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# Meta Reasoning for Large Language Models

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**Peizhong Gao<sup>1\*</sup>** **Ao Xie<sup>1</sup>** **Shaoguang Mao<sup>2</sup>** **Wenshan Wu<sup>2</sup>**

**Yan Xia<sup>2</sup>** **Haipeng Mi<sup>1</sup>** **Furu Wei<sup>2</sup>**

<sup>1</sup> Tsinghua University

<sup>2</sup> Microsoft Research

<https://aka.ms/GeneralAI>

## Abstract

We introduce Meta-Reasoning Prompting (MRP), a novel and efficient system prompting method for large language models (LLMs) inspired by human meta-reasoning. Traditional in-context learning-based reasoning techniques, such as Tree-of-Thoughts, show promise but lack consistent state-of-the-art performance across diverse tasks due to their specialized nature. MRP addresses this limitation by guiding LLMs to dynamically select and apply different reasoning methods based on the specific requirements of each task, optimizing both performance and computational efficiency. With MRP, LLM reasoning operates in two phases. Initially, the LLM identifies the most appropriate reasoning method using task input cues and objective descriptions of available methods. Subsequently, it applies the chosen method to complete the task. This dynamic strategy mirrors human meta-reasoning, allowing the model to excel in a wide range of problem domains. We evaluate the effectiveness of MRP through comprehensive benchmarks. The results demonstrate that MRP achieves or approaches state-of-the-art performance across diverse tasks. MRP represents a significant advancement in enabling LLMs to identify cognitive challenges across problems and leverage benefits across different reasoning approaches, enhancing their ability to handle diverse and complex problem domains efficiently. Every LLM deserves a Meta-Reasoning Prompting to unlock its full potential and ensure adaptability in an ever-evolving landscape of challenges and applications.

## 1 Introduction

Large language models (LLMs) have shown remarkable capabilities in natural language understanding and generation, making significant strides in various reasoning tasks. However, the diversity and complexity of real-world problems require advanced reasoning methods that surpass the capabilities of a single, static approach. While existing reasoning techniques, such as Chain-of-Thoughts [27], Tree-of-Thoughts [32], Analogical Prompting [33], and Solo Performance Prompting [26], offer valuable tools for enhancing reasoning, they often fall short in consistently achieving state-of-the-art performance across different tasks.

These challenges highlight the need for a more adaptive and flexible approach to reasoning in LLMs. In human cognition, meta-reasoning involves monitoring and regulating reasoning and problem-solving activities, adjusting strategies based on the context and specific task requirements [5, 4]. This adaptive capability allows humans to efficiently allocate cognitive resources, balancing trade-offs between accuracy, complexity, and computational cost. Inspired by this, we propose Meta-Reasoning Prompting (MRP) to endow LLMs with similar adaptive reasoning capabilities.

Meta-Reasoning Prompting (MRP) is a simple yet effective system prompt designed to guide LLMs in dynamically selecting and applying the most suitable reasoning method for a specific task. By incorporating meta-reasoning principles, MRP transforms task-specific prompt engineering into a

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\*Work was done when interning at Microsoft Research Asia.

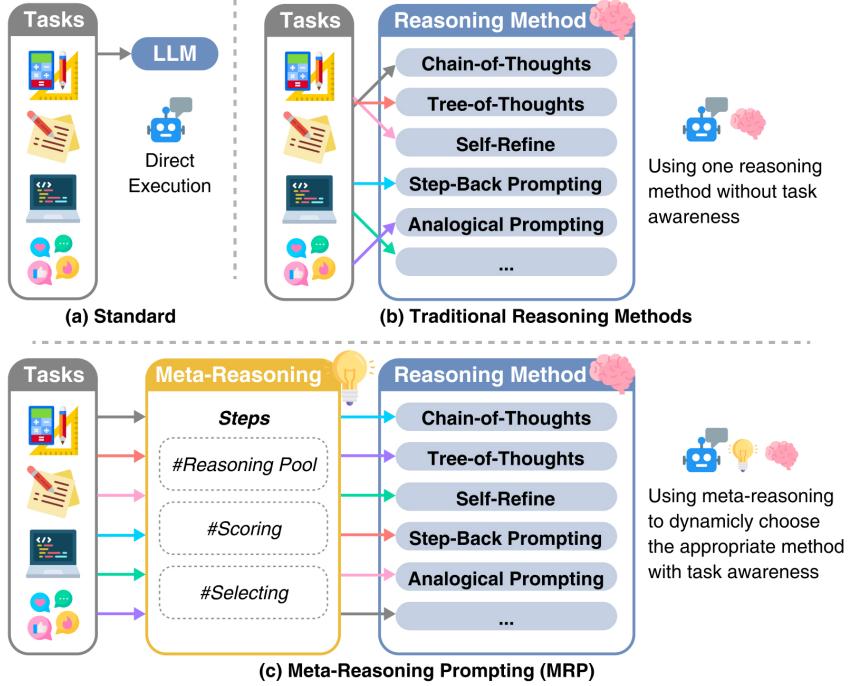


Figure 1: Illustration of Meta-Reasoning Prompting (MRP) and the difference compared to standard reasoning and traditional reasoning methods.

more general and flexible approach. Under the guidance of MRP, the LLM evaluates the task input and selects an appropriate reasoning method from a predefined set (Reasoning Pool). This selection is informed by objective descriptions and evaluations of the available methods. The chosen method is then applied to complete the task, ensuring the model uses the most effective strategy for the given problem.

