

BCSE Game Theory 12-01

Signaling Games: The Spence Labor Market Model

BCSE Game Theory

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The Market for Lemons

Revisiting Adverse Selection

Motivation: Hidden Information

- ▶ In many economic transactions, one party knows more than the other.
- ▶ **Examples:**
 - ▶ **Used Cars:** Seller knows if the car is a "Lemon" (bad) or "Peach" (good). Buyer only sees a car.
 - ▶ **Labor Market:** Worker knows their own ability. Firm only sees a résumé.
 - ▶ **Insurance:** You know your health risks. The insurer only knows statistics.
- ▶ This asymmetry can lead to **Market Failure**.

The Market for Used Cars (Setup)

- ▶ Consider a market with 100 sellers and many potential buyers.
- ▶ **Two types of cars:**
 - ▶ **Peaches (High Quality):** 50% of cars.
 - ▶ **Lemons (Low Quality):** 50% of cars.
- ▶ **Valuations** (Reserve Prices):

	Seller Value (v_S)	Buyer Value (v_B)
Lemon (L)	\$1,000	\$1,200
Peach (H)	\$2,000	\$2,400

- ▶ Note: $v_B > v_S$ for both types. Trade is **always efficient**.

Benchmark: Symmetric Information

- ▶ Suppose quality is **observable** (Buyers can distinguish Peaches from Lemons).
- ▶ **Market for Lemons:**
 - ▶ Price P_L will settle between \$1,000 and \$1,200.
 - ▶ All Lemons are traded.
- ▶ **Market for Peaches:**
 - ▶ Price P_H will settle between \$2,000 and \$2,400.
 - ▶ All Peaches are traded.
- ▶ **Result:** Full Efficiency. Both markets clear.

Asymmetric Information: The Problem

- ▶ Now suppose Buyers **cannot distinguish** types. They only know the prior probability (50/50).
- ▶ There can only be **one market price** P .
- ▶ **Buyer's Expected Value** ($E[v_B]$):

$$E[v_B] = 0.5(\$1,200) + 0.5(\$2,400) = \$600 + \$1,200 = \$1,800$$

- ▶ Buyers are risk-neutral, so they are willing to pay up to \$1,800.

Market Collapse (Adverse Selection)

- ▶ Buyers offer $P = \$1,800$.
- ▶ **Lemon Owners:**
 - ▶ Value $v_S(L) = \$1,000$.
 - ▶ $P > v_S(L)$. They are happy to sell.
- ▶ **Peach Owners:**
 - ▶ Value $v_S(H) = \$2,000$.
 - ▶ $P < v_S(H)$. They **refuse to sell**.
- ▶ **Result:** Peaches exit the market.

The Death Spiral

- ▶ Buyers anticipate that Peach owners will exit.
- ▶ The market actually consists of **100% Lemons**.
- ▶ Revised Expected Value:

$$E[v'_B] = 1.0(\$1,200) + 0.0(\$2,400) = \$1,200$$

- ▶ Buyers drop their offer to \$1,200.
- ▶ **Outcome:**
 - ▶ Only Lemons trade.
 - ▶ High-quality cars (Peaches) are driven out.
 - ▶ **Inefficiency:** Potential gains from trade for Peaches are lost.

Summary of The Lemons Problem

Adverse Selection

Process where "bad" products or customers are more likely to be selected in a market with asymmetric information, driving out "good" products.

How do we fix this?

- ▶ **Spence's Solution:** Signaling (this lecture).
- ▶ **Akerlof's Solution:** Institutions / Screening (next lecture).

Signaling

Spence's Solution

Spence's Solution: Signaling

Michael Spence asked: Can the **informed party** (Seller/Worker) take an action to distinguishing themselves?

- ▶ **Signal:** An observable action taken by the informed player.
- ▶ **Key Requirement:** The signal must be **costly** to mimic.
- ▶ Specifically, it must be *more costly* for the Low type than for the High type.

Example: The Peacock's Tail

- ▶ **Biological Signaling** (Zahavi's Handicap Principle).
- ▶ Why does a male peacock have a giant, heavy, colorful tail?
 - ▶ It attracts predators.
 - ▶ It makes flying difficult.
 - ▶ It consumes energy to grow.
- ▶ **Logic:** Only a **very fit, healthy** bird can survive with such a burden.
- ▶ A weak bird trying to mimic this tail would die.
- ▶ **Conclusion:** The tail signals genetic quality **because** it is wasteful.

