GENERAL ASSEMBLY

# INTRO TO DATA SCIENCE

## LECTURE 7: PROBABILITY & LOGISTIC REGRESSION

# LAST TIME:

- LINEAR REGRESSION
- REGULARIZATION

QUESTIONS?

I. REVIEW OF REGULARIZATION

II. LOGISTIC REGRESSION

*These regularization problems can also be expressed as:*

**OLS:** $\min_\beta(\|y - X\beta\|_2^2)$

**L1 regularization:** $\min_\beta(\|y - X\beta\|_2^2 + \alpha\|\beta\|_1)$

**L2 regularization:** $\min_\beta(\|y - X\beta\|_2^2 + \alpha\|\beta\|_2^2)$

*We are no longer just minimizing error but also an additional term to penalize model complexity.*

# II. LOGISTIC REGRESSION

| | continuous | categorical |
|---|---|---|
| supervised | regression | classification |
| unsupervised | dimension reduction | clustering |

*Q: What is* **logistic regression***?*

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*A: A generalization of the linear regression model to classification problems.*

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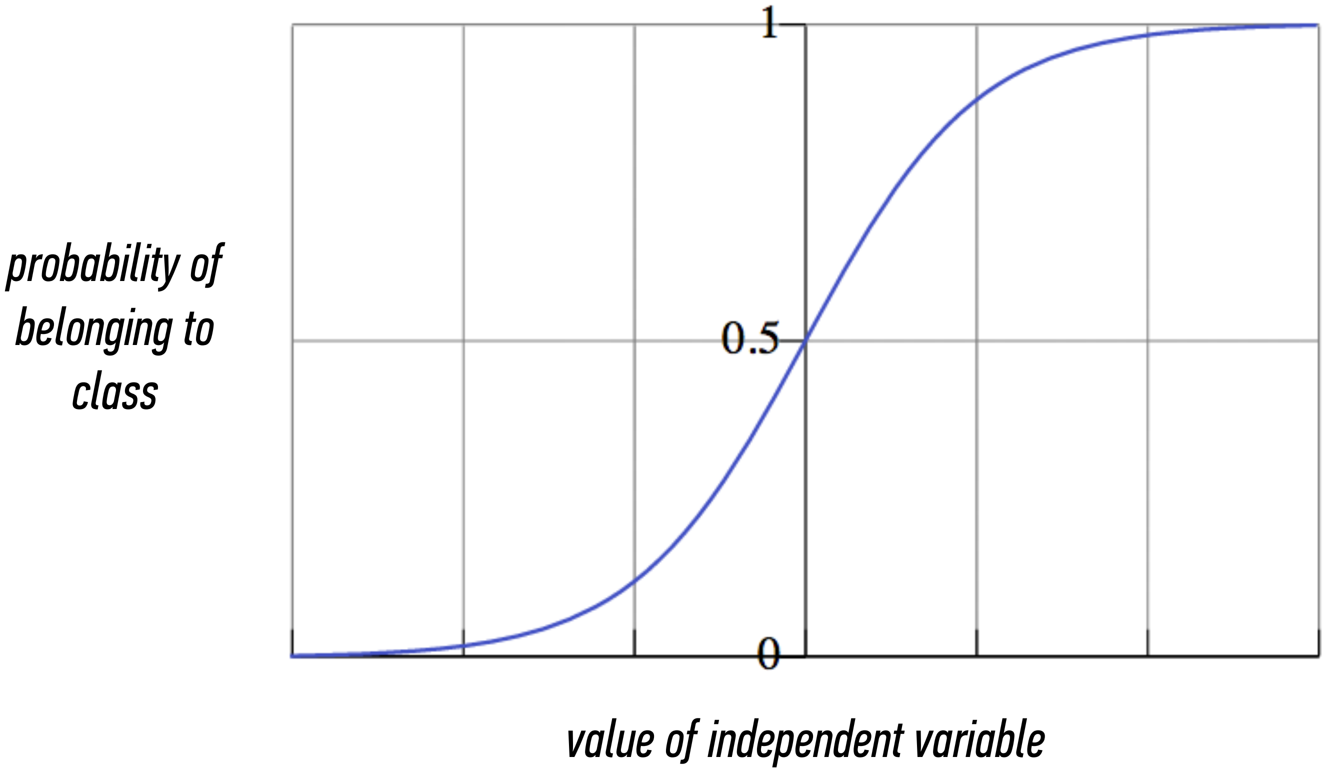
*In logistic regression, we use a set of covariates to predict probabilities of class membership.*

