A New Dragon in the Den:
Fast Conversion From Floating-Point Numbers

CASSIO NERI

Cppcon
The C++ Conference
2024
September 15 - 20

WARNING:

This PDF poorly reproduces the slides shown at the talk.

You are strongly encourage to watch the talk, with its proper slides, when it becomes available on YouTube.

Hercules slaying the Hydra

Hercules slaying the Hydra
Louis Chéron
Oil painting, ca. 1690-1725
Victoria & Albert Museum, London, UK

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Bicho de sete cabeças.

Seven-headed beast.

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Seven-headed beast.

Medieval French Apocalypse Tapestry, Produced between 1377 and 1382.
Château d'Angers, France
Credits: Jean-Pierre Dalbéra, CC BY 2.0, via Wikimedia Commons.

Scream for me Aurora !!!

Album The Number of the Beast
Iron Maiden, 1982.

Bruce Dickinson in Brno, Czech Republic, June 8, 2014
Credits: Vaclav Salek/CTK Photo/Alamy Live News

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Bicho de sete cabeças.

Seven-headed beast.

Não é um bicho de sete cabeças.

It's not a seven-headed beast.

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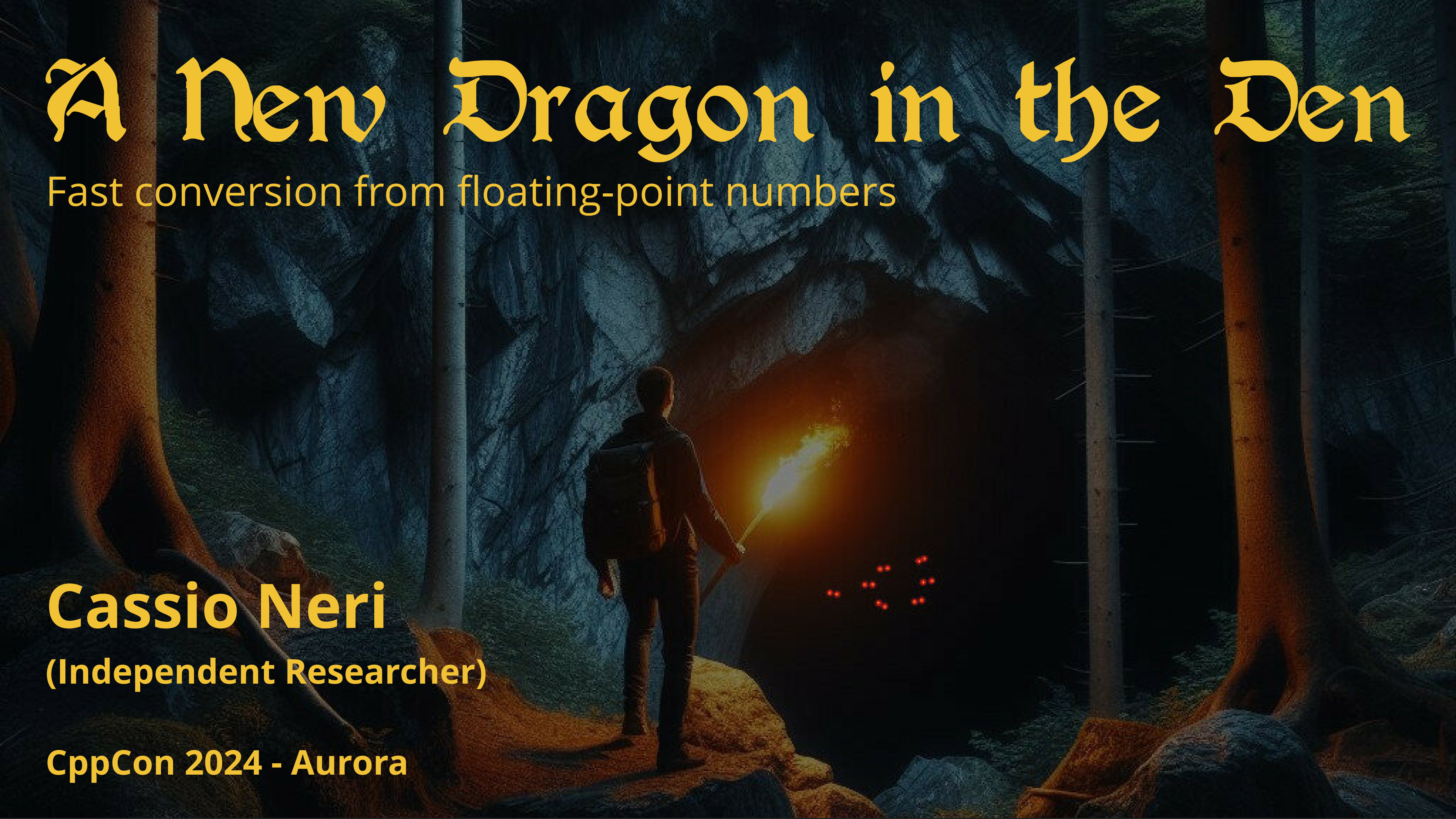
Bicho de sete cabeças.

Seven-headed beast.

Não é um bicho de sete cabeças.

It's not a seven-headed beast.

It's not rocket science.



A New Dragon in the Den

Fast conversion from floating-point numbers

Cassio Neri

(Independent Researcher)

CppCon 2024 - Aurora

Me

Me

1️⃣ Neri's equation

Annales de l'Institut Henri Poincaré C, Analyse non linéaire

Volume 21, Issue 3, May–June 2004, Pages 381-399

Statistical mechanics of the *N*-point vortex system with random intensities on a bounded domain

Cassio Neri [1]

$$\begin{cases} -\Delta u\,(x) = \left[\int\int e^{-\beta r u(y)}\,\mathrm{d}y P\,(\mathrm{d}r)\right]^{-1} \int r_1 e^{-\beta r u(x)} P\,(\mathrm{d}r)\,, & \forall x \in \Omega, \\ u\,(x) = 0, & \forall x \in \partial\Omega. \end{cases}$$

Me

1 Neri's equation

2 Neri-Schneider algorithms

Journal of: Software: Practice and Experience

RESEARCH ARTICLE | Open Access

Euclidean affine functions and their application to calendar algorithms

Cassio Neri, Lorenz Schneider

First published: 03 December 2022 | https://doi.org/10.1002/spe.3172

Me

1 Neri's equation 2 Neri-Schneider algorithms 3 Neri's hack

arXiv > cs > arXiv:1602.06426

Computer Science > Data Structures and Algorithms

[Submitted on 20 Feb 2016 (v1), last revised 19 Mar 2018 (this version, v2)]

A loopless and branchless $O(1)$ algorithm to generate the next Dyck word

Cassio Neri

```
uint64_t next_dyck_word(uint64_t w) {
    uint64_t a = w & -w;
    uint64_t b = w + a;
    uint64_t c = w ^ b;
             c = (c / a >> 2) + 1;
             c = ((c * c - 1) & 0xaaaaaaaaaaaaaaaa) | b;
    return c;
}
```

Me

1️⃣ Neri's equation 2️⃣ Neri-Schneider algorithms 3️⃣ Neri's hack

π

https://godbolt.org/z/a8qETqzbj

```cpp
string s3 = (stringstream{} << pi).str();           // 3.14159
string s4 = to_string(pi);                          // 3.141593
auto r5 = to_chars(s5.data(), s5.data() + s5.size(), pi); // 3.141592653589793
string s6 = format("{}", pi);                       // 3.141592653589793
```

π

https://godbolt.org/z/a8qETqzbj

```
string s3 = (stringstream{} << pi).str();                // 3.14159
string s4 = to_string(pi);                               // 3.141593
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string s6 = format("{}", pi);                            // 3.141592653589793
```

.NET https://godbolt.org/z/czhrT1hhh

GO https://godbolt.org/z/qh4zdzqG6

https://godbolt.org/z/8M483Kvba

JS https://godbolt.org/z/nG88rvv5f

https://godbolt.org/z/5aseWr7fz

https://godbolt.org/z/Y46WY4ooY

π

https://godbolt.org/z/a8qETqzbj

```
string s3 = (stringstream{} << pi).str();                    // 3.14159
string s4 = to_string(pi);                                   // 3.141593
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string s6 = format("{}", pi);                                // 3.141592653589793
```

.NET https://godbolt.org/z/czhrT1hhh

GO https://godbolt.org/z/qh4zdzqG6

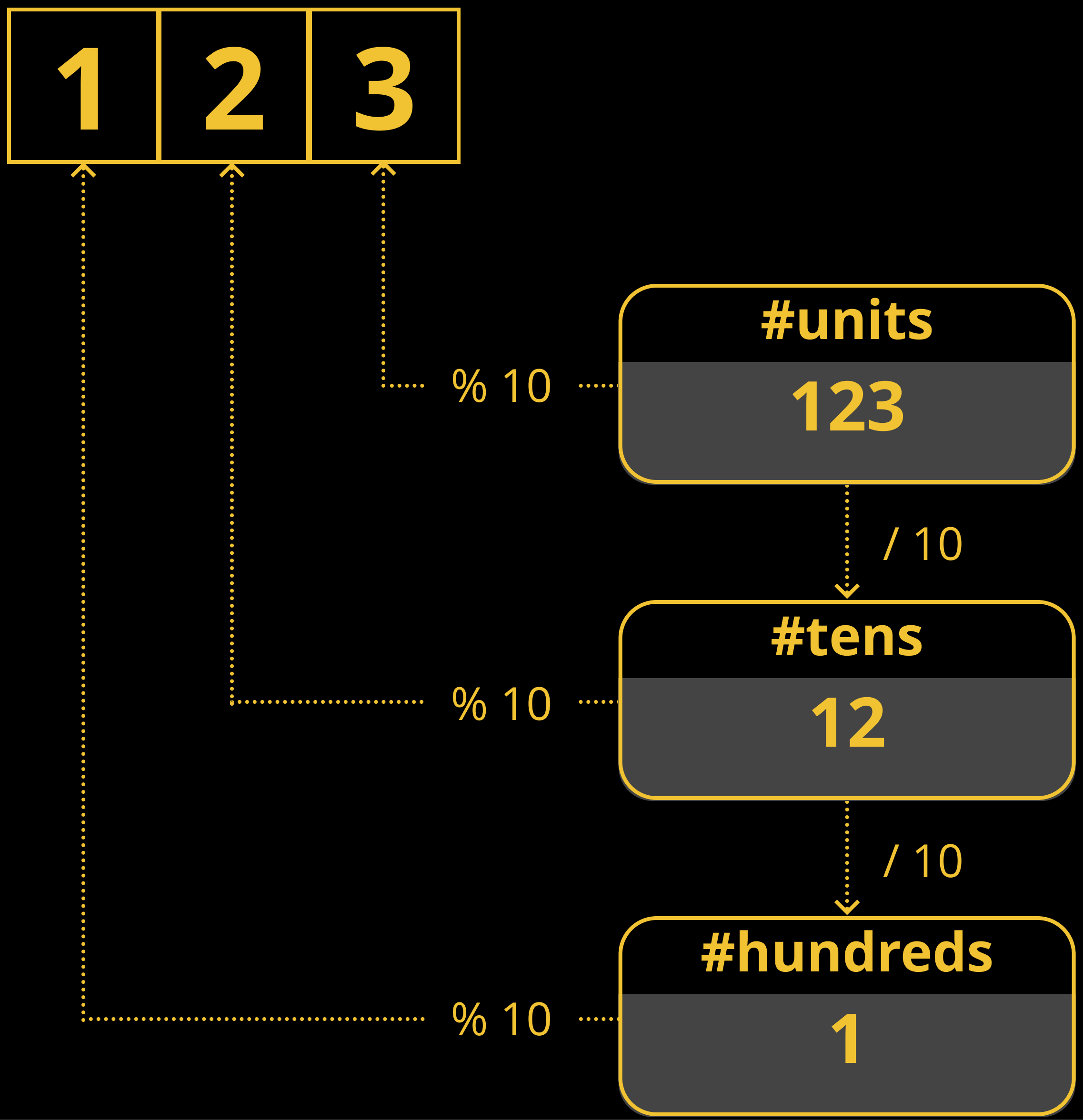
https://godbolt.org/z/8M483Kvba

JS https://godbolt.org/z/nG88rvv5f

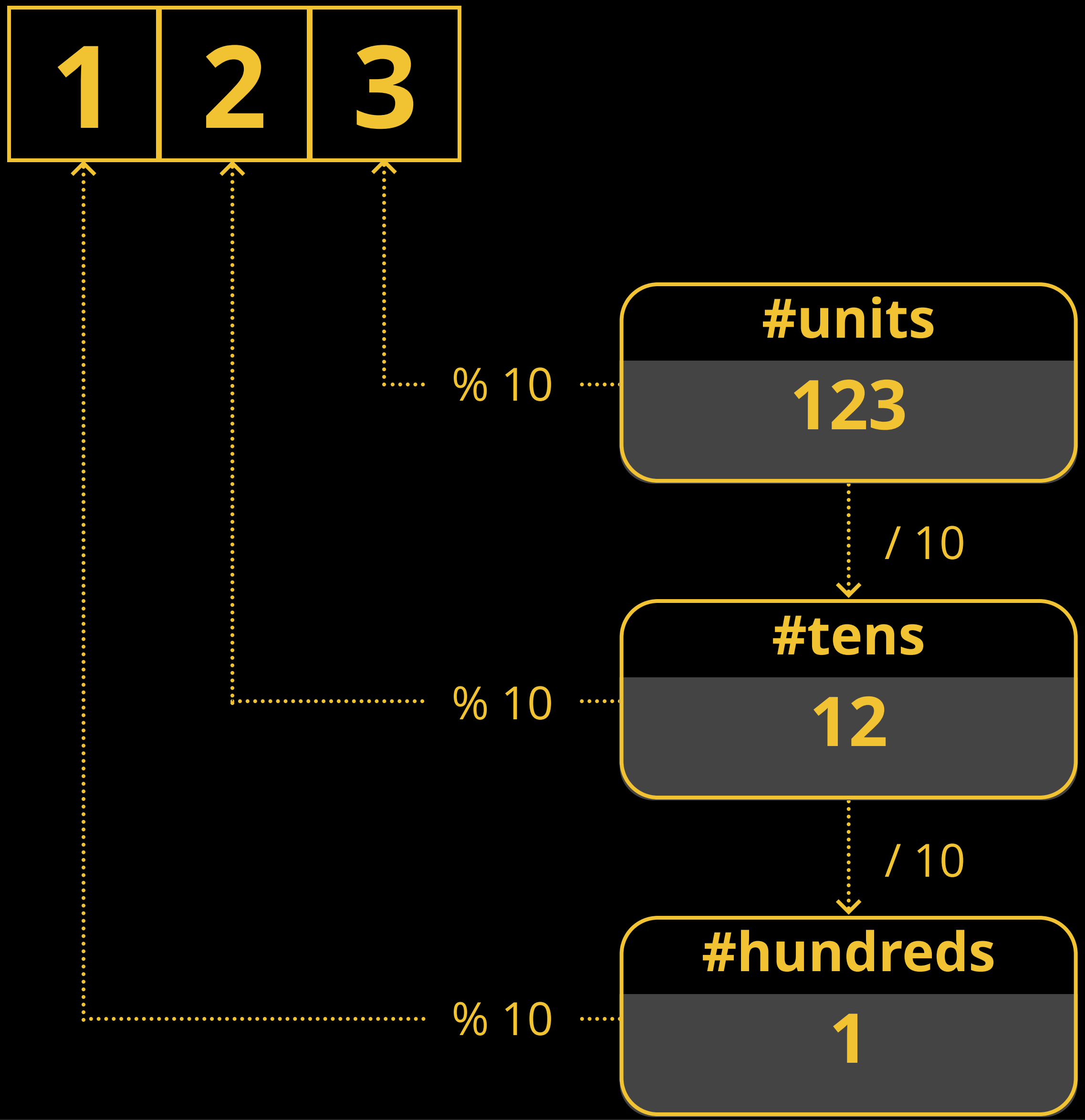
https://godbolt.org/z/5aseWr7fz

https://godbolt.org/z/Y46WY4ooY

Integer to string



Integer to string



https://godbolt.org/z/1WWGM81Ko

```
string convert(unsigned int n) {

  size_t size = number_of_digits(n);
  string str(size, '\0');
  char* p = &str.back();

  do {

    *p = n % 10 + '0';
    n /= 10;

    --p;

  } while(n);

  return str;
}
```

https://godbolt.org/z/Gdo61Yh4o

`FLT_MIN`

$$2^{-126} = \begin{aligned}0.&000,000,000,000,000,000,000,000,000,000,\\ &000,000,011,754,943,508,222,875,079,687,\\ &365,372,222,456,778,186,655,567,720,875,\\ &215,087,517,062,784,172,594,547,271,728,\\ &515,625\end{aligned}$$

(126 digits after the dot) 🙁

https://godbolt.org/z/Gdo61Yh4o

FLT_MIN

$$2^{-126} =$$

0.000,000,000,000,000,000,000,000,000,000,
000,000,011,754,943,508,222,875,079,687,
365,372,222,456,778,186,655,567,720,875,
215,087,517,062,784,172,594,547,271,728,
515,625

(126 digits after the dot) 🙁

$$\cong 1.1754944 \times 10^{-38}$$ 🙂

https://godbolt.org/z/Gdo61Yh4o

FLT_MIN

2^{-126} = 0.000,000,000,000,000,000,000,000,000,000,
000,000,011,754,943,508,222,875,079,687,
365,372,222,456,778,186,655,567,720,875,
215,087,517,062,784,172,594,547,271,728,
515,625

(126 digits after the dot) 🙁

≅ 1.1754944 x 10^{-38} 🙂

Decimal fixed-point representation

1234

Decimal fixed-point representation

12.34

Decimal floating-point representation

exponent

mantissa

$$10^{1} \times 2.34$$

Decimal floating-point representation

exponent

mantissa

$$10^{1} \times 2.34$$

0,0 0 0,0 0 0,0 2 3.4 0

Decimal floating-point representation

exponent

mantissa

$$10^{1} \times 2.34$$

0,0 0 0,0 0 0,0 2 3.4 0

Decimal floating-point representation

exponent $\neq 0$

$\Longrightarrow$

mantissa's first digit $\neq 0$

(scientific notation)

exponent

mantissa

$$10^{1} \times 2.34$$

0,0 0 0,0 0 0,0 2 3.4 0

Decimal floating-point representation

exponent

mantissa

$$10^{9} \times 9.99 \times 10^{-3}$$

9,9 9 0,0 0 0.0 0 0,0 0

Decimal floating-point representation

biased exponent $\neq 0$

$\Longrightarrow$

mantissa's first digit $\neq 0$

(scientific notation)

biased exponent

mantissa

$$10^{9-3} \times 9.99$$

9 , 9 9 0 , 0 0 0 . 0 0 0 , 0 0

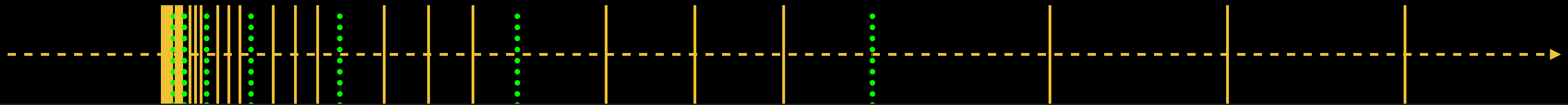
Binary floating-point representation

Exponent and mantissa are in binary.

biased exponent

mantissa

$$2^{000-11} \times 0.00$$



Binary floating-point representation

Exponent and mantissa are in binary.

biased exponent $\neq 0$

$\Longrightarrow$

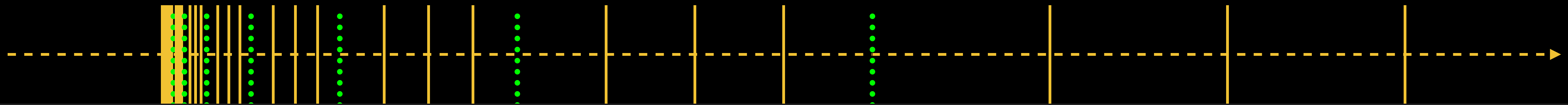
mantissa's first bit $= 1$

(normal)

biased exponent

mantissa

$$2^{000-11} \text{x}\, 0.00$$



Binary floating-point representation

Exponent and mantissa are in binary.