The Spence Labor Market Model

Setup and Mechanisms

Model Setup (Education): Players and Timeline

1. **Nature** chooses Worker's type $\theta \in \{\theta_L, \theta_H\}$ with probabilities $1 - p$ and p .
 - ▶ θ represents **productivity**. $\theta_H > \theta_L > 0$.
2. **Worker** observes θ and chooses Education level $e \geq 0$.
3. **Firms** observe e (but not θ) and compete for the worker.
4. Firms offer wage $w(e)$.
 - ▶ Assumption: Competitive labor market
 $\Rightarrow w(e) = E[\text{Productivity}|e]$.

Payoffs and Costs

Firm's Payoff:

$$\Pi = \theta - w$$

(Zero profit in expectation due to competition).

Worker's Payoff:

$$U(w, e, \theta) = w - c(e, \theta)$$

- ▶ w : Wage (Benefit).
- ▶ $c(e, \theta)$: Cost of education.

Cost Structure Assumptions

The cost function $c(e, \theta)$ has standard properties:

- ▶ $c(0, \theta) = 0$ (No education is free).
- ▶ $\frac{\partial c}{\partial e} > 0$ (Education is costly).
- ▶ $\frac{\partial^2 c}{\partial e^2} > 0$ (Marginal cost is increasing).

CRITICAL ASSUMPTION:

$$c(e, \theta_L) > c(e, \theta_H) \quad \text{for } e > 0$$

- ▶ Education is **more painful** (or takes longer, is harder) for Low productivity workers.

Single Crossing Property (SCP)

Definition: Single Crossing Property

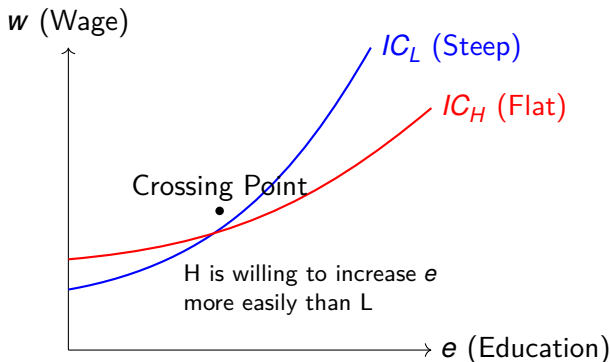
The marginal disutility of the signal is **strictly lower** for the High type.

$$\frac{\partial^2 c(s, \theta)}{\partial s \partial \theta} < 0 \implies MU_L(s) > MU_H(s)$$

where s is the signal (education, advertising, waiting, etc.).

- ▶ This ensures that High types find it “easier” to send the signal.
- ▶ **Education:** Smart workers find studying less painful.
- ▶ **Advertising:** High-quality firms expect repeat customers, so ads pay off more.
- ▶ Without SCP, separation is impossible.

Graphical Intuition: Single Crossing



Curves cross exactly once. At any point, L 's IC is steeper than H 's.

Marginal Rate of Substitution (MRS)

- ▶ Consider the Worker's Indifference Curve (IC) in (e, w) space.
- ▶ Along an IC, utility is constant: $dw - MC \cdot de = 0$.
- ▶ **Slope of IC:**

$$\text{Slope} = \frac{dw}{de} = MC(e, \theta)$$

- ▶ This slope is the **MRS** between Education and Wage.
- ▶ **Implication of SCP** (as seen in the figure):
 - ▶ $MC_L > MC_H \Rightarrow$ **Low type's IC is STEEPER.**
 - ▶ Low type requires a **larger** wage increase to accept same increase in education.

Equilibrium Analysis

Separating Equilibrium

Types of Perfect Bayesian Equilibria

In a Perfect Bayesian Equilibrium (PBE), we look for consistent Strategies and Beliefs.

1. **Separating Equilibrium:**

- ▶ Types choose different education levels ($e_L \neq e_H$).
- ▶ Firms perfectly identify types: $w(e_L) = \theta_L, w(e_H) = \theta_H$.

2. **Pooling Equilibrium:**

- ▶ Types choose the same education level ($e_L = e_H = e^*$).
- ▶ Firms learn nothing: $w(e^*) = E[\theta]$.

