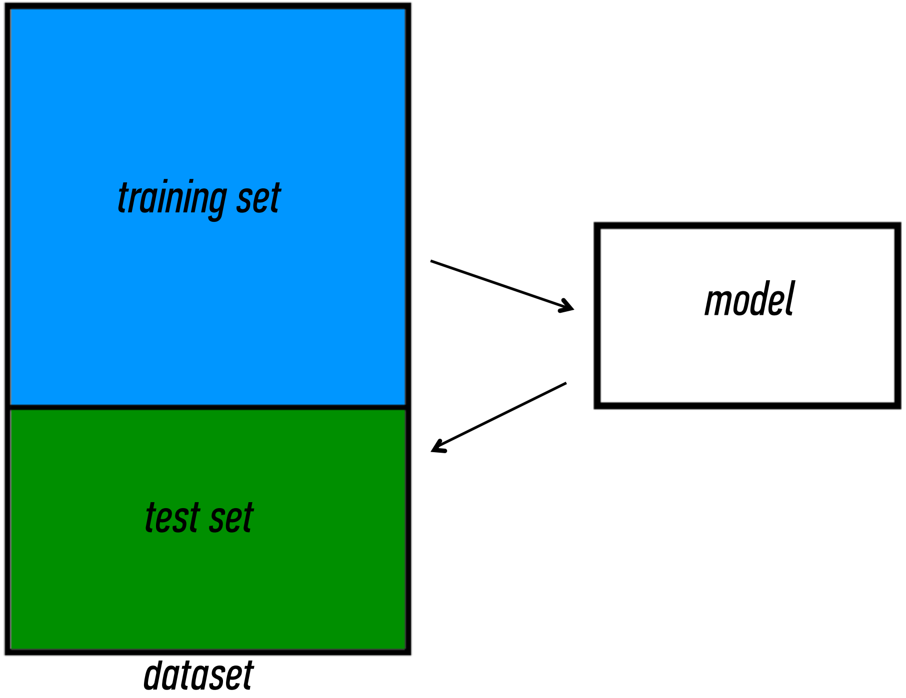
Q: What steps does a classification problem require?

1) *split dataset*
2) *train model*



Q: What steps does a classification problem require?

1) *split dataset*
2) *train model*
3) *test model*



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1) *split dataset*
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3) *test model*
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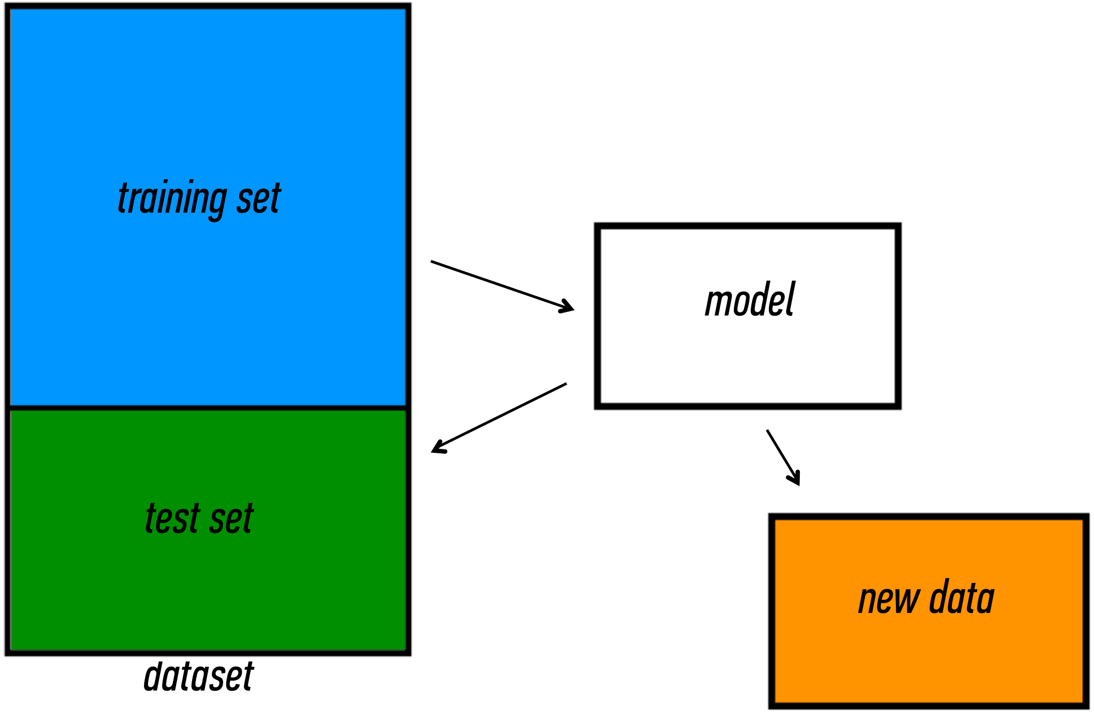