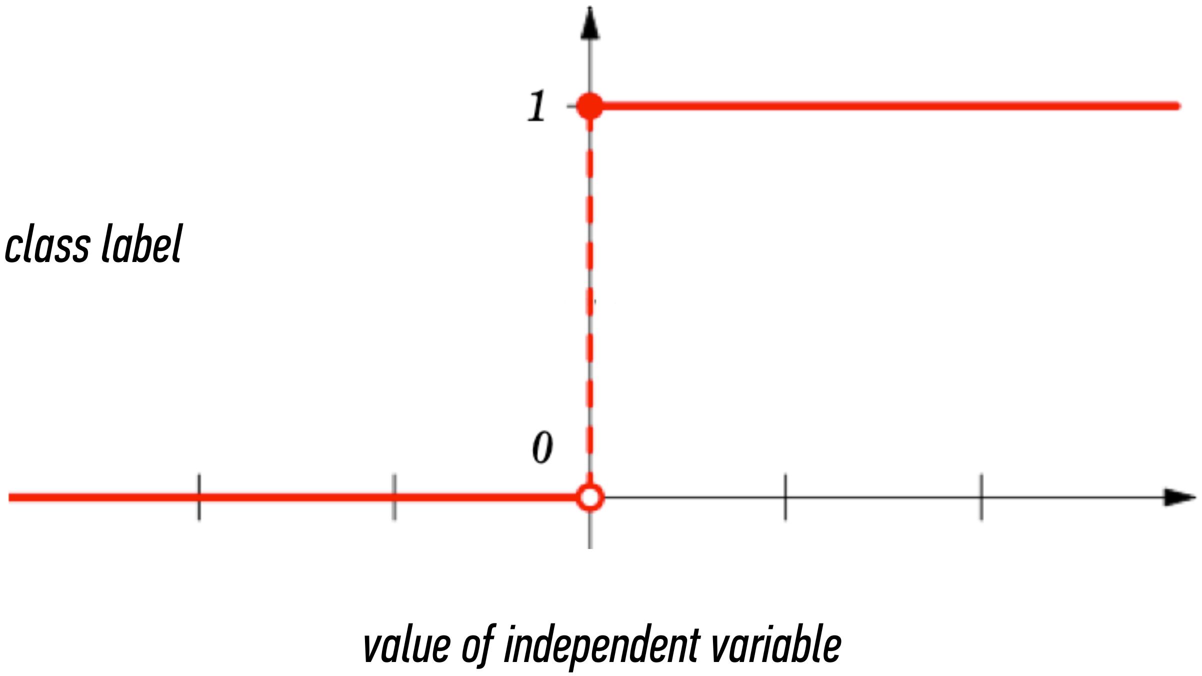
*These* ***probabilities are then mapped to class labels****, thus solving the classification problem.*




**NOTE**

Probability predictions look like this.

# CLASS LABELS




**NOTE**

Probabilities are "snapped" to class labels (eg by threshholding at 50%).

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*The main difference is in the outcome variable.*

*The key variable in any regression problem is the* **response type** *of the outcome variable* y *given the value of the covariate* x*:*

$$E(y|x)$$

*The key variable in any regression problem is the* **conditional mean** *of the outcome variable* y *given the value of the covariate* x*:*

$$E(y|x)$$

*In linear regression, we assume that this conditional mean is a linear function taking values in* $(-\infty, +\infty)$*:*

$$E(y|x) = \alpha + \beta x$$

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*The first step in extending the linear regression model to logistic regression is to map the outcome variable* $E(y|x)$ *into the unit interval.*

*Q: How do we do this?*

*A: By using a transformation called the* **logistic function***:*

$$E(y|x) = \pi(x) = \frac{e^{\alpha+\beta x}}{1+e^{\alpha+\beta x}}$$

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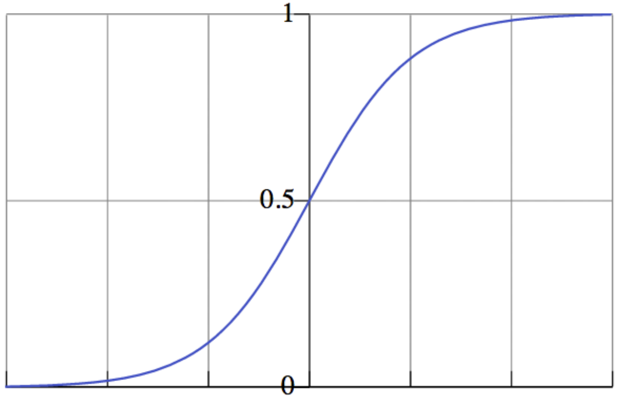

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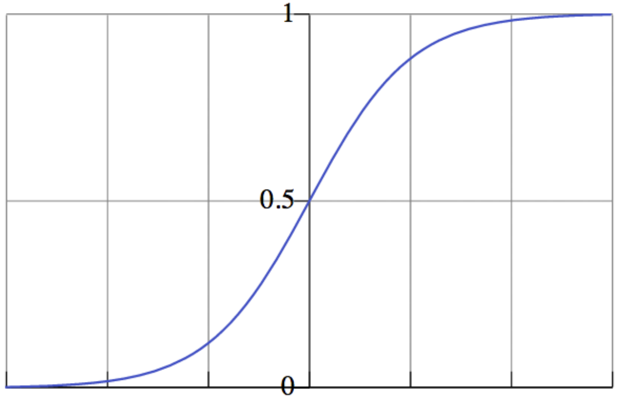


**NOTE**

For any value of x, y is in the interval [0, 1]

This is a nonlinear transformation!

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*The logit function is also called the* **log-odds function***.*