Game Theory 09-03

Exercises: Incomplete Information

BCSE Game Theory

Dec. 3, 2025

Exercise Session

Bayesian Nash Equilibrium & Cournot with Entry

Answer on Google Slides

https://sites.google.com/vju.ac.vn/bcse-gt

- Submit one PDF per team.
- Focus on the logic of multiple equilibria in Q2.
- Show your calculations for the profit comparisons.

Notes

1. For Q2, remember that fixed costs are paid only if $q > 0$.
2. Check the "Exit" condition by verifying if optimal entry yields negative profit.

Q1. Bayesian Nash Equilibrium (Warm-up)

Q1. Asymmetric Information Matrix Game

Consider a game between Player 1 and Player 2.

- Player 1 has one type.
- Player 2 has two types: **Left** type (prob 0.6) and **Right** type (prob 0.4).
- Player 2 knows their own type; Player 1 only knows the probabilities.

Payoffs (P1, P2):

If P2 is Left Type

	L	R
U	(2, 2)	(0, 0)
D	(0, 0)	(1, 1)

If P2 is Right Type

	L	R
U	(0, 0)	(1, 1)
D	(2, 2)	(0, 0)

1. Find the Bayesian Nash Equilibrium (BNE) in pure strategies.
2. (Hint: P2's best response depends on their type. P1 maximizes expected payoff given P2's strategy).

Q2. Cournot with Fixed Costs (Multiple Equilibria)

Q2. Entry Decision with Fixed Costs

Demand $P = 12 - Q$. Firm 1 has cost $c_1 = 0$. Firm 2 has cost $c_2 \in \{0, 4\}$ with equal probability (0.5). **Fixed Cost**: Both firms face a fixed cost $F = 2.5$ if they produce ($q > 0$).

1. **Equilibrium A (Both Enter)**: Assume both types of Firm 2 enter.
   - Calculate the equilibrium quantities (q_1, q_2^L, q_2^H).
   - Calculate the profit of Firm 2's High type. Is it > 0?
2. **Equilibrium B (High Type Exits)**: Assume Firm 2's High type exits ($q_2^H = 0$).
   - Calculate the new equilibrium quantities (q_1, q_2^L).
   - Calculate the profit Firm 2's High type *would* earn if they deviated and entered (best response to q_1). Is it < 0?
3. **Discussion**: Why do both equilibria exist? How does Firm 1's aggressive production in Case B deter the High type?

Q3. Differentiated Bertrand Competition

Q3. Pricing with Unknown Costs

Two firms sell differentiated products. Demand for firm i: $q_i = 10 - p_i + 0.5p_j$.

- Firm 1 has cost $c_1 = 2$.
- Firm 2 has cost $c_2 \in \{0, 4\}$ with equal probability (0.5).
- Firm 1 maximizes expected profit:
  $E[\pi_1] = (p_1 - 2)(10 - p_1 + 0.5E[p_2])$.

1. Write down Firm 1's profit function in terms of p_1 and $E[p_2]$.
2. Derive Firm 1's best response function $p_1(E[p_2])$.
3. (Optional) If Firm 2's best response is $p_2(p_1) = \frac{10+c_2+0.5p_1}{2}$, find the equilibrium price p_1.

Q4. First-Price Auction Strategy

Q4. Optimal Bidding

You are Bidder 1 in a First-Price Auction with valuation $v_1 = 4$. You believe Bidder 2 bids as follows:

- If $v_2 = 2$: Bidder 2 bids $b_2 = 1$.
- If $v_2 = 4$: Bidder 2 bids $b_2 = 3$.

Assume v_2 is 2 or 4 with probability 0.5 each.

1. Calculate your expected payoff if you bid $b_1 = 3.1$. (Win against $b_2 = 1$ and $b_2 = 3$).
2. Calculate your expected payoff if you bid $b_1 = 1.1$. (Win against $b_2 = 1$, lose against $b_2 = 3$).
3. Which bid is better?

Q5. Best Price Guarantee

Q5. Conceptual Analysis

Many retailers offer a "Best Price Guarantee" (BPG): "If you find a lower price, we will match it."

1. Explain intuitively why BPG might lead to **higher** equilibrium prices compared to standard price competition.
2. Does BPG increase or decrease the incentive to undercut prices? (Hint: If opponent lowers price, do you have incentive to lower yours?)