BCSE Game Theory 05-01

Selecting Equilibria in Finite Games

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Today's Agenda

Today's Goals

- Recall strategic-form notation for finite games.
- Review Pareto efficiency as a baseline screen.
- Understand how focal points guide coordination.
- Define payoff and risk dominance rigorously.
- Combine the criteria into a practical selection workflow.

Lecture Roadmap

1. Motivation: why equilibrium selection matters for managers and policymakers.
2. Pareto screening in 2×2 coordination games.
3. Designing and recognising focal points.
4. Payoff dominance, risk dominance, and loss-product logic.
5. Case study in an IT platform pricing game.

Notation Refresher

Strategic-form game $G = \langle N, (S_i), (u_i) \rangle$, $S = \prod_{i \in N} S_i$, Nash set $\mathrm{NE}(G)$.

Why Equilibrium Selection Matters

- Coordination games, procurement, and regulation routinely present multiple Nash equilibria.
- Stakeholders request a recommendation: “Which outcome is efficient?” “Which is safe?”
- Selection criteria translate payoff data into actionable advice or policy design.
- Even when the equilibrium is unique, documenting the criteria builds confidence and audit trails.

Pareto Perspective

Pareto Efficiency

Definition: Pareto Efficiency

A profile $s \in S$ is Pareto efficient if no $s' \in S$ satisfies $u_i(s') \geq u_i(s)$ for all i with strict inequality for at least one player.

- Pareto inefficiency signals missed mutual gains—remove such equilibria immediately.
- Efficiency does not solve distributional concerns; we still need additional filters.

Stag Hunt Illustration

	Stag	Rabbit
Stag	$(3, 3)$	$(0, 2)$
Rabbit	$(2, 0)$	$(2, 2)$

- Both $(\text{Stag}, \text{Stag})$ and $(\text{Rabbit}, \text{Rabbit})$ are Nash equilibria.
- Pareto comparison: $(\text{Stag}, \text{Stag})$ strictly improves both players relative to $(\text{Rabbit}, \text{Rabbit})$.
- Cooperative equilibrium is therefore the only Pareto efficient candidate.

Pareto Lessons

- Efficiency checks are quick and intuitive; they should open every selection memo.
- Managers may use transfers or revenue-sharing to implement Pareto improvements.
- Regulators often treat Pareto dominated equilibria as unacceptable due to fairness concerns.

Focal Points

Focal Points Defined

Definition: Focal Point

A focal point is an equilibrium that players select because of salience—history, culture, or design cues.

- Salient cues align beliefs without explicit communication.
- Focal reasoning often explains real-world coordination when payoffs alone are ambiguous.

Recognising Focal Cues

- Historical anchors: Schelling's "Grand Central at noon" coordination experiment. (in New York City)
- Visual hierarchy: highlighted buttons, colour schemes, or map markers.
- Rabbit narratives (Stag Hunt): slogans, rituals, or brand identities that single out one equilibrium. (Cross-Cultual Understanding game)

Designing Focal Points

- Consistency: repeat the same cue across channels (UI, emails, signage). (Green = OK, Red = Cancel)
- Visibility: make the intended action literally stand out via placement or typography.
- Complementary incentives: modest rewards or penalties reinforce the focal message.
- Supplementary example: Using Nudge to Stop Cigarette Litter (YouTube, start at 0:30) illustrates visual cues shaping behaviour.

Payoff and Risk Criteria

Definition: Payoff Dominance

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In a finite game, an equilibrium s^{PD} payoff dominates equilibrium s' if $u_i(s^{\mathrm{PD}}) \geq u_i(s')$ for all i and $u_j(s^{\mathrm{PD}}) > u_j(s')$ for at least one player j.

- Captures the "best mutual outcome" logic.
- Works well when side payments or trust allow players to prioritise joint gains.

Definition: Risk Dominance

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For a 2×2 coordination game with equilibria (A, A) and (B, B), let

$$L_A = u_i(A, A) - u_i(B, A), \qquad L_B = u_i(B, B) - u_i(A, B)$$

be the unilateral deviation losses, and similarly for the other player. Equilibrium (A, A) risk dominates (B, B) if $L_A L'_A > L_B L'_B$, where L'_A, L'_B are the opponent's deviation losses.

- Emphasises safety against the other's mistakes.
- Often preferred by cautious regulators or risk-averse firms.