biased exponent $\neq 0$

$\implies$

mantissa's first bit $= 1$

(normal)

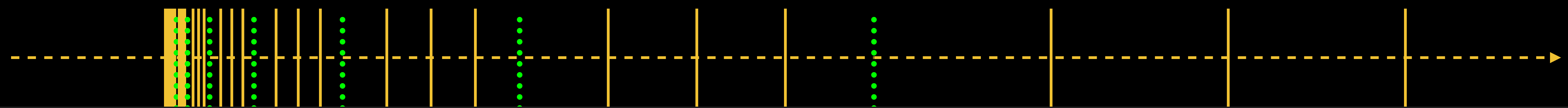
biased exponent

mantissa

$$2^{000-11} \times 0.00$$

Centred value

Uncentred value



IEEE-754 representation

- *Binary16* (`std::float16_t`)
- *Binary32* (`float` and `std::float32_t`)
- *Binary64* (`double` and `std::float64_t`)
- *Binary128* (`__float128` and `std::float128_t`)
- Others

Binary32

0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Binary32

mantissa

23

0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Binary32

<table>
<tr><td></td><td colspan="8">biased exponent</td><td colspan="23">mantissa</td></tr>
<tr><td></td><td colspan="8">8</td><td colspan="23">23</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>

Binary32

<table>
<tr><td>±</td><td colspan="8">biased exponent</td><td colspan="23">mantissa</td></tr>
<tr><td>1</td><td colspan="8">8</td><td colspan="23">23</td></tr>
<tr><td>0</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>1</td><td>0</td><td>0</td><td>1</td><td>0</td><td>1</td><td>1</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0</td></tr>
</table>

Binary32

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<tr><td>±</td><td colspan="8">biased exponent</td><td colspan="23">mantissa</td></tr>
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</table>

0 sign: **+** (**0**) or **–** (**1**)

Binary32

±	biased exponent								mantissa																						
1	8								23																						
0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

Binary32

<table>
<tr><td>±</td><td colspan="8">biased exponent</td><td colspan="23">mantissa</td></tr>
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</table>

$$\begin{array}{cccccccccc} & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ - & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ \hline & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{array}$$

biased exponent $\neq 0 \implies$ normal

bias: **01111111** (127, normal) or **011111110** (126, subnormal)

+

Binary32

±	biased exponent								mantissa																						
1	8								23																						
0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$\text{biased exponent} \neq 0 \implies \text{normal}$$

$$+2^{00000011} \times 1.$$

https://godbolt.org/z/9PGPT9q1E

Binary32

<table>
<tr><td>±</td><td colspan="8">biased exponent</td><td colspan="23">mantissa</td></tr>
<tr><td>1</td><td colspan="8">8</td><td colspan="23">23</td></tr>
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</table>

$$+2^{00000011} \times 1.01011000000000000000000$$

https://godbolt.org/z/9PGPT9q1E

Binary32

±	biased exponent								mantissa																						
1	8								23																						
0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$+2^{00000011} \times 1.01011000000000000000000$$

$$+2^{3} \times 1.3\ 4\ 3,7\ 5$$

https://godbolt.org/z/9PGPT9q1E

Binary32

±	biased exponent	mantissa
1	8	23

0	1	0	0	0	0	0	1	0	0	1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

$$+2^{-00010100} \times 10101100000000000000000.$$

$$\oplus 2^{\boxed{-20}} \times \boxed{1\ 1,2\ 7\ 2,1\ 9\ 2}.0\ 0\ 0,0\ 0$$

Quiz

What's the output?

https://godbolt.org/z/G9j5aj8zY

```
double x = 1.0 / 3.0;
std::print("{}", x);
```

Options:

(a) 0.3

(b) 0.3333333333333333

(c) 0.3333333333333334

(d) One third

(e) *None of the above*

Quiz

What's the output?

https://godbolt.org/z/G9j5aj8zY

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std::print("{}", x);
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Quiz

What's the output?

https://godbolt.org/z/G9j5aj8zY

```cpp
double x = 1.0 / 3.0;
std::print("{}", x);
```

https://godbolt.org/z/1vrE7brfa

```cpp
float x = 1.f / 3.f;
std::print("{}", x);
```

Options:

(a) 0.3

(b) 0.3333333333333333

(c) 0.3333333333333334

(d) One third

(e) *None of the above*

(a) 0.3

(b) 0.33333333

(c) 0.33333334

(d) One third

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Quiz

What's the output?

https://godbolt.org/z/G9j5aj8zY

```cpp
double x = 1.0 / 3.0;
std::print("{}", x);
```

https://godbolt.org/z/1vrE7brfa

```cpp
float x = 1.f / 3.f;
std::print("{}", x);
```

Options:

Left (double):

(a) 0.3

(b) 0.333333333333333**3**

(c) 0.333333333333333**4**

(d) One third

(e) *None of the above*

Right (float):

(a) 0.3

(b) 0.3333333**3**

(c) 0.3333333**4**

(d) One third

(e) *None of the above*

Quiz

What's the output?

https://godbolt.org/z/G9j5aj8zY

```cpp
double x = 1.0 / 3.0;
std::print("{}", x);
```

Options:

(a) 0.3

(b) 0.3333333333333333

(c) 0.3333333333333334

(d) One third

(e) *None of the above*

https://godbolt.org/z/1vrE7brfa

```cpp
float x = 1.f / 3.f;
std::print("{}", x);
```

(a) 0.3

(b) 0.33333333

(c) 0.33333334

(d) One third

(e) *None of the above*

.NET

https://godbolt.org/z/Kf99boffG

https://godbolt.org/z/1hM48xza9

https://godbolt.org/z/nYbYnY6cc

https://godbolt.org/z/3aWxPqdfq

https://godbolt.org/z/xfs9WbfzT

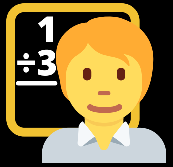
⅓

$0.333,333,333,333,333,333,333,333,\overline{3}$

⅓

https://godbolt.org/z/xfs9WbfzT

$$0.333{,}333{,}333{,}333{,}333{,}333{,}333{,}333{,}\overline{3} = 2^{-25} \times \left(11{,}184{,}810 + \frac{2}{3}\right)$$



⅓

$$0.333{,}333{,}343{,}267{,}440{,}795{,}898{,}437{,}5 = 2^{-25} \times 11{,}184{,}811$$

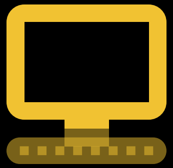
$$0.333{,}333{,}333{,}333{,}333{,}333{,}333{,}333{,}\overline{3} = 2^{-25} \times \left(11{,}184{,}810 + \frac{2}{3}\right)$$

$$0.333{,}333{,}313{,}465{,}118{,}408{,}203{,}125{,}0 = 2^{-25} \times 11{,}184{,}810$$

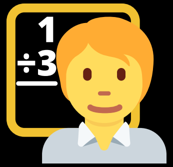
⅓

https://godbolt.org/z/xfs9WbfzT

$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$



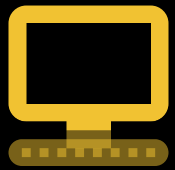
$0.333,333,333,333,333,333,333,333,\overline{3} = 2^{-25} \times (11,184,810 + \frac{2}{3})$



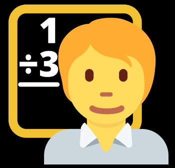
$0.333,333,313,465,118,408,203,125,0 = 2^{-25} \times 11,184,810$

⅓

$$0.333{,}333{,}343{,}267{,}440{,}795{,}898{,}437{,}5 = 2^{-25} \times 11{,}184{,}811$$



$$3{,}333{,}333{,}333{,}333{,}333{,}\bar{3} = 2^{-25} \times \left(11{,}184{,}810 + \frac{2}{3}\right)$$



$$0.333{,}333{,}313{,}465{,}118{,}408{,}203{,}125{,}0 = 2^{-25} \times 11{,}184{,}810$$

⅓

$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$

$3,333,333,333,333,333,\overline{3} = 2^{-25} \times (11,184,810 + \frac{2}{3})$

$0.333,333,313,465,118,408,203,125,0 = 2^{-25} \times 11,184,810$

https://godbolt.org/z/xfs9WbfzT

⅓

$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$

$3,333,333,333,333,333,\bar{3} = 2^{-25} \times \left(11,184,810 + \frac{2}{3}\right)$

$0.333,333,313,465,118,408,203,125,0 = 2^{-25} \times 11,184,810$

https://godbolt.org/z/xfs9WbfzT

⅓

$0.333,333,343,267,440,795,898,437,5 = 2^{-25} \times 11,184,811$

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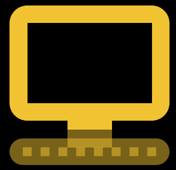

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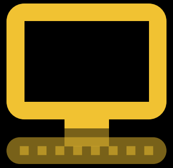

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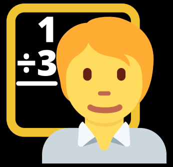

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1 ÷3

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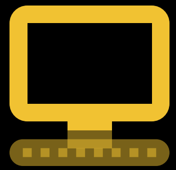

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Jorge Luis Borges, The Book of Imaginary Beings

“There is something in the dragon’s image that fits man’s imagination.”

The dragon's den

1990 - *Dragon*	Guy L. Steele, Jon L. White
1996	Robert G. Burger and R. Kent Dybvig
2010 - *Grisù*	Florian Loitsch
2013	Aubrey Jaffer
2016 - *Errol*	Marc Andrysco, Ranjit Jhala, Sorin Lerner
2018 - *Ryū*	Ulf Adams
2020 - *Schubfach*	Raffaello Giulietti
2020 - *Grisù-Exact*	Junekey Jeon
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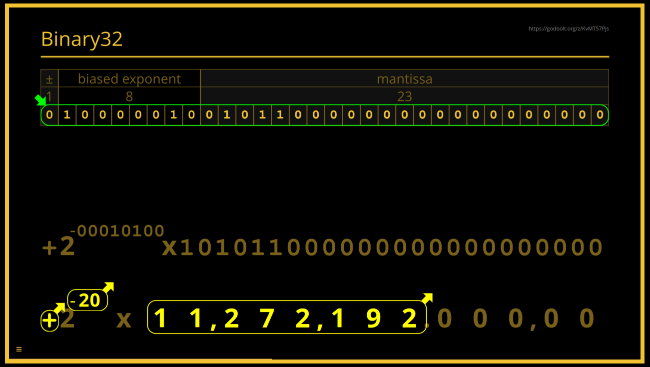
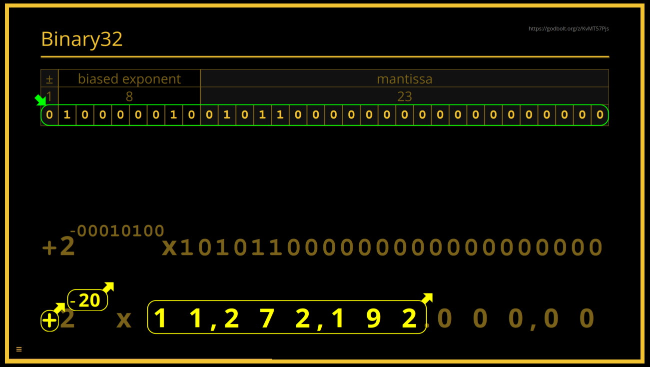
Three steps

Three steps



decode representation

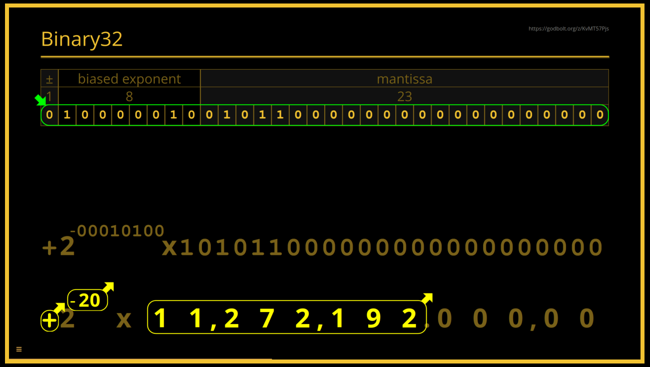
Three steps



decode representation

convert binary to decimal

Three steps



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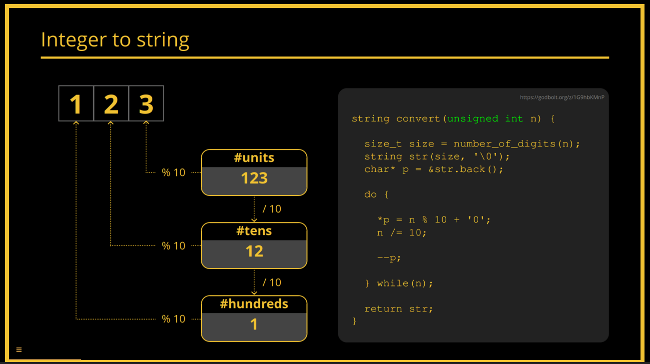

convert exponent and mantissa to string

Dragon's problem

Given $m \in \mathbb{N}$ and $E \in \mathbb{Z}$,

find $n \in \mathbb{N}$ and $F \in \mathbb{Z}$ such that

$$n \times 10^F \cong m \times 2^E.$$

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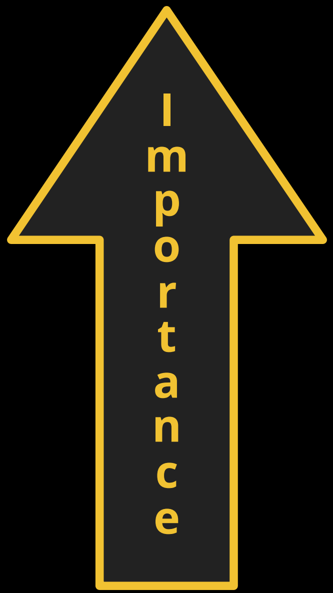
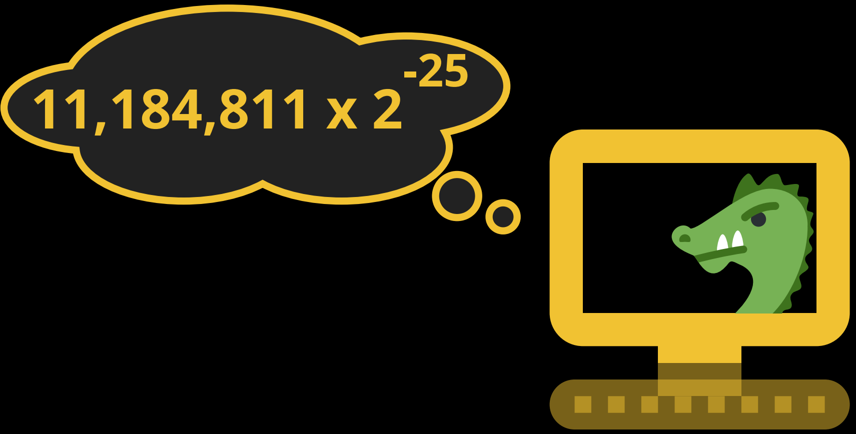
- No information loss
- As short as possible
- As close as possible
- Tiebreak rules



https://godbolt.org/z/j4TEr3eTE

No information loss



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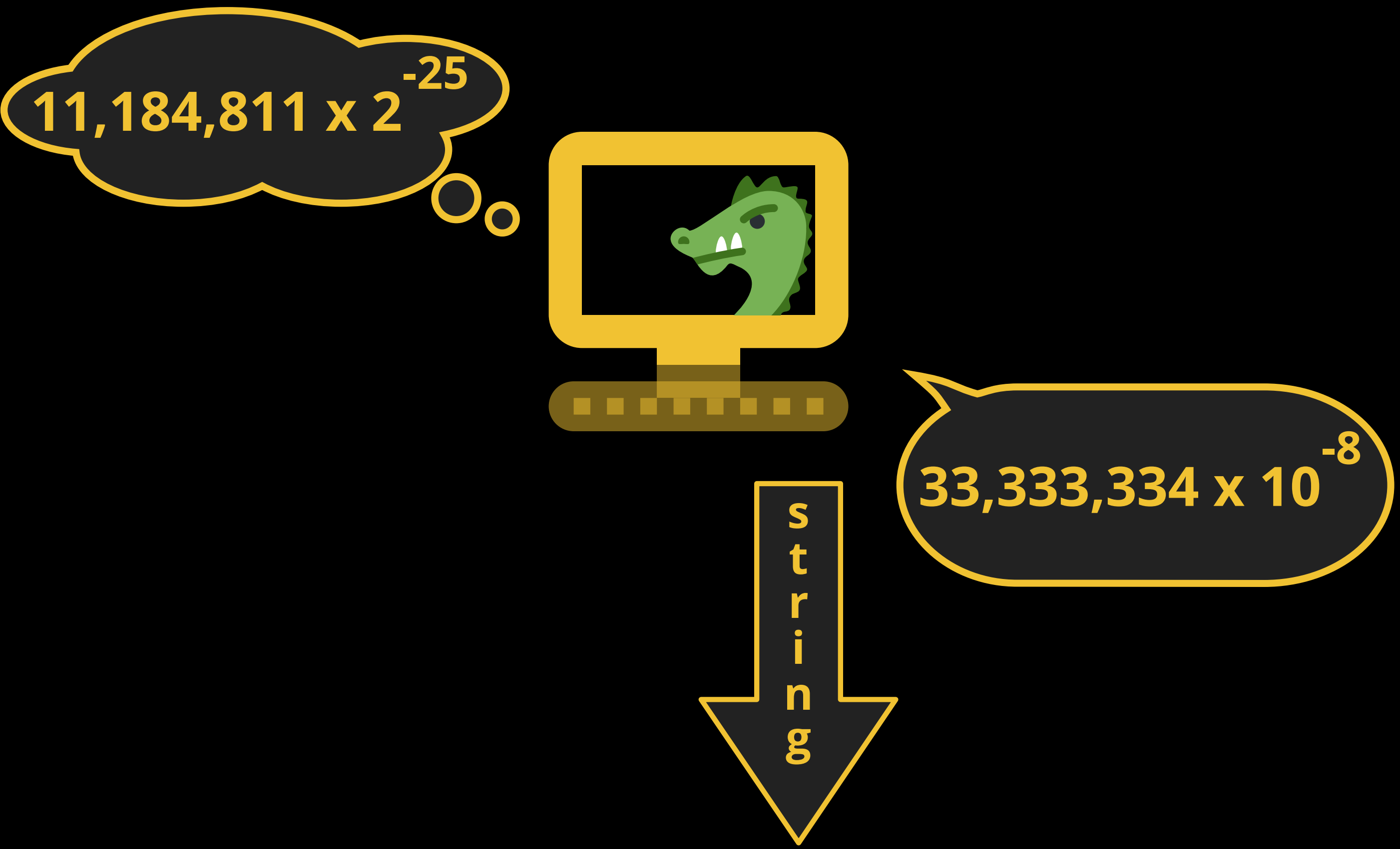

