Unit I: Ax = b and the Four Subspaces

Seesion 1.1: The Geometry of Linear Equations

We have a system of equations:

$$\begin{aligned} 2x - y &= 0 \\ -x + 2y &= 3 \end{aligned}$$

Row Picture

Line $2x - y = 0$ and line $-x + 2y = 0$ intersects at the point $(1, 2)$, so $(1, 2)$ is the solution of the system of equations. > Maybe I should draw a X-Y coordinates here…

Column Picture

We rewrite the system of linear equations as a single equation:

$$x \begin{bmatrix} 2 \\ -1 \end{bmatrix} + y \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

We see x and y as coefficients of column vectors: $\boldsymbol{v_1} = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ and $\boldsymbol{v_2} = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$, and the sum $x\boldsymbol{v_1} + y\boldsymbol{v_2}$ is called a *linear combination* of $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$.

Geometrically, we can find one copy of $\boldsymbol{v_1}$ added to two copies of $\boldsymbol{v_2}$ just equals the vector $\begin{bmatrix} 0 \\ 3 \end{bmatrix}$. Then the solution should be $x = 1, y = 2$. > I will add a figure when time is available >_>

Matrx Picture

We rewrite the equations in our example as a compact form,

$$A\boldsymbol{x} = \boldsymbol{b},$$

that is

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Matrix Multiplication

$$\begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} = 1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} + 2 \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

A matrix times by a vector is just **a linear combination of the column vectors of the matrix**.