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Walking in the Rain, Reconsidered

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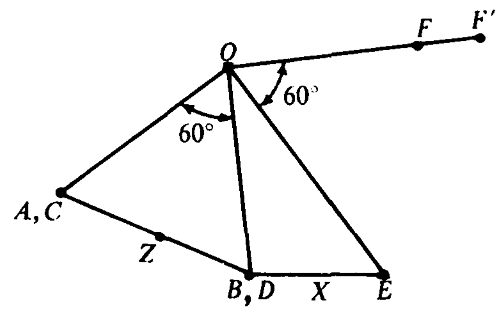
Let points A and C be fixed points coinciding with P and let points B and D be fixed coinciding with Q. OE is an arbitrary radius and $OF' = OE$.



It follows from our first theorem that the midpoint of AF must coincide with that of AF'. Thus, $OF = OF'$ and the curve is one of 6-fold symmetry.

WALKING IN THE RAIN, RECONSIDERED

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Introduction. In an earlier paper [1], one of the authors studied the best strategy according to which a man might walk (or run) in the rain from point A to point B in order to minimize the amount of wetting he undergoes. This paper corrects a technical error in the earlier work and, in doing so, simplifies and extends the earlier results. Notation and terminology follow the earlier paper. However, for completeness, we restate the definitions needed to make this paper self-contained.

The model. Let $\boldsymbol{i}$ be a unit vector in the direction $\overrightarrow{AB}$; $\boldsymbol{k}$ a unit vector pointing upwards; and $\boldsymbol{j} = \boldsymbol{k} \times \boldsymbol{i}$. The rain, assumed a fluid of uniform density, is falling at speed V_T, and being swept along by a horizontal wind of velocity $V_T(w\boldsymbol{i} + W\boldsymbol{j})$. Thus the velocity of the rain is $V_T(w\boldsymbol{i} + W\boldsymbol{j} - \boldsymbol{k})$, as seen from a fixed point on the ground. The man's velocity is $\boldsymbol{i}\, V_T x$, where x is to be determined. Thus, the relative velocity of the rain droplets, as seen by the man is $V_T\{(w-x)\boldsymbol{i} + W\boldsymbol{j} - \boldsymbol{k}\}$. The man is modelled as a cuboid, getting wet on three of his six sides: front or back, right or left, and top. These are taken to have areas A, ηA, and εA respectively.

Wetness function. The amount of rain impinging on the top surface per unit time is proportional to the area εA multiplied by the velocity component of the rain normal to the surface. Thus the amount of rain falling on the man's head is proportional to εA. Similarly, the amount striking the side surface is proportional to $\eta A\,|W|$; and finally the amount impinging on front or back is proportional to $A\,|w - x|$.

The absolute values in the last two expressions arise from the observation that just two of the four vertical sides can be wetted by rain, depending on the signs of W and $(x - w)$.

Thus the total amount of rain falling on the man per unit time is proportional to $\varepsilon + \eta|W| + |w - x| = \phi + |w - x|$, where ϕ is defined as $\varepsilon + \eta|W|$.

The total amount falling on the man in his trip from A to B is directly proportional to the amount landing on him per unit time, and inversely proportional to his speed—i.e., it is proportional to

$$(1) \qquad F(x) = \frac{\phi + |w - x|}{x}$$

which we refer to as the "wetness function".

This differs in form from the corresponding expression in the earlier paper. The earlier paper erred in the inclusion of a factor $\sqrt{(w-x)^2 + W^2 + 1}$ in the denominator.

Heuristic confirmation. The previous (incorrect) form for $F(x)$ had the property that it became arbitrarily small for sufficiently large x. This implied that the man could reduce his soaking to as small a level as he wished if he could attain a great enough speed. Clearly, this is not true. If he goes extremely fast from A to B, the man sweeps out, as an irreducible minimum, all the rain that is in the prism whose long axis is the line segment AB, and whose cross section is the same as the man's own. This quantity of rain is a positive constant; it cannot be made arbitrarily small.

The corrected version of $F(x)$ given above does have a positive limiting value (unity) approached when x becomes large, and hence passes this intuitive test.