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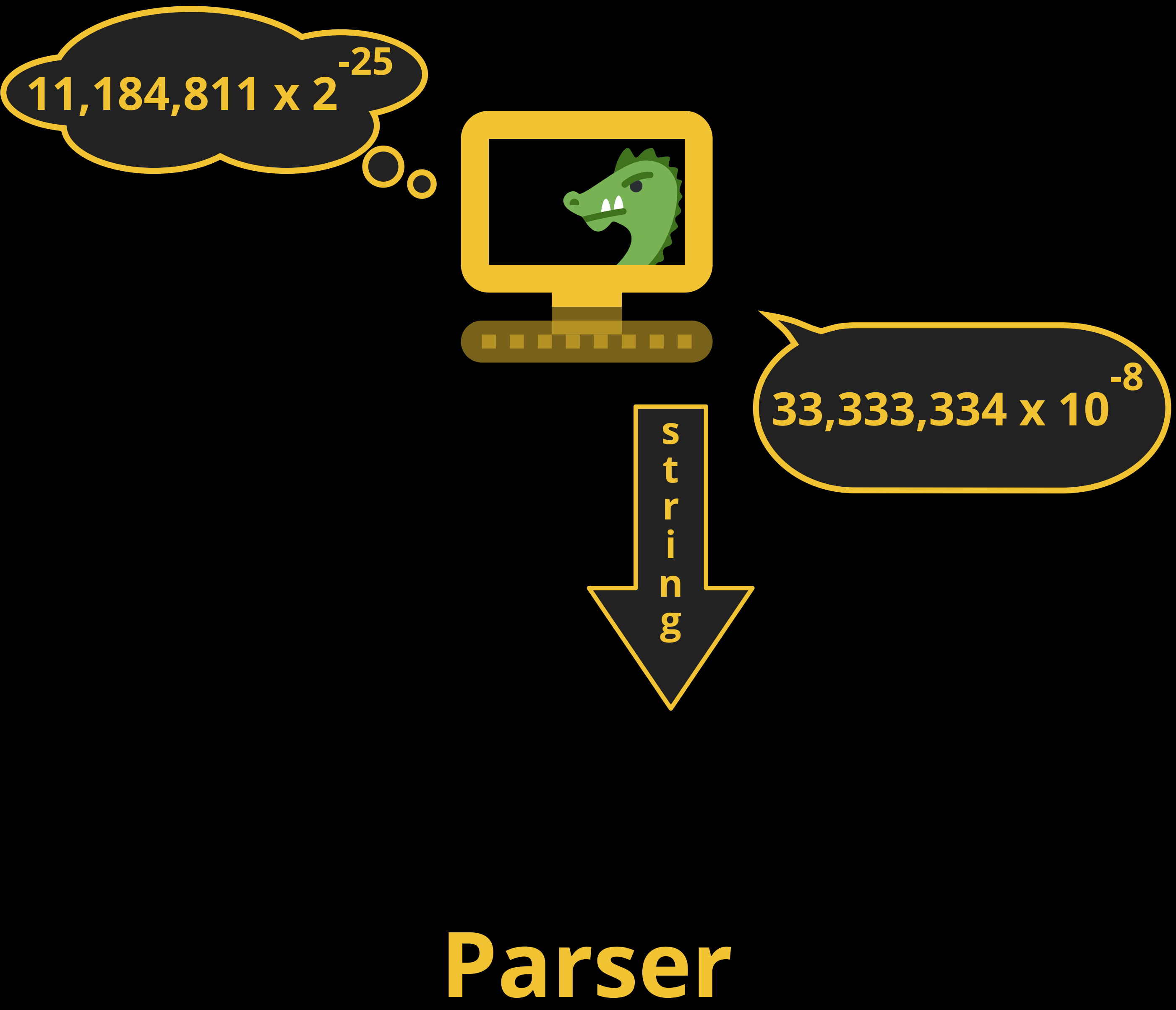

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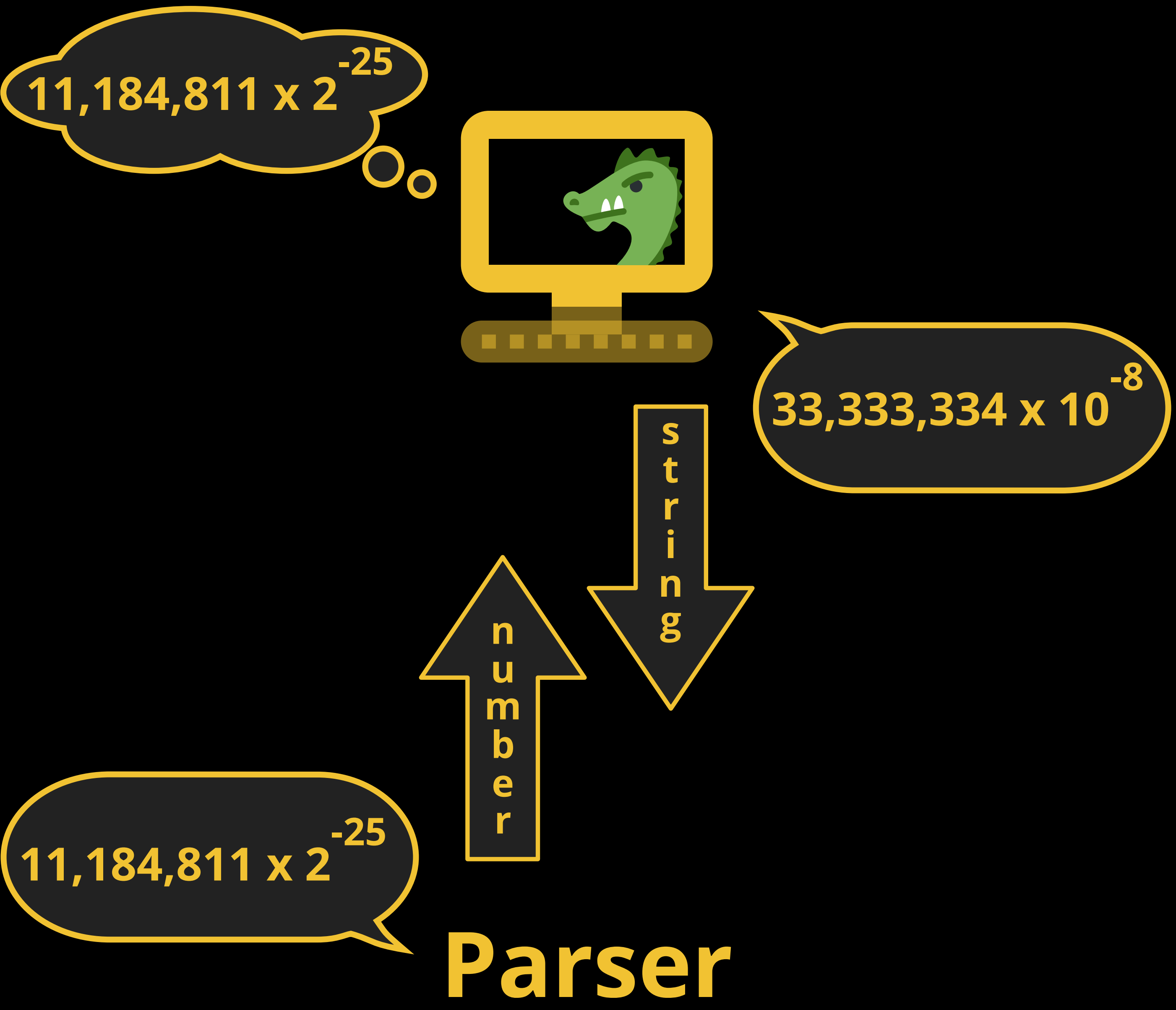

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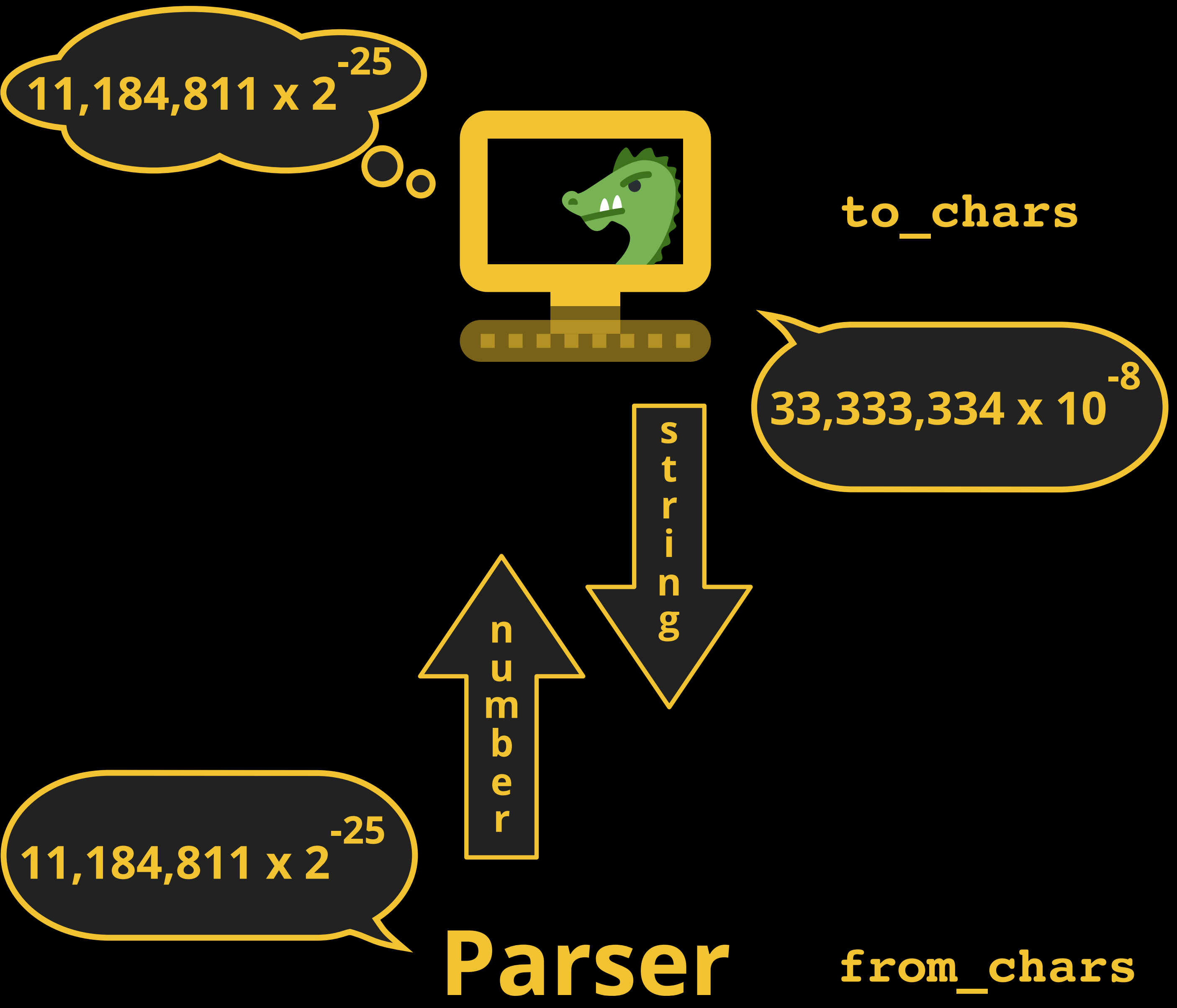

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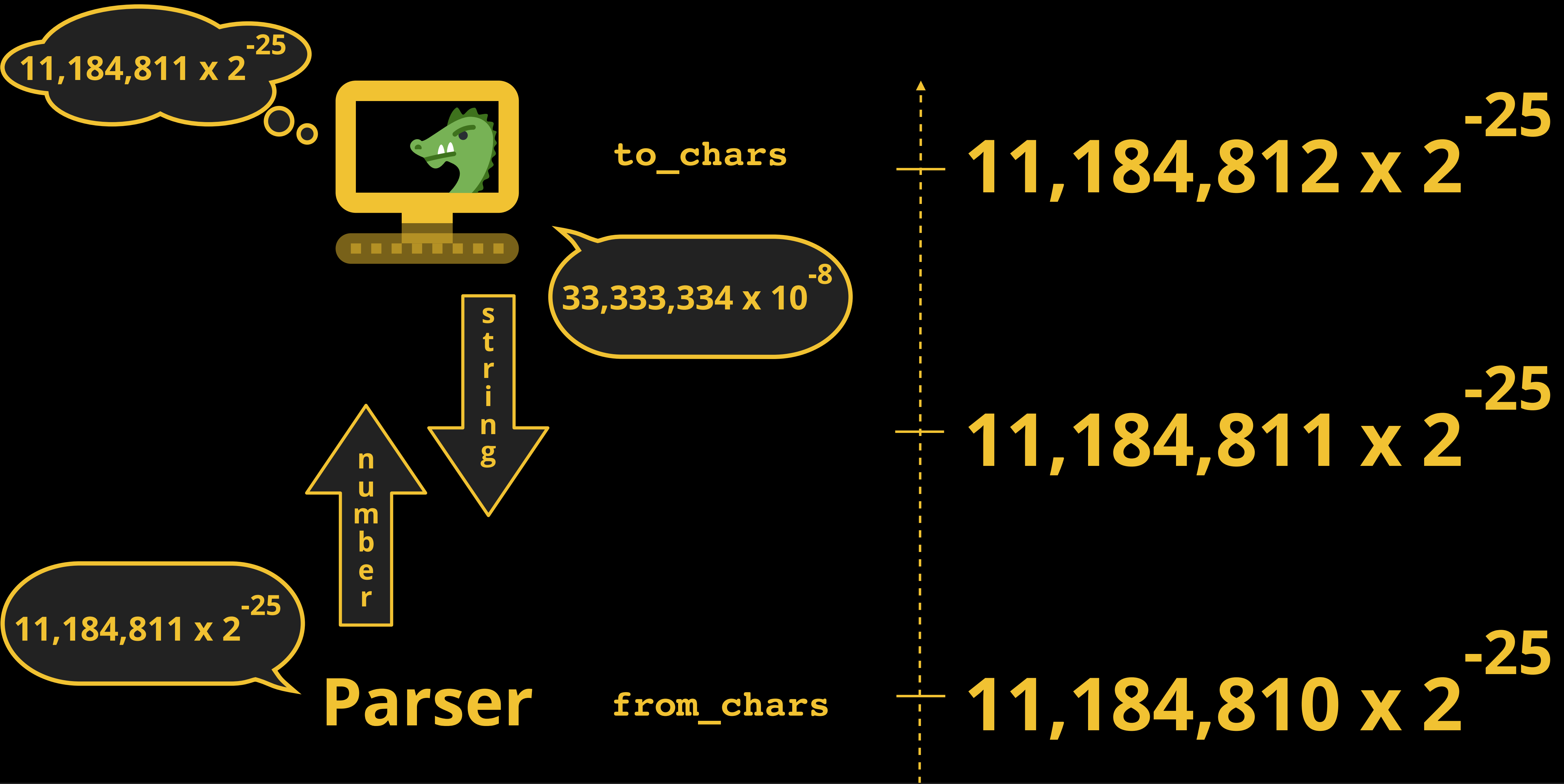

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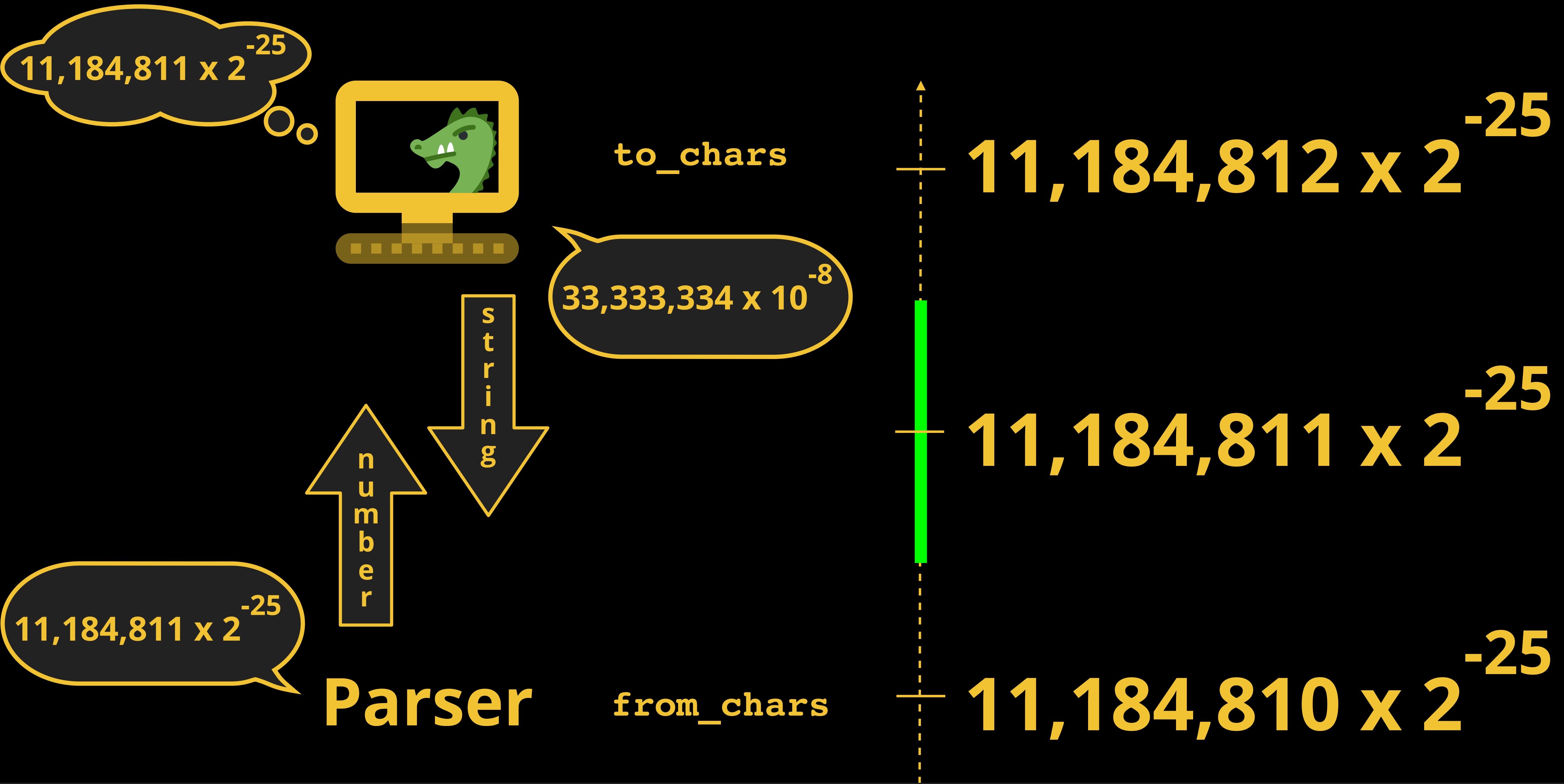

