Former McDonald's Worker Teaches Integrals

(50 integrals solved)

Video: https://youtu.be/X0UwldufY9Y

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I. Know your Derivatives

(Q1.) $\int \sec^2 x\,dx$

(Q2.) $\int \frac{1}{x}\,dx$

(Q3.) $\int \frac{1}{\sqrt{1-x^2}}\,dx$

(Q4.) $\int \sec x \tan x\,dx$

(Q5.) $\int \frac{1}{1+x^2}\,dx$

(Q6.) $\int \cos x\,dx$

(Q7.) $\int \sin x\,dx$

(Q8.) $\int e^x\,dx$

$$\frac{d}{dx}\left(e^x\right)=e^x \qquad \frac{d}{dx}\left(b^x\right)=b^x \ln b$$

$$\frac{d}{dx}(\ln x)=\frac{1}{x} \qquad \frac{d}{dx}\left(\log_b x\right)=\frac{1}{x\ln b}$$

$$\frac{d}{dx}(\sin x)=\cos x \qquad \frac{d}{dx}(\csc x)=-\csc x \cot x$$

$$\frac{d}{dx}(\cos x)=-\sin x \qquad \frac{d}{dx}(\sec x)=\sec x \tan x$$

$$\frac{d}{dx}(\tan x)=\sec^2 x \qquad \frac{d}{dx}(\cot x)=-\csc^2 x$$

$$\frac{d}{dx}\left(\sin^{-1} x\right)=\frac{1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(\csc^{-1} x\right)=\frac{-1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\cos^{-1} x\right)=\frac{-1}{\sqrt{1-x^2}} \qquad \frac{d}{dx}\left(\sec^{-1} x\right)=\frac{1}{x\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\tan^{-1} x\right)=\frac{1}{1+x^2} \qquad \frac{d}{dx}\left(\cot^{-1} x\right)=\frac{-1}{1+x^2}$$

II. Reverse Power Rule

(Q9.) $\int \sqrt{x}(x+4)\,dx$

(Q10.) $\int \frac{1+x^6}{x^2}\,dx$

$$\frac{d}{dx}\left(x^n\right)=nx^{n-1}$$

$$\int x^n\,dx=\frac{1}{n+1}x^{n+1}+C,\quad n\neq -1$$

$$\int x^{-1}\,dx=\int \frac{1}{x}\,dx=\ln|x|+C$$

III. U sub

(Q11.) $\int 4x^3 \sec^2\left(x^4\right)dx$

(Q12.) $\int \frac{x^3}{1+x^4}\,dx$

(Q13.) $\int \frac{x}{1+x^4}\,dx$

(Q14.) $\int \frac{1}{1+\sqrt{x}}\,dx$

$$\frac{d}{dx}\left(f\left(g(x)\right)\right)=f'\left(g(x)\right)g'(x)$$

$$\int f'\left(g(x)\right)g'(x)\,dx=f\left(g(x)\right)+C$$

$$\int f(ax+b)\,dx=\frac{1}{a}\int f(u)\,du$$

$$\int \frac{f'(x)}{f(x)}\,dx=\ln|f(x)|+C$$

IV. Know the famous ones (part 1. famous first step)

(Q15.) $\int \tan x\,dx$

(Q16.) $\int \sec x\,dx$

(Q17.) $\int \frac{1}{x^3+x}\,dx$

V. Say NO to these integral addictions

(Q18.) $\int \sin^3 x\, dx = \cancel{\frac{1}{4}\sin^4 x + C}$

(Q19.) $\int \frac{1}{1+\sqrt{x}}\, dx = \cancel{\ln\left|1+\sqrt{x}\right| + C}$

(Q20.) $\int e^{x^2}\, dx = \cancel{\frac{1}{2x} e^{x^2} + C}$

(Q21.) $\int \tan^{-1} x\, dx = \cancel{\frac{1}{1+x^2} + C}$

(Q22.) $\int x^2 \sin x\, dx = \cancel{-\frac{1}{3} x^3 \cos x + C}$

$$\int x^3\, dx = \frac{1}{4}x^4 + C \qquad \int \sin^3 x \cos x\, dx = \frac{1}{4}\sin^4 x + C$$

$$\int \frac{1}{1+x}\, dx = \ln|1+x| + C \qquad \int \frac{1}{ax+b}\, dx = \frac{1}{a}\ln|ax+b| + C$$

$$\int e^{2x}\, dx = \frac{1}{2}e^{2x} + C \qquad \int f(ax+b)\, dx = \frac{1}{a}\int f(u)\, du$$

$$\frac{d}{dx}\left(\sin^{-1} x\right) = \frac{1}{\sqrt{1-x^2}}$$

$$\int \left(x^2 + \sin x\right) dx = \frac{1}{3}x^3 - \cos x + C$$

VI. Know the famous ones (part2. non-elementary integrals)

$$\int e^{x^2}\, dx \quad \int e^{-x^2}\, dx \quad \int \frac{\sin x}{x}\, dx \quad \int \frac{\cos x}{x}\, dx$$

$$\int \frac{e^x}{x}\, dx \quad \int \frac{1}{\ln x}\, dx \quad \int \sin\left(x^2\right) dx \quad \int \cos\left(x^2\right) dx$$

