BCSE Game Theory 12-02

Applications: Screening, Reputation, and Entry Deterrence

BCSE Game Theory

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Akerlof's Solutions

Institutions and Screening

Akerlof's Solution: Institutions

George Akerlof suggested that "institutions" arise to counteract quality uncertainty:

- **Guarantees / Warranties**: Sellers promise to pay if quality is bad.
  - Costly for Low-quality sellers (they will have to pay).
  - Cheap for High-quality sellers (they rarely pay).
- **Brand Names**: A reputation for quality acts as a bond.
  - Deception destroys the brand value.
- **Licensing / Certification**: Third-party verification.
  - Examples: Mechanics, Doctors, University Accreditation.

From Signaling to Screening

- **Signaling** (Spence): The *informed* party moves first.
  - Worker chooses education → Firms observe and offer wages.
- **Screening** (Rothschild-Stiglitz): The *uninformed* party moves first.
  - Principal designs a menu of contracts → Agents self-select.
- Both rely on **Single Crossing Property** to separate types.

Screening (Monopolist)

Designing Menus of Contracts

Signaling vs Screening

- **Recap**: In **Signaling** (last lecture), the *informed* agent moves first (e.g., Worker gets educated).
- **Screening**: The *uninformed* principal moves first by offering a menu of choices.
- **The Goal**: Design a menu of contracts $\{(q_L, t_L), (q_H, t_H)\}$ so that agents "self-select" into the correct contract.
- **Examples**:
  - **Insurance**: High deductible (cheap) vs Low deductible (expensive).
  - **Airlines**: Economy (cramped, cheap) vs Business (spacious, expensive).

Model Setup (Insurance): Players and Contracts

- **Principal**: Monopolist seller (e.g., Insurance Company).
- **Agent**: Buyer (e.g., Driver) with private type θ.
- **Types**:
  - θ_H (High Willingness to Pay / High Risk). Prob p.
  - θ_L (Low Willingness to Pay / Low Risk). Prob $1 - p$.
  - Assumption: $\theta_H > \theta_L$.
- **Contract**: (q, t).
  - q: Quality (Coverage level).
  - t: Transfer (Premium paid).

Utility and Profits

Agent's Utility:

$$U(q, t, \theta) = \theta V(q) - t$$

- $V(q)$: Concave function ($V' > 0, V'' < 0$).
- High type θ_H values quality **more** (Single Crossing).

Principal's Profit:

$$\Pi = t - C(q)$$

- $C(q)$: Cost of providing quality (Convex).

Benchmark: First Best (Observable Types)

If the Principal can observe types, they extract all surplus (Perfect Price Discrimination).

- For type θ_i ($i \in \{L, H\}$):
- Maximize Total Surplus:

$$V'(q_i^{FB}) = C'(q_i^{FB})/\theta_i$$

- Set Transfer to extract full utility:

$$t_i = \theta_i V(q_i^{FB})$$

- **Result**: Efficient Quality levels (q_L^{FB}, q_H^{FB}), Buyer gets 0 surplus.

Second Best: Unobservable Types

The Principal cannot see θ.

- If the Principal offers the First Best menu $\{(q_L^{FB}, t_L), (q_H^{FB}, t_H)\}$:
- The High Type will look at the Low contract.
- **Why would High Type prefer Low Contract?**
  - Low contract: Less coverage ($q_L < q_H$), but **much cheaper** ($t_L \ll t_H$).
  - High type values coverage more, but the **premium savings** can outweigh the coverage loss.
  - If $\theta_H V(q_L^{FB}) - t_L > \theta_H V(q_H^{FB}) - t_H$, High type will **mimic** Low.
- The Principal must adjust the menu to prevent this mimicking.

The Incentive Constraints

The Principal chooses $\{(q_L, t_L), (q_H, t_H)\}$ subject to 4 constraints.

1. **Participation Constraints (IR)** (Individual Rationality):
   - $IR_L : \theta_L V(q_L) - t_L \geq 0$
   - $IR_H : \theta_H V(q_H) - t_H \geq 0$
2. **Incentive Compatibility Constraints (IC)**:
   - $IC_L : \theta_L V(q_L) - t_L \geq \theta_L V(q_H) - t_H$ (L prefers L)
   - $IC_H : \theta_H V(q_H) - t_H \geq \theta_H V(q_L) - t_L$ (H prefers H)

Which Constraints Bind?