Risk Dominance Intuition

- Think of each player assigning beliefs about the other's move. Risk dominance favours the equilibrium that succeeds under the widest range of imperfect beliefs.
- Loss products $L_A L'_A$ and $L_B L'_B$ measure how costly it is if the opponent "blunders" into the wrong action. Higher products mean the equilibrium forgives larger mistakes.
- With a 50-50 prior on the opponent's action, the risk-dominant strategy yields the higher expected payoff; it is the safe harbour when forecasts are noisy.
- Graphically, replicator dynamics show a larger basin of attraction around the risk-dominant outcome. Small perturbations in play drift back toward that equilibrium.
- In regulatory briefs, highlight risk dominance when decision-makers fear miscoordination more than leaving money on the table.

Loss-Product Logic

- Loss products reflect how painful it is if the other player deviates unexpectedly.
- Symmetric games: evaluate one player's losses and square the result.
- Asymmetries: compute the product for both players explicitly before ranking equilibria.

Stag Hunt Revisited

	Stag	Rabbit
Stag	$(3, 3)$	$(0, 2)$
Rabbit	$(2, 0)$	$(2, 2)$

- Payoff dominance selects $(\text{Stag}, \text{Stag})$.
- Loss products: 1 for Stag, 4 for Rabbit $\Rightarrow$ risk dominance picks $(\text{Rabbit}, \text{Rabbit})$.
- Present both outcomes and explain which criterion suits the client's risk tolerance.

Selection Workflow

Checklist for Advisers

1. Enumerate Nash equilibria (pure first, mixed if relevant).
2. Remove Pareto dominated equilibria.
3. Compute payoff dominance and risk dominance for remaining candidates.
4. Document focal cues, narratives, or institutional supports.

Documenting Recommendations

- Provide a summary table with columns for efficiency, safety, and focal justifications.
- State assumptions (information symmetry, available transfers, enforcement mechanisms).
- Suggest experiments or pilots to validate proposed focal cues when evidence is thin.

Case Study

IT Platform Pricing Matrix

	High Price	Low Price
High Price	$(5, 5)$	$(1, 7)$
Low Price	$(7, 1)$	$(3, 3)$

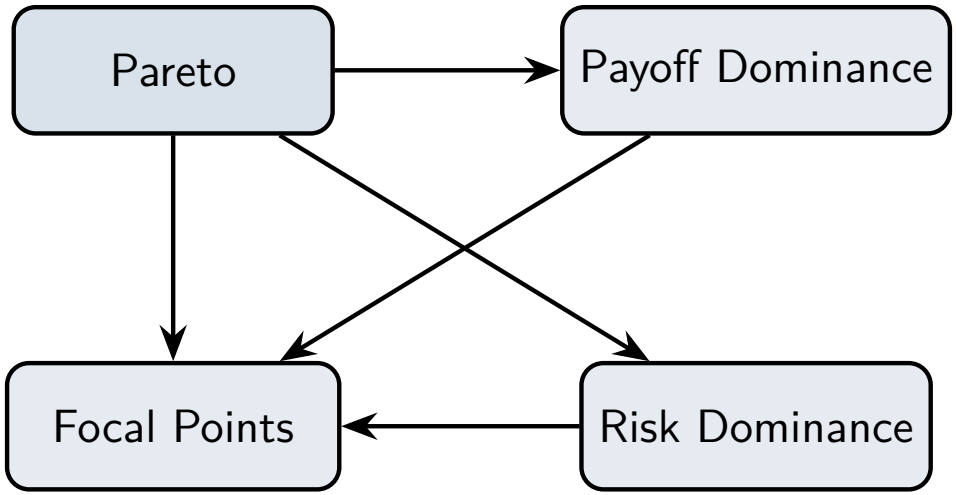
- Two cloud platforms target the same developer base.
- (High, High) maximises joint profit but needs credible coordination.
- (Low, Low) protects against detection risk or aggressive undercutting.

Interpreting the Case

- Payoff dominance ⇒ premium pricing; joint-venture messaging justifies it.
- Risk dominance ⇒ defensive pricing when monitoring is weak.
- Focal cues: synchronised launch dates, shared SDK milestones, coordinated branding.
- Policy lens: regulators test whether these cues verge on collusion.

Visual Summary

Map of Selection Criteria



- Use Pareto filters first, then articulate payoff vs. risk trade-offs.
- Focal design delivers a persuasive recommendation to real decision-makers.

Key Takeaways

- Multiple criteria complement one another; document the rationale for each.
- Highlight both efficiency gains and safety against miscoordination.
- Tailor focal cues to the client's culture, regulation, and risk appetite.