Game Theory 08-03

Exercises: Repetition and Enforcement

BCSE Game Theory

Nov. 25, 2025

Exercise Session

Apply finite + infinite repetition insights

Answer on Google Slides

https://sites.google.com/vju.ac.vn/bcse-gt

- Submit one PDF per team summarising equations, incentive checks, and diagrams.
- Highlight which sections rely on the finite-horizon lessons (08-01) versus the infinite-horizon apparatus (08-02).

Notes

1. Record the history-dependent strategy as a flowchart or bullet list.
2. Use the dynamic programming notation from Lecture 08-02 when describing continuation values.

Q6. Advanced Topics and Private Monitoring

Q6. Conceptual Challenges

- **Private Monitoring**: Explain why standard trigger strategies might fail when players observe private signals instead of a public outcome. (Hint: Do players agree on the history?)
- **Review Strategies**: How does maintaining the same action for a fixed block of periods (review phase) help in an imperfect monitoring environment? What is the trade-off involved?

Q1. Stag Hunt and Equilibrium Paths

Q1. Coordination in Finite Horizon

Consider a Stag Hunt game where players choose Work (W) or Rest (R).

	W	R
W	$(5, 5)$	$(0, 4)$
R	$(4, 0)$	$(2, 2)$

1. Identify the two pure Nash equilibria. Which is Pareto dominant? Which is risk dominant?
2. Explain why the strategy "Play W in every period" is a Subgame Perfect Equilibrium (SPE) for any finite T.
3. Consider $T = 2$. Show that the path "Play R in period 1, then W in period 2" is also an SPE outcome.
4. Contrast this with the finite Prisoner's Dilemma (Q2), where the unique stage Nash equilibrium implies a unique SPE path.

Q2. Cournot Competition with Punishment

Q2. Collusion in Duopoly

Consider a Cournot duopoly with demand $P = 24 - Q$ and zero costs ($MC = 0$).

1. **Cooperative Payoff**: Calculate the profit for each firm if they collude to maximize joint monopoly profit and split it equally.
2. **Punishment Payoffs**: Calculate the profit for each firm in the one-shot Nash equilibrium, and the minmax payoff.
3. **Minimum Punishment Duration**: Suppose $\delta = 0.6$. The firms use a strategy: "Cooperate. If deviation, revert to Nash equilibrium for m periods, then Cooperate."
   - Calculate the one-shot gain from deviating (best response to cooperation).
   - Find the minimum m required to sustain cooperation.
   - (Hint: Incentive constraint is Gain $\leq$ Discounted Loss).
   - **Credibility**: Explain why reverting to the Nash equilibrium for m periods is a credible threat (subgame perfect) even after a deviation.

Q3. Bellman equation in action

Q3. Continuation payoff calculations

You target the payoff vector $v = (4.5, 4.5)$ using continuation payoffs in an infinite repetition of a new stage game where mutual cooperation gives $(5, 5)$, mutual defection $(2, 2)$, and unilateral deviation $(7, 1)$.

1. Write the Bellman equation for player 1 (use a symmetric version for player 2).
2. Let the continuation payoff in cooperation be γ and the punishment payoff be $\gamma_P = 2$. Solve for γ such that $5 + \delta\gamma = v_1$.
3. Use the incentive constraint from Lecture 08-02 to derive the minimum δ that deters a deviation given $\gamma_P = 2$.
4. Explain how the recursive structure (current stage + continuation) ensures that v is enforceable.

Q4. Imperfect monitoring exercise

Q4. Noise and forgiving triggers

Each period, a public signal $y \in \{\text{good}, \text{bad}\}$ is drawn: cooperation produces good with probability 0.9, defection yields bad with probability 0.7. A grim-style trigger interprets one bad signal as defection and punishes forever.

1. Compute the probability that a good path suffers punishment in the first two periods (false alarm).
2. Express the expected loss from the false alarm as a function of δ when punishment is $(2, 2)$ forever.
3. Propose a lenient trigger (e.g. two consecutive bad signals before punishment) and argue how it mitigates the cost as $\delta \to 1$.
4. **Limit Analysis**: Calculate the limit of the expected payoff of the Grim Trigger as $\delta \to 1$. Does it approach the cooperative payoff or the punishment payoff?

Q5. Folk theorem construction

Q5. Crafting individually rational payoffs

A repeated stage game has minmax values $(3, 3)$ and feasible payoffs include $(6, 4)$, $(5, 5)$, and $(7, 2)$.

1. Verify which of these vectors are individually rational.
2. Sketch a punishment scheme that can support $(6, 4)$ using continuation payoffs and triggers designed in Lecture 08-02.
3. Describe how the Folk theorem justifies your scheme when δ is high.
4. Contrast your plan with the limited punishment palette available in 08-01’ s finite repetitions.