BCSE Game Theory 02-01

Dominated strategy

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Today's Plan

Today's Goals

- Explore motivating examples that reveal conflicts between individual and joint incentives.
- Formalise strict and weak dominance and practice eliminating dominated strategies.
- Use IESDS to connect dominance reasoning to broader solution concepts.

Motivating Games

Teamwork Game

- If at least one teammate works hard, the pair earns a good grade in this course.
- If both slack off, the project tanks and so does the course grade.
- Working hard consumes hours that could support other classes, so there is an opportunity cost.
- Slacking off frees those hours and may improve performance elsewhere.

		Binh	
		Work Hard	Slack Off
Anh	Work Hard	(3, 3)	(1, 4)
	Slack Off	(4, 1)	(2, 2)

Teamwork Game Discussion

Tension between incentives

Given these incentives, how would you describe the tension between individual incentives and the joint outcome?

Are Anh and Binh acting cooperatively, or are they trapped by self-interest?

$\frac{2}{3}$ Average Guessing Game

- Four players simultaneously pick an integer in $[0, 10]$.
- Compute the arithmetic mean of the four guesses.
- Multiply that mean by $\frac{2}{3}$.
- Whoever is closest to the target wins 10 points (ties split the prize).

Reasoning Toward the Target

How low do you expect the guesses to go if we keep repeating the game?

What reasoning path takes you there?

	Round 1	Round 2	Round 3
Group 1			
Group 2			
Group 3			
Group 4			
Average			
$\frac{2}{3}$ Average			
Winning Bid			

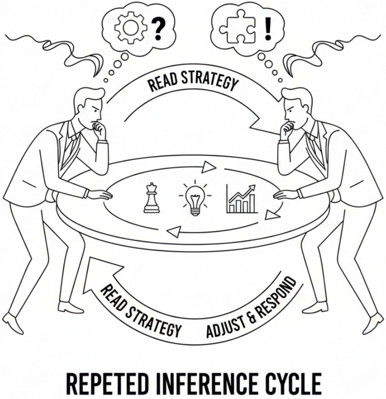
Why Do Outcomes Shift?

Across repetitions we saw the target fall in the guessing game. Players keep updating beliefs about what others will do, so best responses shift accordingly.

How does that iterative reasoning push play toward what looks like an irrational outcome?

Iterative Reasoning in Practice

- We forecast others' moves and adjust our strategies to best respond.
- The forecast itself evolves because others are predicting us as well.



Notation Refresher

Notation for Today

We will use a few pieces of notation from earlier lectures.

- S_i: strategy set for player i.
- $S \stackrel{\text{def}}{=} \prod_{i=1}^{n} S_i$: set of all strategy profiles.
- $S_{-i} \stackrel{\text{def}}{=} \prod_{jei} S_j$: strategies of everyone except i.

$$S \stackrel{\text{def}}{=} S_1 \times \cdots \times S_{i-1} \times S_{i+1} \times \cdots \times S_n$$
$$S_{-i} \stackrel{\text{def}}{=} S_1 \times \cdots \times S_{i-1} \times S_{i+1} \times \cdots \times S_n$$

Dominated Strategies

Strongly Dominated Strategies

Definition: Strongly dominated strategy

Fix player i and two strategies $s_{i,1}, s_{i,2} \in S_i$ of player i. We say that $s_{i,1}$ is strongly dominated by $s_{i,2}$ if $s_{i,2}$ yields a strictly higher payoff no matter what the opponents play.

$$U_i(s_{i,1}, s_{-i}) < U_i(s_{i,2}, s_{-i}) \quad \text{for all } s_{-i} \in S_{-i}.$$

Teamwork Game Revisited

We revisit the teamwork game to reason about dominated strategies.

		Binh: Work Hard	Binh: Slack Off
Anh	Work Hard	(3, 3)	(1, 4)
	Slack Off	(4, 1)	(2, 2)

Teamwork Game Revisited

We revisit the teamwork game to reason about dominated strategies.

		Binh: Work Hard	Binh: Slack Off
Anh	Work Hard	(3,)	(1,)
	Slack Off	(4,)	(2,)

- Consider Anh's decision making.
- When Binh chooses Work Hard, which action leaves Anh better off? $\boxed{}$
- When Binh chooses Slack Off, which action leaves Anh better off? $\boxed{}$
- Therefore the strategy $\boxed{}$ is strongly dominated.

Implications of Strong Dominance

Claim. A rational decision maker never plays a strongly dominated strategy.

If a strategy strictly dominates every alternative a player has, that strategy is the obvious choice. When each player possesses such a strictly dominant strategy, the resulting profile is called a strongly dominant-strategy equilibrium.

Definition: Strongly dominant-strategy equilibrium

A strategy profile $s \stackrel{\text{def}}{=} (s_1, \ldots, s_n)$ is a strongly dominant-strategy equilibrium if, for every player i, the strategy s_i strongly dominates every other strategy in S_i.

