Game Theory 04-03

Exercise

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Answer on Google Slides

Use the shared Google Slides deck to upload your answers. Summarise your reasoning and cite any references you consult.

https://sites.google.com/vju.ac.vn/bcse-gt

Notes

1. Record teammates who collaborated on the submission.
2. Handwritten work is welcome—take clear photos and upload them.

Q1. Indifference Method Drill

		Column	
Row	Left	(4,-4)	(1,-1)
	Right	(0,0)	(3,-3)

1. Let Row mix p on Left and Column mix q on Left. Write each player's expected payoff from their pure strategies.
2. Solve the indifference conditions and probability constraints to obtain (p^*, q^*).
3. Verify that each player is indifferent over the support and state the equilibrium value for Row.

Q2. Best-Response Geometry

For the penalty-kick game normalised in 04-01, the best-response correspondences are

Keeper:

$p < \frac{1}{2}$: Dive Right

$p = \frac{1}{2}$: Dive Left or Right

$p > \frac{1}{2}$: Dive Left

Kicker:

$q < \frac{1}{2}$: Shoot Left

$q = \frac{1}{2}$: Either side

$q > \frac{1}{2}$: Shoot Right

1. Sketch the unit square with these best responses and mark the intersection corresponding to the mixed equilibrium.
2. Compute the expected scoring probability at that intersection and explain why deviating increases the opponent's payoff.
3. Briefly discuss how the lines shift if the kicker's left shot is saved with probability 0.7 while the right shot is saved with probability 0.4.

Q3. Asymmetric Rock–Paper–Scissors

Consider the payoff matrix from 04-02 where Player 1 values Rock, Scissors, Paper wins as $(1, 2, 5)$.

1. Replicate the indifference equations for Player 2's mix (x_R, x_S, x_P) and solve for the probabilities.
2. Derive Player 1's optimal mix that keeps Player 2 indifferent and compute the game value.
3. Comment on which action Player 1 now plays most often and relate this to the support conditions.

Q4. Volunteer Dilemma Variations

Suppose n identical players face benefit $b = 6$ and cost $c = 2$.

1. Solve for the symmetric mixed-strategy equilibrium probability $p^*(n)$.
2. Evaluate $p^*(n)$ at $n = 2, 3, 6$ and interpret how the incentive to volunteer changes.
3. Compute $\lim_{n\to\infty} p^*(n)$ and explain the economic intuition for the limit.

Q5. Checking Kakutani's Assumptions

Let a finite two-player game have mixed-strategy profile space $X = \Delta(S_1) \times \Delta(S_2)$ and best-response correspondence BR.

1. Specify which Kakutani condition is guaranteed by the fact that $\Delta(S_i)$ is closed and bounded.
2. Explain why linearity of expected payoffs implies convex-valued best responses.
3. Describe how upper hemicontinuity of BR follows from Berge's maximum theorem or the closed-graph argument covered in 04-02.