HW4 Solution

3.37 → 문제 해석에 따라 차이가 있어 틀에 벗어나지 않는한 정답으로 인정.

ⓐ X : # of students who took the SAT

$P(X=x) = \binom{100}{x}(0.45)^x(0.55)^{100-x}$

∴ $n=100$, $p=0.45$

ⓑ Not a binomial r.v

ⓒ We don't know all of them took a test and probability of scored above avg.

ⓓ Not a binomial r.v

ⓔ we don't know the gender of students.

3.40 $n=20$, $p=0.8$.

ⓐ $P(X=14) = \binom{20}{14}(0.8)^{14}(0.2)^6 = 0.1091$

ⓑ $P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.001$

$= 0.999$

ⓒ $P(14 \leq X \leq 18) = P(X \leq 18) - P(X \leq 13)$

$= 0.931 - 0.087$

$= 0.844$

ⓓ $P(X \leq 16) = 0.589$

3.42 $n=15$ $p=\frac{1}{5}$

ⓐ $p=\frac{1}{4}$

$P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 1.0\ (0.998)$

ⓑ $p=\frac{1}{3}$

$P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.986\ (0.992)$

3.43 $p=0.7$ $n=5$

ⓐ $P(X=5) = \binom{5}{5}(0.7)^5(0.3)^0 = 0.1681$

ⓑ $P(X \geq 4) = P(X=5) + P(X=4)$

$= 1 - P(X \leq 3)$

$= 1 - 0.472$

$= 0.528$

3.49 $p=0.25$, $n=3$

ⓐ $P(X=3) = \binom{3}{3}(0.25)^3(0.75)^0 = 0.016$

ⓑ $P(X=1) = \binom{3}{1}(0.25)^1(0.75)^2 = 0.4219$

ⓒ $P(\text{3rd develop} \mid \text{1st, 2nd not}) = \frac{0.75 \cdot 0.75 \cdot 0.25}{0.75 \cdot 0.75}$

$= 0.25$

3.53 $n=15$ $p=0.5$

ⓐ X : prefer A

$P(X \geq 10) = 1 - P(X \leq 9)$

$= 1 - 0.849$

$= 0.151$

ⓑ Y : prefer B

$P(X \geq 10) + P(Y \geq 10)$

$= 0.151 + 0.151$

$= 0.302$

3.58 X : # Survive , $n=20$, $p=0.8$

ⓐ $P(X=14) = \binom{20}{14}(0.8)^{14}(0.2)^{6} = 0.1091$

ⓑ $P(X\geq 10) = 1 - P(X\leq 9)$

$= 1 - 0.001$

$= 0.999$

ⓒ $P(X \leq 16) = 0.589$

ⓓ $E(X) = np = 20 \cdot 0.8 = 16$

$V(X) = npq = 20 \cdot 0.8 \cdot 0.2 = 3.2$

3.60 $n=4$, $p=0.1$

$E(Y) = n \cdot p = 4 \cdot 0.1 = 0.4$

$V(Y) = npq = 0.4 \cdot 0.9 = 0.36$

$E(Y^2) = V(Y) + E^2(Y)$

$= 0.36 + 0.16$

$= 0.52$

$\Rightarrow E(C) = E[3Y^2 + Y + 2]$

$= 3E(Y^2) + E(Y) + 2$

$= 3 \cdot 0.52 + 0.4 + 2$

$= 3.96$

3.67 $p=0.3$

$\Rightarrow (0.7)^4(0.3) = 0.07203$

3.76 $p=0.6$

$\Rightarrow P(X=5) = (0.4)^4(0.6) = 0.01536$

$\Rightarrow P(X\geq 5) = \sum_{x=5}^{\infty}(0.4)^{x-1}(0.6)$

$= 0.6 \times \left(\frac{(0.4)^4}{1-0.4}\right)$

$= (0.4)^4$

$= 0.0256$

3.78 $p=0.3$

$P(Y > y_0) \geq 0.1$

$\Rightarrow 1 - \sum_{y=0}^{y_0}(0.7)^{y-1}(0.3) \geq 0.1$

$\Rightarrow \sum_{y=0}^{y_0}(0.7)^{y-1}(0.3) \leq 0.9$

$\Rightarrow \frac{0.3(1-(0.7)^{y_0})}{1-0.7} \leq 0.9$

$\Rightarrow (0.7)^{y_0} \geq 0.1$

$\Rightarrow y_0 \ln 0.7 \geq \ln 0.1$

$\Rightarrow y_0 \leq \frac{\ln 0.1}{\ln 0.7}$

$\Rightarrow y_0 \leq 6.46$

$\therefore y_0 = 6$

3.79 $p = 0.5$

$E(X) = \frac{1}{p} = \frac{1}{0.5} = 2$

3.91 $X \sim NB(3, 0.4)$, $r=3$, $p=0.4$

$E(20X) = 20 \cdot E(X)$

$= 20 \cdot \frac{3}{0.4}$

$= 150$

$V(20X) = 20^2 V(X)$

$= 400 \cdot \frac{3(1-0.4)}{(0.4)^2}$

$= 4500$

3.95

$p(\text{defective}) = 0.1$

$p(\text{nondefective}) = 0.9$ ✓

$X \sim \text{Geom}(0.9)$

$\Rightarrow P(X \geq 4 \mid X > 2)$

$= P(X > 3 \mid X > 2)$

$= \dfrac{P(X>3)}{P(X>2)}$ — memoryless property

$= \dfrac{(0.1)^3}{(0.1)^2}$

$= (0.1)^{3-2}$

$= 0.1$

3.96

ⓐ $X \sim \text{Geom}(0.4)$

$P(X=1) = 0.4$

$P(X=2) = 0.6 \times 0.4 = 0.24$

$P(X=3) = (0.6)^2 \times 0.4 = 0.144$

ⓑ $Y \sim NB(2, 0.4)$

$P(Y=4) = \binom{3}{1}(0.4)^2(0.6)^2$

$= 0.1728$