Statement

Definition : true or false but not both

Compound Statement :

"but" "However" " , "	means p and q
"neither" "nor"	means ~p and ~q
$x \le a$	means $x < a$ or $x = a$
$a \le x \le b$	means $a \le x$ and $x \le b$

Conjunction (AND) : $T \wedge T = T$

Disjunction (OR) : $F \vee F = F$

Exclusive OR (XOR) : $\sim(p \vee q) \equiv (p \vee q) \wedge \sim(p \wedge q)$

* ถ้ามี ∧, ∨ อยู่ในประโยคเดียวกันให้ใส่ ( ) ด้วย

Testing Logically Equipvalent

(เหมือนกันทุกอย่าง)

p	q	$p \wedge q$	$q \wedge p$
T	T	T	T
T	F	F	F
F	T	F	F
F	F	F	F

$p \wedge q$ and $q \wedge p$ always have the same truth values, so they are logically equivalent

Conditional Statement : $T \to F = F$

$p \to q$

p is hypothesis / antecedant

q is conclusion / consequent

De Morgan's Law (กระจาย ~)

$\sim(p \wedge q) \equiv \sim p \vee \sim q$

$-1 < x$ and $x \le 4$.

$-1 \not< x$ or $x \not\le 4$,

$-1 \ge x$ or $x > 4$.

Tautologies & Contradictions

tautology : always true

contradication : always false

Theorem 2.1.1 Logical Equivalences

Given any statement variables p, q, and r, a tautology $\mathbf{t}$ and a contradiction $\mathbf{c}$, the following logical equivalences hold.

1. *Commutative laws:*	$p \wedge q \equiv q \wedge p$	$p \vee q \equiv q \vee p$
2. *Associative laws:*	$(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	$(p \vee q) \vee r \equiv p \vee (q \vee r)$
3. *Distributive laws:*	$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$
4. *Identity laws:*	$p \wedge \mathbf{t} \equiv p$	$p \vee \mathbf{c} \equiv p$
5. *Negation laws:*	$p \vee \sim p \equiv \mathbf{t}$	$p \wedge \sim p \equiv \mathbf{c}$
6. *Double negative law:*	$\sim(\sim p) \equiv p$	
7. *Idempotent laws:*	$p \wedge p \equiv p$	$p \vee p \equiv p$
8. *Universal bound laws:*	$p \vee \mathbf{t} \equiv \mathbf{t}$	$p \wedge \mathbf{c} \equiv \mathbf{c}$
9. *De Morgan's laws:*	$\sim(p \wedge q) \equiv \sim p \vee \sim q$	$\sim(p \vee q) \equiv \sim p \wedge \sim q$
10. *Absorption laws:*	$p \vee (p \wedge q) \equiv p$	$p \wedge (p \vee q) \equiv p$
11. *Negations of* $\mathbf{t}$ *and* $\mathbf{c}$:	$\sim\mathbf{t} \equiv \mathbf{c}$	$\sim\mathbf{c} \equiv \mathbf{t}$

Logical Equipvalent Involving →

$p \vee q \rightarrow r \equiv (p \rightarrow r) \wedge (q \rightarrow r)$

$p \rightarrow q \equiv \sim p \vee q.$ ***

Only If & Biconditional :

T ↔ T = T
F ↔ F = T
(เหมือน T ต่าง F)

p only if q means if ~q then ~p
if p then q

$p \leftrightarrow q$ ≡ when it is tautology

$p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$

$p \leftrightarrow q \equiv (\sim p \vee q) \wedge (\sim q \vee p)$

Order of logical operators

~ → ∧ → ∨ → ⟶ → ⟷

Negation of conditional statement

$\sim(p \rightarrow q) \equiv p \wedge \sim q$

Contrapositive : p→q is ~q→~p

* conditional statement ≡ contrapositive

Converse : p→q is q→p

Inverse : p→q is ~p→~q

Converse & Inverse

* conditional statement ≢ converse
* conditional statement ≢ inverse
* converse ≡ inverse

Necessary Condition & Sufficient Condition

r is a sufficient condition for s means if r then s
r is a necessary condition for s means if ~r then ~s
if s then r
r is not a sufficient condition for s means ~sufficient condition
r is not a necessary condition for s means ~neccessary condition

r is a necessary and sufficient condition for s means r if, and only if, s
r unless s means if ~s then r
*หลัง for เป็น s

