Game Theory 11-03

Exercises: Dynamic Games with Incomplete Information

BCSE Game Theory

Dec. 30, 2025

Exercise Session

Dynamic Games & Beliefs

Answer on Google Slides

https://sites.google.com/vju.ac.vn/bcse-gt

- Submit one PDF per team.
- Show all steps for Bayes' Rule calculations.
- Explain your reasoning for equilibrium checks.

Notes

1. Focus on the logic of **beliefs** and **incentives**.
2. Remember: PBE = Strategies + Beliefs.
3. For Q4, consider what the Receiver would do if they knew the type.

Q1. Bayesian Belief Updating

Q1. Calculating Posteriors

Consider a game where Player 1 (Nature) selects a Type $t \in \{t_1, t_2\}$ with prior probabilities $P(t_1) = 0.3$ and $P(t_2) = 0.7$. Player 1 then chooses an action $m \in \{L, R\}$.

1. Suppose the strategy of Player 1 is:
   - If t_1, play L with probability 1.
   - If t_2, play L with probability 0.5 and R with probability 0.5.
2. Player 2 observes action L. Using Bayes' Rule, calculate the posterior belief $\mu(t_1|L)$.
3. Player 2 observes action R. Calculate $\mu(t_1|R)$.
4. **Hint**: $\mu(t_1|L) = \frac{P(L|t_1)P(t_1)}{P(L|t_1)P(t_1)+P(L|t_2)P(t_2)}$.

Q2. Checking Consistency

Q2. Consistent Beliefs

Consider a game with priors $P(t_1) = 0.5, P(t_2) = 0.5$. Proposed Strategy Profile:

- Type t_1 plays X.
- Type t_2 plays X.

Observation: Receiver observes action X.

1. If the Receiver's belief at the information set following X is $\mu(t_1|X) = 0.9$, is this belief **consistent** with the strategy profile?
2. If not, what should the consistent belief $\mu(t_1|X)$ be?
3. What is the belief $\mu(t_1|Y)$ if action Y is **off-the-equilibrium-path** (never played)? Can it be set freely in PBE?

Q3. Incentives in Pooling

Q3. Mimicking Logic

Imagine a "Job Market" game.

- **High (H)**: Value 100, Cost 10. **Low (L)**: Value 50, Cost 30.
- Wage $w = E[\text{Value}]$.

Scenario: Both types choose "Get Education" (Pooling).

1. If prior $P(H) = 0.5$, what is wage w upon observing "Education"?
2. Calculate payoffs: High $(w - 10)$ and Low $(w - 30)$.
3. Without education, firm pays $w = 50$.
4. Does Low type deviate to "No Education"? (Compare $w - 30$ vs 50).
5. Is this Pooling sustainable?

Q4. Payoff Alignment

Q4. Conflict of Interest

Consider a communication game (Cheap Talk framework).

- State $\theta = 10$. Receiver takes action a.
- Receiver's Payoff: $U_R = -(a - \theta)^2$. (Detailed: Wants $a = \theta$).
- Sender's Payoff: $U_S = -(a - (\theta + 5))^2$. (Detailed: Wants $a = \theta + 5$).

1. What is the Receiver's ideal action a_R^* given $\theta = 10$?
2. What is the Sender's ideal action a_S^* given $\theta = 10$?
3. Calculate the bias $b = a_S^* - a_R^*$.
4. If the Sender says "The state is 10", and the Receiver believes it perfectly, the Receiver plays $a = 10$. Does the Sender have an incentive to lie (e.g., say "The state is 15") if the Receiver is naive?