BCSE Game Theory 02-02

IESDS

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Rational Choices

Rational vs. Apparently Irrational Choices

Today we revisit rational decision making when games appear to induce “irrational” behaviour. We focus on iterative elimination and best responses using richer business scenarios.

IESDS Practice: Price Competition

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We study a Bertrand-style price competition game involving homogeneous high-performance computing (HPC) systems.

- You represent an HPC manufacturer bidding for an AI infrastructure project.
- Products are essentially identical across vendors; the customer selects the lowest price among three bids.
- Opponent: a large incumbent vendor labelled **Competitor**.

Available Price Options

You must submit one of three price quotes:

- High: 450 billion VND
- Standard: 440 billion VND
- Low: 430 billion VND (equal to production cost; zero profit before fees)

Both firms have a production cost of 420 billion VND.

Question

Which price should you quote to win the contract?

Additional Bid Preparation Costs

- High quote: requires detailed specifications and proposals costing an extra 2 billion VND.
- Standard quote: requires standard proposal material costing 1 billion VND.
- Low quote: minimal paperwork; zero additional cost.

These costs help create strict dominance relations for this exercise.

		Competitor		
		High	Standard	Low
You	High	(13, 13)	(-2, 19)	(-2, 10)
	Standard	(19, -2)	(9, 9)	(-1, 10)
	Low	(10, -2)	(10, -1)	(5, 5)

Entries show (your profit, competitor profit) in billions of VND after costs and demand split rules.

How the Payoff Table Is Constructed

1. **Price menu**: $p_H = 450$, $p_S = 440$, $p_L = 430$ (billions of VND).
2. **Revenue**: If you win at price p, revenue is p; otherwise it is 0 (ties split demand).
3. **Production cost**: Subtract the common marginal cost $c = 420$ regardless of p.
4. **Proposal cost**: Subtract the additional preparation cost $k_H = 2$, $k_S = 1$, $k_L = 0$.
5. **Opponent symmetry**: The competitor faces the same menu, so the same calculations populate the other entries.

$$\pi_i(p_i, p_j) = \begin{cases} (p_i - c) - k_{p_i} & (p_i < p_j), \\ \frac{1}{2}(p_i - c) - k_{p_i} & (p_i = p_j), \\ -k_{p_i} & (p_i > p_j) \end{cases}$$

Applying IESDS

- We can iteratively remove strategies that are strictly dominated.
- Identify dominated strategies for both you and the competitor.
- Does IESDS lead to a unique prediction?

First Elimination Step

High is strictly dominated by Standard for both players; compare each column to see that $(13, 13)$ vs $(19, -2)$ and $(-2, 19)$ vs $(9, 9)$ favour Standard in every contingency.

		Competitor	
		Standard	Low
You	Standard	(9, 9)	(-1, 10)
	Low	(10, -1)	(5, 5)

Question. Are there any further dominated strategies?

Second Elimination Step

Standard is strictly dominated by Low for both players once High has been removed. The payoff pairs $(-1, 10)$ vs $(5, 5)$ and $(9, 9)$ vs $(10, -1)$ confirm that both you and the competitor prefer Low.

		Competitor
		Low
You	Low	(5, 5)

Only the profile (Low, Low) survives; this is the IESDS equilibrium.

Notes on the Outcome

- (Low, Low) is not a strongly dominant-strategy equilibrium in the original game.
- In the original matrix, Low does not strictly dominate Standard; the dominance emerges after the first elimination.
- Because only strictly dominated strategies were removed, the order of deletion does not affect the outcome.
- Still, iterative elimination yields a clear prediction consistent with Bertrand competition intuition.

Bertrand Competition

Price competition with homogeneous goods and simultaneous pricing is known as **Bertrand competition**.

- Customers buy exclusively from the lowest-price firm; ties split demand evenly.
- Even with only two firms, prices tend to fall toward marginal cost.
- The extra proposal costs were introduced solely to produce strict dominance for illustration.

Bertrand Competition (Continued)

- The intensity of price competition drives equilibrium prices toward cost.
- The qualitative conclusion holds without the additional bidding costs.

Best Responses: Quantity Competition

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We now consider quantity competition between two semiconductor manufacturers.

- Firms choose production quantities determined by plant capacity.
- Products are homogeneous; price cannot exceed the market-clearing level.
- Given total output Q, the market price adjusts so that demand equals supply.

Demand Concepts

- A **demand function** gives quantity demanded at each price.
- The **inverse demand function** gives the market price associated with a given total quantity.
- Producers sell at that price; charging more leads to unsold inventory and price cuts.

Quantity Game Parameters

Let

- q_i: output of firm i ($i = 1, 2$)
- q_j: output of the rival
- $Q \stackrel{\text{def}}{=} q_1 + q_2$: total market supply
- Inverse demand: $P(Q) \stackrel{\text{def}}{=} a - bQ$ with $a > c$
- Marginal cost: c for each firm (constant)

Profit for firm i:

$$\pi_i(q_i, q_j) = (P(Q) - c)q_i = (a - b(q_i + q_j) - c)q_i.$$

Later we set $\frac{a-c}{2b} = 3$ so the reaction-curve graph matches the notes.