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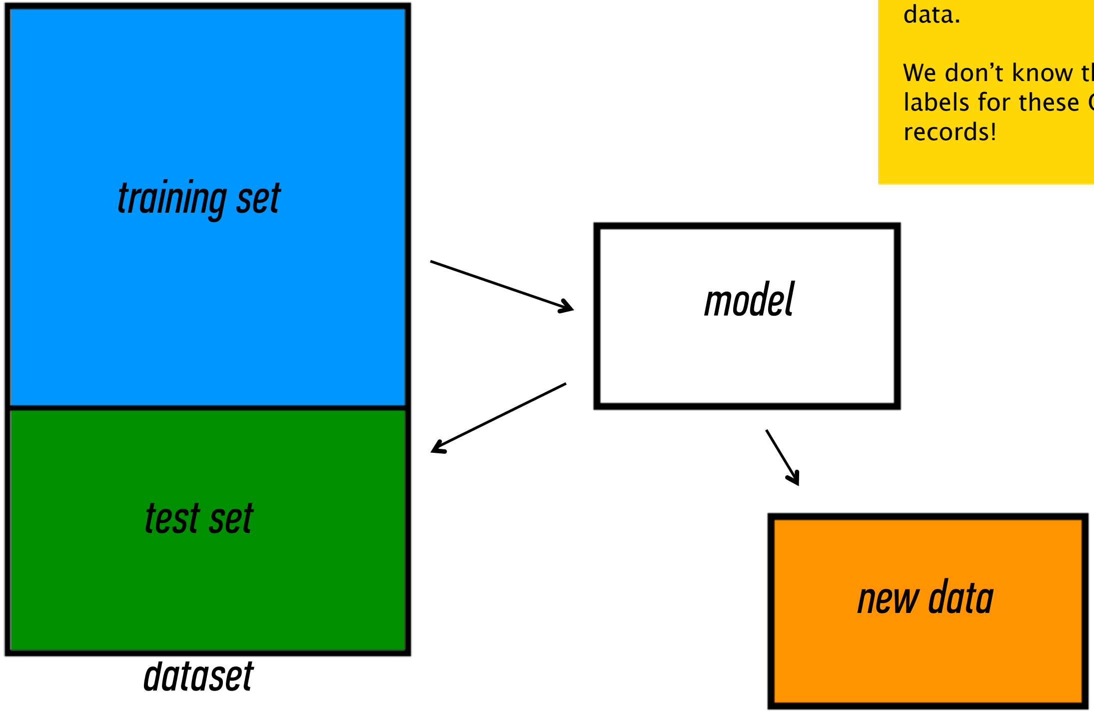
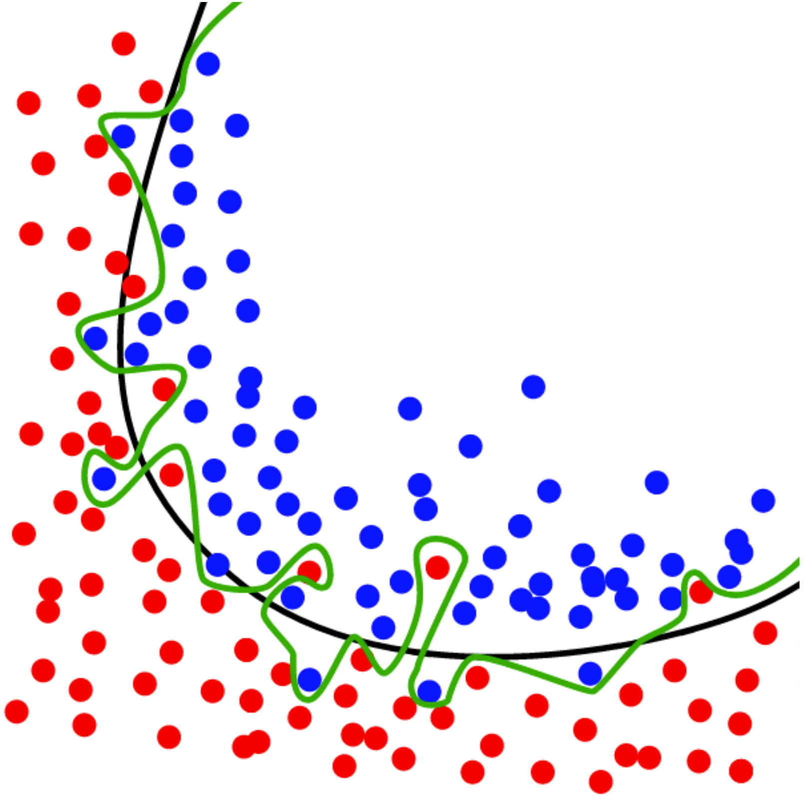


