BCSE Game Theory 11-02

Advanced Signaling & Refinements

BCSE Game Theory

Dec. 30, 2025

Recap & Framework

Recap from 11-01

- **PBE** = Strategies + Beliefs.
- **Sequential Rationality**: Optimization at every information set.
- **Consistency**: Beliefs derived via Bayes' Rule (where possible).
- **4-Step Algorithm**: Conjecture → Beliefs → R's BR → S's IC.
- **Gift Game**: Separating failed, Pooling succeeded with off-path beliefs.
- Today: Deep dive into **Steak-Pasta Game** and **Refinements**.

Signaling Game Structure

1. **Nature** picks Sender's type $t \in T$ with prior $p(t)$.
2. **Sender** observes t, chooses message $m \in M$.
3. **Receiver** observes m (not t), chooses action $a \in A$.
4. **Payoffs**: $U_S(t, m, a)$, $U_R(t, m, a)$.

Strategy Types

- **Separating**: Different types send different messages. $(t_1 \to m_1, t_2 \to m_2)$.
- **Pooling**: Different types send the SAME message. $(t_1 \to m_1, t_2 \to m_1)$.
- **Semi-Separating (Hybrid)**: Randomization used, partial information revealed.

Steak-Pasta Game: Complete Analysis

The Steak-Pasta Game Setup

Scenario

- **Player 1 (Row)**: Either **Tough** (Strong) or **Timid** (Weak).
- Prior: $P(\text{Tough}) = 0.1$, $P(\text{Timid}) = 0.9$.
- **Action**: Choose lunch: **Steak (St)** or **Pasta (Pa)**.
- **Player 2 (Column)**: Observes lunch choice, decides to **Challenge (C)** or **Not Challenge (N)**.

Preferences:

- **Tough**: Prefers Steak (bold/hearty), doesn't mind confrontation.
- **Timid**: Prefers Pasta (comforting), avoids confrontation.
- **Player 2**: Wants to Challenge Timid, avoid Challenging Tough.

Payoff Matrices

Tough chooses Steak:

	C	N
Steak	(1, 0)	(3, 0)

Timid chooses Steak:

	C	N
Steak	(0, 1)	(2, 1)

Tough chooses Pasta:

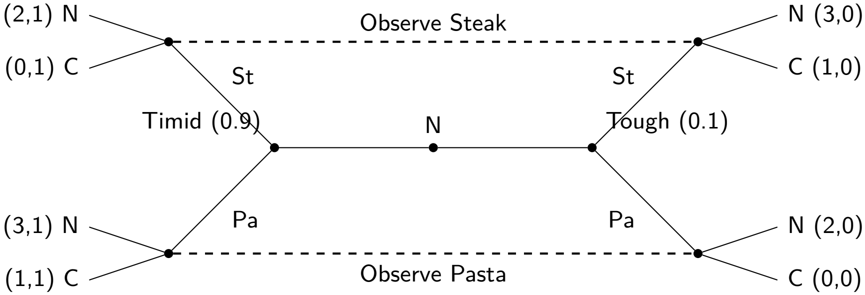
	C	N
Pasta	(0, 0)	(2, 0)

Timid chooses Pasta:

	C	N
Pasta	(1, 1)	(3, 1)

- Tough: Steak $\succ$ Pasta. Payoff higher with Steak regardless of P2's action.
- Timid: Pasta $\succ$ Steak. Payoff higher with Pasta regardless of P2's action.
- Both: Not Challenge $\succ$ Challenge (first component increases).

The Game Tree



Candidate Equilibrium 1: Separating

Conjecture: Tough → Steak, Timid → Pasta.

- **Beliefs**: $\mu(\text{Tough}|St) = 1$, $\mu(\text{Timid}|Pa) = 1$.
- **P2's Best Response**:
  - If Steak: Believes Tough. Not Challenge (0 vs 0, indifferent, assume N).
  - If Pasta: Believes Timid. Challenge (1 vs 1, indifferent, assume C).
- **Check P1's Incentives**:
  - Tough: Steak → N gives 3. Pasta → C gives 0. OK.
  - Timid: Pasta → C gives 1. Steak → N gives 2. **DEVIATION!**

Conclusion

Separating equilibrium **FAILS**. Timid wants to mimic Tough to avoid Challenge.