Recent advances in reasoning techniques, such as those described in [30, 24], introduce a meta-buffer for storing high-level thoughts or use ensemble mechanisms to improve model generalizability. While some of these approaches align with the inherent logic of meta-reasoning, our proposed MRP achieves simple and efficient meta-cognitive effects by directly leveraging the meta-reasoning capabilities of LLMs through prompts, without introducing complex mechanisms.

To evaluate the effectiveness of MRP, we conducted experiments using multiple widely used benchmarks. These benchmarks cover different knowledge and reasoning abilities, providing a comprehensive test of the LLM’s performance across various reasoning tasks. Our findings demonstrate that MRP not only approaches state-of-the-art performance across these benchmarks but also excels in tasks requiring a blend of different reasoning strategies. Additionally, we observe that larger models, such as GPT-4, exhibit superior meta-reasoning capabilities compared to smaller models like GPT-3.5.

As models improve, their understanding of problems and methods—i.e., their meta-reasoning abilities—also enhances. MRP utilizes the inherent meta-cognitive abilities of LLMs, providing a straightforward and effective method that enhances their generality across different tasks. Experimental and analytical results indicate the significant potential of MRP in boosting LLM performance. Future work could explore the broader application of MRP, such as constructing training data to enhance the meta-cognitive and general reasoning abilities of LLMs during the training process.

Our key contributions are as follows:

1. We propose Meta-Reasoning Prompting (MRP), a system prompt that enables LLMs to dynamically select the most suitable reasoning method for specific tasks, enhancing their flexibility and effectiveness.

## Meta-Reasoning Prompt

### Instructions :

You are an adaptive reasoning method with meta-reasoning abilities, capable of selecting the most appropriate reasoning method based on the task provided by the user. Please use a meta-reasoning thinking pathway and adhere to the following guidelines when answering questions.

### Reasoning method pool:

These reasoning methods are available for your tasks. Understand their applications for various tasks.

#### 1. Method 1:

{Method\_description}

#### 2. Method 2:

{Method\_description}

.....

.....

### Analyzing giving task:

Evaluate the problem's difficulty carefully. Avoid underestimating the complexity and make a considered decision.

### Anticipate the mistakes you might make:

Identify possible errors in method selection, such as underestimating the problem's difficulty.

### Meta reasoning:

Apply meta-reasoning to choose the appropriate pathway.

### Grading:

Rate the suitability of each solution on a scale of 1-7, selecting the method with the highest score as your preferred choice.

### Choosing reasoning method:

Proceed methodically, taking a deep breath and thinking step-by-step. Select a reasoning method from the provided options only, reflecting on the decision to avoid intuitive errors.

### The question is:

{user\_query}

### Output:

>> FINAL CHOICE:[final method choice]

Figure 2: Meta-Reasoning Prompt.

2. Experiments on multiple benchmarks show that MRP approaches state-of-the-art performance and excels in tasks requiring diverse reasoning strategies, particularly in larger models like GPT-4.
3. MRP leverages LLMs' inherent meta-cognitive abilities, improving their generality and performance across tasks. Future work could further enhance these abilities through targeted training data.

## 2 Meta-Reasoning Prompting

The Meta-Reasoning Prompting (MRP) is designed to guide a Language Learning Model (LLM) in selecting the most suitable reasoning method from a pool of available methods, thereby enhancing the overall reasoning performance of the model. Detailed prompts can be found in Fig. 2.

Guided by the Meta-Reasoning Prompting, the LLM ( $M$ ) begins with an input  $x_0$  and a set of available reasoning methods  $\alpha_1, \alpha_2, \dots, \alpha_n$ . A reasoning pool contains descriptions of each reasoning method in the form of prompts  $p_1, p_2, \dots, p_n$ , with these descriptions extracted from the abstracts of

corresponding papers. A Meta-Reasoning Prompting  $p_{MR}$  is defined to guide the selection process. For each reasoning method  $\alpha_i$  ( $i$  ranging from 1 to  $n$ ), the model  $M$  evaluates the combined prompt  $(p_i | p_{MR} | x_0)$ . This evaluation yields a score  $s_i$  indicating the effectiveness of method  $\alpha_i$  for the given input  $x_0$ .

$$s_i = M(p_i \| p_{MR} \| x_0) \quad \text{for } i = 1, 2, \dots, n. \quad (1)$$

The algorithm identifies the reasoning method  $\alpha_k$  that receives the highest score  $s_i$  by finding the index  $k$  that maximizes the set  $s_1, s_2, \dots, s_n$ .

$$k = \arg \max_i \{s_1, s_2, \dots, s_n\} \quad (2)$$

Once the best reasoning method  $\alpha_k$  is determined, it is executed on the input  $x_0$ . The model  $M$  generates the final output  $y_0$  using the prompt  $(p_k | x_0)$ , which combines the description of the chosen reasoning method with the original input.

$$y_0 = \alpha_k(x_0) \quad (3)$$

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**Algorithm 1** Process Under MRP Guidelines

**Require:** Input  $x_0$ , model  $M$ , reasoning methods  $\alpha_1, \alpha_2, \dots, \alpha_n$ , a set of prompts of reasoning method descriptions  $p_1, p_2, \dots, p_n$ , the prompt for meta reasoning process denoted as  $p_{MR}$ . Evaluation score is denoted as  $s_1, s_2, \dots, s_n$ .