Constructing a Separating Equilibrium (Step 1)

Low Type's Strategy:

- ▶ In a separating equilibrium, L is identified.
- ▶ Wage will be $w_L = \theta_L$.
- ▶ L maximizes $w_L - c(e_L, \theta_L) = \theta_L - c(e_L, \theta_L)$.
- ▶ Optimal choice: $e_L^* = 0$. (Minimal cost).
- ▶ L gets utility $U_L^* = \theta_L$.

Constructing a Separating Equilibrium (Step 2)

High Type's Strategy:

- ▶ H wants to choose e_H to signal "I am High".
- ▶ Wage will be $w_H = \theta_H$.
- ▶ **Incentive Compatibility (IC) Constraint for Low:**
 - ▶ L must NOT want to mimic H.
 - ▶ Utility of being L \geq Utility of mimicking H.

$$\theta_L \geq \theta_H - c(e_H, \theta_L)$$

▶ IC Constraint for High:

- ▶ H must prefer signaling over being thought of as Low.

$$\theta_H - c(e_H, \theta_H) \geq \theta_L - c(0, \theta_H)$$

Finding the Signal Level e_H

Rearranging the IC constraints:

Required Range for e_H

$$c(e_H, \theta_H) \leq \theta_H - \theta_L \leq c(e_H, \theta_L)$$

- ▶ **LHS:** e_H must be "cheap enough" for H.
- ▶ **RHS:** e_H must be "expensive enough" for L.
- ▶ Does such an e_H exist?
- ▶ **Yes**, because of Single Crossing ($c(\cdot, \theta_L) > c(\cdot, \theta_H)$).

The Riley Outcome (Least Cost Separation)

- ▶ There are typically many education levels e_H that satisfy the range.
- ▶ H prefers the **smallest valid** e_H (lowest cost).
- ▶ The most "efficient" separating equilibrium is where L's IC binds:

$$c(e_H^*, \theta_L) = \theta_H - \theta_L$$

- ▶ e_H^* is just high enough to discourage L.

Numerical Example: Setup

Parameters

- ▶ **Types:** $\theta_L = 10, \theta_H = 20$.
- ▶ **Probabilities:** $P(\theta_L) = 0.5, P(\theta_H) = 0.5$.
- ▶ **Wage:** $w = E[\theta]$.
- ▶ **Cost Function:** $c(e, \theta) = \frac{e^2}{\theta}$.

Check Single Crossing:

- ▶ $MC_L = \partial c / \partial e = 2e/10 = 0.2e$.
- ▶ $MC_H = \partial c / \partial e = 2e/20 = 0.1e$.
- ▶ $MC_L > MC_H$ for all $e > 0$. **Satisfied.**

Numerical Example: Analyzing Low Type

Step 1: Determine Low Type's Action

- ▶ In a separating equilibrium, L is revealed.
- ▶ Wage $w_L = \theta_L = 10$.
- ▶ L chooses e_L to maximize $10 - \frac{e_L^2}{10}$.
- ▶ Optimal choice: $e_L^* = 0$.
- ▶ L's Utility: $U_L^* = 10 - 0 = 10$.

Numerical Example: Analyzing High Type

Step 2: Determine High Type's Action

- ▶ H wants to choose e_H to signal $\theta_H = 20$.
- ▶ **Condition:** L must NOT want to mimic H.
- ▶ L's Utility from Mimicking ($e = e_H, w = 20$):

$$U_L^{\text{mimic}} = 20 - \frac{e_H^2}{10}$$

- ▶ **IC Constraint:** $U_L^* \geq U_L^{\text{mimic}}$

$$10 \geq 20 - \frac{e_H^2}{10}$$

Numerical Example: Finding the Signal

Solving for e_H :

$$\frac{e_H^2}{10} \geq 20 - 10 = 10$$

$$e_H^2 \geq 100 \Rightarrow e_H \geq 10$$

Step 3: Riley Outcome

- ▶ H chooses the lowest valid signal to minimize cost.
- ▶ $e_H^* = 10$.