Testing an Argument Form for Validity

If all premise true but conclusion false then invalid

If all premise true but conclusion true then valid

p	q	r	$\sim r$	$q \vee \sim r$	$p \wedge r$	$p \to q \vee \sim r$ (premises)	$q \to p \wedge r$ (premises)	$p \to r$ (conclusion)
T	T	T	F	T	T	T	T	T
T	T	F	T	T	F	T	F	
T	F	T	F	F	T	F	T	
T	F	F	T	T	F	T	T	F
F	T	T	F	T	F	T	F	
F	T	F	T	T	F	T	F	
F	F	T	F	F	F	T	T	T
F	F	F	T	T	F	T	T	T

This row shows that an argument of this form can have true premises and a false conclusion. Hence this form of argument is invalid.

Converse Error

$$\begin{array}{l} p \to q \\ q \\ \therefore p \end{array}$$

Inverse Error

$$\begin{array}{l} p \to q \\ \sim p \\ \therefore \sim q \end{array}$$

Knight and Knaves

Knight : always tell the truth

Knaves : always tell the lie

Modus Ponens	$p \to q$, p, $\therefore q$	**Elimination**	a. $p \vee q$, $\sim q$, $\therefore p$ b. $p \vee q$, $\sim p$, $\therefore q$
Modus Tollens	$p \to q$, $\sim q$, $\therefore \sim p$	**Transitivity**	$p \to q$, $q \to r$, $\therefore p \to r$
Generalization	a. p, $\therefore p \vee q$ b. q, $\therefore p \vee q$	**Proof by Division into Cases**	$p \vee q$, $p \to r$, $q \to r$, $\therefore r$
Specialization	a. $p \wedge q$, $\therefore p$ b. $p \wedge q$, $\therefore q$		
Conjunction	p, q, $\therefore p \wedge q$	**Contradiction Rule**	$\sim p \to c$, $\therefore p$

*อะไรก็ตามที่นำไปสู่ข้อขัดแย้งจะเป็นF

The Universal Quantifier : $\forall$

Formal form:

"$\forall x$, if ______ then ______"

$\forall x$, if x is a square then x is a rectangle.

"$\forall$ ______ x, ______"

$\forall$ squares x, x is a rectangle.

Informal form: ใช้ every, any, each, all แทนเครื่องหมาย

universal condition statement :

- Formal : $\forall$ ______, if ______ then ______.
- Informal : If a real number is greater than 2 then its square is greater than 4.

The Existential Quantifier : $\exists$

Formal Form :

$\exists n$ such that ______ $\wedge$ ______.

$\exists$ ______ n such that ______.

Informal form: ใช้ there exists, there is, at least one, for some แทน เครื่องหมาย

“$\forall x \in D,\ Q(x)$.”

1 : quantifiers

2 : predicate

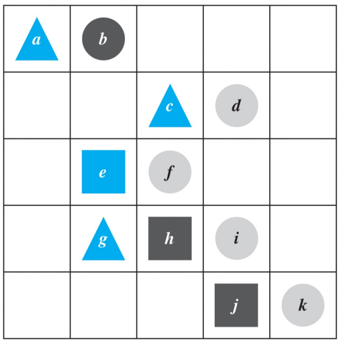
Q → predicate symbol

(x) → predicate variable

Trailing Quantifier

Quantifier A, B → B quantifier A

Tarski's World



Note!
method of exhaustion : check every element

Universal Modus Ponens

Formal Version	Informal Version
$\forall x$, if $P(x)$ then $Q(x)$.	If x makes $P(x)$ true, then x makes $Q(x)$ true.
$P(a)$ for a particular a.	a makes $P(x)$ true.
$\therefore Q(a)$.	$\therefore a$ makes $Q(x)$ true.

Negation of Quantifier Statement

$\sim\forall \equiv \exists$ *อย่าลืมเปลี่ยน such that / ,

$\sim\exists \equiv \forall$

Universal Modus Tollens

Formal Version	Informal Version
$\forall x$, if $P(x)$ then $Q(x)$.	If x makes $P(x)$ true, then x makes $Q(x)$ true.
$\sim Q(a)$, for a particular a.	a does not make $Q(x)$ true.
$\therefore \sim P(a)$.	$\therefore a$ does not make $P(x)$ true.