Candidate Equilibrium 2: Pooling on Steak

Conjecture: Both → Steak.

- **Beliefs on-path**: $\mu(\text{Tough}|St) = 0.1$ (prior).
- **P2's Best Response to Steak**:
  - $E[U(C)] = 0.1(0) + 0.9(1) = 0.9$.
  - $E[U(N)] = 0.1(0) + 0.9(1) = 0.9$.
  - Indifferent. Assume **Not Challenge**.
- **Off-path beliefs for Pasta**: Assume $\mu(\text{Timid}|Pa) = 1$. P2 Challenges.
- **Check P1's Incentives**:
  - Tough: Steak → N gives 3. Pasta → C gives 0. OK.
  - Timid: Steak → N gives 2. Pasta → C gives 1. OK.

Result

Pooling on Steak **IS** a PBE.

Candidate Equilibrium 3: Pooling on Pasta

Conjecture: Both → Pasta.

- **Beliefs on-path**: $\mu(\text{Tough}|Pa) = 0.1$ (prior).
- **P2's Best Response to Pasta**:
  - $E[U(C)] = 0.1(0) + 0.9(1) = 0.9$.
  - $E[U(N)] = 0.1(0) + 0.9(1) = 0.9$.
  - Indifferent. Assume **Not Challenge**.
- **Off-path beliefs for Steak**: Assume $\mu(\text{Timid}|St) = 1$. P2 Challenges.
- **Check P1's Incentives**:
  - Tough: Pasta → N gives 2. Steak → C gives 1. OK.
  - Timid: Pasta → N gives 3. Steak → C gives 0. OK.

Result

Pooling on Pasta **IS** a PBE.

Summary of PBEs

We found **two** Pooling PBEs:

1. **Pooling on Steak**: Both choose Steak. Off-path: Pasta → Challenge.
2. **Pooling on Pasta**: Both choose Pasta. Off-path: Steak → Challenge.

- Both are sustained by ”punishing” off-path beliefs.
- But are these beliefs **reasonable**?
- **Problem**: Pooling on Pasta forces Tough to eat Pasta (which they dislike) to avoid a Challenge they don't fear!
- This seems implausible. We need a **refinement**.

Refinements: The Intuitive Criterion

The Problem with Arbitrary Off-Path Beliefs

- PBE allows **any** belief off-path.
- This can sustain implausible equilibria.
- **Example**: In Pooling on Pasta, if Steak is observed, P2 believes it's Timid.
- But who would rationally deviate to Steak?
  - **Timid**: Eq Payoff (Pasta $\rightarrow$ N) is 3. Max from Steak (Steak $\rightarrow$ N) is 2.
  - $3 > 2 \implies$ Steak is **equilibrium dominated** for Timid!
  - **Tough**: Eq Payoff (Pasta $\rightarrow$ N) is 2. Max from Steak (Steak $\rightarrow$ N) is 3.
  - $2 < 3 \implies$ Steak is **NOT** equilibrium dominated for Tough.
- **Conclusion**: Rational belief should attribute Steak to Tough, not Timid.

The Intuitive Criterion (Cho & Kreps 1987)

Definition: Equilibrium Domination

A message m is **equilibrium dominated** for type t if, even with the best possible response from the Receiver, type t cannot improve upon their equilibrium payoff by sending m.

Definition: Intuitive Criterion

If a deviation message m' is equilibrium dominated for type t_1 but NOT for type t_2, then the Receiver should believe that m' came from t_2 (i.e., $\mu(t_2|m') = 1$).

- **Intuition**: Don't attribute deviations to types who have nothing to gain.