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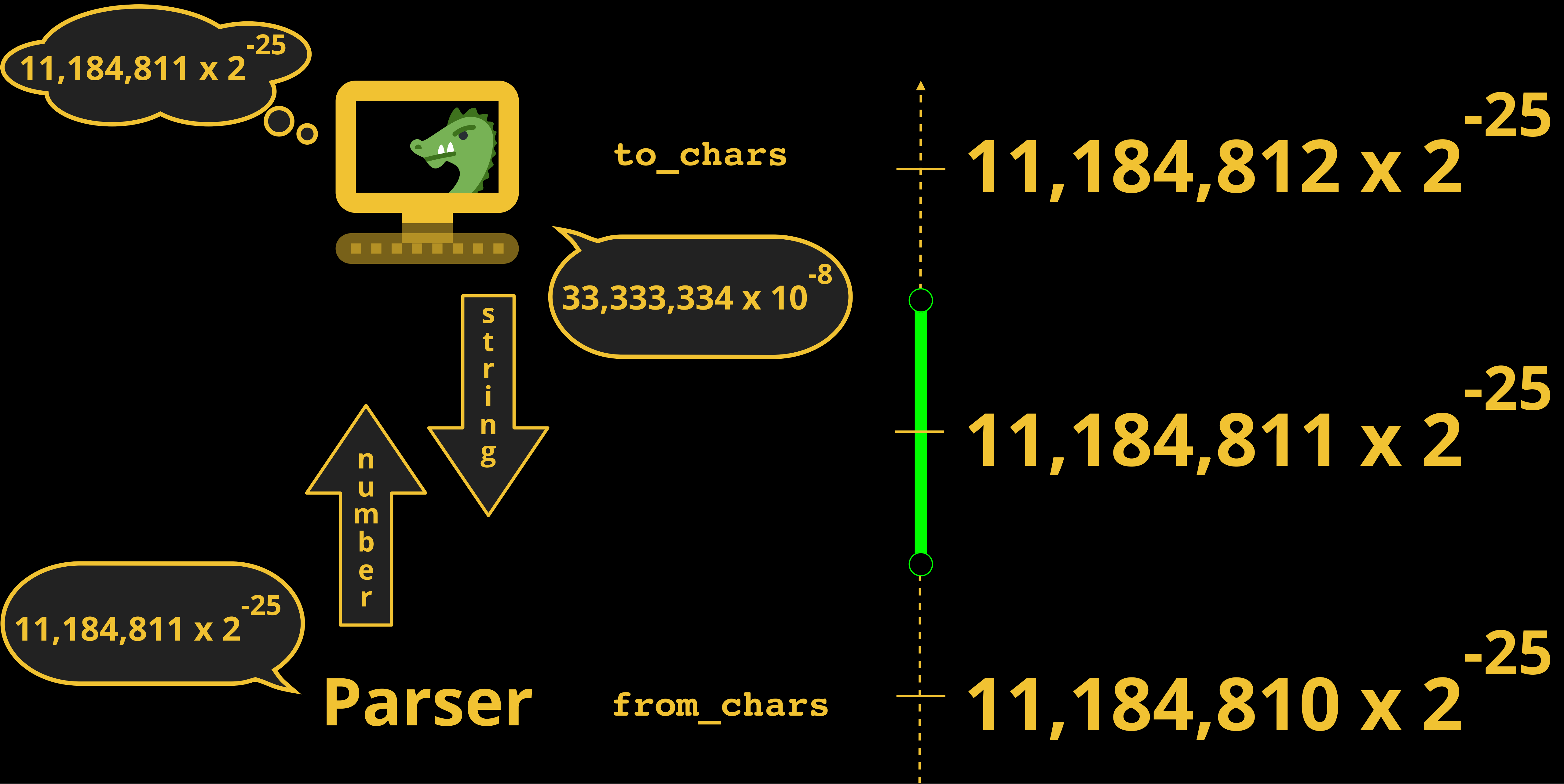

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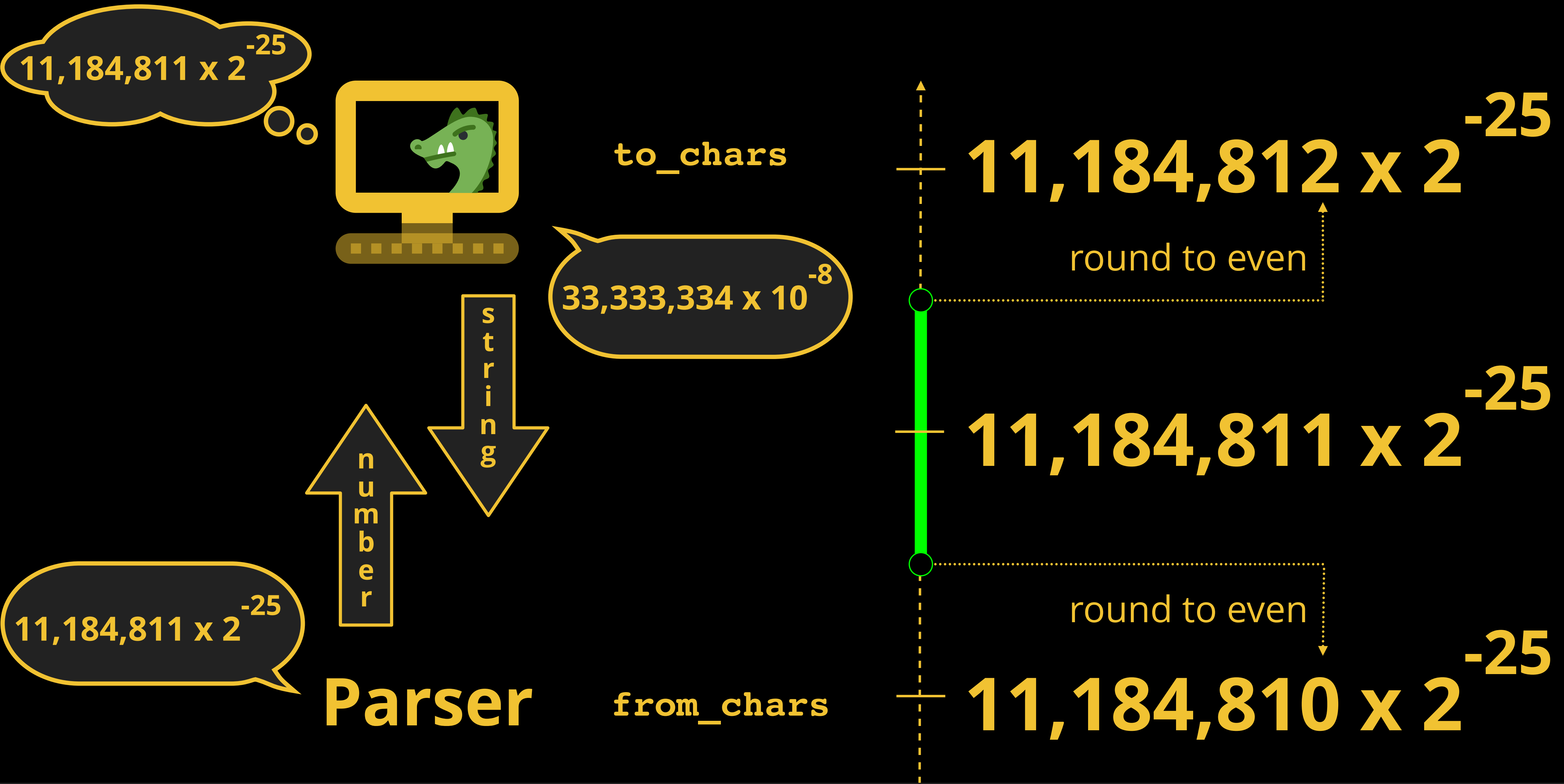

https://godbolt.org/z/3czzTzsTs

As short as possible

$$9{,}765{,}625 \times 2^{10} = 10 \times 10^{9}$$

$$= 1 \times 10^{10}$$

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Only the mantissa matters.

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Only the mantissa matters.

The larger the exponent,
the shorter the mantissa.

As short as possible

$$13{,}421{,}773 \times 2^{-27} \cong 100 \times 10^{-3}$$

$$\cong 10 \times 10^{-2}$$

$$\cong 1 \times 10^{-1}$$

As short as possible

https://godbolt.org/z/5xKzvfzfx

$$13{,}421{,}773 \times 2^{-27} \cong 100 \times 10^{-3}$$

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Only representations of the form $n \times 10^F$ matter.

As short as possible

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$\cong$ 

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As short as possible

https://godbolt.org/z/hc514Y9TM

$$30{,}000{,}002 \times 10^{-8}$$

$$10{,}066{,}330 \times 2^{-25}$$

$$30{,}000{,}001 \times 10^{-8}$$

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As short as possible

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What do we wish to see?

As short as possible

https://godbolt.org/z/hc514Y9TM

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What do we wish to see?

closest: **0.30000001**

As short as possible

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What do we wish to see?

closest: **0.30000001**

shortest: **0.3**

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$30{,}000{,}000 \times 10^{-8}$

$\boxed{3 \times 10^{-1}}$

If a mantissa is multiple of 10, then we can do better.

Pythagoras

“Leave the roads;

take the trails.”

Guaraní people

Guaraní mythology

https://tinyurl.com/75xhk7ef

Guaraní mythology

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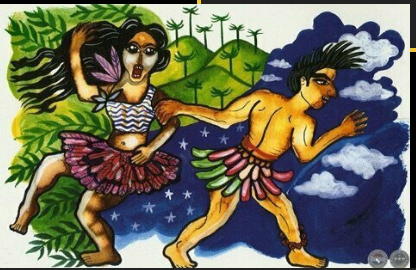

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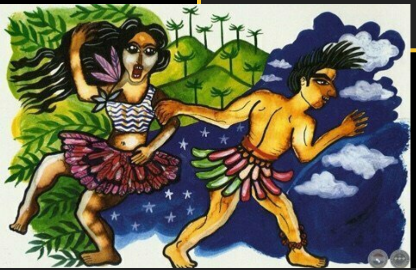

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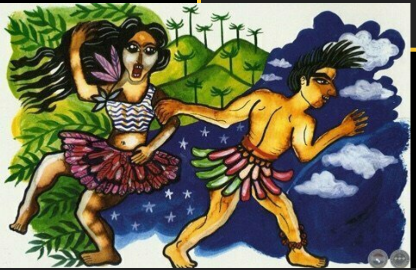

6️⃣ Arasy, placed a curse upon Taú's seven sons, who were born as terrifying beasts.

7️⃣ Tejú Jaguá was the first legendary monster.

Tejú Jaguá - I

$$n \times 10^F \cong m \times 2^E$$

Tejú Jaguá - I

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Tejú Jaguá - I

$$n \times 10^{F} \cong m \times 2^{E}$$

$$(m-1) \times 2^{E} \qquad m \times 2^{E} \qquad (m+1) \times 2^{E}$$

Tejú Jaguá - I

$$n \times 10^{F} \cong m \times 2^{E}$$

$$(m-1)\times 2^{E} \qquad \left(m-\tfrac{1}{2}\right)\times 2^{E} \qquad m\times 2^{E} \qquad \left(m+\tfrac{1}{2}\right)\times 2^{E} \qquad (m+1)\times 2^{E}$$

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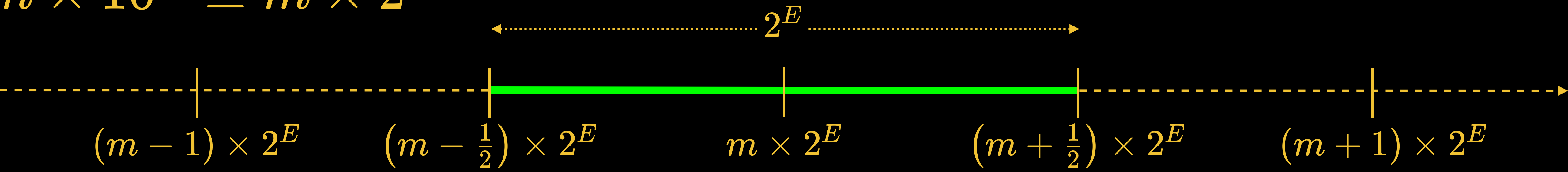

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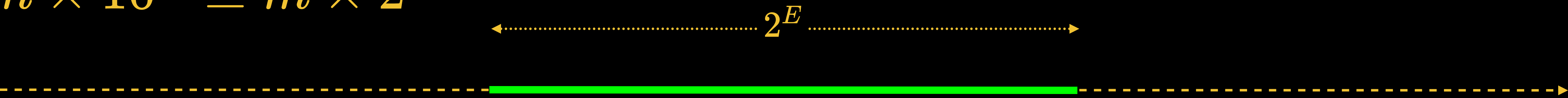

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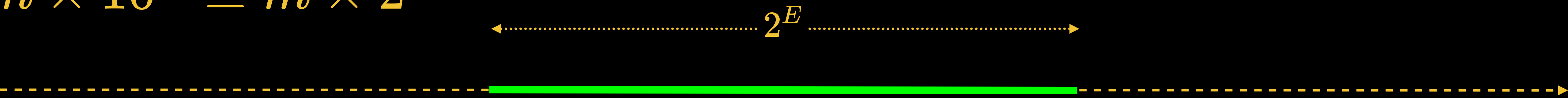

- There must be some $n \times 10^{F}$ inside the permissible interval.

Tejú Jaguá - I

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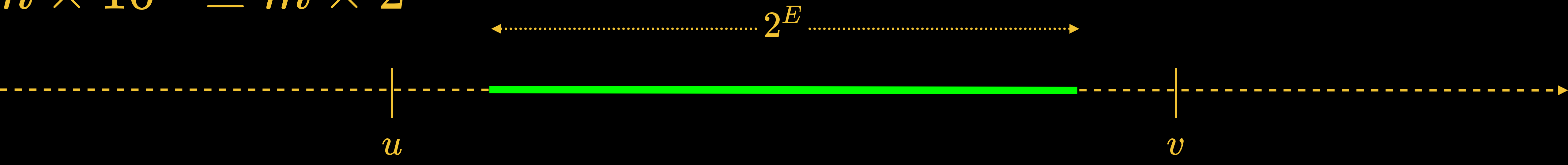

- There must be some $n \times 10^F$ inside the permissible interval.
- If F is bad, then two **consecutive** numbers u and v of this form fall before and after the permissible interval and thus, $v - u = 10^F > 2^E$.

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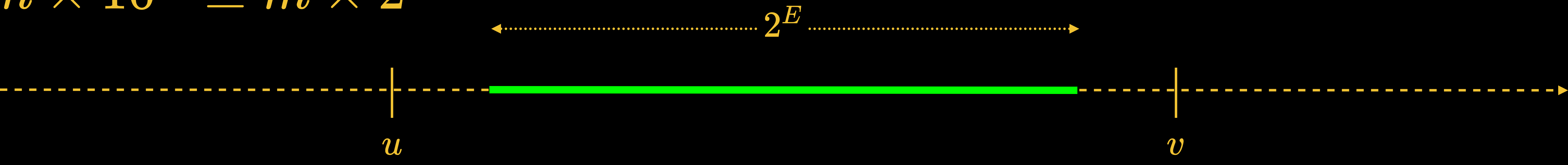

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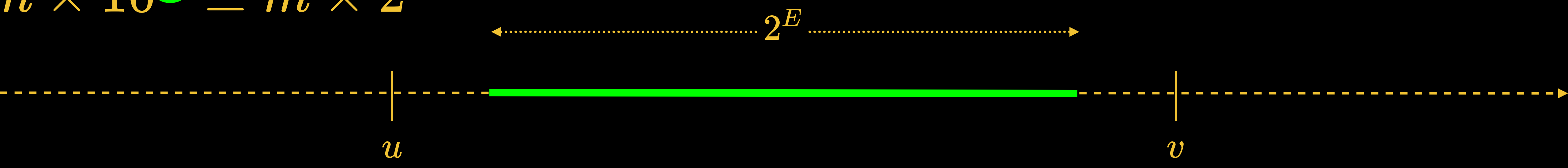

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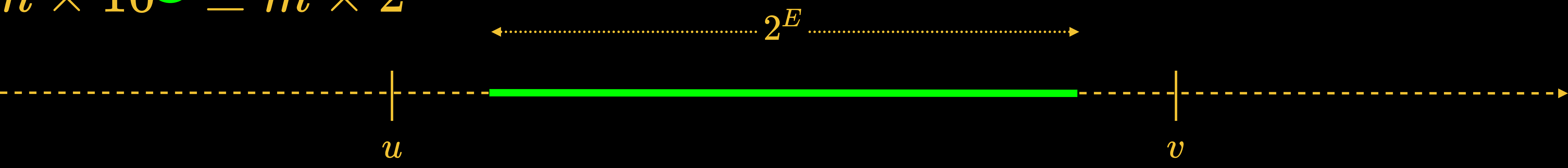

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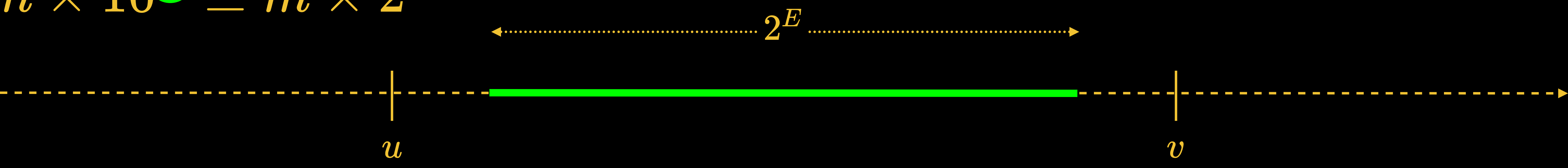

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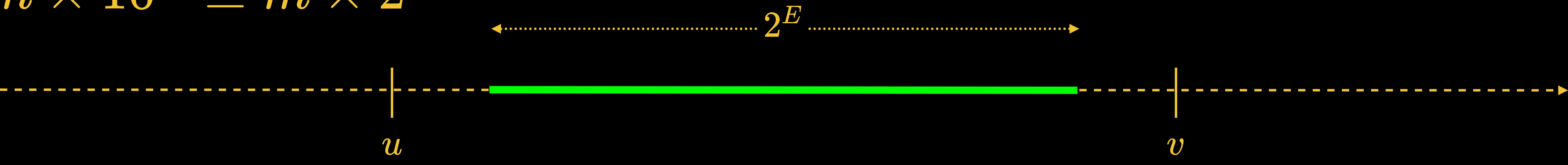
$$10^F \leq 2^E < 10^{F+1} \iff F \leq E \times \log(2) < F + 1$$

Tejú Jaguá - I

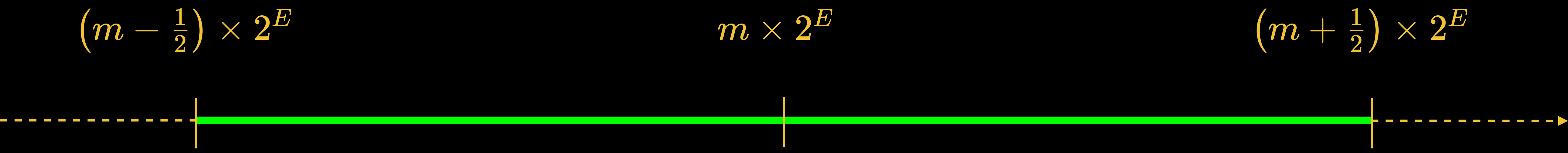
$$n \times 10^{F} \cong m \times 2^{E}$$



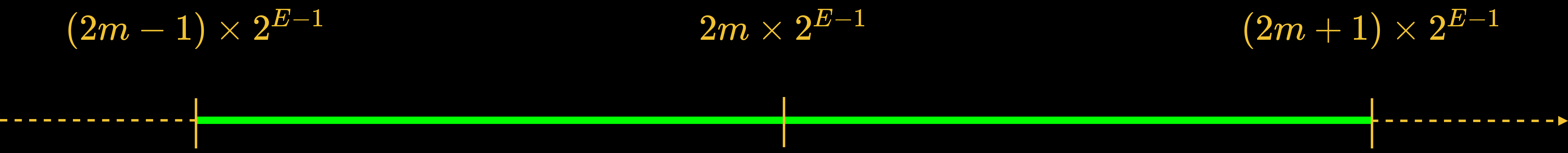
- There must be some $n \times 10^F$ inside the permissible interval.
- If F is bad, then two **consecutive** numbers u and v of this form fall before and after the permissible interval and thus, $v - u = 10^F > 2^E$.
- The larger F, the shorter n.
- We set F to be the largest integer such that $10^F \leq 2^E$, *i.e.*,

$$10^F \leq 2^E < 10^{F+1} \iff F \leq E \times \log(2) < F + 1 \iff \boxed{F = \lfloor E \times \log(2) \rfloor}$$