1. IC_H **binds**: The High type is the one who wants to mimic the Low. (Rich people want cheap insurance).
2. IR_L **binds**: The Principal squeezes the Low type until they are indifferent to leaving.
3. IR_H **is slack**: The High type gets "Information Rent" (positive surplus) to prevent mimicking.
4. IC_L **is slack**: The Low type has no desire to pay the High price.

Solving the Problem

From Binding Constraints:

1. $t_L = \theta_L V(q_L)$ (From IR_L).
2. $t_H = \theta_H V(q_H) - (\theta_H V(q_L) - t_L)$ (From IC_H).
    - Substitute t_L:
    - $t_H = \theta_H V(q_H) - (\theta_H - \theta_L)V(q_L)$.

Interpretation:

- High type pays for their value $\theta_H V(q_H)$ minus a **discount**.
- Discount $= (\theta_H - \theta_L)V(q_L)$.
- This discount is the **Information Rent**.

Optimal Quality Levels

Principal maximizes Expected Profit.

$$\max_{q_L, q_H} p(t_H - C(q_H)) + (1-p)(t_L - C(q_L))$$

Recall from binding constraints:

- $t_L = \theta_L V(q_L)$
- $t_H = \theta_H V(q_H) - (\theta_H - \theta_L) V(q_L)$

FOC with respect to q_H:

$$\theta_H V'(q_H) = C'(q_H) \implies \mathbf{q_H^{SB} = q_H^{FB}}$$

Result 1: No Distortion at the Top.

- The "good" type gets efficient quality. Why distort efficiency?

Distortion at the Bottom: Derivation

Substitute t_L, t_H into Profit:

$$\begin{aligned}\Pi = & \ p\big[\theta_H V(q_H) - (\theta_H - \theta_L)V(q_L) - C(q_H)\big] \\ & + (1-p)\big[\theta_L V(q_L) - C(q_L)\big]\end{aligned}$$

FOC with respect to q_L:

$$\frac{\partial \Pi}{\partial q_L} = -p(\theta_H - \theta_L)V'(q_L) + (1-p)[\theta_L V'(q_L) - C'(q_L)] = 0$$

Rearranging:

$$V'(q_L)\left[(1-p)\theta_L - p(\theta_H - \theta_L)\right] = (1-p)C'(q_L)$$

Distortion at the Bottom: Result

Comparing to First Best:

- First Best: $\theta_L V'(q_L^{FB}) = C'(q_L^{FB})$
- Second Best: $V'(q_L)\left[\theta_L - \frac{p}{1-p}(\theta_H - \theta_L)\right] = C'(q_L)$

The term $\frac{p}{1-p}(\theta_H - \theta_L)$ is **positive**, so:

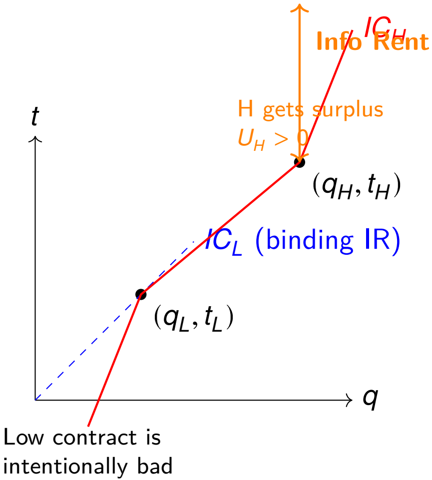
$$\text{Effective marginal benefit} < \theta_L V'(q_L)$$

$$\implies \mathbf{q_L^{SB} < q_L^{FB}}$$

Economic Intuition:

- Raising q_L increases Information Rent paid to High type (via $V(q_L)$ in t_H).
- Principal deliberately **degrades** Low contract to reduce this rent.

Visualizing Information Rent (Graph)



Screening: Numerical Example

Calculating the Optimal Menu

Example Setup

- **Agent Types**: $\theta_L = 2, \theta_H = 4$.
- **Probabilities**: $P(\theta_L) = 0.5, P(\theta_H) = 0.5$.
- **Utility**: $U = \theta q - t$.
- **Cost**: $C(q) = \frac{1}{2}q^2$.
- **First Best** (Reference):
  - $MB = MC \implies \theta = q$.
  - $q_L^{FB} = 2, q_H^{FB} = 4$.
  - Surplus $S^{FB} = \theta q - \frac{1}{2}q^2$.
  - $S_L^{FB} = 2(2) - 2 = 2$. $S_H^{FB} = 4(4) - 8 = 8$.