Applying Intuitive Criterion to Steak-Pasta

Test Pooling on Pasta:

- **Equilibrium payoffs**: Tough gets 2, Timid gets 3.
- **Deviation**: Steak.
- **Check Timid**:
  - Best possible from Steak: $U_T(St, N) = 2 < 3$. Cannot improve.
  - Steak is **equilibrium dominated** for Timid.
- **Check Tough**:
  - Best possible from Steak: $U_{To}(St, N) = 3 > 2$. Can improve!
  - Steak is **NOT** equilibrium dominated for Tough.
- **Conclusion**: If Steak is observed, it must be Tough. P2 should Not Challenge.
- But then Tough *will* deviate to Steak!

Result

Pooling on Pasta **FAILS** the Intuitive Criterion.

Test Pooling on Steak

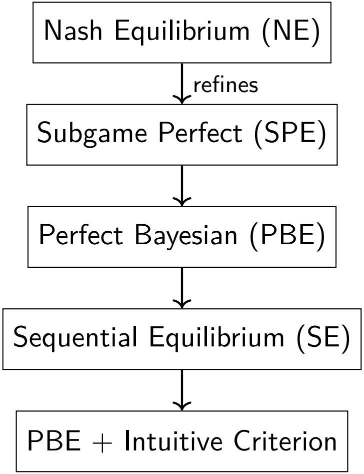
Test Pooling on Steak:

- **Equilibrium payoffs**: Tough gets 3, Timid gets 2.
- **Deviation**: Pasta.
- **Check Tough**:
  - Best possible from Pasta: $U_{To}(Pa, N) = 2 < 3$. Cannot improve.
  - Pasta is **equilibrium dominated** for Tough.
- **Check Timid**:
  - Best possible from Pasta: $U_T(Pa, N) = 3 > 2$. Can improve!
  - Pasta is **NOT** equilibrium dominated for Timid.
- **Conclusion**: If Pasta is observed, it must be Timid. P2 should Challenge.
- Timid gets $U_T(Pa, C) = 1 < 2$. No incentive to deviate.

Result

Pooling on Steak **PASSES** the Intuitive Criterion.

Equilibrium Concept Hierarchy



- Each refinement eliminates "unreasonable" equilibria.
- **Trade-off**: Stronger refinements → Fewer equilibria, but harder to compute.

Cheap Talk

What if Signals are Costless?

Cheap Talk: Communication without direct cost or commitment.

- **Examples**:
  - Pre-election promises by politicians.
  - Advertising claims without verification.
  - Verbal assurances in negotiations.
- **Key Question**: Can information be credibly transmitted when messages are costless?

Contrast with Costly Signaling

- In Spence/Steak-Pasta: Signal has differential cost across types.
- This cost difference enables separation.
- In Cheap Talk: No cost → Anyone can send any message.

Crawford-Sobel Framework (1982)

Setup:

- **Sender**: Has private information (type θ).
- **Message**: Costless signal m.
- **Receiver**: Takes action $a \in \mathbb{R}$ based on m.
- **Payoffs (Quadratic Loss)**:
  - Receiver wants $a = \theta$: $U_R(a, \theta) = -(a - \theta)^2$.
  - Sender wants $a = \theta + b$: $U_S(a, \theta) = -(a - (\theta + b))^2$.
- **Bias $b > 0$**: Measures the degree of conflict.

Key Parameter: **Preference Alignment**

- **Perfectly aligned**: Both want the same action for each type.
  - Example: Doctor-Patient (both want correct treatment).
- **Misaligned**: Sender prefers different action than Receiver.
  - Example: Salesperson-Customer (seller wants high price, buyer wants low).

The Babbling Equilibrium

When preferences are misaligned:

- **Babbling Equilibrium**: Messages convey no information.
  - All types send the same message (or randomize uniformly).
  - Receiver ignores the message, uses prior beliefs.
  - Action is based only on $E[\theta]$.

Why does this happen?

1. Suppose Sender tries to separate: "High type says m_H, Low type says m_L".
2. Receiver responds: a_H to m_H, a_L to m_L.
3. But messages are costless! Low type can just say m_H to get a_H.
4. Separation unravels. Only Babbling remains.

