BCSE Game Theory 05-02

Continuous Strategy Games and Selection

Author

Nov. 4, 2025

Today's Agenda

Today's Goals

- Review Cournot, Bertrand, and Hotelling models.
- Interpret equilibria and welfare in continuous strategy settings.
- Extend payoff/risk/focal reasoning beyond discrete games.
- Summarise the Harsanyi–Selten selection procedure.

Lecture Outline

1. Continuous action spaces and best-response functions.
2. Cournot quantity competition and reaction curves.
3. Bertrand pricing with softened competition.
4. Hotelling location choice and focal reasoning.
5. Applying selection criteria and the Harsanyi–Selten steps.

Management Refresher

- **Marginal cost (MC)**: additional cost of producing one more unit; in Cournot, drives best responses.
- **Price elasticity**: sensitivity of demand to price changes; key for Bertrand adjustments.
- **Differentiation levers**: product features, timing, location—capture how firms soften price competition.
- **Customer surplus** vs. **producer surplus**: welfare split that regulators monitor when selecting equilibria.
- **Zoning & capacity policy**: managerial/legal tools to tilt Hotelling or Cournot outcomes toward desired equilibria.

Continuous Games Overview

Representative Models

- **Cournot**: quantities on an interval, price adjusts via inverse demand.
- **Bertrand**: prices on an interval, quantity splits based on demand.
- **Hotelling**: locations on a line, consumers choose the nearest provider.
- Continuous strategy equilibria may be unique yet still require interpretation.

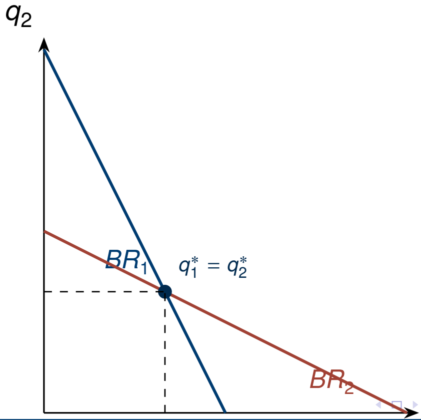
Cournot Competition

Model Setup

- Inverse demand $P(Q) = a - bQ$, $Q = \sum_{i=1}^{n} q_i$, $a > c \geq 0$, $b > 0$.
- Profit $\pi_i(q_i, q_{-i}) = q_i(a - b(q_i + q_{-i}) - c)$.
- Strategic variable: quantity $q_i \geq 0$ chosen simultaneously.
- Best response solves $a - 2bq_i - bq_{-i} - c = 0$ when interior.

Reaction Functions

- Symmetric duopoly: $\hat{q}_i(q_j) = \max\left\{0, \frac{a-c}{2b} - \frac{1}{2}q_j\right\}$.
- Strategic substitutes: higher rival output reduces optimal response.
- Entry of n firms yields $q_i^* = \frac{a-c}{b(n+1)}$.



Equilibrium and Welfare

- Duopoly equilibrium: $q_i^* = \frac{a-c}{3b}$, price $P^* = \frac{a+2c}{3}$.
- Consumer surplus $\frac{(a-c)^2}{6b}$; industry profit $\frac{2(a-c)^2}{9b}$.
- Compare with monopoly (higher price) and perfect competition (price = c).
- Selection questions shift to policy instruments and robustness to shocks.

Bertrand Competition

Baseline Bertrand

- Firms set prices $p_i \in [0, \bar{p}]$ simultaneously.
- Homogeneous good, marginal cost c: unique equilibrium $p_1^* = p_2^* = c$.
- "Bertrand paradox": duopoly profits collapse to zero despite two firms.

Softening Price Competition

- Capacity constraints: firms cannot serve total demand ⇒ mixed strategies or Edgeworth cycles.
- Product differentiation: e.g. $q_1 = a - bp_1 + dp_2$, $q_2 = a - bp_2 + dp_1$ with $0 < d < b$ delivers positive margins ⇒.
- Menu costs or search frictions: create inertia and pricing dispersion.
- Selection: regulators weigh consumer surplus; firms worry about safe price corridors.

Hotelling Competition

Hotelling Model Setup

- Two firms choose locations $x_A, x_B \in [0, 1]$ on a line with unit mass of consumers.
- Consumers incur transportation cost $t > 0$ per unit distance and pay price p (margin $p - c$).
- Firms earn $(p - c)$ times the measure of consumers who choose them.
- Location choice therefore targets market share given the rival's location.