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1: for  $i = 1$  to  $n$  do
2:    $s_i = M(p_i \| p_{MR} \| x_0)$ 
3: end for
4:  $k = \arg \max_i \{s_1, s_2, \dots, s_n\}$ 
5: Determine  $k$  for which  $\alpha_k$  is executed and reason with the chosen method.
6:  $y_0 = \alpha_k(x_0)$ 
Return  $y_0$ 
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### 3 Experiments

#### 3.1 Setup

**Implementation of Meta-Reasoning Prompting** We implement MRP with seven popular and distinct in-context learning reasoning methods, which also serve as our baseline for comparison. We prompt descriptions for each method, allowing the LLM to understand.

**Tasks** We experiment with seven diverse tasks, Details about the dataset and its construction are provided in Appendix A.1:

1. **Arithmetic Reasoning:** GSM8K [3], 1319 basic math questions.
2. **Complex Mathematical Reasoning:** Game of 24 [32], a game using 4 numbers and basic arithmetic four operations to obtain 24.
3. **Creative Writing:** Trivia Creative Writing (Trivia CW) [26, 14], necessitating the model to assimilate and combine heterogeneous information from multiple domains internally.
4. **Multi-Hop Reasoning:** HotpotQA, [31], requiring models to connect pieces of information from multiple documents to answer a question.
5. **Social Reasoning:** BigToM [8], to evaluate social situations understanding and the theory of mind.
6. **Computer Code:** Code Readability (Code) [19], to enhance the readability of given code snippets.
7. **STEM:** MMLU [11], Physics, Chemistry, Biology, and Math problems of high school domain.

Table 1: Experiments with GPT4: Comparison of performance on benchmarks using Meta-Reasoning Prompting versus using other methods independently. **Bold** represents the best performance, and underline represents the second-best performance.

Method	GSM8K	Gameof24	Trivia CW	HotpotQA	BigToM	Code	MMLU	Macro Avg.
COT	0.914	0.050	0.762	0.800	0.470	0.685	0.894	0.654
TOT	<b>0.942</b>	<b>0.410</b>	0.786	0.716	0.430	0.765	0.815	<u>0.725</u>
Analogical	0.924	0.040	0.735	0.777	0.500	0.614	<b>0.947</b>	0.648
Self-Refine	0.929	0.080	0.764	0.763	0.470	<b>0.872</b>	0.861	0.677
SPP	0.929	0.170	<b>0.861</b>	0.763	0.550	0.672	0.874	0.688
STEP-BACK	0.933	0.090	0.787	<b>0.810</b>	0.420	0.809	0.841	0.670
SimTom	<u>0.938</u>	0.040	0.739	0.667	<b>0.590</b>	0.694	0.815	0.640
<b>MRP (our)</b>	0.921	<u>0.310</u>	<u>0.796</u>	0.797	<u>0.570</u>	<u>0.867</u>	0.854	<b>0.772</b>

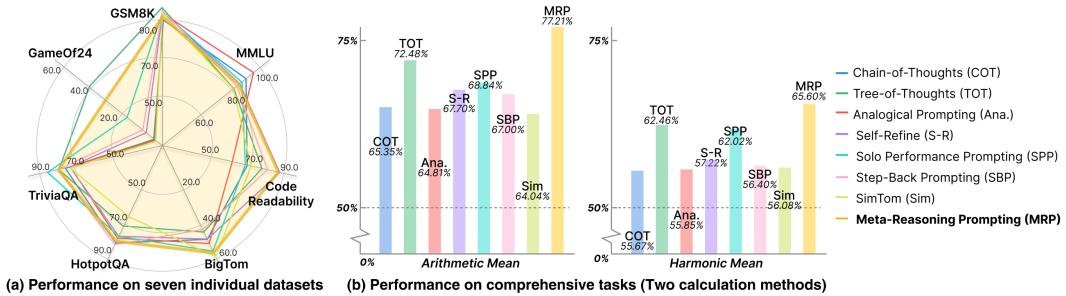


Figure 3: (a) Comparison of methods on different benchmarks reveals that guiding LLM to dynamically choose the appropriate reasoning method enables MRP to achieve consistently better performance across all tasks. (b) The arithmetic and harmonic average performances of applying a specific reasoning approach to all benchmarks demonstrate that MRP consistently excels in overall evaluation.

**Metrics** To prevent any method from skewing the results due to exceptional performance on a specific task, we reported both the arithmetic mean accuracy and the harmonic mean accuracy of each method across all benchmarks.

**Models** We used gpt-3.5-turbo<sup>2</sup> and gpt-4-turbo<sup>3</sup> with identical prompts to compare the effect of model size on meta-reasoning ability.