NOTE
This new data is called out of sample data.

We don't know the labels for these OOS records!

OVERFITTING – EXAMPLE



source: http://www.dtreg.com

II. LINEAR REGRESSION

	continuous	categorical
supervised	???	???
unsupervised	???	???

	continuous	categorical
supervised	regression	classification
unsupervised	dimension reduction	clustering

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The **simple linear regression** *model captures a linear relationship between a single input variable* x *and a response variable* y*:*

$$y = \alpha + \beta x + \varepsilon$$

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x = **input variable** *(the one we use to train the model)*

α = **intercept** *(where the line crosses the y-axis)*

β = **regression coefficient** *(the model "parameter")*

ε = **residual** *(the prediction error)*

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$$y = \alpha + \beta_1 x_1 + \dots + \beta_n x_n + \varepsilon$$

Linear regression involves several technical assumptions and is often presented with lots of mathematical formality.

The math is not very important for our purposes, but you should check it out if you get serious about solving regression problems.

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In practice, any respectable piece of software will do this for you.

But again, if you get serious about regression, you should learn how this works!

V. POLYNOMIAL REGRESSION

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$$y = \alpha + \beta_1 x + \beta_2 x^2 + \varepsilon$$

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Q: This represents a nonlinear relationship. Is it still a linear model?

A: Yes, because it's linear in the β's!

Polynomial regression allows us to fit very complex curves to data.

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Q: Does anyone know what it is?

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But there is one problem with the model we've written down so far.

Q: Does anyone know what it is?

A: This model violates one of the assumptions of linear regression!

This model displays **multicollinearity***, which means the predictor variables are highly correlated with each other.*

$$y = \alpha + \beta_1 x + \beta_2 x^2 + \ldots + \beta_n x^n + \varepsilon$$

```
> x <- seq(1, 10, 0.1)
> cor(x^9, x^10)
[1] 0.9987608
```

This model displays **multicollinearity***, which means the predictor variables are highly correlated with each other.*

$$y = \alpha + \beta_1 x + \beta_2 x^2 + \dots + \beta_n x^n + \varepsilon$$

Multicollinearity causes the linear regression model to break down, because it can't tell the predictor variables apart.