$$\int x^x\, dx \qquad \int \sqrt{1+x^3}\, dx$$

VII. Integration by Parts

(Q23.) $\int x \cos(x^2)\, dx$

(Q24.) $\int x \cos x\, dx$

(Q25.) $\int x^3 \ln x\, dx$

(Q22*.) $\int x^2 \sin x\, dx$

(Q25*.) $\int x^3 \ln x\, dx$

(Q26.) $\int e^x \sin(2x)\, dx$

(Q21*.) $\int \tan^{-1} x\, dx$

(Q27.) $\int \frac{\ln x}{\sqrt{x}}\, dx$

(Q28.) $\int x^2 e^{3x}\, dx$

(Q29.) $\int x \sec x \tan x\, dx$

$$d(uv) = u\,dv + v\,du$$

$$\int u\, dv = uv - \int v\, du$$

	D	I
$+$	u	dv
$-$	du	v

VIII. Use Trig Identities

(Q30.) $\int \sin^2 x \cos x\, dx$

(Q31.) $\int \sin^2 x\, dx$

(Q18*.) $\int \sin^3 x\, dx$

(Q32.) $\int \sec^4 x\, dx$

(Q33.) $\int \sec^4 x \tan x\, dx$

(Q34.) $\int \tan^3 x\, dx$

(Q35.) $\int \sec^3 x\, dx$

$\int \left(\begin{array}{c} \text{an expression} \\ \text{in terms of } \sin x \end{array} \right) \cos x\, dx$ $\quad \int \left(\begin{array}{c} \text{an expression} \\ \text{in terms of } \cos x \end{array} \right) \sin x\, dx$

$\int \left(\begin{array}{c} \text{an expression} \\ \text{in terms of } \tan x \end{array} \right) \sec^2 x\, dx$ $\quad \int \left(\begin{array}{c} \text{an expression} \\ \text{in terms of } \sec x \end{array} \right) \sec x \tan x\, dx$

$\sin^2\theta = 1 - \cos^2\theta \qquad \cos^2\theta = 1 - \sin^2\theta$

$\sin^2\theta = \frac{1}{2}(1 - \cos(2\theta)) \qquad \cos^2\theta = \frac{1}{2}(1 + \cos(2\theta))$

$\sin(2\theta) = 2\sin\theta\cos\theta \qquad \cos(2\theta) = \cos^2\theta - \sin^2\theta$

$\cos(2\theta) = 2\cos^2\theta - 1 \qquad \cos(2\theta) = 2\cos^2\theta - 1$

$\tan^2\theta = \sec^2\theta - 1 \qquad \sec^2\theta = \tan^2\theta + 1$

$\sin\alpha\cos\beta = \frac{1}{2}(\sin(\alpha - \beta) + \sin(\alpha + \beta))$

$\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$

$\cos\alpha\cos\beta = \frac{1}{2}(\cos(\alpha - \beta) + \cos(\alpha + \beta))$

IX. Trig Sub

(Q36.) $\int \sqrt{x^2 - 6x + 9}\, dx$

(Q37.) $\int \sqrt{x^2 + 9}\, dx$

(Q38.) $\int \frac{1}{x\sqrt{x^2 - 4}}\, dx$

(Q39.) $\int \sqrt{1 - x^2}\, dx$

(Q40.) $\int \frac{1}{(25 + x^2)^{\frac{3}{2}}}\, dx$

(Q41.) $\int \frac{x}{(25 + x^2)^{\frac{3}{2}}}\, dx$

(Q42.) $\int \frac{1}{x^2 + a^2}\, dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$

you see	you let	you use
$\sqrt{x^2 + a^2}$	$x = a\tan\theta$	$\tan^2\theta + 1 = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$	$\sec^2\theta - 1 = \tan^2\theta$
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$	$1 - \sin^2\theta = \cos^2\theta$

X. Partial Fractions

(Q43.) $\int \frac{x^3}{x^2+9}dx$

(Q44.) $\int \frac{8x-17}{x^2-5x+4}dx$

(Q45.) $\int \frac{4x^2-9x+2}{(x+3)(x^2+4)}dx$

(Q46.) $\int \frac{1}{x^2+6x+13}dx$

(Q47.) $\int \frac{2x-5}{x^3+x^2}dx$

(Q48.) $\int \frac{2x^2+8x+5}{x^2+4x+13}dx$

(Q49.) $\int \frac{6x^2+31x+45}{x^3+6x^2+9x}dx$

(Q50.) $\int \frac{1}{x^2-a^2}dx$

$$\int \frac{1}{(ax+b)^n}dx = \frac{1}{a(1-n)}(ax+b)^{1-n}+C, \quad n \neq 1$$

$$\int \frac{1}{ax+b}dx = \frac{1}{a}\ln|ax+b|+C$$

$$\int \frac{x}{ax^2+b}dx = \frac{1}{2a}\ln|ax^2+b|+C$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

$$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right|+C$$

linear factors

$$\frac{8x-17}{x^2-5x+4} = \frac{A}{x-1}+\frac{B}{x-4}$$

Irreducible Quadratic Factors

$$\frac{4x^2-9x+2}{(x+3)(x^2+4)} = \frac{A}{x+3}+\frac{Bx+C}{x^2+4}$$

Repeated Factors

$$\frac{2x-5}{x^2(x+1)} = \frac{Ax+B}{x^2}+\frac{C}{x+1} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x+1}$$

$$\frac{2x-5}{x^3(x+1)^2} = \frac{A}{x}+\frac{B}{x^2}+\frac{C}{x^3}+\frac{D}{x+1}+\frac{E}{(x+1)^2}$$

$$\frac{6x^2+31x+45}{x(x+3)^2} = \frac{A}{x}+\frac{B}{x+3}+\frac{C}{(x+3)^2}$$

$$\frac{2x-5}{(x+1)(x^2+4)^2} = \frac{A}{x+1}+\frac{Bx+C}{x^2+4}+\frac{Dx+E}{\left(x^2+4\right)^2}$$