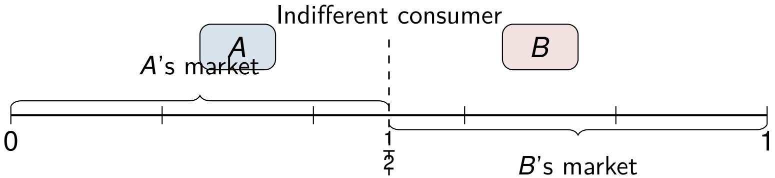
Consumer Allocation

- Assume w.l.o.g. $x_A \leq x_B$; indifferent consumer m solves $p + t|m - x_A| = p + t|m - x_B|$.
- Solution: $m = \frac{x_A + x_B}{2}$, so A serves $[0, m]$ and B serves $[m, 1]$.
- Market shares: $q_A(x_A, x_B) = \frac{x_A + x_B}{2}$ and $q_B(x_A, x_B) = 1 - q_A$.
- Profits: $\pi_A(x_A, x_B) = (p - c)\,\frac{x_A + x_B}{2}$ and $\pi_B(x_A, x_B) = (p - c)\,\frac{2 - x_A - x_B}{2}$.

Best Responses and Nash Equilibrium

- Interior derivative: $\frac{\partial \pi_A}{\partial x_A} = \frac{p-c}{2} > 0$ when $x_A \le x_B$ (mirror result for B when $x_B \ge x_A$).
- Firm A prefers to move right until it meets B: best response $BR_A(x_B) = x_B$ (allowing ties).
- Symmetrically $BR_B(x_A) = x_A$, so mutual best responses require $x_A = x_B$.
- Symmetry of the interval forces the unique Nash equilibrium $(x_A^*, x_B^*) = (\frac{1}{2}, \frac{1}{2})$.
- Any unilateral deviation shifts the indifferent consumer and reduces the deviator's market share below $\frac{1}{2}$.

Hotelling Diagram



- Dashed line marks the indifferent consumer $m = \frac{x_A + x_B}{2}$ splitting market share.
- Midpoint clustering is a Nash outcome; zoning or subsidies are required for dispersion.

Applying Selection Criteria

Cournot and Selection

- Unique equilibrium simplifies selection but still requires welfare commentary.
- Risk analysis highlights sensitivity to demand shocks or cost spikes.
- Focal cues include announced capacity targets or industry benchmarks.

Hotelling and Selection

- Midpoint equilibrium is focal yet may be Pareto dominated by differentiation.
- Regulators can tilt payoffs with zoning or subsidies to encourage dispersion.
- When discretised (e.g. $\{0, \frac{1}{2}, 1\}$), apply payoff/risk comparisons to the reduced game.

Harsanyi–Selten Procedure

Selection Steps

Definition: Harsanyi–Selten Procedure (Sketch)

1. **Dominance elimination**: remove strictly dominated strategies (after discretising if needed).
2. **Risk dominance**: among remaining equilibria, compare loss products.
3. **Payoff refinement**: select the payoff dominant equilibrium if ties remain.

- Continuous problems require discretisation or representative grid points.
- Record the grid, step size, and sensitivity checks in the advisory note.

Summary

Key Takeaways

- Continuous strategy games rely on calculus tools but invite the same selection criteria.
- Payoff, risk, and focal reasoning remain complementary in policy briefs.
- The Harsanyi–Selten steps provide a disciplined way to narrow multiple equilibria.

Group Homework

Group Homework: Gale–Shapley

- **Read**: Gale & Shapley (1962) "College Admissions and the Stability of Marriage", The American Mathematical Monthly, 68(1):9–15, focusing on stability, deferred acceptance, and key propositions.
- **Summary (A4, one page)**: capture the motivation, algorithm flow, main results, and any assumptions or limitations you judge critical.
- **Application insight (A4, one page)**: design a context where you would deploy the mechanism, outlining necessary adaptations, risks, and implementation roadmap.
- **Contribution log**: close each section with a short paragraph naming who led the work for that part so individual responsibilities are clear.
- **Submission**: email a single PDF per group to `y.hino@vju.ac.vn` by Nov. 9, Sunday 23:59; assume every teammate will present highlights in class.

Group Homework: Gale–Shapley

Evaluation focus

1. **Content Comprehension**: Accurately capture the paper's context, definitions, and main results, and explain the takeaways in your own words.
2. **Organization & Clarity**: Keep the one-page brief well structured so the summary and application proposal are easy to follow.
3. **Application / Originality**: Propose a deployment scenario that is both inventive and attentive to feasibility and stakeholder concerns.
4. **Critical Insight**: Identify limitations, risks, or adjustments needed when the original assumptions are relaxed, and reflect on how they affect implementation.

Peer Review Next Week

- Read every submitted report and produce a ranked list ($1 =$ strongest) covering all groups.
- Prepare at least half an A4 page (up to one full page) per group to justify your ranking with headline strengths, key questions, and concrete improvement advice.
- Submit the compiled peer review packet together with your ranking sheet by the start of the following class.
- Peer rankings feed into each group's grade, and the clarity and constructiveness of your review packet is graded separately.
- Peer review quality: Your final grade also reflects how fairly and insightfully you rank other groups and how concretely you justify those rankings against the evaluation criteria.
- Keep feedback professional: align comments with the evaluation focus and reference specific arguments or evidence.