

# Game Theory 09-03

## Exercises: Incomplete Information

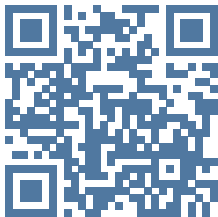
BCSE Game Theory

Dec. 3, 2025

# Exercise Session

## Bayesian Nash Equilibrium & Cournot with Entry

# Answer on Google Slides



<https://sites.google.com/vju.ac.vn/bcse-gt>

- ▶ Submit one PDF per team.
- ▶ Focus on the logic of multiple equilibria in Q2.
- ▶ Show your calculations for the profit comparisons.

## Notes

1. For Q2, remember that fixed costs are paid only if  $q > 0$ .
2. Check the "Exit" condition by verifying if optimal entry yields negative profit.

# Q1. Bayesian Nash Equilibrium (Warm-up)

## Q1. Asymmetric Information Matrix Game

Consider a game between Player 1 and Player 2.

- ▶ Player 1 has one type.
- ▶ Player 2 has two types: **Left** type (prob 0.6) and **Right** type (prob 0.4).
- ▶ Player 2 knows their own type; Player 1 only knows the probabilities.

**Payoffs (P1, P2):**

If P2 is Left Type			If P2 is Right Type		
	L	R		L	R
U	(2, 2)	(0, 0)	U	(0, 0)	(1, 1)
D	(0, 0)	(1, 1)	D	(2, 2)	(0, 0)

1. Find the Bayesian Nash Equilibrium (BNE) in pure strategies.
2. (Hint: P2's best response depends on their type. P1 maximizes expected payoff given P2's strategy).

## Q2. Cournot with Fixed Costs (Multiple Equilibria)

### Q2. Entry Decision with Fixed Costs

Demand  $P = 12 - Q$ . Firm 1 has cost  $c_1 = 0$ . Firm 2 has cost  $c_2 \in \{0, 4\}$  with equal probability (0.5). **Fixed Cost:** Both firms face a fixed cost  $F = 2.5$  if they produce ( $q > 0$ ).

- Equilibrium A (Both Enter):** Assume both types of Firm 2 enter.
  - ▶ Calculate the equilibrium quantities  $(q_1, q_2^L, q_2^H)$ .
  - ▶ Calculate the profit of Firm 2's High type. Is it  $> 0$ ?
- Equilibrium B (High Type Exits):** Assume Firm 2's High type exits ( $q_2^H = 0$ ).
  - ▶ Calculate the new equilibrium quantities  $(q_1, q_2^L)$ .
  - ▶ Calculate the profit Firm 2's High type *would* earn if they deviated and entered (best response to  $q_1$ ). Is it  $< 0$ ?
- Discussion:** Why do both equilibria exist? How does Firm 1's aggressive production in Case B deter the High type?

## Q3. Differentiated Bertrand Competition

### Q3. Pricing with Unknown Costs

Two firms sell differentiated products. Demand for firm  $i$ :  $q_i = 10 - p_i + 0.5p_j$ .

- ▶ Firm 1 has cost  $c_1 = 2$ .
  - ▶ Firm 2 has cost  $c_2 \in \{0, 4\}$  with equal probability (0.5).
  - ▶ Firm 1 maximizes expected profit:  
 $E[\pi_1] = (p_1 - 2)(10 - p_1 + 0.5E[p_2])$ .
1. Write down Firm 1's profit function in terms of  $p_1$  and  $E[p_2]$ .
  2. Derive Firm 1's best response function  $p_1(E[p_2])$ .
  3. (Optional) If Firm 2's best response is  $p_2(p_1) = \frac{10+c_2+0.5p_1}{2}$ , find the equilibrium price  $p_1$ .

## Q4. First-Price Auction Strategy

### Q4. Optimal Bidding

You are Bidder 1 in a First-Price Auction with valuation  $v_1 = 4$ . You believe Bidder 2 bids as follows:

- ▶ If  $v_2 = 2$ : Bidder 2 bids  $b_2 = 1$ .
- ▶ If  $v_2 = 4$ : Bidder 2 bids  $b_2 = 3$ .

Assume  $v_2$  is 2 or 4 with probability 0.5 each.

1. Calculate your expected payoff if you bid  $b_1 = 3$ . (Win against  $b_2 = 1$  and  $b_2 = 3$ ).
2. Calculate your expected payoff if you bid  $b_1 = 1$ . (Win against  $b_2 = 1$ , lose against  $b_2 = 3$ ).
3. Which bid is better?

## Q5. Best Price Guarantee

### Q5. Conceptual Analysis

Many retailers offer a "Best Price Guarantee" (BPG): "If you find a lower price, we will match it."

1. Explain intuitively why BPG might lead to **higher** equilibrium prices compared to standard price competition.
2. Does BPG increase or decrease the incentive to undercut prices? (Hint: If opponent lowers price, do you have incentive to lower yours?)