**Baselines** We select seven popular reasoning methods as baselines. These methods include:

1. **Chain-of-Thoughts:** breaking down problems into a series of coherent reasoning steps [27].
2. **Tree-of-Thoughts:** exploring multiple reasoning paths and self-evaluating choices to solve complex problems [32].
3. **Analogical prompting:** self-generating few-shots based on past experiences and related problems [33].
4. **Self-Refine:** self-evaluating for refinement and continuously improving the output [17].
5. **Solo Performance Prompting:** simulating multiple personas to collaboratively solve complex tasks [26].
6. **Step-Back Prompting:** abstract high-level concepts and principles to guide the reasoning process [38].
7. **SimToM:** enabling perspective-taking to understand the character’s beliefs and goals [28]

<sup>2</sup>Azure OpenAI, Model Name: gpt-35-turbo, API Version: 0301

<sup>3</sup>Azure OpenAI, Model Name: gpt-4, API Version: 1106-Preview

Table 2: Experiments with GPT3.5: Comparison of performance on benchmarks using Meta-Reasoning Prompting versus using other methods independently. **Bold** represents the best performance, and underline represents the second-best performance.

Method	GSM8K	Gameof24	Trivia CW	HotpotQA	BigToM	Code	MMLU	Macro Avg.
COT	<b>0.831</b>	0.030	0.414	0.187	0.610	0.578	0.675	0.416
TOT	0.810	<u>0.100</u>	0.155	<b>0.360</b>	0.430	<b>0.797</b>	<u>0.735</u>	0.352
Analogical	<u>0.825</u>	0.060	0.324	0.197	<b>0.660</b>	0.729	<u>0.721</u>	0.433
Self-Refine	0.716	0.030	0.213	0.167	<u>0.650</u>	<u>0.796</u>	0.543	0.372
SPP	0.823	<b>0.160</b>	<b>0.536</b>	<u>0.217</u>	0.540	0.684	0.689	<b>0.469</b>
STEP-BACK	0.817	0.010	<b>0.536</b>	0.190	0.570	0.642	<b>0.788</b>	<u>0.452</u>
SimTom	0.586	0.040	0.240	0.177	0.460	0.599	0.503	0.315
<b>MRP (our)</b>	0.781	0.050	0.346	0.187	0.600	0.759	0.722	0.433

### 3.2 Main Results

**Meta-Reasoning Prompting performs best on comprehensive tasks** As shown in table 1, MRP consistently exhibits robust performance across multiple benchmarks. MRP achieves the **second-best** in **4 of 7** tasks, including **Gameof24**, **TriviaQA**, **BigToM** and **Code**. This impressive performance across a wide range of tasks demonstrates MRP’s ability to effectively select and apply appropriate reasoning methods tailored to the specific requirements of each task. In terms of overall performance, MRP attains the highest across the 7 tasks, with an average of 0.772. In contrast, although TOT excels in certain tasks such as GSM8K and Gameof24, it performs less impressively in others. We observe noticeable performance gaps compared with MRP in tasks such as BigToM (**0.43 VS 0.57**) and Code (**0.765 VS 0.867**). This consistent excellence across all benchmarks underscores MRP’s advantages, demonstrating its ability to maintain impressive performance across diverse task domains (as shown in figure 4).

**Meta-reasoning capability is influenced by the base model capability** As illustrated in table 2, while the performance with GPT-4 is satisfactory, the experimental results with GPT-3.5 indicate that the effectiveness of MRP is suboptimal. Error analysis revealed the main issues: *Scoring Error*, *Self-opinion*, *Factual Error*, and *Reasoning Error*. This indicates that when the model’s capabilities are limited, it cannot have sufficient awareness of its own reasoning abilities and the meta-issues behind the reasoning problems. This performance drop also appears in other reasoning methods, which also indicates that the capability of meta-reasoning, like other reasoning abilities, improves as the model becomes more powerful.

**Meta-Reasoning Prompting is less effective for simple tasks but significantly improved for more differentiated tasks** From the experimental results (see figure 5), it can be seen that MRP and other methods show equal competitiveness on GSM8K, the accuracy of all the reasoning methods is above 90%, but the differentiation between the accuracy of each method is not very high, it can be seen that when the task is simpler, it is harder for MRP to reflect its own advantages, but MRP method is better than each method on the more difficult and comprehensive But the MRP method is significantly better than the other methods in the more difficult and comprehensive tasks.

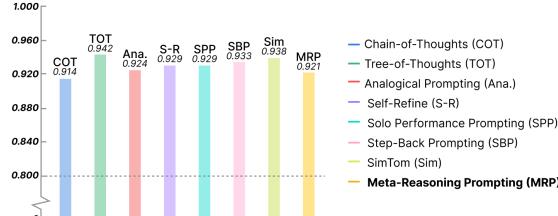


Figure 5: Performance of methods on GSM8K benchmark

## 4 Related Works

### 4.1 Reasoning with LLMs

Prompt-based reasoning methods have become a key technology for enhancing the capabilities of pretrained large language models (LLMs). The Chain-of-Thought (CoT) prompting [27], and its

## Inference Process of Meta-Reasoning Prompting

Given the objective-task of solving a specific instance of the **game24** with the input numbers 1, 2, 4, and 7, the task requires a method that can efficiently explore multiple combinations of arithmetic operations and numbers while also ensuring the correctness of the solution. Considering the nature of the task and the reasoning methods available, let's evaluate the suitability of each method:

