Game Theory 12-03 Solutions

Solutions: Signaling, Screening, and Dynamic Games

BCSE Game Theory

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Q1. Signaling (Solution)

Answer: Costs and Incentives

1. **Single Crossing Property**:
   - $C_L = c(1, 5) = 4$.
   - $C_H = c(1, 10) = 2$.
   - $C_L > C_H$ $(4 > 2)$. Yes, the **SCP is satisfied**.
2. **Separating Equilibrium Check**:
   - Candidate Profile: High $\rightarrow e = 1$, Low $\rightarrow e = 0$.
   - Beliefs/Wages: $w(1) = 10$, $w(0) = 5$.
   - **Check High (IC)**:
     - Signal: $10 - 2 = 8$.
     - Deviate: $5 - 0 = 5$.
     - $8 > 5$. High type prefers to signal. (OK)
   - **Check Low (IC)**:
     - Don't Signal: $5 - 0 = 5$.
     - Mimic High: $10 - 4 = 6$.
     - $6 > 5$. **Low type prefers to MIMIC**.
   - **Conclusion**: This is **NOT** an equilibrium. The cost of education is too low

Q2. Screening (Solution)

Answer: Optimal Menu

1. **Constraints**:
   - IR_T: $10(5) - t_T \geq 0 \implies 50 - t_T \geq 0$.
   - IC_B: $20(10) - t_B \geq 20(5) - t_T \implies 200 - t_B \geq 100 - t_T$.
2. **Solving (Binding Constraints)**:
   - From Binding IR_T: $\mathbf{t_T = 50}$.
   - From Binding IC_B:

$$200 - t_B = 100 - 50 = 50 \implies \mathbf{t_B = 150}$$

3. **Utility Comparison**:
   - Business Type Utility (Second Best): $U_B = 20(10) - 150 = \mathbf{50}$.
   - First Best (Perfect Info): Airline would charge t = Value = 200. Benefit would be $200 - 200 = 0$.
   - **Result**: The Business type is **better off** than in the First Best case (Positive Surplus).

Q3. Reputation (Solution)

Answer: Backward Induction vs Reputation

1. $N = 1$ **(Endgame)**:
   - Rational Incumbent prefers Accommodate (Payoff usually higher) over Fight.
   - Since there is no future, there is no benefit to building a reputation.
   - Entrant enters. Incumbent accommodates.
2. $N = 100$ **(Long Horizon)**:
   - **Cost of Fighting**: Short-term loss (Fighting is costly/suboptimal statically).
   - **Benefit of Fighting**: If Incumbent fights, Entrants update beliefs: "Maybe this guy IS Crazy".
   - If $P(Crazy)$ stays high, future entrants ($N = 99, ..., 2$) will stay OUT.
   - **Investment**: Spending small cost now to gain monopoly profits in 90+ future markets.
   - Thus, Rational Incumbent mimics the Crazy type.

Q4. Information Cascade (Solution)

Answer: Bayes Update

1. **Person 1**: $s_1 = A$. Pr(Good=A) = 0.7. **Chooses A**.
2. **Person 2**: Observable History $\{A\}$. Private Signal $s_2 = A$.
   - Infers $s_1 = A$. Total info: $\{A, A\}$.
   - Probability of A is very high. **Chooses A**.
3. **Person 3**: History $\{A, A\}$. Private Signal $s_3 = B$.
   - Infers $s_1 = A, s_2 = A$.
   - Total Information: $\{A, A, B\}$.
   - Majority is A. (Two As vs One B).
   - Posterior $P(A) > 0.5$.
   - **Action**: **Chooses A**, ignoring his own signal B.
   - **Cascade**: Person 4 will see AAA. Even if $s_4 = B$, they will choose A.

Q5. Signaling vs Screening (Solution)

Answer: Who Moves First?

Scenario A: Student takes Course (Signaling)

- **Informed Party (Student)** moves first.
- Student knows their ability and takes action to reveal it.

Scenario B: Employer requires TOEIC (Screening)

- **Uninformed Party (Employer)** moves first.
- Employer sets a test/hurdle (mechanism) that applicants must jump over to separate themselves.

Key Difference:

- Signaling: Sender (Informed) initiates.
- Screening: Receiver (Uninformed) initiates.