Best Response Derivation

$$\max_{q_i \geq 0} (a - b(q_i + q_j) - c)q_i \quad \Rightarrow \quad \frac{\partial \pi_i}{\partial q_i} = a - c - 2bq_i - bq_j = 0,$$

$$\therefore BR_i(q_j) = \frac{a-c}{2b} - \frac{1}{2}q_j.$$

The second derivative $\frac{\partial^2 \pi_i}{\partial q_i^2} = -2b < 0$ confirms we have found the profit-maximising response.

Strategic substitutes

A larger q_j reduces your best response. This negative slope mirrors the reaction curves plotted in the notes.

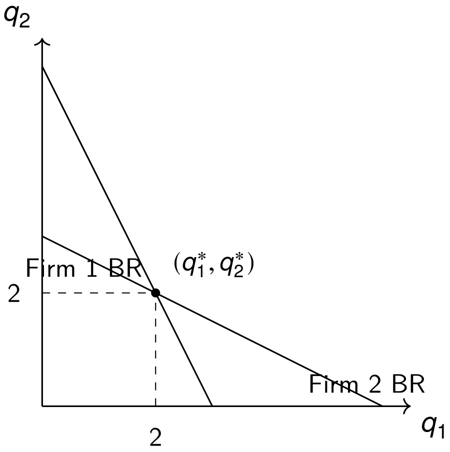
Iterating Best Responses

While this differs from literal IESDS, we can still narrow the rival's plausible choices by iterating the best-response logic.

- Symmetry implies both firms use $BR(q) = \frac{a-c}{2b} - \frac{1}{2}q$.
- Substituting $q_j = BR(q_i)$ repeatedly shrinks the feasible interval and the process converges.
- The fixed point satisfies $q_i = \frac{a-c}{3b}$, giving the Cournot quantities (q_1^*, q_2^*).
- For the numerical illustration $\frac{a-c}{2b} = 3$, we obtain $q_1^* = q_2^* = 2$ as shown in the reaction-curve figure.

Thus the intersection of the two reaction curves reproduces the Cournot equilibrium discussed in the notes.

Reaction Curves (Example)



- Each reaction curve slopes downward: higher rival output lowers your best response.
- The dashed guides show how to read the equilibrium quantities from the axes.

Cournot Competition

Quantity competition of this form is known as **Cournot competition**.

- Firms choose quantities simultaneously without knowing rivals' choices.
- With few firms, prices remain above cost; competition is softer than in Bertrand settings.

Best Responses

Why Best Responses?

IESDS may stop before identifying a unique outcome. We therefore analyse players' best responses to beliefs about opponents' strategies.

Definition: Belief

A belief of player i is a conjecture about opponents' strategies: $s_{-i} \in S_{-i}$.

Definition: Best response

A strategy $s_i \in S_i$ is player i's best response to (beliefs about) opponents' strategies $s_{-i} \in S_{-i}$ if

$$u_i(s_i, s_{-i}) \geq u_i(s_i', s_{-i}) \quad \text{for all } s_i' \in S_i.$$

- People form beliefs about what others will do.
- They typically choose a best response to those beliefs.

Example: Teamwork

Neither player has a strictly dominated strategy. Best responses depend on expectations about the opponent.

		Binh	
		Work Hard	Slack Off
Anh	Work Hard	(3, 3)	(1, 4)
	Slack Off	(4, 1)	(2, 2)

- If Binh chooses Work Hard, Anh's best response is Slack Off.
- If Binh chooses Slack Off, Anh's best response is Slack Off.

Most people choose a best response.

Example: Cross-Cultural Understanding

When best responses are unique, prediction is straightforward. Here each player has more than one best response depending on expectations.

		Binh: Vietnam Style	Binh: Japan Style
Anh	Vietnam Style	(3, 3)	(1, 1)
	Japan Style	(1, 1)	(2, 2)

- If Binh chooses Vietnam Style, Anh's best response is Vietnam Style.
- If Binh chooses Japan Style, Anh's best response is Japan Style.

Best Responses: Golden Balls

The Golden Balls Game

A televised game show illustrates belief-dependent best responses.

- Two players (A and B) compete for USD 10 million.
- Each simultaneously chooses between **Split** and **Steal**.

If the opponent chooses Steal, both Split and Steal are best responses; if the opponent chooses Split, Steal is strictly better.

		Player B	
		Split	Steal
Player A	Split	(5, 5)	(0, 10)
	Steal	(10, 0)	(0, 0)

Observations from the Show

- Initially the contestant Ibraham hesitates between Split and Steal.
- After Nick credibly promises to choose Steal, Split becomes Ibraham's only best response.
- Behaviour hinges on beliefs about the opponent's move.

See the video:
https://www.youtube.com/watch?v=S0qjK3TWZE8.

Golden Balls: Remark

		Player B	
		Split	Steal
Player A	Split	(5, 5)	(0, 10)
	Steal	(10, 0)	(0, 0)

- The best response to Split is not Split.
- Both (Split, Steal) and (Steal, Split) are mutual best responses.

Wrap-Up

Key Takeaways

- Strict dominance gives an order-independent elimination path; once weak dominance is allowed we must document the deletion order and its rationale.
- Bertrand competition pushes prices down to cost, whereas Cournot quantity competition keeps prices above cost because capacity choices soften rivalry.
- When IESDS stalls, spell out the beliefs that justify each survivor as a best response; multiple best responses hinge on those expectations.
- Credible communication (as in the Golden Balls example) can shift beliefs and therefore the best responses players are willing to choose.