Q: What can we do about this?

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A: Replace the correlated predictors with uncorrelated predictors.

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$$y = \alpha + \beta_1 f_1(x) + \beta_2 f_2(x^2) + \ldots + \beta_n f_n(x^n) + \varepsilon$$

So far, we've seen how polynomial regression allows us to fit complex nonlinear relationships, and even to avoid multicollinearity (by using basis functions).

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Q: Can a regression model be too complex?

V. REGULARIZATION

Recall our earlier discussion of **overfitting**.

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When we talked about this in the context of classification, we said that it was a result of matching the training set too closely.

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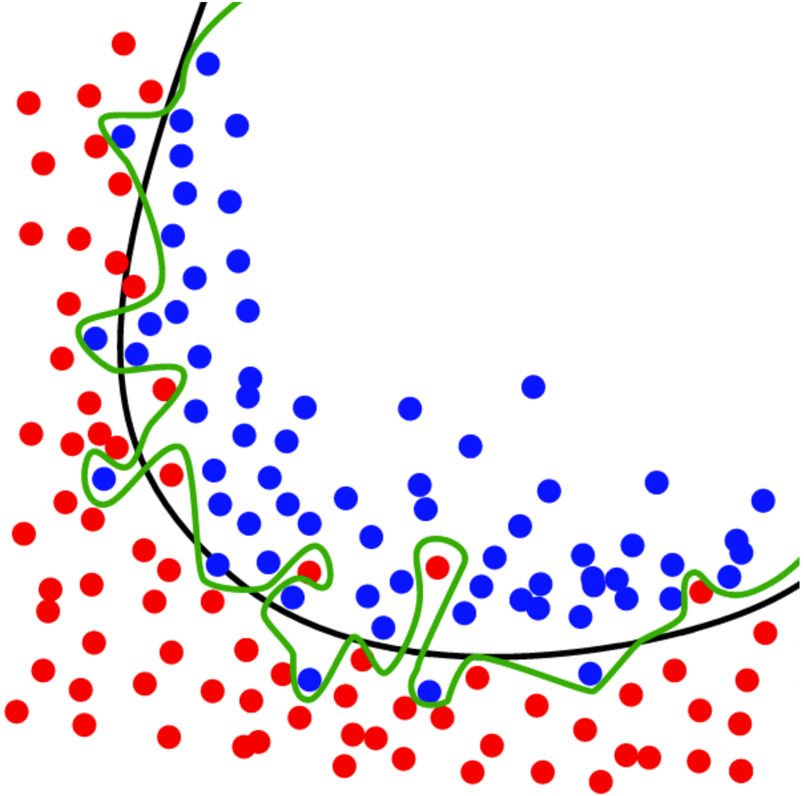
When we talked about this in the context of classification, we said that it was a result of matching the training set too closely.

In other words, an overfit model matches the **noise** *in the dataset instead of the* **signal***.*

OVERFITTING EXAMPLE (CLASSIFICATION)