Second Best: Binding Constraints

- Principal solves:

$$\max_{q_L, q_H, t_L, t_H} 0.5(t_L - \frac{1}{2}q_L^2) + 0.5(t_H - \frac{1}{2}q_H^2)$$

- Subject to:
  - $IR_L : 2q_L - t_L = 0 \implies t_L = 2q_L$.
  - $IC_H : 4q_H - t_H = 4q_L - t_L$.
- Substitute t_L into IC_H:

$$4q_H - t_H = 4q_L - 2q_L = 2q_L$$

$$t_H = 4q_H - 2q_L$$

Second Best: Solving for Quality

Substitute t_L, t_H into Profit:

$$\Pi = 0.5(2q_L - 0.5q_L^2) + 0.5((4q_H - 2q_L) - 0.5q_H^2)$$

Optimize q_H:

$$\frac{\partial \Pi}{\partial q_H} = 0.5(4 - q_H) = 0 \implies \mathbf{q_H^{SB} = 4}$$

(Same as First Best! No distortion at top).
Optimize q_L:

$$\frac{\partial \Pi}{\partial q_L} = 0.5(2 - q_L) + 0.5(-2) = 0$$

$$1 - 0.5q_L - 1 = 0 \implies 0.5q_L = 0 \implies \mathbf{q_L^{SB} = 0}$$

(Corner solution in this specific linear-quadratic example implies L is excluded).

Alternative Example (Non-Exclusion)

Let $\theta_L = 3, \theta_H = 4$. $P = 0.5$.

- $\Pi = 0.5(3q_L - 0.5q_L^2) + 0.5((4q_H - 1q_L) - 0.5q_H^2)$.
- FOC q_L:

$$0.5(3 - q_L) - 0.5(1) = 0$$

$$1.5 - 0.5q_L - 0.5 = 0 \implies 1 = 0.5q_L \implies \mathbf{q_L^{SB} = 2}$$

- Compare to First Best $q_L^{FB} = 3$.
- **Result**: $q_L^{SB} < q_L^{FB}$. Quality is degraded.

Real World Screening

- **Economy Class Seats**: Airlines make them uncomfortable not just to save space, but to force business travelers to pay for Business Class.
- **Software**: "Student Versions" disable key features to force Pros to buy the full license.
- **Coupons**: Clipping coupons takes time. Low opportunity cost people clip; High opportunity cost people pay full price.

Reputation in Dynamic Games

Building Credibility

Reputation Concept

- In repeated games, players care about the future.
- **Reputation**: The set of beliefs other players hold about your type.
- If you are a **Strategic** player, you might mimic a "Good" type early on to reap benefits later.
- **Trade-off**:
  - Short-term cost: Mimicking "Good" usually costs money/effort.
  - Long-term gain: Trust, higher prices, deterrence.

General Structure: Long-Run vs Short-Run

Most Reputation models share this structure:

1. **Long-Run Player** (e.g., Incumbent, Monopolist, Restaurant).
   - Plays the game N times (or infinitely).
   - Cares about the sum of discounted payoffs.
2. **Sequence of Short-Run Players** (e.g., Entrants, Consumers).
   - Each plays only ONCE.
   - They only care about the current period payoff.
   - They cannot "punish" the Long-Run player in the future (they won't be there).

Classic Examples:

- **Entry Deterrence**: Chain Store vs Entrants.
- **Product Quality**: Restaurant vs Customers (if I serve bad food today, today's customer is unhappy, but future customers won't come).

The Chain Store Paradox (Selten 1978)

Scenario

- An Incumbent (Chain Store) exists in N markets.
- A potential Entrant appears in market 1, then market 2, ..., N.
- In each market k:
  1. Entrant k chooses In or Out.
  2. If In, Incumbent chooses **Fight** (Predatory Price) or **Accommodate**.

Payoffs (Entrant, Incumbent):

- Out: $(0, 5)$.
- In, Accommodate: $(2, 2)$.
- In, Fight: $(-1, 0)$.