Negation of a Universal Conditional Statement

$$\sim(\forall x, \text{ if } P(x) \text{ then } Q(x)) \equiv \exists x \text{ such that } P(x) \text{ and } \sim Q(x).$$

Converse Error (Quantified Form)

Formal Version	Informal Version
$\forall x$, if $P(x)$ then $Q(x)$.	If x makes $P(x)$ true, then x makes $Q(x)$ true.
$Q(a)$ for a particular a.	a makes $Q(x)$ true.
$\therefore P(a)$. ← invalid conclusion	$\therefore a$ makes $P(x)$ true. ← invalid conclusion

Two Different Quantifiers

$\forall x$ in D, $\exists y$ in E such that $P(x, y)$

$\exists x$ in D such that $\forall y$ in E, $P(x, y)$

* change order = change meaning

Inverse Error (Quantified Form)

Formal Version	Informal Version
$\forall x$, if $P(x)$ then $Q(x)$.	If x makes $P(x)$ true, then x makes $Q(x)$ true.
$\sim P(a)$, for a particular a.	a does not make $P(x)$ true.
$\therefore \sim Q(a)$. ← invalid conclusion	$\therefore a$ does not make $Q(x)$ true. ← invalid conclusion

Universal Transitivity

Formal Version	Informal Version
$\forall x\, P(x) \to Q(x)$.	Any x that makes $P(x)$ true makes $Q(x)$ true.
$\forall x\, Q(x) \to R(x)$.	Any x that makes $Q(x)$ true makes $R(x)$ true.
$\therefore \forall x\, P(x) \to R(x)$.	$\therefore$ Any x that makes $P(x)$ true makes $R(x)$ true.

Direct Prove & Counter Example

n is even $\Leftrightarrow$ $\exists$ an integer k such that $n = 2k$.
n is odd $\Leftrightarrow$ $\exists$ an integer k such that $n = 2k + 1$.

n is prime $\Leftrightarrow$ $\forall$ positive integers r and s, if $n = rs$ then either $r = 1$ and $s = n$ or $r = n$ and $s = 1$. ไม่ r ก็ s เป็น 1

n is composite $\Leftrightarrow$ $\exists$ positive integers r and s such that $n = rs$ and $1 < r < n$ and $1 < s < n$.

r is rational $\Leftrightarrow$ $\exists$ integers a and b such that $r = \dfrac{a}{b}$ and $b \neq 0$.

Zero Product Property

If neither of two real numbers is zero, then their product is also not zero.

Theorem 4.2.1

Every integer is a rational number.

Theorem 4.2.2

The sum of any two rational numbers is rational.

Corollary 4.2.3

The double of a rational number is rational.

Definition: A number c is called a **root** of a polynomial $p(x)$ if, and only if, $p(c) = 0$.

• Definition

If n and d are integers and $d \neq 0$ then

n **is divisible by** d if, and only if, n equals d times some integer.

Instead of "n is divisible by d," we can say that

n **is a multiple of** d, or
d **is a factor of** n, or
d **is a divisor of** n, or
d **divides** n.

The notation **d | n** is read "d divides n." Symbolically, if n and d are integers and $d \neq 0$:

$d \mid n \quad \Leftrightarrow \quad \exists$ an integer k such that $n = dk$.

Proof:

Theorem: The sum of any even integer and any odd integer is odd.

Proof: Suppose m is any even integer and n is $\underline{\text{(a)}}$. By definition of even, $m = 2r$ for some $\underline{\text{(b)}}$, and by definition of odd, $n = 2s + 1$ for some integer s. By substitution and algebra,

$$m + n = \underline{\text{(c)}} = 2(r + s) + 1.$$

Since r and s are both integers, so is their sum $r + s$. Hence $m + n$ has the form twice some integer plus one, and so $\underline{\text{(d)}}$ by definition of odd.

Counterexample : ใช้ได้กับ $\forall$ เท่านั้น

Method of Direct Proof

1. Express the statement to be proved in the form "$\forall x \in D$, if $P(x)$ then $Q(x)$." (This step is often done mentally.)
2. Start the proof by supposing x is a particular but arbitrarily chosen element of D for which the hypothesis $P(x)$ is true. (This step is often abbreviated "Suppose $x \in D$ and $P(x)$.")
3. Show that the conclusion $Q(x)$ is true by using definitions, previously established results, and the rules for logical inference.

Definition: An integer n is called a **perfect square** if, and only if, $n = k^2$ for some integer k.