- **Chain-of-Thoughts:** While this method allows for decomposing problems into intermediate steps, it may not efficiently explore all possible combinations of operations and numbers due to its linear nature.  
**Suitability:** 3/7.
- **Sim-Tom Prompting:** This method focuses on understanding mental states and is not directly applicable to mathematical problem-solving.  
**Suitability:** 1/7.
- **Tree-of-Thoughts:** This method is designed to explore multiple reasoning paths, which is crucial for a problem like the game of 24 where different combinations of operations and numbers need to be considered. It allows for strategic look ahead and backtracking, which can be beneficial in finding a solution that achieves the total of 24.  
**Suitability:** 6/7.
- **Step-Back Prompting:** This method abstracts the problem to a higher level, which may not be the most effective approach for a task that requires specific numerical manipulations and combinations.  
**Suitability:** 2/7.
- **Analogical Prompting:** While recalling relevant problems and solutions could be useful, the game of 24 with specific inputs demands a more direct exploration of arithmetic combinations rather than relying on analogies.  
**Suitability:** 3/7.
- **Solo-Performance Prompting:** Engaging multiple personas to tackle complex tasks might not be the most efficient way to solve a structured mathematical problem like the game of 24, where the solution space is defined by arithmetic operations.  
**Suitability:** 2/7.
- **Self-Refine:** This method focuses on iterative refinement of solutions. While it can improve the quality of outputs, the initial exploration of possible combinations to reach 24 might not be its strength.  
**Suitability:** 4/7.

Considering the task's requirements and the strengths of each method, **Tree-of-Thoughts** emerges as the most suitable approach. It provides a framework for exploring multiple reasoning paths and making strategic decisions, which aligns well with the task's demand for finding a specific combination of numbers and operations to achieve a total of 24.

>> FINAL CHOICE:Tree-of-Thoughts

Figure 4: The inference process of large language models (LLMs) under meta-reasoning prompting.

variants [37, 39, 2, 12, 25], such as Tree of thoughts (TOT) [32], Graph of thoughts (GOT) [1], enhances LLMs' ability to decompose complex tasks into smaller, manageable tasks, utilizing structured approaches to explore problem-solving pathways. Numerous studies have demonstrated the exceptional performance of prompt-based reasoning methods across various domains and benchmarks. [17, 28, 38, 20, 23] Some researchers have even employed analogical reasoning [34, 7, 33], enabling large models to generate similar questions based on user queries and subsequently summarize solutions based on the answers to these questions. While independent reasoning methods have been proven to improve LLM performance from different perspectives, they still fail to meet integrated problems.

There are also some methods to enhance LLM reasoning through ensemble mechanisms or tuning. X-of-Thoughts improves the success rate of LLM on arithmetic problems by integrating three methods [15]. It proposes a trial-and-error iterative mechanism that allows LLM to autonomously

repeat attempts to find a final solution. Ni et al. blending off-the-shelf benchmarks to create a comprehensive, integrated LLM assessment [18]. Mixtural-Of-Prompts (MoP) dynamically manage and optimize prompt tuning across heterogeneous tasks and data distributions, significantly reducing perplexity and mitigating interference in multi-task scenarios [6]. Some researchers fine-tune smaller models with a well-prepared dataset inspired by preference learning to achieve reasoning power comparable to a larger model [35, 22, 29]. They present problem-method coupled datasets and show how to improve the model’s grasp of inference skills at the data level. However, there is still a lack of research to explore the meta-reasoning ability of LLMs to choose reasoning methods.

## 4.2 Meta Reasoning

Meta-reasoning is a crucial cognitive process in human intelligence, involving the recognition and interpretation of reasoning to select optimal methods based on past experiences [9]. In artificial intelligence, it refers to efficiently deploying computational resources for informed decision-making in specific situations [4, 5]. Recently, some works develop routing or buffer systems to improve performance, using supervised learning algorithms [21], reward model-based techniques, and other methods [10, 16, 24]. Hu et al. created a benchmark to evaluate these methods’ effectiveness [13]. Zeng et al. noted the neglect of meta-reasoning in independent LLMs and proposed a benchmark to evaluate reasoning rationality [36]. In [30], the authors introduce a meta-buffer to store a series of high-level thoughts distilled from problem-solving processes across various tasks. This approach aligns with the inherent logic of meta reasoning. However, MRP achieves simple and efficient meta-cognitive effects by directly unleashing the meta reasoning capabilities of LLM through prompts, without introducing complicated mechanisms.

## 5 Conclusions and Outlook

This paper introduces Meta-Reasoning Prompting (MRP), a novel and efficient approach inspired by human meta-reasoning, designed to enhance the adaptability and efficiency of large language models (LLMs). By dynamically selecting and applying the most suitable reasoning method for each task, MRP enables LLMs to optimize performance across diverse problem domains, achieving near state-of-the-art results in comprehensive benchmarks.

Our experiments demonstrate that MRP significantly improves LLMs’ ability to handle tasks requiring a blend of different reasoning strategies, particularly in larger models like GPT-4. This dynamic adaptability highlights MRP’s potential to address the limitations of traditional reasoning techniques, offering a more flexible and effective solution for varied and complex tasks.

