

Extrema/Concavity Practice FRQ:

Let $f(x)$ be a twice-differentiable function defined for all real numbers, with its first derivative given by:

$$f'(x) = x^3 - 6x^2 + 9x$$

1. Find all critical points of $f(x)$. Justify your answer.
2. Determine whether each critical point found in part (a) corresponds to a local maximum, local minimum, or neither. Justify your answer using the First or Second Derivative Test.
3. Find the intervals where $f(x)$ is concave up and concave down. Determine the coordinates of any points of inflection.
4. Given that $f(3) = 5$, find the equation of the tangent line to $f(x)$ at $x = 3$.