Numerical Example: Verification

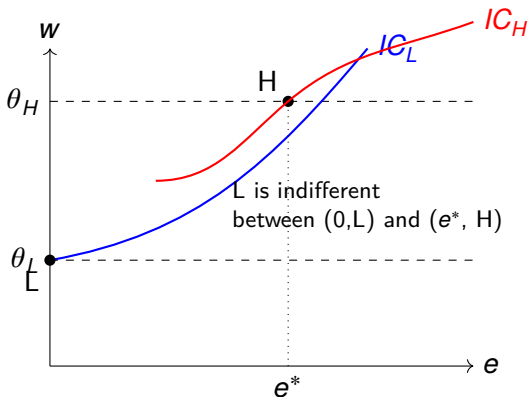
Step 4: Check High Type's IC

- ▶ H chooses $e = 10$, gets $w = 20$.
- ▶ Cost for H: $c(10, 20) = \frac{100}{20} = 5$.
- ▶ H's Utility: $20 - 5 = 15$.
- ▶ Alternative (Pool with L): $e = 0, w = 10$. Utility = 10.
- ▶ Since $15 > 10$, H is happy to signal.

Result:

- ▶ $e_L = 0, w_L = 10$.
- ▶ $e_H = 10, w_H = 20$.
- ▶ Full Separation achieved.

Separating Equilibrium: Graph



Pooling Equilibrium

Brief Overview

Pooling Equilibrium Idea

- ▶ Can both types choose the same e_p ?
- ▶ If so, firm belief stays at prior: $w = E[\theta]$.
- ▶ Need:
 - ▶ L prefers pooling ($E[\theta] - c(e_p)$) over separating ($L - 0$).
 - ▶ H prefers pooling ($E[\theta] - c(e_p)$) over... separating?
- ▶ Pooling often requires firms to threaten "If you deviate to something slightly higher than e_p , I will still think you are Low."
- ▶ Such beliefs often fail refinements like the **Intuitive Criterion**.
- ▶ (We focus on Separating Equilibrium in this course).

Welfare Implications

Is Signaling Good?

Private vs. Social Returns

▶ **Private Return:**

- ▶ High type gains $w_H - w_L = \theta_H - \theta_L$ by signaling.
- ▶ They pay cost $c(e^*, \theta_H)$.
- ▶ Net Private Gain > 0 .

▶ **Social Return:**

- ▶ Does the signal increase productivity?
- ▶ **In this model:** NO. Education is purely wasteful.
- ▶ Total Output = θ (same as without signaling).
- ▶ Total Costs = Education costs are “burned”.

▶ **Example: Advertising** (Lecture 11):

- ▶ High-quality firms burn money on ads to signal quality.
- ▶ Ads themselves provide no direct value to consumers.
- ▶ A clear case where signaling is **socially wasteful**.

Pareto Analysis

- ▶ Compare **Separating Eq** vs **Pooling** (No Signal).
- ▶ **Low Types:**
 - ▶ Separation: Get θ_L .
 - ▶ Pooling: Get $E[\theta] > \theta_L$.
 - ▶ **Worse off** under signaling.
- ▶ **High Types:**
 - ▶ Separation: Get θ_H – Cost.
 - ▶ Pooling: Get $E[\theta]$.
 - ▶ Better off **only if** $\theta_H - \text{Cost} > E[\theta]$.
- ▶ Often, High types are forced into a "Rat Race" where they signal just to prove they are good, but the cost eats up most of the gain.

Alternative View: Human Capital

Human Capital Theory (Becker)

- ▶ Education **increases** productivity (θ).
- ▶ You go to college to learn skills that make you more valuable.

Signaling Theory (Spence)

- ▶ Education **reveals** productivity.
- ▶ You go to college to demonstrate you have the discipline/intelligence to graduate.

Reality: Probably a mix of both.

Summary

Summary

1. **Adverse Selection** destroys markets (Lemons Problem) when quality is unobservable.
2. **Signaling** restores markets by allowing High types to "burn money" in a way Low types cannot mimic.
3. **Single Crossing Property** (SCP) is the engine: Marginal cost of signal must be lower for High type.
4. **Separating Equilibrium**:
 - ▶ Low types take $e = 0$.
 - ▶ High types take e^* such that Low types are indifferent.
 - ▶ Wage differentiates: $w_L = \theta_L, w_H = \theta_H$.
5. **Social Welfare**: Signaling reveals information but is **costly**. It can be pareto-worse than random hiring!