Looking ahead, future research could explore the integration of MRP into training datasets to further enhance LLMs’ general reasoning abilities. Additionally, combining MRP with other advanced reasoning techniques could yield further improvements in model performance. Overall, MRP represents a significant step forward in developing more intelligent, efficient, and adaptable AI systems, capable of meeting the diverse demands of real-world problem-solving.

## 6 Limitations

Our study investigates the meta-reasoning mechanisms of LLMs by dynamically selecting suitable methods to enhance their performance across various reasoning tasks without introducing new knowledge or training efforts. Currently, Meta-Reasoning Prompting (MRP) selects the highest-scoring method for each task. However, drawing from human cognitive processes, tackling complex problems often involves combining multiple reasoning methods. Future research will explore mechanisms such as Top-Probability (Top-P) or Top-K to allow models to ensemble relevant methods, potentially achieving better performance.

Our experimental results indicate that the meta-reasoning ability of LLMs is influenced by the capabilities of the models themselves. For instance, GPT-4’s Meta-Reasoning Prompting shows significantly greater improvement compared to GPT-3.5, which aligns with our expectations. Nonetheless, we can further enhance the smaller model’s meta-reasoning capabilities through instruction tuning in future works.

Due to space constraints and limited resources, our experiments primarily tested the most representative LLMs (GPT-4 and GPT-3.5). We did not fully cover the performance of other open-source or closed-source models. However, we believe that the experimental results on these representative LLMs provide sufficient insights and implications.

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## A Implementation Details

### A.1 Dataset Details

Table 3 shows the split and number of examples used for evaluations in GSM8K, Game of 24, Trivia Creative Writing, HotpotQA, BigTOM, Code Readability and MMLU. The dataset sizes of GSM8K, Gameof24, Trivia Creative Writing are consistent with the size used in the references. To control cost, we randomly tested 100-300 sample of data from HotpotQA, BigTOM, and Code Readability and MMLU. Despite of the economic consideration, we found that on this data scale, MRP has achieved significant results.

Table 3: Dataset Split and Number of Examples

Domain	Dataset	Number of Examples
Arithmetic Reasoning	GSM8K	1319
Complex Mathematical Reasoning	Game of 24	100
Creative Writing	Trivia Creative Writing	100
Multi-hop Reasoning	HotpotQA	300
Social Reasoning	BigTOM	100
Computer Code	Code Readability	300
STEM	MMLU	151

### A.2 Source Prompts of the Reasoning Methods Used in This Paper

### Full prompts of COT

Task Description:  
{description}

Input:  
{input}

Let's think step by step.

Figure 6: Prompt of COT

### Full prompts of SIMTOM for general task

**STEP 1:**

Task Description:  
{description}

Input:  
{input}

What does you know about as an AI assistant to solve this {domain} problem?

**STEP 2:**

Task Description:  
{description}

Input:  
{input}

{principle}

### Full prompts of SIMTOM for BigTom

**STEP 1:**

Imagine you are {coi}, and consider this story that has an unexpected event.

If the last sentence of the story says {coi} notices, sees or realizes the unexpected event in this story, simply output the original story with nothing changed.

However, if the sentence says you are not aware of the changes in this story, output only the events you know, i.e., the sentences before the unexpected event happens.

Output either the original story or the edited story, nothing else.

Format your answer as follows: Sees/Notices/Realizes: (Yes/No)

Story:

**STEP 2:**  
{principle}

You are {coi}. Based on the above information, answer the following question:

{input}

Answer the questions based on the context. Keep your answer concise, few words are enough, maximum one sentence.

Figure 7: Prompt of Sim-Tom Prompting

### Full prompts of TOT

**Proposal:**

Task Description:  
{description}

Input:  
{input}

Make a strategy then write. Your output should be of the following format:

Strategy:

Your strategy about how to answer the question.

Answer:

Your answer to the question. It should end with {format}.

**Vote:**

Given an instruction and several choices, decide which choice is most promising.  
Analyze each choice in detail, then conclude in the last line  
"The best choice is {s}", where s is the integer id of the choice.

Figure 8: Prompt of TOT

### Full prompts of Analogical Prompting

Your task is to tackle mathematical problems. When presented with a math problem, recall relevant problems as examples. Afterward, proceed to solve the initial problem.

# Initial Problems  
## Task Description:  
{description}

## Input:  
{input}

# Instructions :  
Make sure that your response follows the instructions below.

## Analogous Problems :  
Offer five diverse examples of math problems that are relevant or analogous to the initial problem. For each problem, elaborate on the solution and conclude with the ultimate answer.

For each problem :  
- After " Q : ", describe the problem.  
- After " A : ", explain the solution and enclose the ultimate answer in here.

## Solve the Initial Problem :  
Q : Copy and paste the initial problem here.  
A : Explain the solution and enclose the ultimate answer use the format that system required.

Figure 9: Prompt of Analogical Prompting

### Full prompts of SELF-REFINE

**STEP 1:**

You are an expert in {domain}.

Task Description:  
{description}

Input:  
{input}

Once you think you have answered the question, please summarize your final answer strictly using the format below:{format} without other word."