The Paradox: Unravelling

Backward Induction:

- **Last Market (N)**:
  - If Entrant enters, Incumbent chooses between Fight (0) and Accommodate (2).
  - Rational Choice: **Accommodate**.
  - Entrant N knows this $\implies$ **Enters**.
- **Market ($N - 1$)**:
  - Incumbent knows that fighting today won't change Market N (since N is determined by short-run incentives).
  - Rational Choice: **Accommodate**.
  - Entrant $N - 1$ **Enters**.
- **Result**: Entry in ALL markets. No Fighting ever.

The Paradox

Theory vs Reality

Theory predicts the Incumbent never fights. Real firms often fight aggressively early on to "teach a lesson" to future entrants.

How do we explain this?

- **Kreps and Wilson (1982)**: Incomplete Information.
- Introduce a small probability that the Incumbent is **"Crazy"** (or "Tough").

The "Crazy" Type Solution

- **Types of Incumbent**:
    1. **Rational**: Payoffs as before. Prefers to Accommodate ($2 > 0$). (Prob $1 - \epsilon$).
    2. **Tough**: Gets joy from fighting. Prefers Fight ($1 > -1$). (Prob ϵ).
- Now, "Fighting" is a **Signal**.
- If Entrant sees "Accommodate", they know for sure Incumbent is Rational.
- If Entrant sees "Fight", Incumbent *might* be Tough.

Equilibrium Dynamics

Rational Incumbent's Strategy:

- **Early Markets**: Fight!
- Why? To mimic the Tough type.
- By Fighting, the Rational Incumbent prevents the belief $P(\text{Tough})$ from dropping to zero.
- If $P(\text{Tough})$ stays high enough, future entrants stay Out.
- **Condition**: Cost of fighting today $<$ Gain from monopoly in future markets.

Unravelling vs Reputation

- With $\epsilon > 0$, the backward induction logic breaks.
- Why? Because behavior in $N - 1$ **change beliefs** in time N.
- **Result**:
  - For most of the game, Rational Incumbent fights.
  - Entrants stay out.
  - Only near the very end (last few markets), the Rational Incumbent stops fighting ("Endgame Effect").

Mechanism: Updating Beliefs

1. **Prior**: $P(\text{Tough}) = \delta_0$.
2. **Observation**: Incumbent Fights.
3. **Update**:
   - If Rational type *always* Fights (pooling): No information revealed. $\delta_1 = \delta_0$.
   - If Rational type *sometimes* Accommodates (mixing): Seeing Fight increases prob of Tough.
4. Rational Incumbent fights to keep δ_t effectively non-zero.

Limit Pricing vs Predatory Pricing

Limit Pricing

- Charging a low price **before** entry to signal low costs.
- "I'm so efficient, you can't compete."
- **Signaling Model**.

Predatory Pricing

- Charging a low price **after** entry (Fighting).
- "I will bleed you dry even if it hurts me."
- **Reputation Model** (Chain Store Paradox).

Connection to Folk Theorem

- In Repeated Games, the Folk Theorem says "Anything is an equilibrium if δ is high enough."
- Reputation models are a **refinement**.
- Instead of "anything goes", we explain **specific** behaviors (like fighting) arising from **specific** types (tough type).
- However, the result depends heavily on *what* crazy types exist.
- If there was an "Altruist" type (loves sharing), reputation would work in reverse!

Summary

Course Wrap-Up

1. **Screening**: Uninformed party moves first. Designing menus (Second-Degree Price Discrimination).
   - Efficient quality for High types.
   - "Damaged" quality for Low types (to extract rent from Highs).
2. **Reputation**:
   - Requires **Repeated Interaction** + **Incomplete Information**.
   - A tiny "grain of doubt" (ϵ crazy type) can drastically change equilibrium behavior.
   - Rational players act "irrationally" (fight, pay costs) to build a valuable reputation.

Theory Recap

We have covered:

- **Static Games**: Nash Equilibrium, Mixed Strategies.
- **Dynamic Games**: Subgame Perfect Equilibrium, Backward Induction.
- **Repeated Games**: Folk Theorems, Cooperation.
- **Incomplete Information**: Bayes Nash Equilibrium, Auctions.
- **Dynamic Incomplete Info**: Perfect Bayesian Equilibrium, Signaling, Screening.

Game Theory is a language to model strategic incentives in complex worlds.