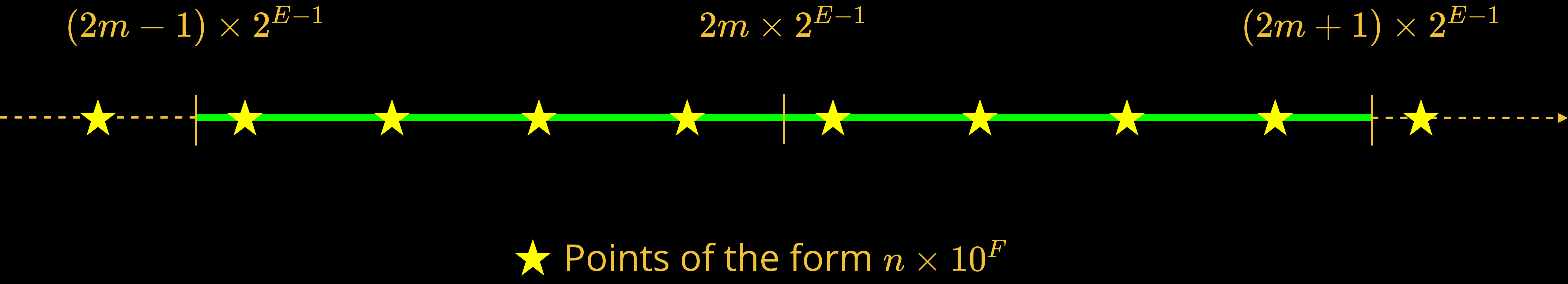
Tejú Jaguá - II



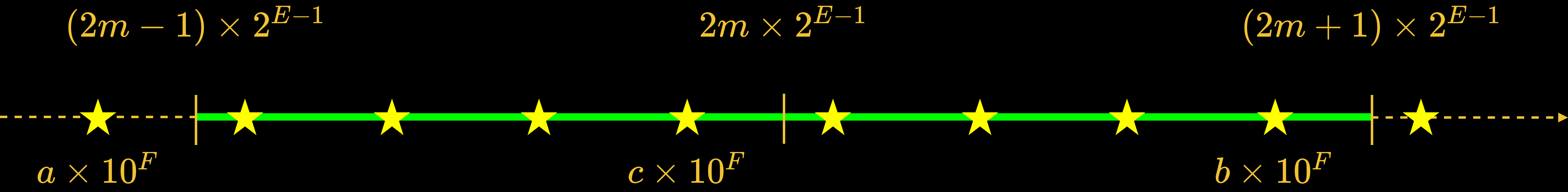
Tejú Jaguá - II



Tejú Jaguá - II

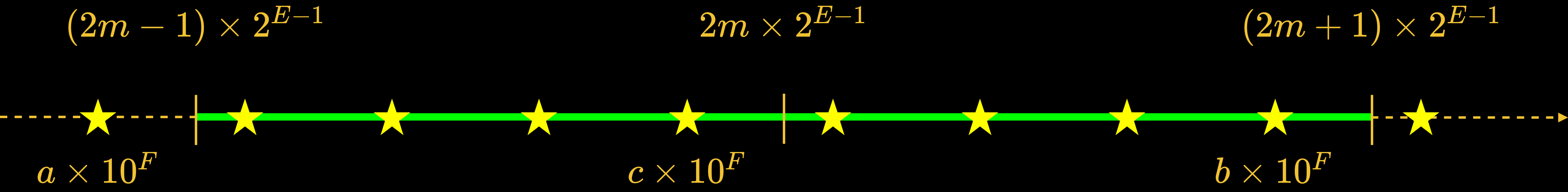


Tejú Jaguá - II



★ Points of the form $n \times 10^F$

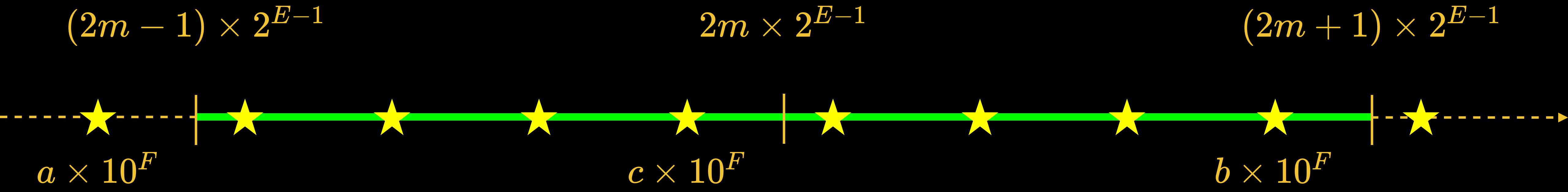
Tejú Jaguá - II



★ Points of the form $n\times 10^F$

$$a\times 10^F \le (2m-1)\times 2^{E-1} < (a+1)\times 10^F$$

Tejú Jaguá - II

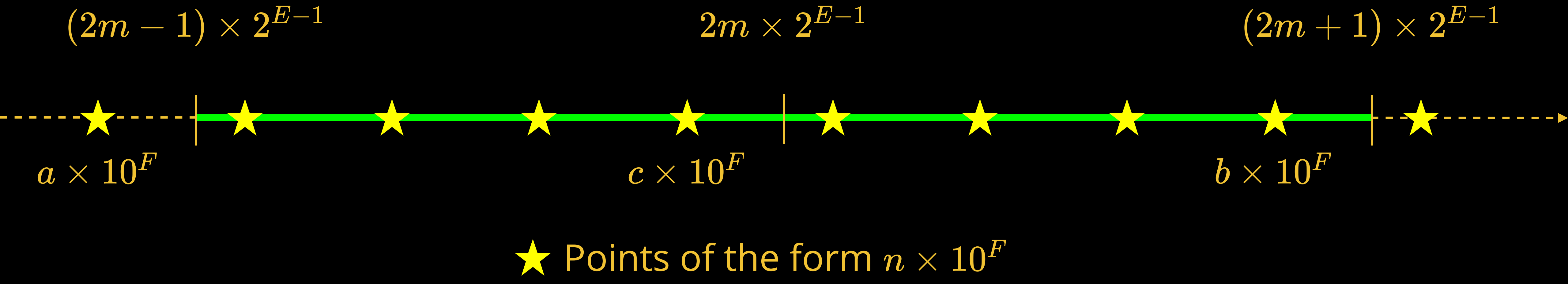


★ Points of the form $n\times 10^F$

$$a\times 10^F \le (2m-1)\times 2^{E-1} < (a+1)\times 10^F$$

$$a \le (2m-1)\times \frac{2^{E-1}}{10^F} < a+1$$

Tejú Jaguá - II

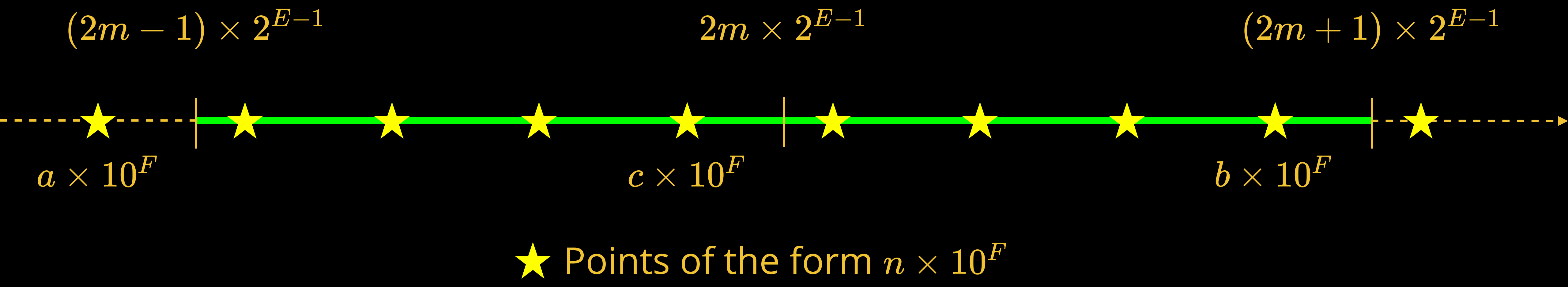


$$a\times 10^F \le (2m-1)\times 2^{E-1} < (a+1)\times 10^F$$

$$a \le (2m-1)\times \frac{2^{E-1}}{10^F} < a+1$$

$$a = \left\lfloor (2m-1)\times \frac{2^{E-1}}{10^F} \right\rfloor$$

Tejú Jaguá - II



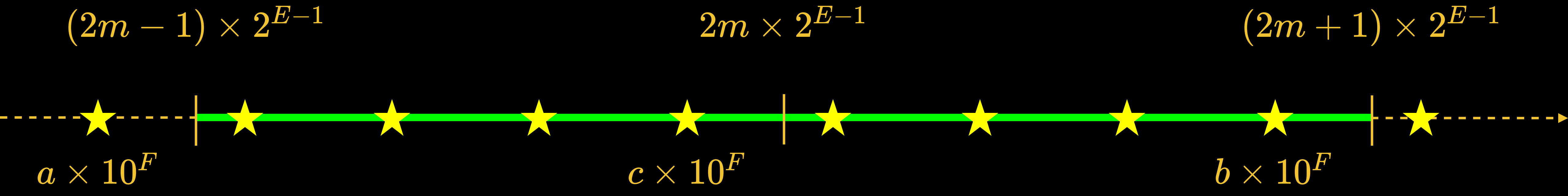
$$a\times 10^F \le (2m-1)\times 2^{E-1} < (a+1)\times 10^F$$

$$b = \left\lfloor (2m+1)\times \frac{2^{E-1}}{10^F} \right\rfloor$$

$$a \le (2m-1)\times \frac{2^{E-1}}{10^F} < a+1$$

$$a = \left\lfloor (2m-1)\times \frac{2^{E-1}}{10^F} \right\rfloor$$

Tejú Jaguá - II



★ Points of the form $n \times 10^F$

$$a \times 10^F \leq (2m-1) \times 2^{E-1} < (a+1) \times 10^F$$

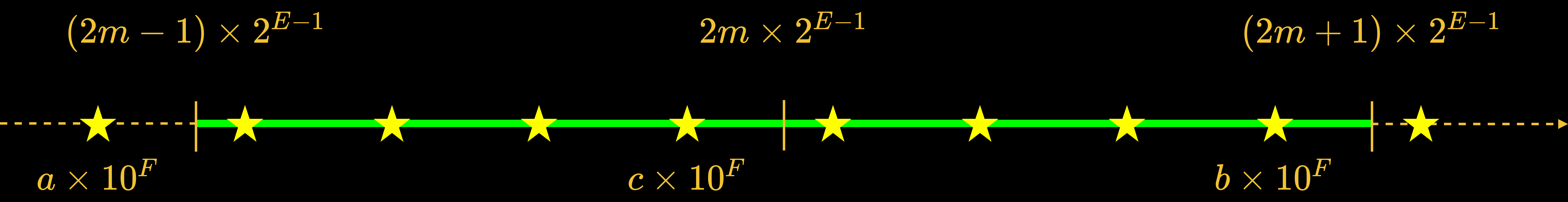
$$b = \left\lfloor (2m+1) \times \frac{2^{E-1}}{10^F} \right\rfloor$$

$$a \leq (2m-1) \times \frac{2^{E-1}}{10^F} < a + 1$$

$$a = \left\lfloor (2m-1) \times \frac{2^{E-1}}{10^F} \right\rfloor$$

$$c = \left\lfloor 2m \times \frac{2^{E-1}}{10^F} \right\rfloor$$

Tejú Jaguá - II



★ Points of the form $n \times 10^F$

$$a \times 10^F \le (2m-1)\times 2^{E-1} < (a+1)\times 10^F$$

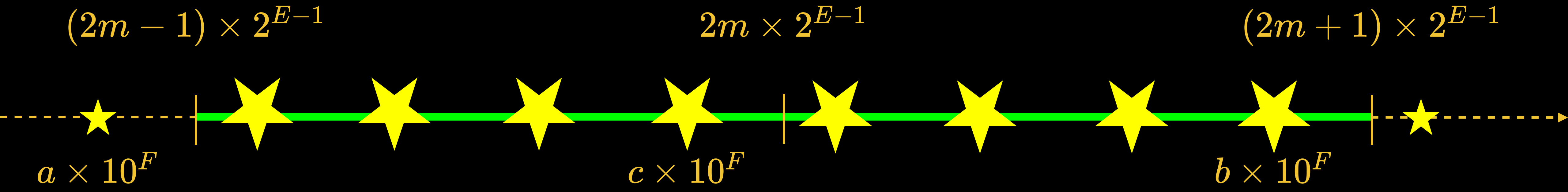
$$\boxed{b = \left\lfloor (2m+1)\times \frac{2^{E-1}}{10^F} \right\rfloor}$$

$$a \le (2m-1)\times \frac{2^{E-1}}{10^F} < a+1$$

$$\boxed{a = \left\lfloor (2m-1)\times \frac{2^{E-1}}{10^F} \right\rfloor}$$

$$\boxed{c = \left\lfloor 2m \times \frac{2^{E-1}}{10^F} \right\rfloor}$$

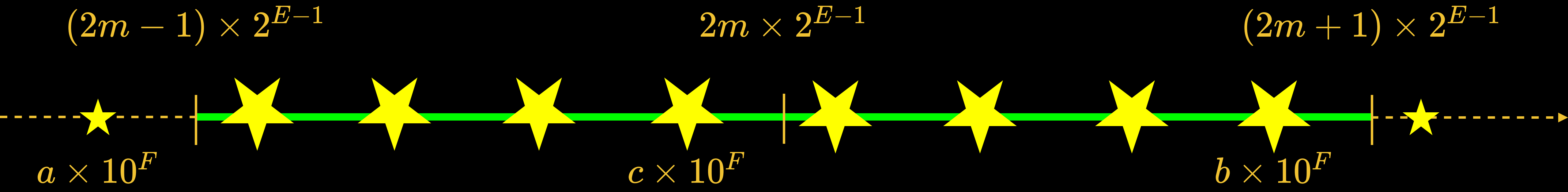
Tejú Jaguá - III



★ Points of the form $n \times 10^F$

Is any ★ $= s \times 10^F$ where s is a multiple of 10?

Tejú Jaguá - III

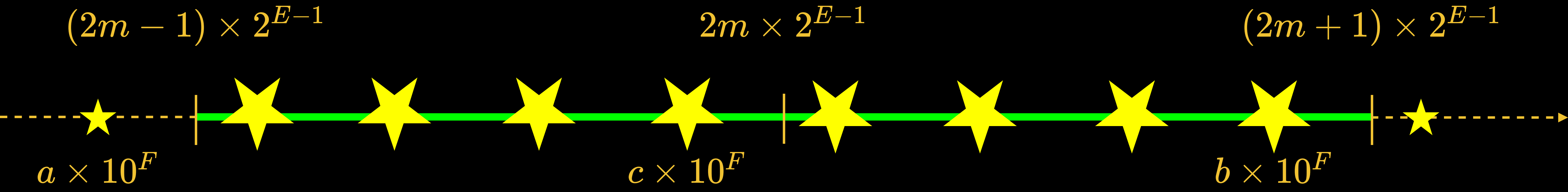


★ Points of the form $n \times 10^F$

Is any ★ $= s \times 10^F$ where s is a multiple of 10?

If so, then we can do better.

Tejú Jaguá - III



★ Points of the form $n \times 10^F$

Is any ★ $= s \times 10^F$ where s is a multiple of 10?

If so, then we can do better.

A good **candidate** is $s = 10\left\lfloor \dfrac{b}{10} \right\rfloor$.

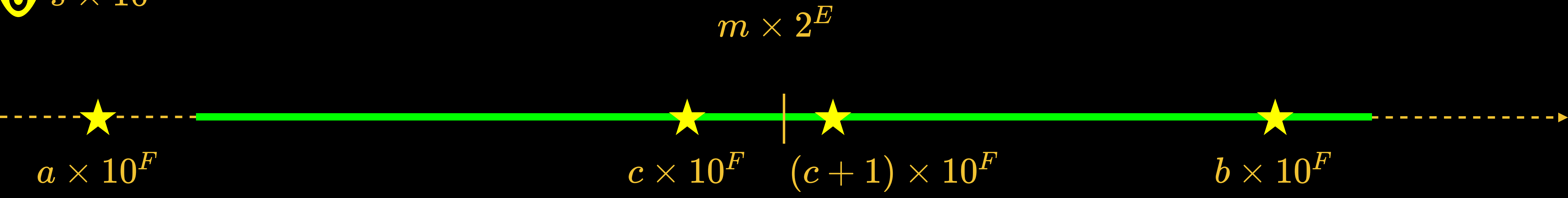
Returns

- the shortest[*], $s \times 10^F$, if it **can**; or
- the closest, $c \times 10^F$ or $(c + 1) \times 10^F$, if it **must**.

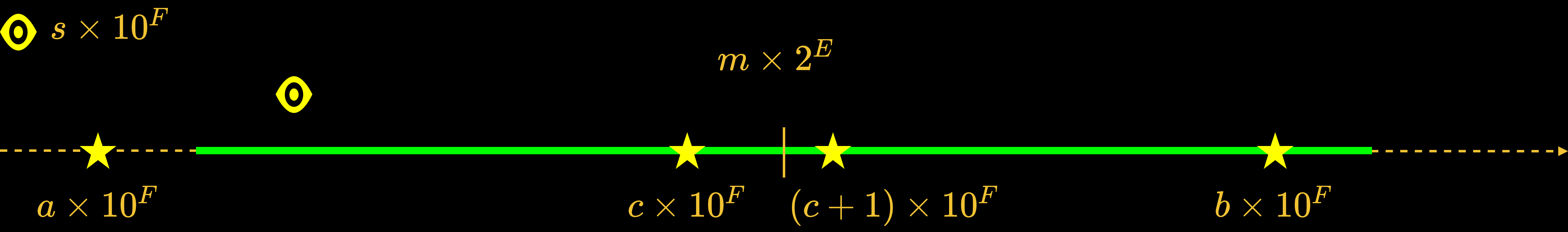
(*) After removing trailing zeros and adjusting exponent.