**STEP2:**

Task Description:  
{description}

Input:  
{input}

Origin Solution:  
{principle}

There is maybe an error in the problem solving process above because of lack of understanding of the question.  
you need to go through complete blocks of the context, check if everything looks good and give your revised answer in the end with the prescribed format.  
please answer with the format below:  
Judge: tell me whether the problem solving process has error here and explain.  
Answer: give your revised solution, and summarize the final answer in the end with the prescribed format without other word.

Figure 10: Prompt of SELF-REFINE

### Full prompts of STEP-BACK Prompting

#### STEP 1:

You are an expert at {domain}.  
You are given a {domain} problem.  
Your task is to extract the {domain} concepts and principles involved in solving the problem.

Here are a few examples:

Question:

Griffin had 24 french fries, but Kyle took 5 of them. Billy took twice as many as Kyle. Ginger gave Griffin a handful of her fries, and then Colby took from Griffin 3 less than the number of fries that Kyle had taken. If in the end Griffin had 27 fries, how many fries did Ginger give Griffin?

Principles Involved:

- 1.Basic Arithmetic Operations: This problem primarily involves addition and subtraction. We use these operations to calculate the changes in the number of fries Griffin has after each event.
- 2.Multiplication and Doubling: The problem states that Billy took twice as many fries as Kyle. This involves the principle of multiplication or doubling a number.
- 3.Comparative Subtraction: The statement that Colby took 3 fries less than Kyle introduces the concept of comparative subtraction, where one value is reduced by a relative amount compared to another.
- 4.Variable Isolation in Equations: To find out how many fries Ginger gave Griffin, we first calculate the total number of fries taken or given and then isolate the variable representing the fries Ginger gave.
- 5.Equation Setup and Solving: The entire scenario can be conceptualized as an equation where the final number of fries is affected by a series of additions and subtractions. Setting up an equation with these operations and solving for the unknown (the fries Ginger gave) is key.

Task Description:{description}

Question:{input}

Principles Involved:

#### STEP 2:

You are an expert at {domain}.  
You are given a {domain} problem and a set of principles involved in solving the problem.  
Solve the problem step by step by following the principles.

Here are a few examples:

Question:

Griffin had 24 french fries, but Kyle took 5 of them. Billy took twice as many as Kyle. Ginger gave Griffin a handful of her fries, and then Colby took from Griffin 3 less than the number of fries that Kyle had taken. If in the end Griffin had 27 fries, how many fries did Ginger give Griffin?

Principles Involved:

- 1.Basic Arithmetic Operations: This problem primarily involves addition and subtraction. We use these operations to calculate the changes in the number of fries Griffin has after each event.
- 2.Multiplication and Doubling: The problem states that Billy took twice as many fries as Kyle. This involves the principle of multiplication or doubling a number.
- 3.Comparative Subtraction: The statement that Colby took 3 fries less than Kyle introduces the concept of comparative subtraction, where one value is reduced by a relative amount compared to another.
- 4.Variable Isolation in Equations: To find out how many fries Ginger gave Griffin, we first calculate the total number of fries taken or given and then isolate the variable representing the fries Ginger gave.
- 5.Equation Setup and Solving: The entire scenario can be conceptualized as an equation where the final number of fries is affected by a series of additions and subtractions. Setting up an equation with these operations and solving for the unknown (the fries Ginger gave) is key.

Answer:

To solve this problem, let's follow the principles and the order of events to find out how many fries Ginger gave Griffin.

Start with the initial number of fries Griffin had, which is 24.

Kyle took 5 fries from Griffin:

Griffin's fries =  $24 - 5 = 19$  fries.

Billy took twice as many as Kyle, so Billy took  $5 * 2 = 10$  fries:

Griffin's fries =  $19 - 10 = 9$  fries.

Colby took 3 fries less than Kyle took. Kyle took 5 fries, so Colby took  $5 - 3 = 2$  fries:  
Griffin's fries =  $9 - 2 = 7$  fries.

Finally, Ginger gave Griffin some fries. We know that at the end of all these transactions, Griffin had 27 fries. Therefore, we set up an equation based on the fries left after Billy and Colby took theirs:

Griffin's fries after Ginger gave some =  $7 + \text{fries given by Ginger} = 27$  fries.

Solving for the fries Ginger gave:

Fries given by Ginger =  $27 - 7 = 20$  fries.

Therefore, Ginger gave Griffin 20 fries.

>> FINAL ANSWER:

\\"\\\"\\\"

Ginger gave Griffin 20 fries.

\\"\\\"\\\"

Task Description:{description}

Question:

{input}

Principles Involved:{principle}

Answer:

Figure 11: Prompt of STEP-BACK Prompting

### Full prompts of Solo Performance Prompting (SPP)

When faced with a task, begin by identifying the participants who will contribute to solving the task. Provide profiles of the participants, describing their expertise or needs. Then, initiate a multi-round collaboration process until a final solution is reached. The participants will give critical comments and detailed suggestions whenever necessary.

Here are some examples:

---

Example #1: Use numbers and basic arithmetic operations (+ - \* /) to obtain 24. You need to use all numbers, and each number can only be used once.  
Input: 6 12 11

Participants: AI Assistant (you); Math Expert

Profiles:

- AI Assistant (you): A super-intelligent AI assistant capable of performing tasks more effectively than humans.
- Math expert: A person who is good at math games, arithmetic calculation, and long-term planning.

Start collaboration!