Key Insight

Without cost or commitment, cheap talk fails to transmit information when interests conflict.

Partial Information Transmission

Crawford-Sobel Result: Even with misalignment, *some* information can be transmitted if misalignment is not too severe.

- **Partition Equilibria**: Types are grouped into intervals.
  - Example: "High", "Medium", "Low" categories.
  - Within each category, types pool.
  - Across categories, separation occurs.
- **Number of partitions** depends on degree of misalignment.
  - Perfect alignment → Full separation possible.
  - Strong misalignment → Only Babbling.
  - Moderate misalignment → Coarse information (2-3 partitions).

Cheap Talk vs Costly Signaling

	Costly Signaling	**Cheap Talk**
Signal Cost	Differential	Zero
Separation	Possible (via cost)	Requires alignment
Efficiency	Wasteful (burning money)	No waste
Credibility	Cost enforces truth	Alignment enforces truth

- **Costly Signaling**: Works even with conflict, but inefficient.
- **Cheap Talk**: Efficient, but requires some common interest.

Practice Problem

Practice: Job Market Signaling

Setup

- **Worker**: High ability (H) or Low ability (L). $P(H) = 0.4$.
- **Signal**: Education level $e \in \{0, 2\}$.
- **Firm**: Observes e, offers wage w.
- **Productivity**: $\theta_H = 10$, $\theta_L = 4$.
- **Cost of education**: $c_H(2) = 1$, $c_L(2) = 3$.
- **Utility**: $U = w - c(e)$.
- **Competition**: Firms offer $w = E[\theta|e]$.

Question: Find all Separating PBEs.

Solution Step 1: Conjecture

Conjecture Separating: $H \to e = 2$, $L \to e = 0$.

- **Beliefs**: $\mu(H|e = 2) = 1$, $\mu(L|e = 0) = 1$.
- **Wages**:
  - $w(e = 2) = E[\theta|H] = 10$.
  - $w(e = 0) = E[\theta|L] = 4$.

Solution Step 2: Check Incentive Constraints

High type:

- Equilibrium: $e = 2 \to w = 10$. Utility: $10 - 1 = 9$.
- Deviation: $e = 0 \to w = 4$. Utility: $4 - 0 = 4$.
- $9 > 4$. **OK**.

Low type:

- Equilibrium: $e = 0 \to w = 4$. Utility: $4 - 0 = 4$.
- Deviation: $e = 2 \to w = 10$. Utility: $10 - 3 = 7$.
- $7 > 4$. **DEVIATION!**

Conclusion

This Separating equilibrium **FAILS**. Low type wants to mimic High type.

Solution Step 3: Try Pooling

Conjecture Pooling on $e = 2$:

- **Beliefs**: $\mu(H|e = 2) = 0.4$ (prior).
- **Wage**: $w(e = 2) = 0.4(10) + 0.6(4) = 6.4$.
- **Off-path**: $w(e = 0) = 4$ (assume Low).
- **Check IC**:
  - High: $e = 2$ gives $6.4 - 1 = 5.4$ vs $e = 0$ gives $4 - 0 = 4$. OK.
  - Low: $e = 2$ gives $6.4 - 3 = 3.4$ vs $e = 0$ gives $4 - 0 = 4$. **DEVIATION!**

Result

No pure strategy PBE exists in this game! (Semi-separating may exist.)

Summary

Summary

- **Steak-Pasta Game**: Two Pooling PBEs exist, but only one passes Intuitive Criterion.
- **Intuitive Criterion**: Eliminates equilibria sustained by "unreasonable" off-path beliefs.
  - Don't attribute deviations to types who are equilibrium dominated.
- **Equilibrium Hierarchy**: NE → SPE → PBE → SE / IC.
- **Practice**: Job market signaling shows that not all games have pure strategy PBEs.
- **Next Lecture**: Spence Model (Signaling with continuous education choice).