Dominance: Key Definitions

Definition: Strict dominance

Strategy $s_{i,1} \in S_i$ strictly dominates $s_{i,0} \in S_i$ when

$$u_i(s_{i,1}, s_{-i}) > u_i(s_{i,0}, s_{-i}) \quad \text{for all } s_{-i} \in S_{-i}.$$

It yields a higher payoff regardless of opponents' actions.

Definition: Weak dominance

Strategy $s_{i,1}$ weakly dominates $s_{i,0}$ if

$$\begin{aligned} u_i(s_{i,1}, s_{-i}) &\geq u_i(s_{i,0}, s_{-i}) && \text{for all } s_{-i} \in S_{-i}, \\ u_i(s_{i,1}, s'_{-i}) &> u_i(s_{i,0}, s'_{-i}) && \text{for some } s'_{-i} \in S_{-i}. \end{aligned}$$

Dominance: Key Definitions

Rationalisability

Eliminating strictly dominated strategies leaves the strategy profiles consistent with rational play that is commonly known. Weak dominance can aid the process, but the order of deletions must be recorded.

Dominated Strategy Warm-Up

Warm-up A: 2×2

Identify any strictly dominated strategy in the game below and state which profile survives once the dominated option is removed.

		Player 2 L	Player 2 R
Player 1	U	(4, 3)	(6, 2)

Dominated Strategy Warm-Up (cont.)

Warm-up B: 3×2

Repeat the dominance check for Player 1's three strategies. Record the order in which you eliminate dominated strategies and the survivor.

Player 2

Player 1		L	R
	U	(4, 3)	(6, 2)
	M	(2, 1)	(3, 6)
	D	(3, 0)	(2, 8)

Solution Sketch: Two Warm-Up Tables

- **First table**: Column L strictly dominates R for Player 2; with R removed Player 1 keeps U, so (U, L) remains.
- **Two-by-three table**: Compare columns. Row U yields higher payoffs than M in both columns, so M is strictly dominated and removed. Then column C strictly dominates L for Player 2. Finally U strictly dominates D, leaving (U, R).
- **Strategy**: Write down each elimination step and check dominance from the reduced table to avoid overlooking options.

Dominated Strategy Exercise

Warm-up B: 3 × 3

1. Start by comparing Player 2's columns and eliminate any column that is strictly dominated.
2. Re-evaluate dominance for Player 1 in the reduced matrix.
3. Continue until only undominated strategies remain. Write the surviving profile on the board.

		Player 2		
		L	C	R
	U	(4, 3)	(5, 1)	(6, 2)
Player 1	M	(2, 1)	(8, 4)	(3, 6)
	D	(3, 0)	(9, 6)	(2, 8)

Solution Sketch: Three-by-Three Matrix

1. **Column comparison**: Column C strictly dominates L for Player 2 because $5 > 4$, $8 > 2$, $9 > 3$.
2. **Row comparison in reduced game**: With L removed, row U strictly dominates D (compare columns C and R).
3. **Remaining strategies**: The surviving profile is (U, R) with payoffs $(6, 2)$.
4. **Reflection**: Unlike weak dominance, these steps are order-independent—students should verify by swapping the first two steps and confirming the same survivor set.

Weakly Dominated Strategies

Definition: Weakly dominated strategy

Strategy $s_{i,0} \in S_i$ is weakly dominated by $s_{i,1} \in S_i$ if $s_{i,1}$ never yields a lower payoff and is strictly better for at least one opponent profile.

$$U_i(s_{i,1}, s_{-i}) \geq U_i(s_{i,0}, s_{-i}) \quad \text{for all } s_{-i} \in S_{-i},$$
$$U_i(s_{i,1}, s'_{-i}) > U_i(s_{i,0}, s'_{-i}) \quad \text{for some } s'_{-i} \in S_{-i}.$$

In today's exercises we focus on strong dominance, but weak dominance will reappear when we discuss equilibrium selection.

Iterated Elimination

Definition: IESDS

A strongly dominant-strategy equilibrium, if it exists, is unique.

But many games have no such profile. We can still simplify them by iteratively deleting strictly dominated strategies—this is the idea of IESDS.

Definition: IESDS

Iterated elimination of strictly dominated strategies (IESDS) is the process of repeatedly removing strategies that are strictly dominated for some player, updating the game after each removal until no additional strictly dominated strategies remain.

Assumptions Behind IESDS

- In finite normal-form games, eliminating strictly dominated strategies is order independent; any sequence of deletions yields the same survivor set.
- The surviving strategies coincide with the rationalizable strategies—those consistent with common knowledge of rationality.
- With infinite strategy spaces or when weak dominance is allowed, order independence can fail, so we must specify the deletion order explicitly (see next example).

When Order Matters

Even though IESDS with strictly dominated strategies is order independent, allowing weakly dominated strategies can break that property.