Tejú Jaguá - V

$s \times 10^F$



Tejú Jaguá - V



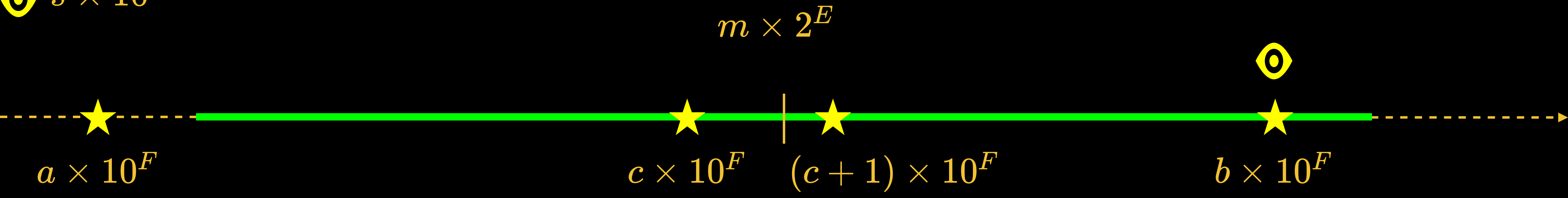
if $a < s$

  if $s < b$

    return the shortest, $s \times 10^F$

Tejú Jaguá - V

$s \times 10^F$



if $a < s$

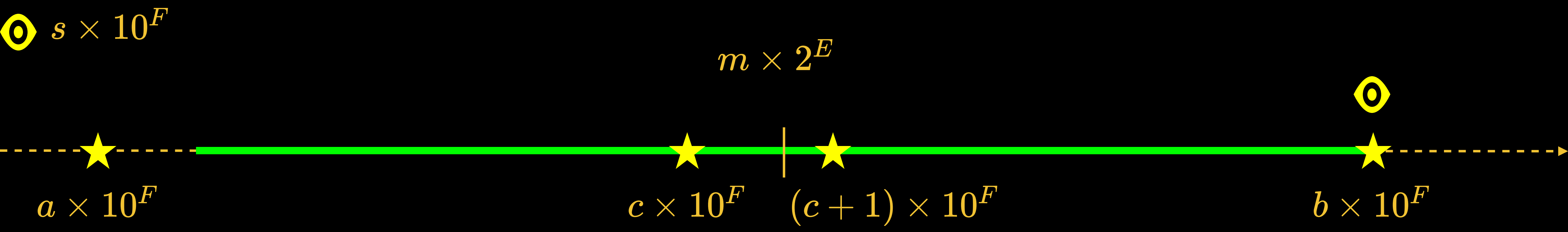
    if $s < b$

        return the shortest, $s \times 10^F$

    if b is not a tie

        return the shortest, $s \times 10^F$

Tejú Jaguá - V



if $a < s$

if $s < b$

return the shortest, $s \times 10^F$

if b is not a tie

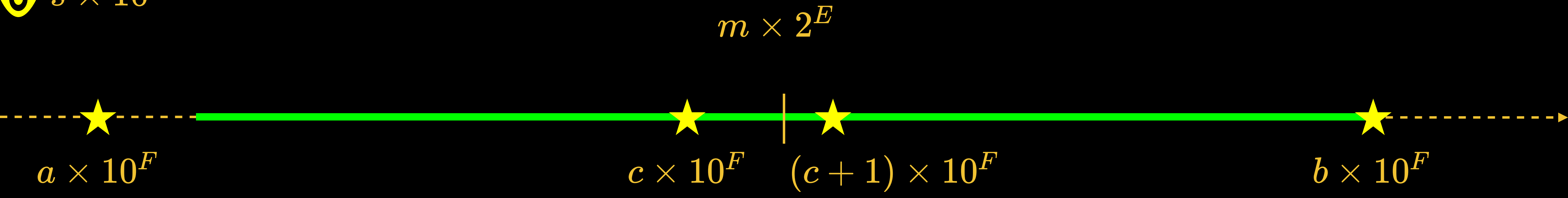
return the shortest, $s \times 10^F$

if m wins against $m + 1$

return the shortest, $s \times 10^F$

Tejú Jaguá - V

$s \times 10^F$



if $a < s$

if $s < b$

return the shortest, $s \times 10^F$

if b is not a tie

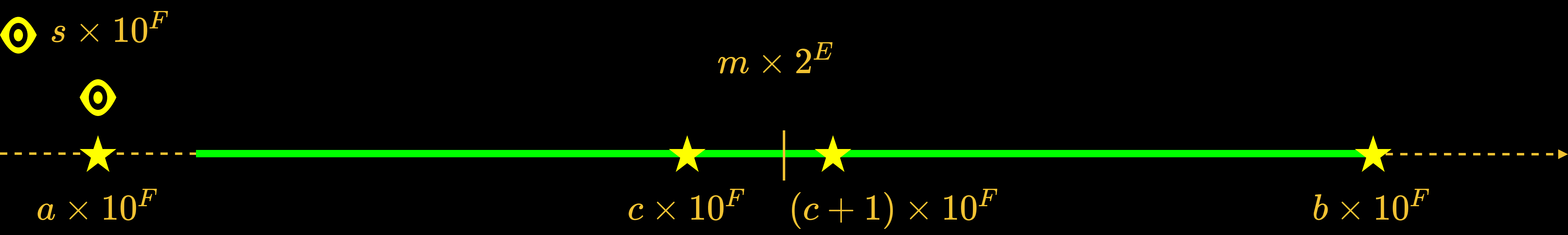
return the shortest, $s \times 10^F$

if m wins against $m+1$

return the shortest, $s \times 10^F$

return the closest, $c \times 10^F$ or $(c+1) \times 10^F$

Tejú Jaguá - V

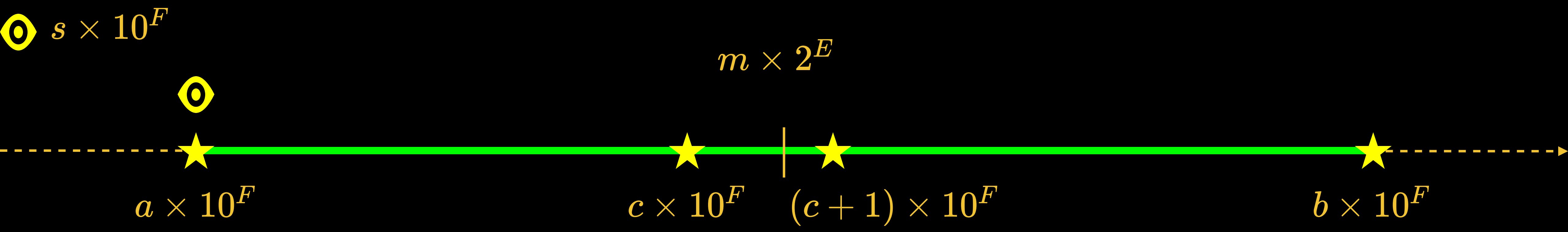


if $a < s$

- if $s < b$
  - return the shortest, $s \times 10^F$
- if b is not a tie
  - return the shortest, $s \times 10^F$
- if m wins against $m + 1$
  - return the shortest, $s \times 10^F$
- return the closest, $c \times 10^F$ or $(c+1) \times 10^F$

if $s = a$

Tejú Jaguá - V



if $a < s$

if $s < b$

return the shortest, $s \times 10^F$

if b is not a tie

return the shortest, $s \times 10^F$

if m wins against $m + 1$

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return the closest, $c \times 10^F$ or $(c + 1) \times 10^F$

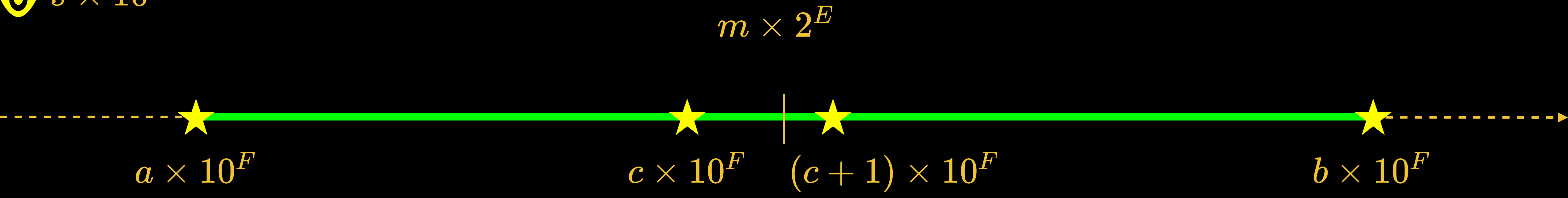
if $s = a$

if a is a tie and m wins against $m - 1$

return the shortest, $s \times 10^F$

Tejú Jaguá - V

$s \times 10^F$



if $a < s$

- if $s < b$
    - return the shortest, $s \times 10^F$
- if b is not a tie
    - return the shortest, $s \times 10^F$
- if m wins against $m+1$
    - return the shortest, $s \times 10^F$
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if $s = a$

- if a is a tie and m wins against $m-1$
    - return the shortest, $s \times 10^F$

return the closest, $c \times 10^F$ or $(c+1) \times 10^F$

The dragon is in the details

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$$F = \lfloor E \times \log(2) \rfloor = \lfloor E \times 0.301... \rfloor$$

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$$\boxed{F = \left\lfloor \frac{1,292,913,987 \times E}{2^{32}} \right\rfloor}$$

$$\forall E \in [-112,815\ ,\ 112,815]$$

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$$\forall E \in [-112,815\ ,\ 112,815]$$

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$$\forall E \in [-112,815\ ,\ 112,815]$$

$$a = \left\lfloor m' \times \frac{2^{E-1}}{10^F} \right\rfloor$$

$$\boxed{a = \left\lfloor \frac{m' \times N(E)}{2^K} \right\rfloor}$$

K = twice the size of floating-point type

The dragon is in the details

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K = twice the size of floating-point type

$N(E) \cong 2^K \times \dfrac{2^{E-1}}{10^F}$ can be precomputed and stored in a look-up table

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$$\forall E \in [-112,815\ ,\ 112,815]$$

Detecting ties becomes a divisibility check by 5^F using the *minverse* algorithm

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Optimisation for c

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$$\forall E \in [-112,815\ ,\ 112,815]$$

Detecting ties becomes a divisibility check by 5^F using the *minverse* algorithm

Optimisation for integers

$$a = \left\lfloor m' \times \frac{2^{E-1}}{10^F} \right\rfloor$$

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K = twice the size of floating-point type

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$$F = \lfloor E \times \log(2) \rfloor = \lfloor E \times 0.301... \rfloor$$

$$F = \left\lfloor \frac{1,292,913,987 \times E}{2^{32}} \right\rfloor$$

$$\forall E \in [-112,815\ ,\ 112,815]$$

Detecting ties becomes a divisibility check by 5^F using the *minverse* algorithm

Optimisation for integers

$$a = \left\lfloor m' \times \frac{2^{E-1}}{10^F} \right\rfloor$$

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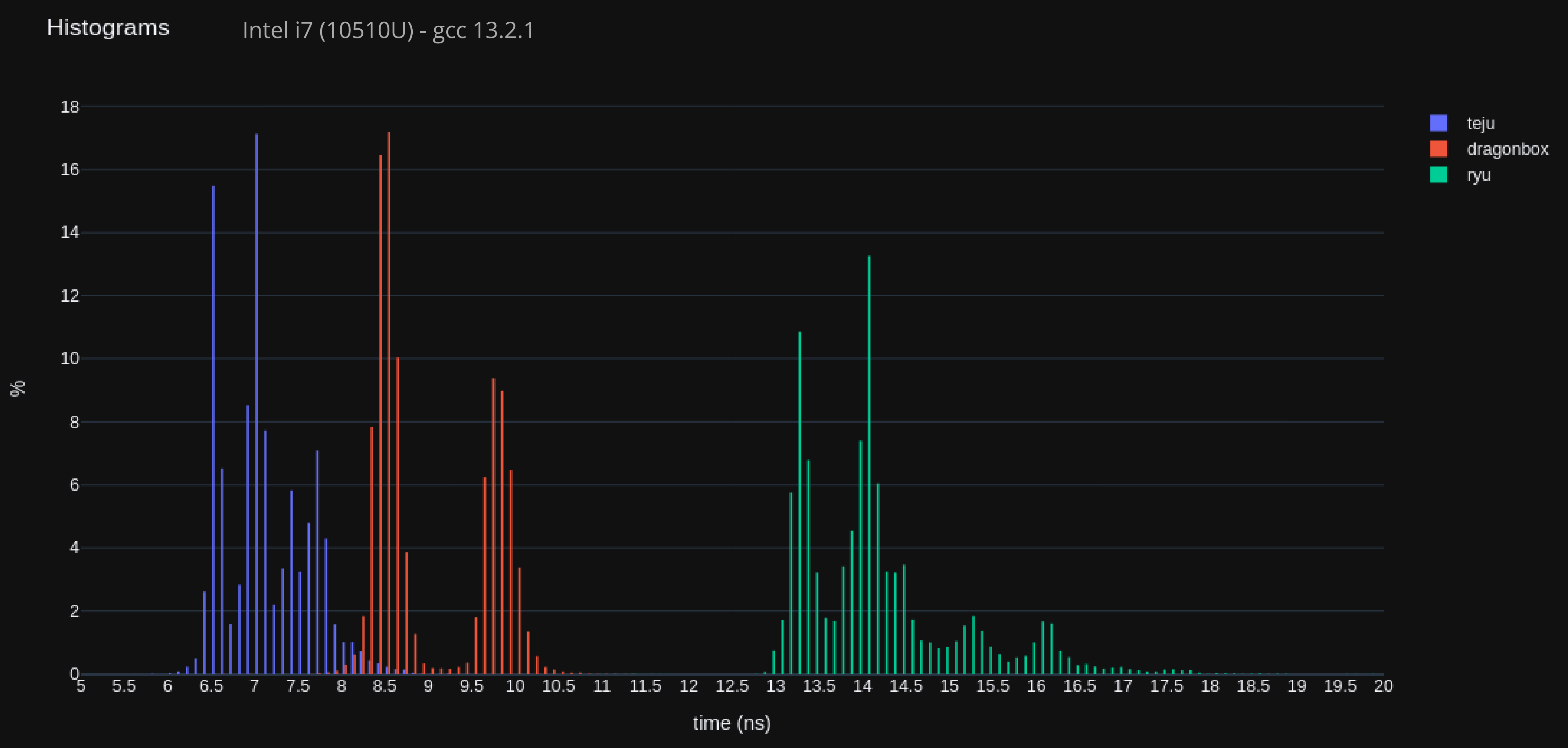
K = twice the size of floating-point type

$N(E) \cong 2^K \times \dfrac{2^{E-1}}{10^F}$ can be precomputed and stored in a look-up table