Math Expert: Let's analyze the task in detail. You need to make sure that you meet the requirement, that you need to use exactly the four numbers (6 12 11) to construct 24. To reach 24, you can think of the common divisors of 24 such as 4, 6, 8, 3 and try to construct these first. Also you need to think of potential additions that can reach 24, such as 12 + 12.

AI Assistant (you): Thanks for the hints! Here's one initial solution:  $(12 / (1 + 1)) * 6 = 24$

Math Expert: Let's check the answer step by step.  $(1+1) = 2$ ,  $(12 / 2) = 6$ ,  $6 * 6 = 36$  which is not 24! The answer is not correct. Can you fix this by considering other combinations? Please do not make similar mistakes.

AI Assistant (you): Thanks for pointing out the mistake. Here is a revised solution considering 24 can also be reached by  $3 * 8$ :  $(6 + 1 + 1) * (12 / 4) = 24$ .

Math Expert: Let's first check if the calculation is correct.  $(6 + 1 + 1) = 8$ ,  $12 / 4 = 3$ ,  $8 * 3 = 24$ . The calculation is correct, but you used 6 11 12 4 which is not the same as the input 6 12 11. Can you avoid using a number that is not part of the input?

AI Assistant (you): You are right, here is a revised solution considering 24 can be reached by 12 + 12 and without using any additional numbers:  $6 * (1 - 1) + 12 = 24$ .

Math Expert: Let's check the answer again.  $1 - 1 = 0$ ,  $6 * 0 = 0$ ,  $0 + 12 = 12$ . I believe you are very close, here is a hint: try to change the "1 - 1" to "1 + 1".

AI Assistant (you): Sure, here is the corrected answer:  $6 * (1+1) + 12 = 24$

Math Expert: Let's verify the solution.  $1 + 1 = 2$ ,  $6 * 2 = 12$ ,  $12 + 12 = 24$ . You used 11 6 12 which is identical to the input 6 12 11. Everything looks good!

Finish collaboration!

>> FINAL ANSWER:  
6 \* (1 + 1) + 12 = 24  
---

Example #2: Write a poem that meets the following requirements: (1) the poem has seven lines and the first letters of each line forms the word "CHATGPT"; (2) the poem is about explaining what is a quantum computer. (3) the poem needs to be easy to understand by a ten years old kid.

Participants: AI Assistant (you); Poet; Computer Scientist; Ten year old child

Profiles:

- AI Assistant (you): A super-intelligent AI assistant capable of performing tasks more effectively than humans.
- Poet: A person who studies and creates poetry. The poet is familiar with the rules and formats of poetry and can provide guidance on how to write a poem.
- Computer Scientist: A scholar who specializes in the academic study of computer science. The computer scientist is familiar with the concept of a quantum computer and can provide guidance on how to explain it.
- Ten year old child: A child with a limited English vocabulary and little knowledge about complicated concepts, such as a quantum computer.

Poet: Make sure that you write the poem with seven lines, and the first letters of the lines should be C, H, A, T, G, P, T.

Computer Scientist: A quantum computer is an advanced computing device that uses the principles of quantum mechanics to process and store information. Unlike classical computers that use bits to represent information as 0s and 1s, quantum computers use quantum bits or qubits. Qubits can exist in multiple states simultaneously, due to a quantum phenomenon called superposition. You can consider using these information for the poem.

Ten year old child: I hope the poem to be fun and easy to understanding. I don't want to see a lot of jargons or complicated concepts.

AI Assistant (you): Thanks for the guidance! Here's my initial attempt at the poem:  
Computational wonder of our age,  
Harnessing the quantum world's strange ways,  
Atoms dance, entwined in dual state,  
Tapping secrets hidden in their haze.

Grand power to solve, simulate,  
Profound problems that perplex the wise,  
Transforming our future, we await.

Figure 12: Prompt of SPP Prompting

### Full prompts of Solo Performance Prompting (SPP)

Poet: Let's verify if the poem meets the requirements. The first letters are CHATGPT which is correct! And the poem rhymes well. Good job!

Computer Scientist: Everything looks good to me!

Ten year old child: I don't know what does perplex mean. Can you make the use of words easier to understand?

AI Assistant (you): Sure, let me revise the poem by using more common words. Check out the revised version:

Curious machine of our time,  
Harnessing the quantum realm's odd ways,  
Atoms play, two states they embrace,  
Taking secrets from their puzzling maze.

Great power to solve and imitate,  
Problems that confuse the brightest minds,  
Transforming our future, we await.

Poet: Let's check again if the poem meets the requirements. The first letters are C H A T G P T. And now the poem is more accessible to children. Everything looks good to me.

Computer Scientist: Looking good!

Ten year old child: I like this version a lot!

Finish collaboration!

>> FINAL ANSWER:

'''

Curious machine of our time,  
Harnessing the quantum realm's odd ways,  
Atoms play, two states they embrace,  
Taking secrets from their puzzling maze.

Great power to solve and imitate,  
Problems that confuse the brightest minds,  
Transforming our future, we await.  
'''

---

Task Description:  
{description}

Input:

{input}

Now, identify the participants, provide their profiles, and collaboratively solve the following task step by step. Remember to provide the final solution with the format that 'system' requires:

Figure 13: Prompt of SPP Prompting