

Game Theory 10-03

Exercises: Auctions and Matching Markets

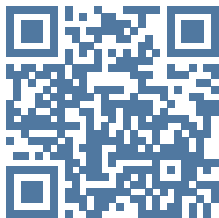
BCSE Game Theory

Dec. 9, 2025

Exercise Session

Auction Theory & Matching Markets

Answer on Google Slides



<https://sites.google.com/vju.ac.vn/bcse-gt>

- ▶ Submit one PDF per team.
- ▶ Show all calculations for auction problems.
- ▶ For matching, verify stability by checking all pairs.

Notes

1. For Q1-Q3, remember to check equilibrium conditions.
2. For Q4, compare profits with and without reserve price.
3. For Q5, check all blocking pairs systematically.

Q1. First-Price Auction Equilibrium

Q1. Bidding Strategy Derivation

Consider a First-Price Auction with $n = 3$ bidders. Valuations are i.i.d. uniform on $[0, 10]$.

1. Using the equilibrium bidding formula $b^*(v) = \frac{n-1}{n}v$, calculate the bid for a bidder with valuation $v = 6$.
2. If this bidder wins, what is their expected profit?
3. Calculate the expected revenue for the seller (use $E[v_{(1)}] = \frac{n}{n+1} \cdot 10$ for the highest valuation).
4. **Bonus:** If the distribution were Uniform $[0, 20]$ instead, how would the equilibrium bid change for $v = 12$?

Q2. Revenue Equivalence Theorem

Q2. Comparing Auction Revenues

Consider $n = 2$ bidders with valuations $v_i \sim \text{Uniform}[0, 1]$.

1. First-Price Auction:

- ▶ Equilibrium bid: $b^*(v) = \frac{1}{2}v$.
- ▶ Calculate expected revenue: $E[R_{1st}] = E[\frac{1}{2}v_{(1)}]$ where $E[v_{(1)}] = \frac{2}{3}$.

2. Second-Price Auction:

- ▶ Winner pays second-highest valuation.
- ▶ Calculate expected revenue: $E[R_{2nd}] = E[v_{(2)}]$ where $E[v_{(2)}] = \frac{1}{3}$.

3. Verify that $E[R_{1st}] = E[R_{2nd}]$.

4. **Bonus:** Explain intuitively why the revenues are equal despite different payment rules.

Q3. Optimal Reserve Price

Q3. Reserve Price Analysis

A seller faces $n = 2$ bidders with valuations $v_i \sim \text{Uniform}[0, 1]$. The seller considers setting a reserve price r .

1. **Without Reserve** ($r = 0$):

▶ Expected revenue: $E[R] = \frac{1}{3}$ (from Q3).

2. **With Reserve** ($r = 0.5$):

▶ Probability of sale: $\Pr(\text{sale}) = 1 - r^n = 1 - 0.25 = 0.75$.

▶ Expected revenue (given formula):

$$E[R] = \frac{n-1}{n+1} (1 - r^{n+1}) + r(1 - r^n).$$

▶ Calculate $E[R]$ for $r = 0.5$.

3. Compare the two revenues. Does the reserve price increase revenue?

4. **Discussion:** Why might a seller prefer $r = 0.5$ even though it reduces the probability of sale?

Q4. Matching Market Stability

Q4. Gale-Shapley Algorithm

Three students $\{S_1, S_2, S_3\}$ and three schools $\{A, B, C\}$ have the following preferences:

| Students | 1st | 2nd | 3rd | Schools | 1st | 2nd | 3rd |
|----------|-----|-----|-----|---------|-------|-------|-------|
| S_1 | A | B | C | A | S_2 | S_1 | S_3 |
| S_2 | B | A | C | B | S_1 | S_3 | S_2 |
| S_3 | A | B | C | C | S_2 | S_1 | S_3 |

1. Run the Gale-Shapley algorithm with students proposing. Show each round.
2. Verify the resulting matching is stable (no blocking pairs).
3. **Bonus:** What matching would result if schools proposed instead?