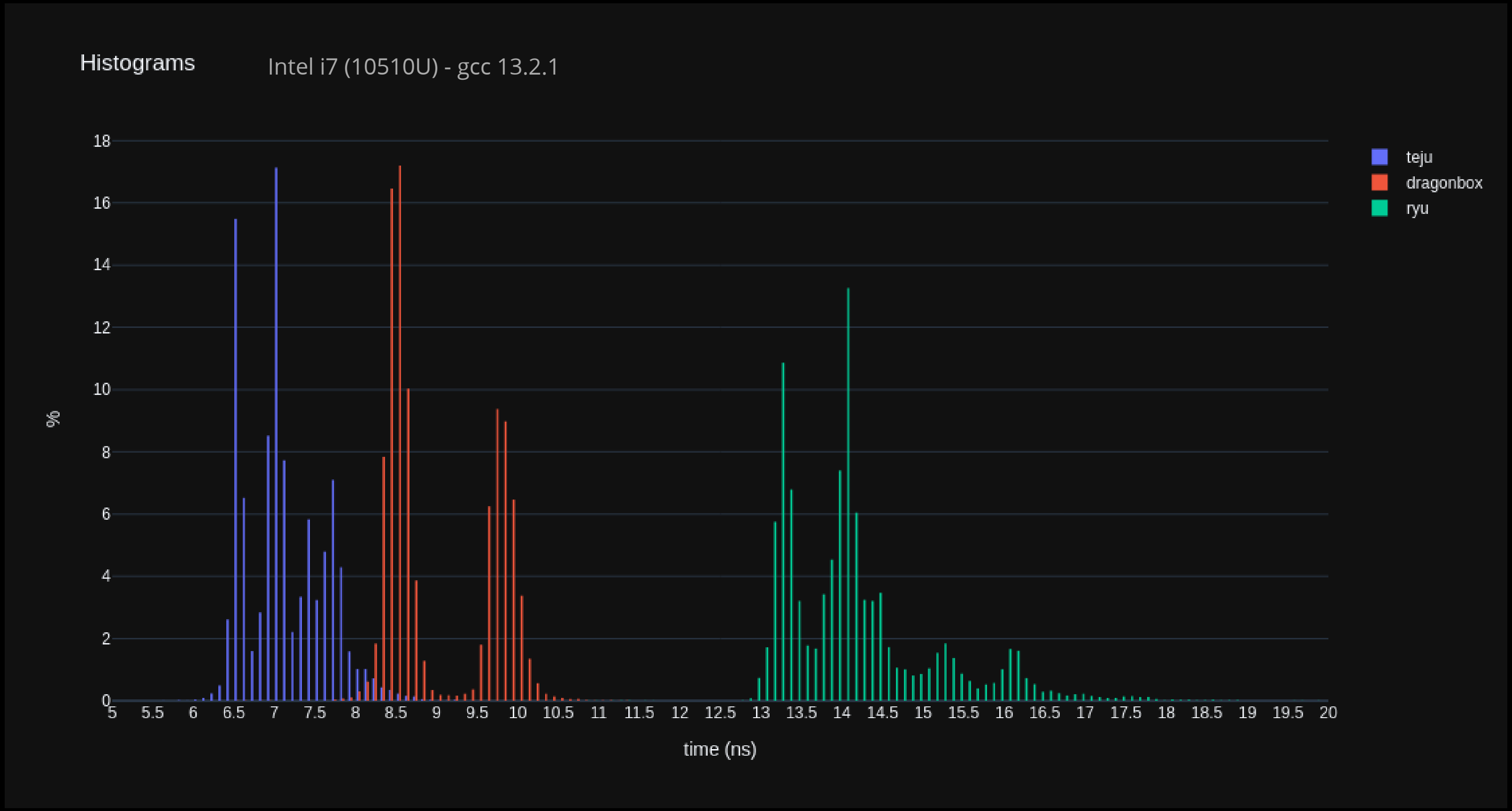
Optimisation for c

Other tricks

Benchmarks - centred

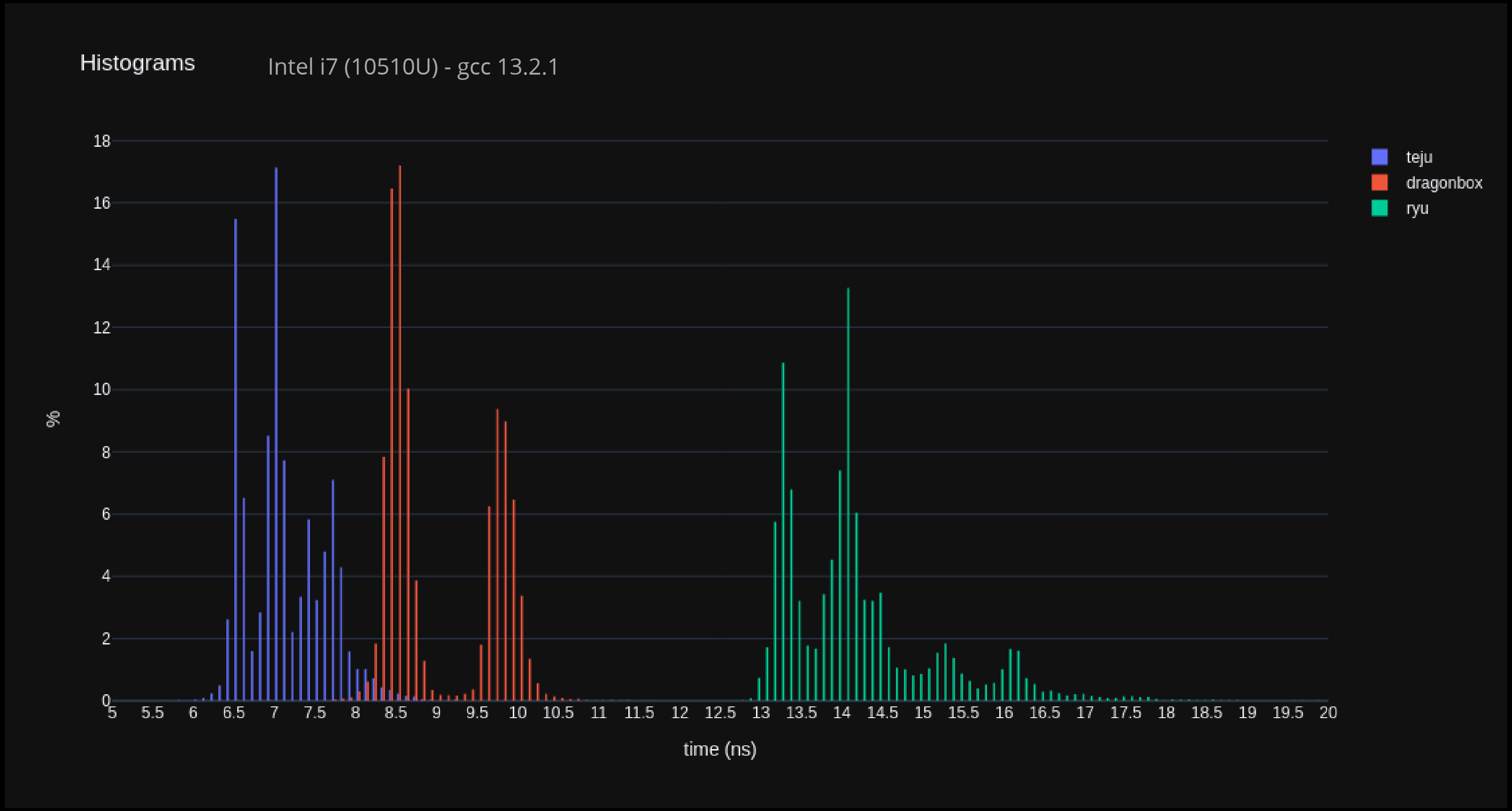


Benchmarks - centred



teju x dragonbox	
wins	99.5%
ties	0.0%
losses	0.5%

Benchmarks - centred



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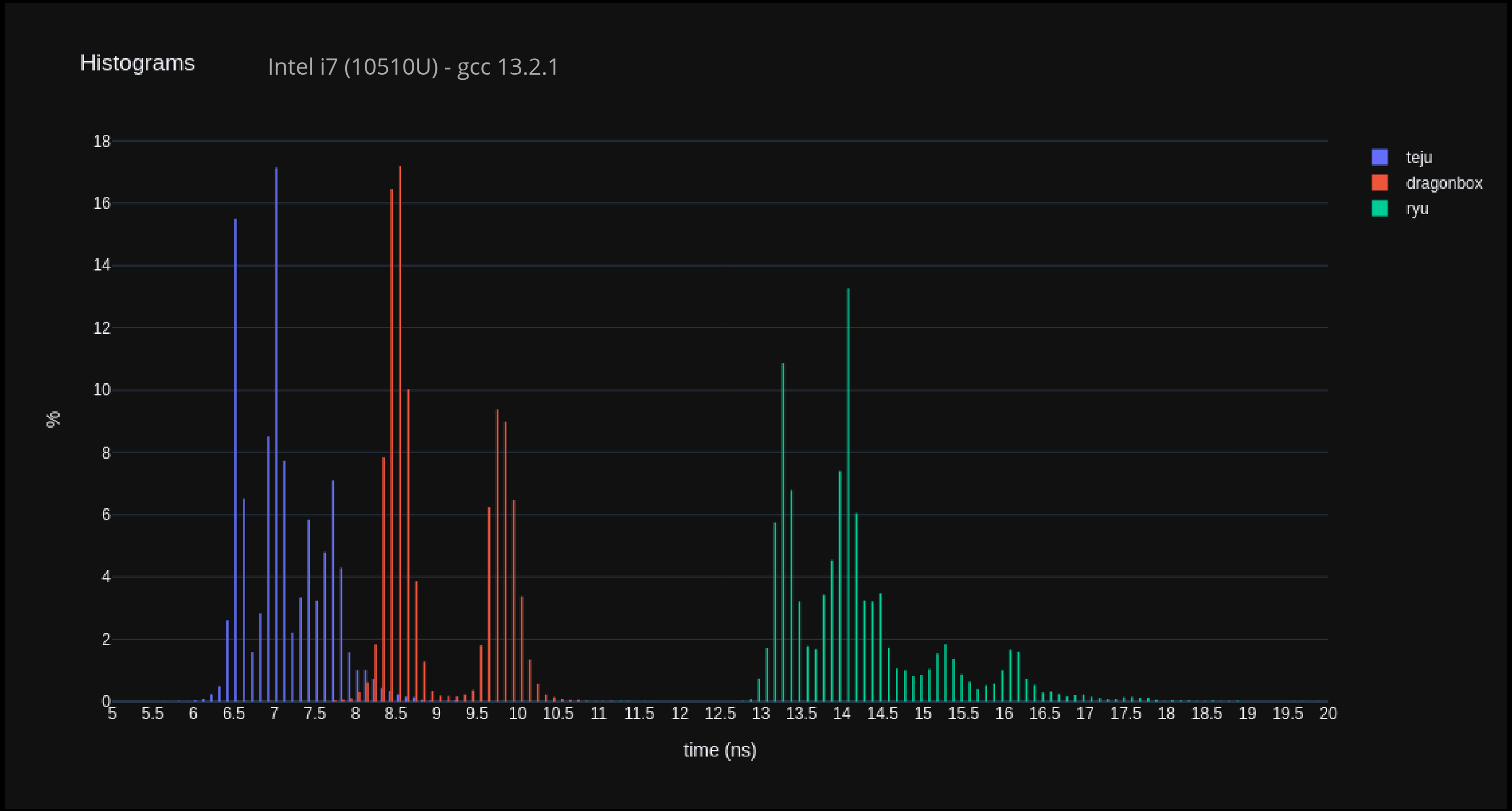
ties	0.0%
losses	0.5%

teju x ryu	
wins	100.0%
ties	0.0%
losses	0.0%

Benchmarks - centred

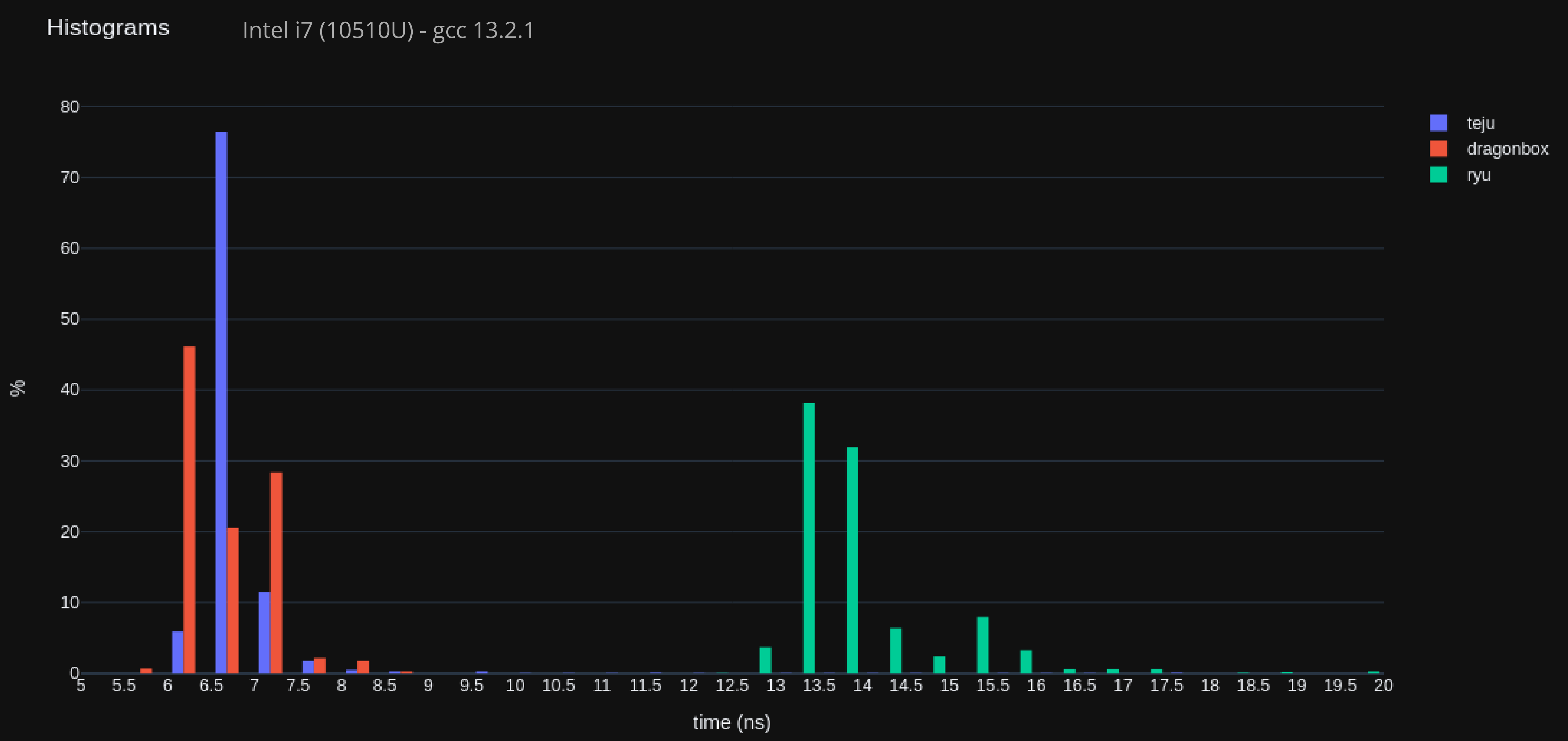


teju x dragonbox	
wins	99.5%
ties	0.0%
losses	0.5%

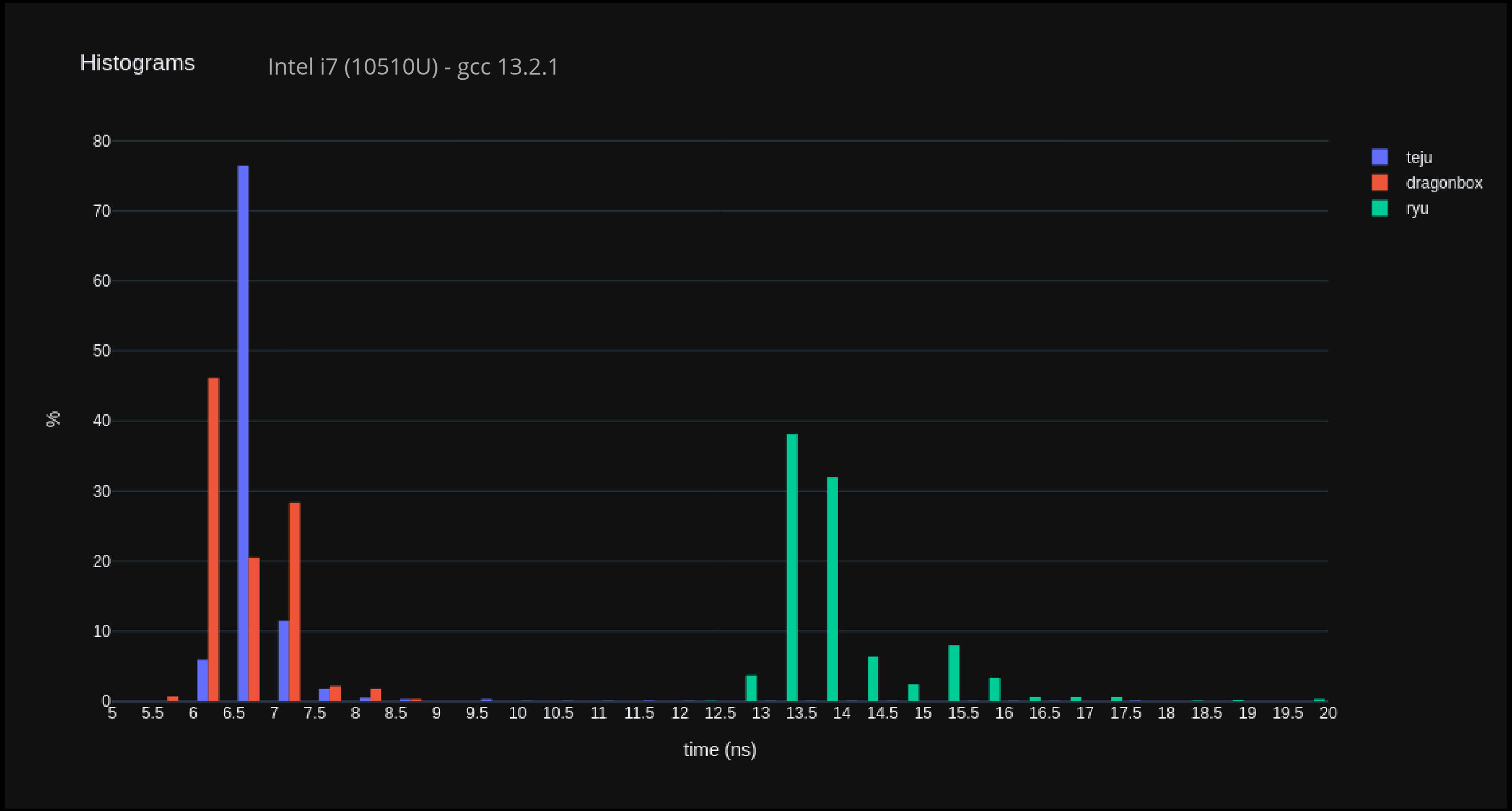
teju x ryu	
wins	100.0%
ties	0.0%
losses	0.0%

	mean	σ	min	median	max	relative
teju	7.14	0.50	2.84	7.06	9.63	1.00
dragonbox	9.04	0.66	6.52	8.64	11.39	1.27
ryu	14.16	0.98	3.40	14.01	20.50	1.98

Benchmarks - uncentred

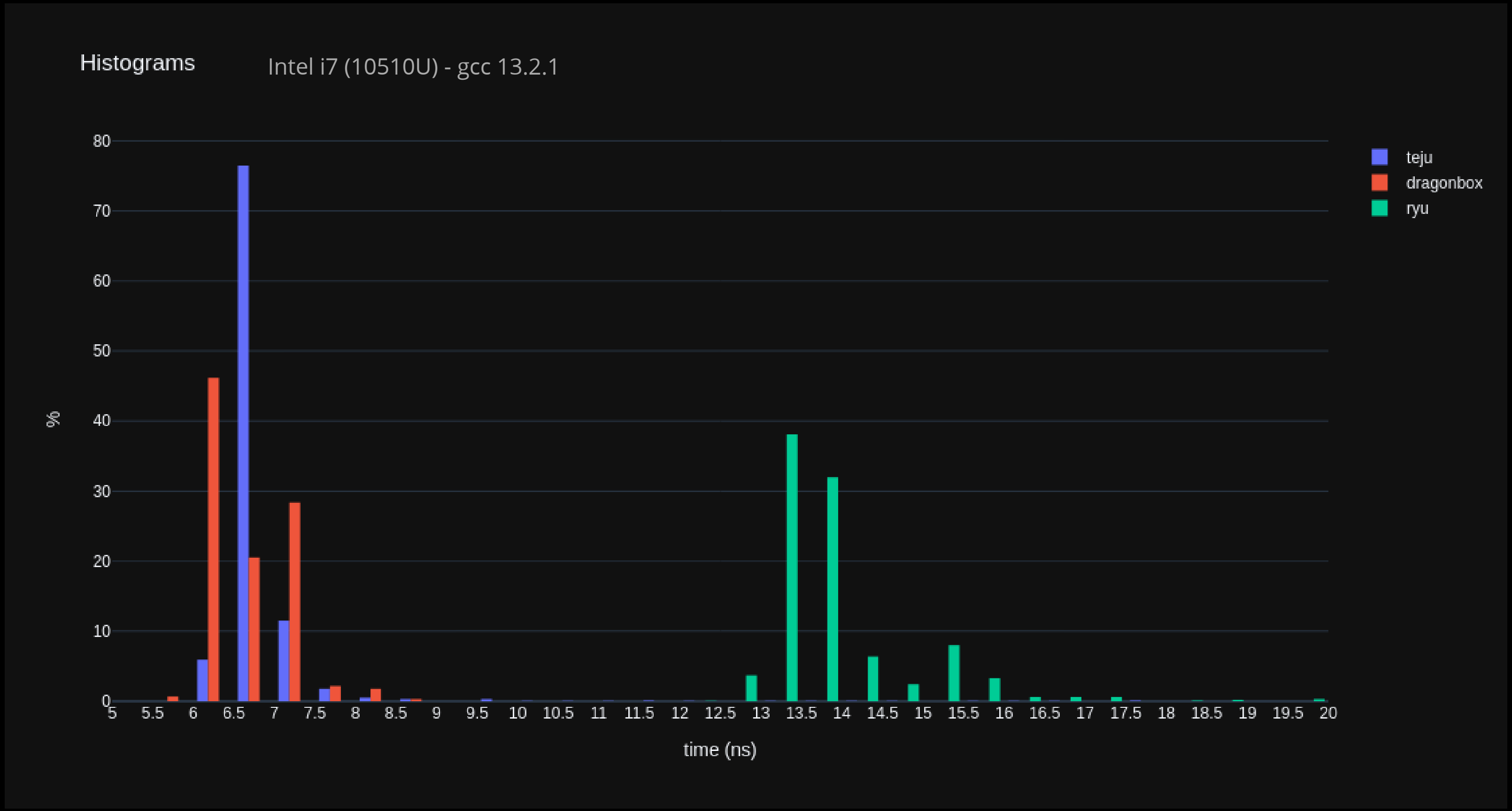


Benchmarks - uncentred



teju x dragonbox	
wins	38.2%
ties	0.0%
losses	61.8%

Benchmarks - uncentred

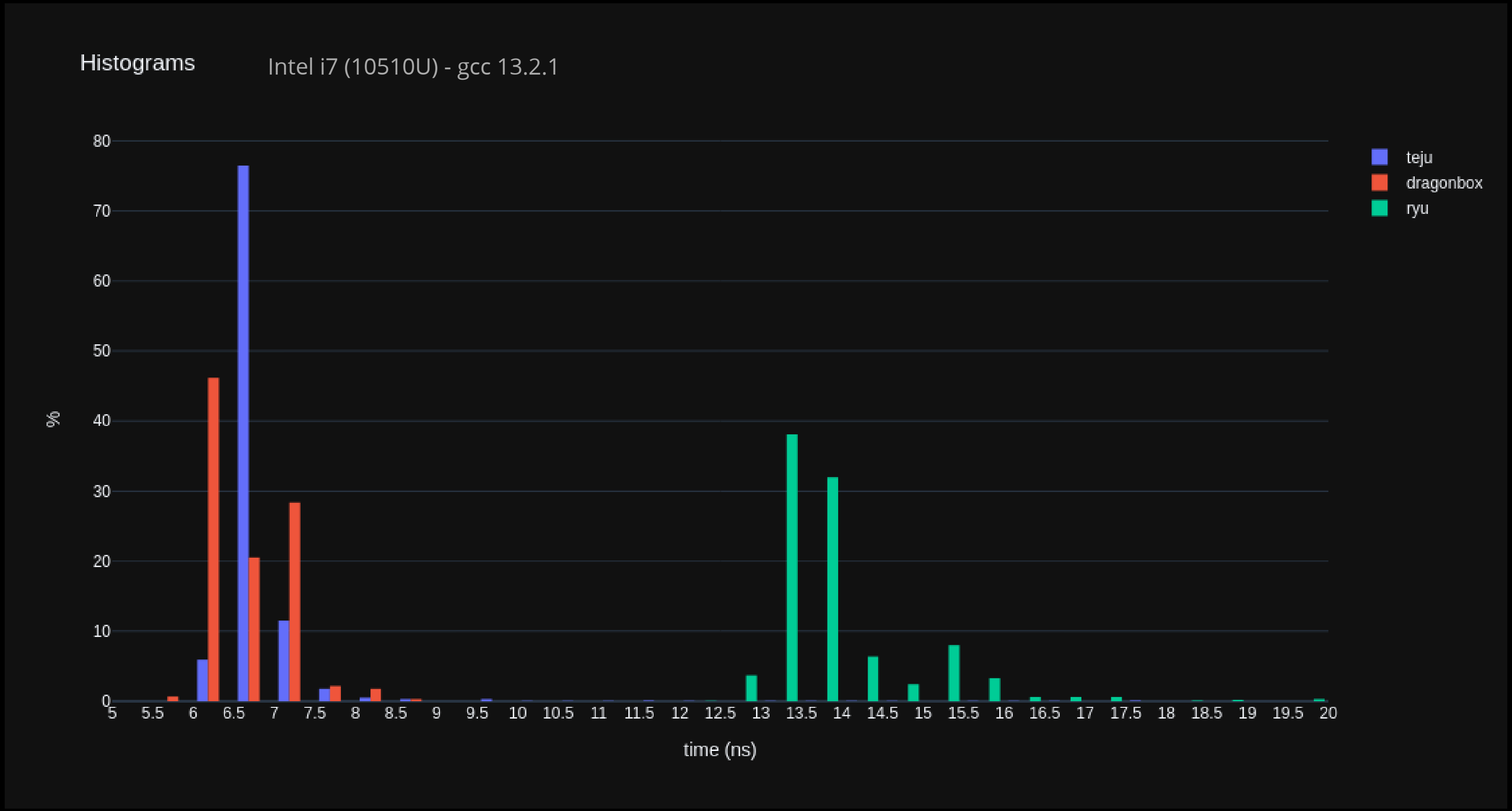


teju x dragonbox	
wins	38.2%
ties	0.0%
losses	61.8%

Only 0.000,000,000,000,01%

of `doubles` are uncentred!