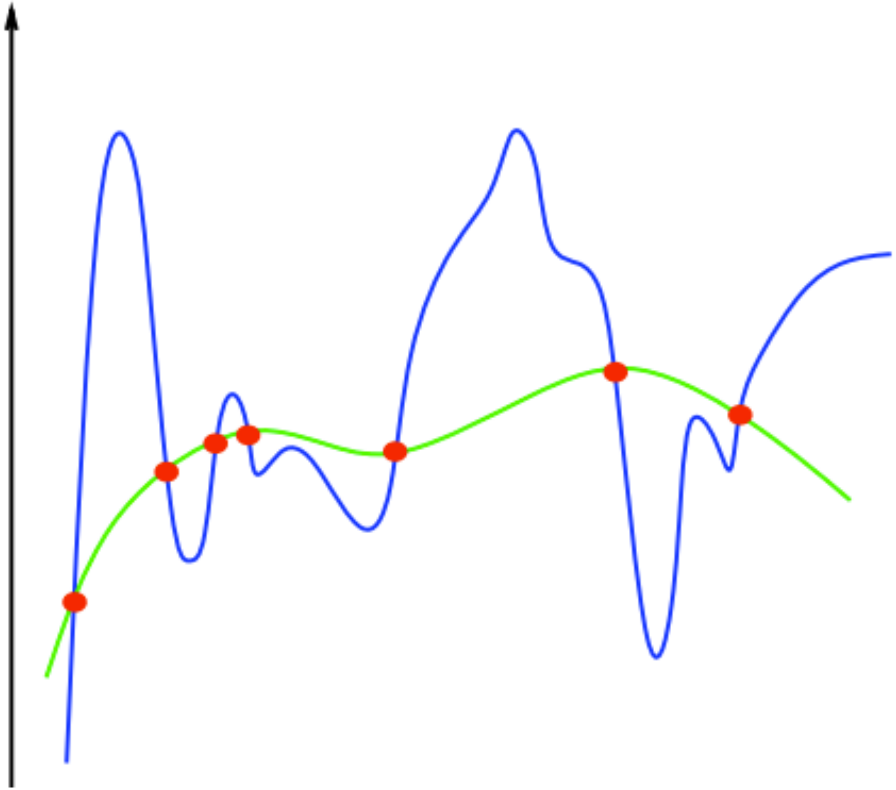
source: http://upload.wikimedia.org/wikipedia/commons/1/19/Overfitting.svg

The same thing can happen in regression.

It's possible to design a regression model that matches the noise in the data instead of the signal.

This happens when our model becomes too complex for the data to support.

OVERFITTING EXAMPLE (REGRESSION)



source: http://www.mit.edu/~9.520/spring12/slides/class02/class02.pdf

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Ex 1: $\Sigma |\beta_i|$

Ex 2: $\Sigma \beta_i^2$

Q: How do we define the **complexity** *of a regression model?*

A: One method is to define complexity as a function of the size of the coefficients.

Ex 1: $\Sigma |\beta_i|$ *this is called the* **L1-norm**

Ex 2: $\Sigma \beta_i^2$ *this is called the* **L2-norm**

These measures of complexity lead to the following **regularization** *techniques:*

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L1 regularization*:* $y = \Sigma\, \beta_i x_i + \varepsilon \quad \text{st.} \quad \Sigma\, |\beta_i| < s$

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L2 regularization*:* $y = \Sigma\, \beta_i x_i + \varepsilon \quad \text{st.} \quad \Sigma\, \beta_i^2 < s$

These measures of complexity lead to the following **regularization** *techniques:*

L1 regularization*:* $y = \sum \beta_i x_i + \varepsilon$ st. $\sum |\beta_i| < s$

L2 regularization*:* $y = \sum \beta_i x_i + \varepsilon$ st. $\sum \beta_i^2 < s$

Regularization *refers to the method of preventing* **overfitting** *by explicitly controlling model* **complexity**.

These measures of complexity lead to the following **regularization** *techniques:*

Lasso **regularization***:* $y = \Sigma\, \beta_i x_i + \varepsilon$ st. $\Sigma\, |\beta_i| < s$

Ridge **regularization***:* $y = \Sigma\, \beta_i x_i + \varepsilon$ st. $\Sigma\, \beta_i^2 < s$

Regularization *refers to the method of preventing* **overfitting** *by explicitly controlling model* **complexity***.*

Q: What problems have we seen?

A:

1) *Correlated predictor variables*
2) *Large number of parameters allow us to overfit*

Q: What can we do about this?

A: If prediction is our only goal – nothing.

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Otherwise,

1) *Drop correlated predictors*
2) *Get more data*