		Player 2		
		L	C	R
	U	(3, 1)	(2, 1)	(0, 0)
Player 1	M	(3, 2)	(1, 0)	(0, 0)
	D	(0, 0)	(0, 0)	(1, 2)

M is weakly dominated by U, and column C is weakly dominated by column L because of row M.

Order-Dependent Outcome

1. **Delete M first.** Treating M as strongly dominated removes it immediately, leaving rows $\{U, D\}$ and predicting the IESDS equilibrium within $\{U, D\} \times \{L, C, R\}$.
2. **Delete C first.** If we instead discard C before reconsidering M, the reduced game keeps rows $\{U, M, D\}$ and columns $\{L, R\}$, delivering a different surviving set of strategy profiles.

Belief Matters

Expectations about the opponent's behaviour determine which strategies we mark as strongly dominated, so different forecasts can change the IESDS equilibrium we report.

Weak Dominance Counterexample

Allowing weakly dominated strategies into IESDS can produce order-dependent predictions.

	L	**C**	**R**
T	(1,1)	(1,1)	(0,0)
B	(0,0)	(1,2)	(1,2)

C weakly dominates both L and R.

- Delete L first: Under weak dominance Player 1 chooses B. Whether Player 2 selects C or R, the resulting payoff is (2,1).
- Delete R first: Under weak dominance Player 1 chooses T. Whether Player 2 selects L or C, the resulting payoff is (1,1).
- Strict dominance avoids this ambiguity; weak dominance makes the survivor order dependent.

IESDS Order Independence

Strict vs. Weak

With strictly dominated strategies the IESDS equilibrium is independent of the deletion order. Allowing weakly dominated strategies destroys that guarantee.

- Strict dominance uses a strict inequality, so removing one strategy never makes another previously undominated strategy become dominant.
- Hence any strictly dominated strategy could have been deleted first; order only changes the pace, not the surviving set.
- Once weak dominance is permitted, equal-payoff ties break this argument and different deletion orders can yield different survivor sets.

Common Pitfalls

- **Skipping steps**: Document each elimination. If you jump straight to the answer it is easy to miss a remaining strategy.
- **Mixing strong and weak dominance**: Mark clearly whether an inequality is strict; only strict dominance preserves order independence.
- **Ignoring beliefs**: A strategy can be undominated yet implausible—state the belief about opponents when you declare a strategy "never played."
- **Confusing payoffs**: Check whose payoff each number represents before comparing rows or columns.

IESDS Step 1: Teamwork Game

		Binh: Work Hard	Binh: Slack Off
Anh	Work Hard	(3, 2)	(1, 4)
Anh	Slack Off	(4, 4)	(2, 2)

IESDS Step 2: Teamwork Game

		Binh	
		Work Hard	Slack Off
Player 1	Slack Off	(4, 4)	(2, 2)

IESDS Step 3: Teamwork Game

		Binh
		Slack Off
Player 1	Slack Off	(4, 4)

Definition: IESDS Equilibrium

Definition: IESDS equilibrium

A strategy profile that survives iterated elimination of strictly dominated strategies is called an IESDS equilibrium. If more than one profile survives, the game does not yield a unique IESDS equilibrium.

Every player's surviving strategy remains after deleting their strictly dominated options, so the surviving strategies are mutually rational under the elimination process.

IESDS Exercise — Setup

Instructions

Use **iterated elimination of strictly dominated strategies**. The next slides walk through the board after each deletion. Before advancing:

1. Decide which column Player 2 should delete first.
2. Remove any strictly dominated row that appears after that deletion.
3. Repeat until no further strictly dominated strategies remain.

		Player 2		
		L	C	R
	U	(4, 3)	(5, 1)	(6, 2)
Player 1	M	(2, 1)	(8, 4)	(3, 6)
	D	(3, 0)	(9, 6)	(2, 8)

IESDS Exercise — Step 1

Step 1: Record the first strictly dominated strategy you eliminate (one valid solution removes Player 2's column C). Update the payoff table accordingly.

		Player 2		
		L		R
Player 1	U	(4, 3)		(6, 2)
	M	(2, 1)		(3, 6)
	D	(3, 0)		(2, 8)

IESDS Exercise — Step 2

Step 2: After the first deletion, remove any newly dominated row. Note which player loses a strategy at this step.

Player 2

		L		R
Player 1	U	(4, 3)		(6, 2)

IESDS Exercise — Step 3

Step 3: Carry the eliminations through to the end. The final slide should display only the surviving profile.

Player 2

		L		
Player 1	U	(4, 3)		

Wrap-Up

Key Takeaways

- Dominance compares payoffs strategy-by-strategy; strictly dominated options should never be played.
- Iterated elimination can reveal predictions even when no dominant-strategy equilibrium exists, but the IESDS equilibrium you report depends on expectations about opponents' behaviour when labelling strategies strongly dominated.
- Practice with payoff tables and notation helps spot dominated strategies quickly during exams.
- Always make your beliefs explicit—different expectations can lead to different IESDS equilibria.