

## Unit I: $Ax = b$ and the Four Subspaces

### Session 1.1: The Geometry of Linear Equations

We have a system of equations:

$$2x - y = 0$$

$$-x + 2y = 3$$

#### Row Picture

Line  $2x - y = 0$  and line  $-x + 2y = 0$  intersects at the point  $(1, 2)$ , so  $(1, 2)$  is the solution of the system of equations.

Maybe I should draw a X-Y coordinates here...

#### Column Picture

We rewrite the system of linear equations as a single equation:

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

We see  $x$  and  $y$  as coefficients of column vectors:  $\mathbf{v}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$  and  $\mathbf{v}_2 = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$ , and the sum  $x\mathbf{v}_1 + y\mathbf{v}_2$  is called a *linear combination* of  $\mathbf{v}_1$  and  $\mathbf{v}_2$ .

Geometrically, we can find one copy of  $\mathbf{v}_1$  added to two copies of  $\mathbf{v}_2$  just equals the vector  $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$ . Then the solution should be  $x = 1, y = 2$ .

I will add a figure when time is available >\_>

#### Matrix Picture

We rewrite the equations in our example as a compact form,

$$A\mathbf{x} = \mathbf{b},$$

that is

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

### Matrix Multiplication

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

A matrix times by a vector is just **a linear combination of the column vectors of the matrix.**