Benchmarks - uncentred



teju x dragonbox	
wins	38.2%
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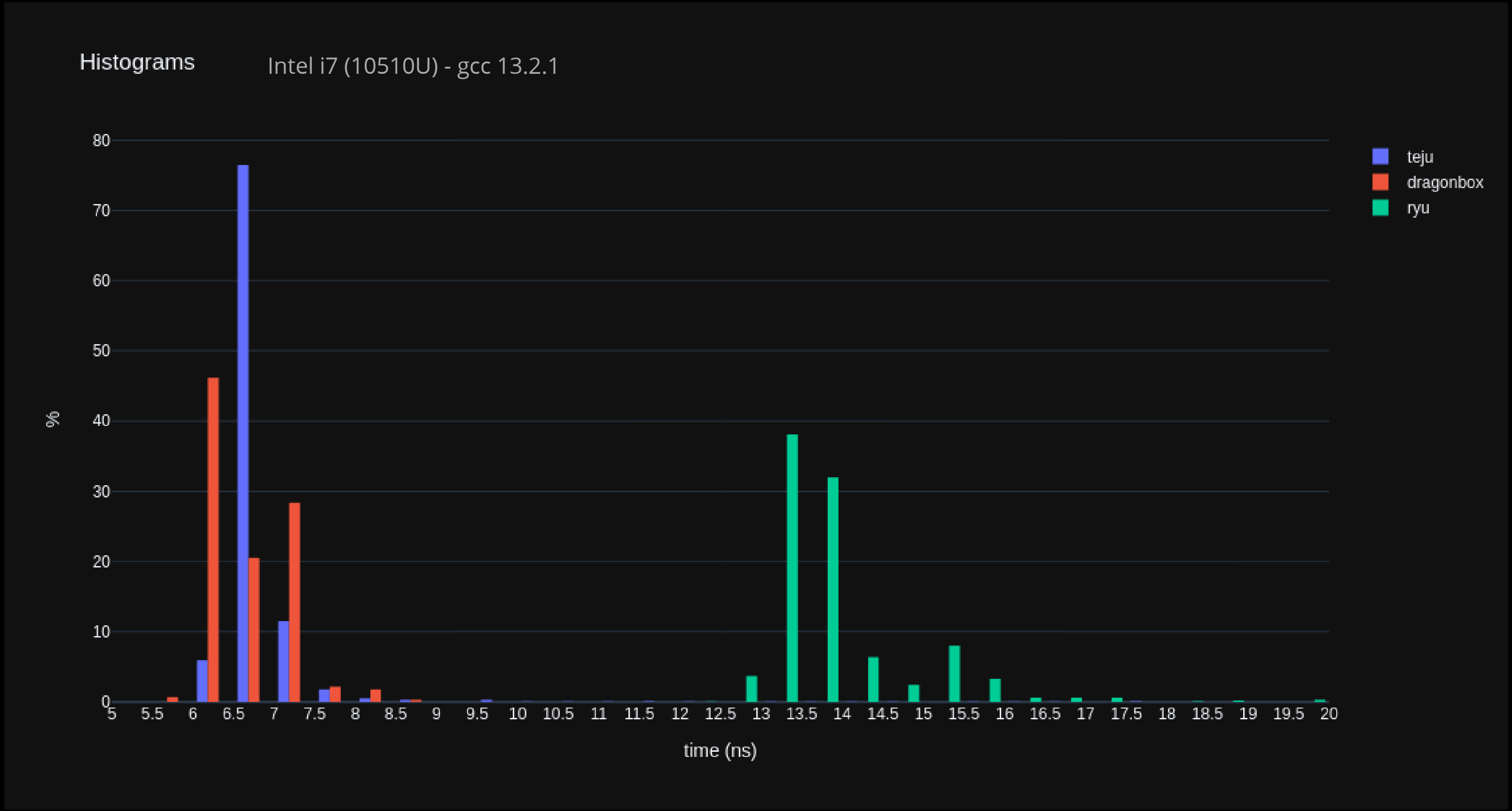
Benchmarks - uncentred

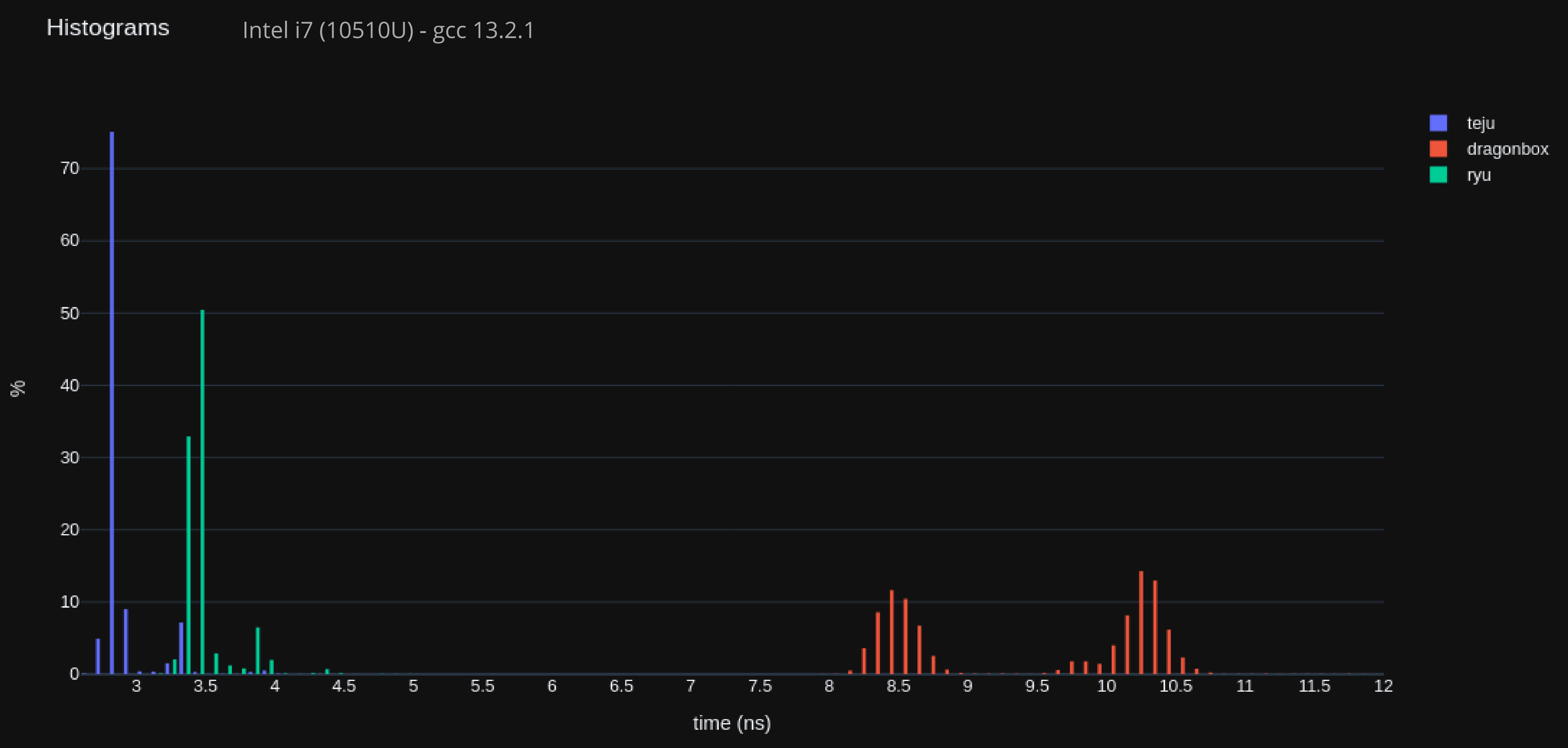


teju x dragonbox	
wins	38.2%
ties	0.0%
losses	61.8%

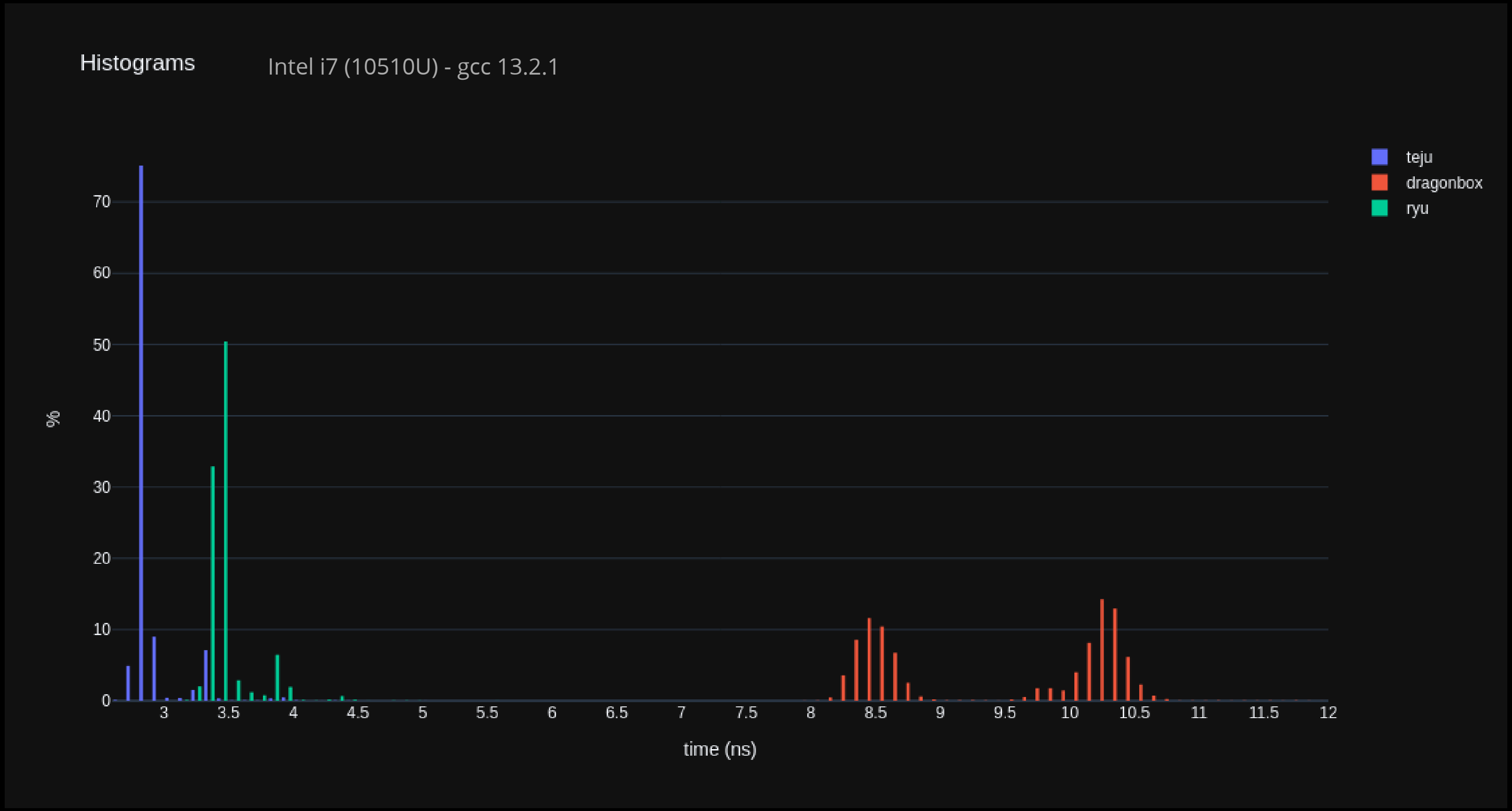
teju x ryu	
wins	100.0%
ties	0.0%
losses	0.0%

	mean	σ	min	median	max	relative
teju	6.78	0.95	2.79	6.79	17.69	1.00
dragonbox	6.72	0.47	5.95	6.53	8.69	0.99
ryu	13.76	2.79	3.34	13.58	45.12	2.03

Benchmarks - integers



Benchmarks - integers



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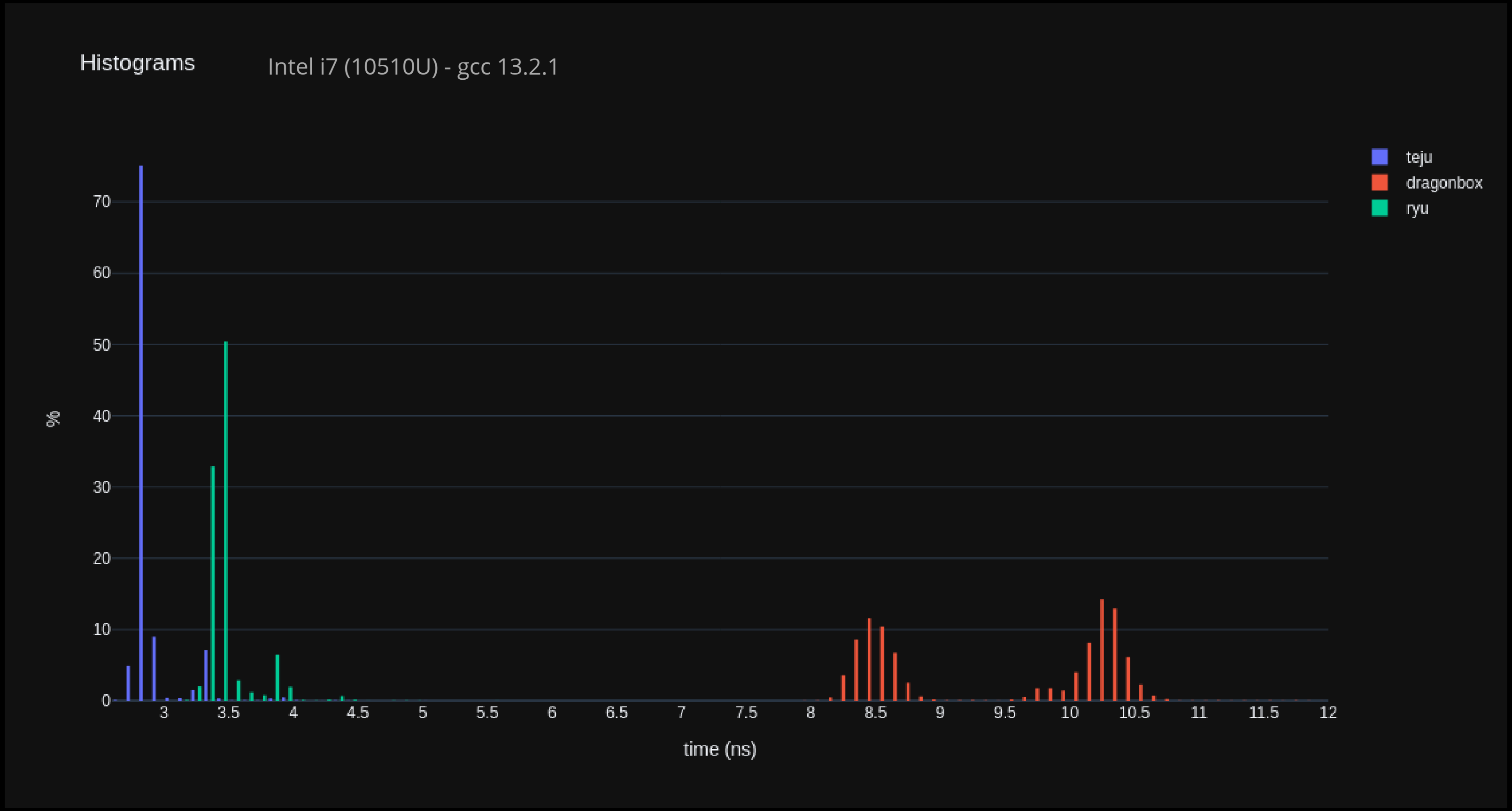
losses	0.0%

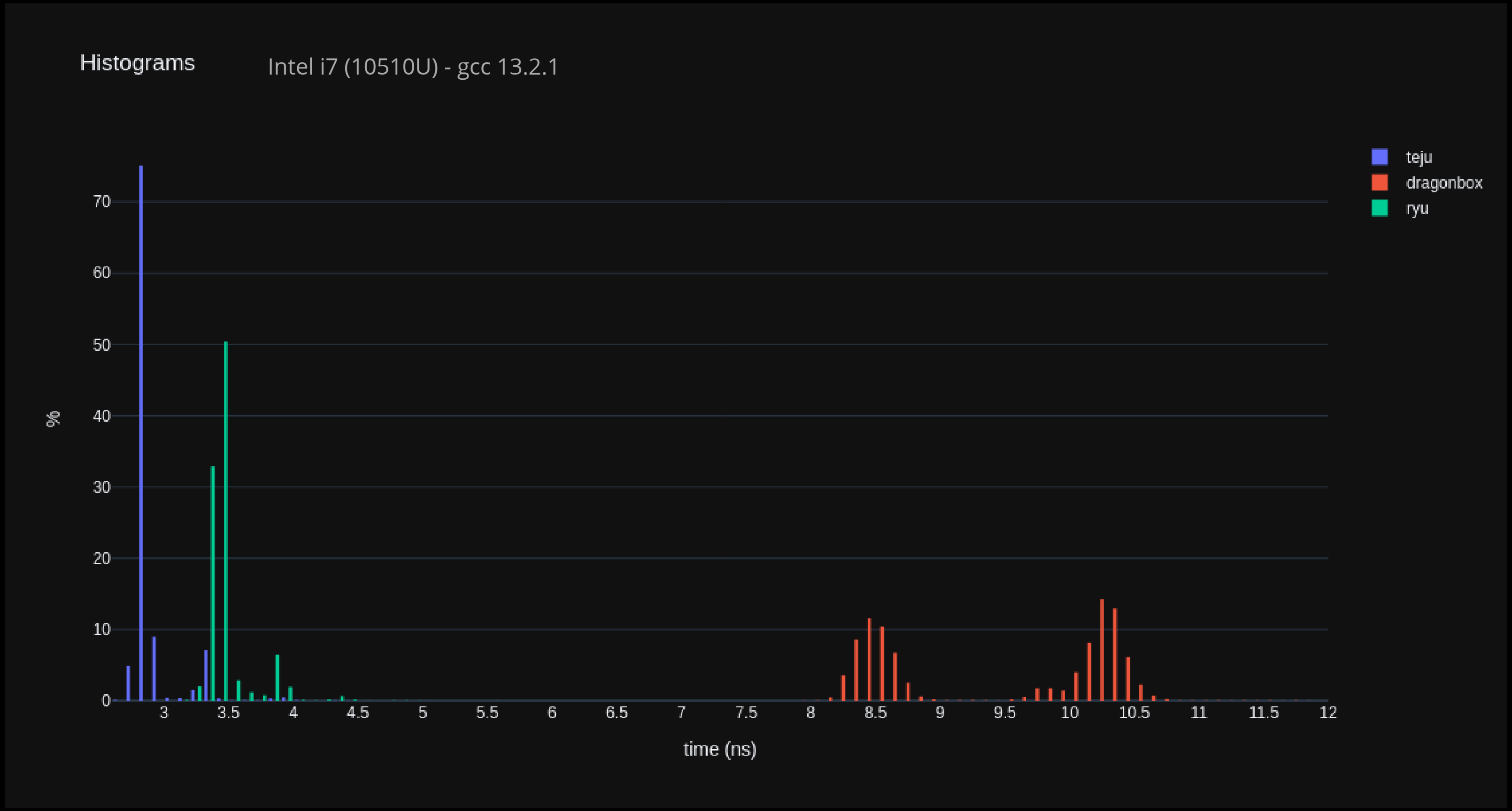
Benchmarks - integers



teju x dragonbox	
wins	100.0%
ties	0.0%
losses	0.0%

teju x ryu	
wins	100.0%
ties	0.0%
losses	0.0%

Benchmarks - integers



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losses	0.0%

teju x ryu	
wins	100.0%
ties	0.0%
losses	0.0%

	mean	σ	min	median	max	relative
teju	2.92	0.18	2.61	2.86	5.05	1.00
dragonbox	9.46	0.90	7.92	10.00	12.58	3.25
ryu	3.47	0.18	3.13	3.42	5.51	1.19

Joseph Campbell

“The very cave you are afraid to enter turns out to be the source of what you are looking for.”

Tejú Jaguá

Tejú Jaguá

Thank you

Cristina Acosta
Aldo Galeano
Jody Hagins
Lorenz Schneider
Manuel Caicoya
Victor Bogado
Felipe Brandão