BCSE Game Theory 07-03

Exercise Session: Asymmetric Extensive-Form Practice

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Answer on Google Slides

Upload your slides with algebra, diagrams, and the inequalities that justify each equilibrium. Assign one recorder per team and keep proofs concise but legible.

https://sites.google.com/vju.ac.vn/bcse-gt

Notes

1. Each team handles exactly one question; cite teammates and highlight any remaining doubts.
2. Prioritise backward-induction diagrams or best-response plots over prose explanations.

Q1. Moral Hazard Contract with Numbers

Principal P offers a wage pair (W_S, W_F) contingent on the publicly observed signal: S (success) or F (failure). Agent A then chooses effort $e \in \{e_H, e_L\}$, incurs cost $c(e)$, and enjoys utility $\ln W$ from wage W. The technology matches Lecture 07-01:

- $p_H = 0.8$, $p_L = 0.5$.
- $R_S = 22$, $R_F = 6$.
- $c_H = 4$, $c_L = 1$.
- $\bar{U} = \ln 6$.

1. Compute $\Pi_H(W_S, W_F) = 0.8(22 - W_S) + 0.2(6 - W_F)$ and $\Pi_L(W_S, W_F) = 0.5(22 - W_S) + 0.5(6 - W_F)$.
2. Use $U_H \geq U_L$ to obtain the minimum bonus ratio $\frac{W_S}{W_F}$.
3. Bind the high-effort participation constraint to solve explicitly for (W_S, W_F).
4. Compare Π_H at those wages with the best low-effort contract that satisfies $U_L \geq \bar{U}$; state which effort P prefers.

Q2. Asymmetric Entry Deterrence

Incumbent I can invest in excess capacity at cost $K = 2$ before facing entrant E. After observing I's decision, E chooses Enter or Stay Out. Payoffs (I, E) are:

	Enter	Stay Out
Invest	$(3 - K,\ 1)$	$(7 - K,\ 0)$
Stay Lean	$(1,\ 3)$	$(6,\ 0)$

1. Put the game into extensive form and mark the subgames.
2. Use backward induction to find the SPNE for $K = 2$ and describe the intuition relative to the symmetric benchmark from 07-01.
3. Solve for the investment threshold K^* that exactly deters entry and interpret what happens for $K > K^*$.

Q3. Cournot with Unequal Marginal Costs

Market demand is $P(Q) = 120 - Q$ with $Q = q_1 + q_2$; firm 1 has marginal cost $c_1 = 20$, firm 2 has $c_2 = 40$. Strategies are quantities chosen simultaneously.

1. Derive each firm's best-response function and sketch them on the (q_1, q_2) plane.
2. Solve for the Cournot equilibrium (q_1^*, q_2^*) and the corresponding payoffs.
3. Compare the outcome with the symmetric-cost case and explain how cost asymmetry shifts the equilibrium away from the 45-degree line.

Q4. Bertrand with Capacity-Constrained Low-Cost Firm

Demand is $D(p) = 100 - p$. Firm A (low cost) has marginal cost 12 but capacity limited to 20 units; firm B (high cost) has marginal cost 20 and unlimited capacity. Firms simultaneously choose prices.

1. Consumers purchase entirely from the lower-priced firm (ties: A serves up to 20 units and B handles the residual demand).
2. Characterise all pure-strategy Nash equilibria, paying attention to who serves residual demand.
3. Discuss how A's capacity cap creates a kinked best-response set for B.
4. Suppose A could expand capacity to 40 units at fixed cost $F = 50$. Determine whether doing so would change the equilibrium price enough to justify paying F.

Q5. Stackelberg with a Cost Advantage Follower

Inverse demand is $P(Q) = 100 - (q_1 + q_2)$. Leader (firm 1) has constant marginal cost $c_1 = 30$, follower (firm 2) has $c_2 = 10$. The follower observes q_1 before choosing q_2.

1. Derive the follower's best-response function $q_2(q_1)$.
2. Substitute into the leader's profit to obtain q_1^{SPNE} and q_2^{SPNE}.
3. Evaluate consumer surplus and total profit under this asymmetric Stackelberg outcome and contrast it with the Cournot benchmark.