Optimal policy. We seek to minimize the function $F(x)$. Observe that $F(\,\cdot\,)$ has a discontinuous derivative at $x = w$.

$$(2) \qquad F'(x) = (\phi + w)/x^2 \text{ for } x < w$$

and

$$(3) \qquad F'(x) = -(\phi - w)/x^2 \text{ for } x > w.$$

Thus we cannot minimize $F(\,\cdot\,)$ by cookbook calculus. The shape of the graph of $F(x)$ depends on the relative values of ϕ and w. If $\phi > w$, then $F(x)$ is monotone decreasing for all x (see Figure 1). Hence the optimal strategy for the man is to run as fast as he can.

On the other hand, if $\varphi \leqq w$, then $F(x)$ has a minimum at $x = w$ (see Figure 2). In this case, the man should travel at the speed $x = w$, if he can, so as to avoid encountering moisture on front or back. He will get wet only on the top and side. If he cannot attain w, the speed of the wind, then he is operating in the portion of the $F(x)$ curve that is monotone decreasing, and hence again should run as fast as he can.

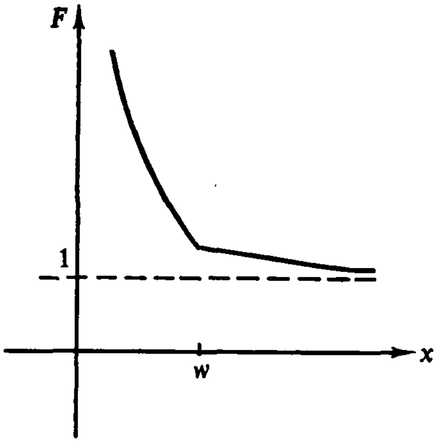

FIG. 1

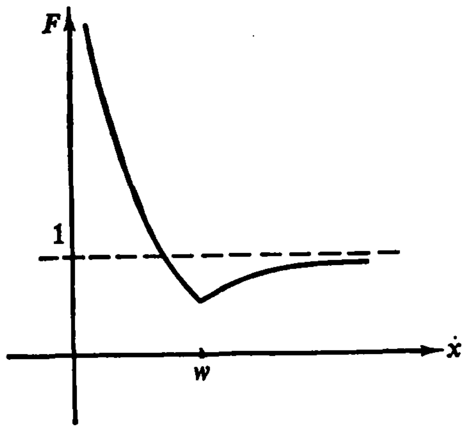

FIG. 2

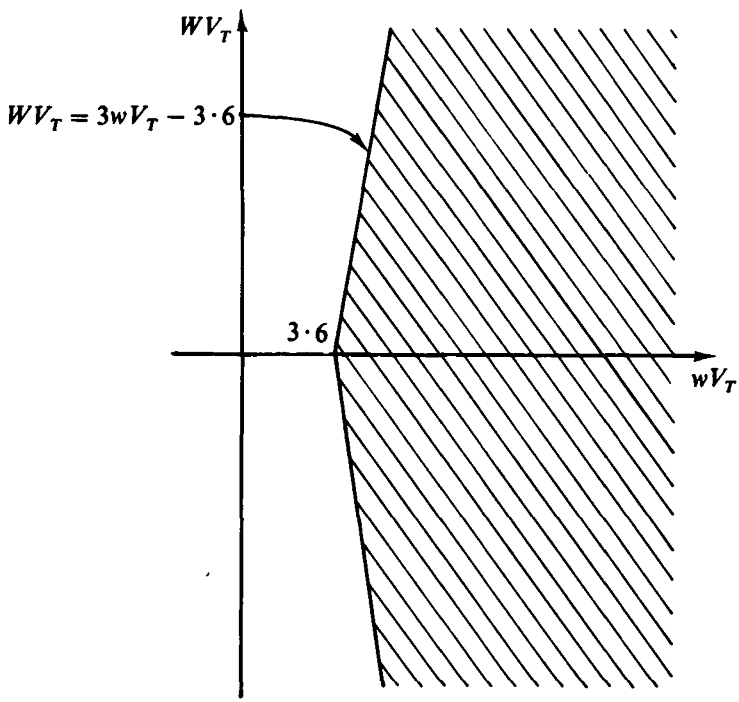

FIG. 3

Decision procedure. The decision as to whether or not to run at top speed is dependent upon whether $\phi > w$. It is best to match one's speed to that of the relevant component of the wind if $\phi < w$, i.e., if

$$\varepsilon + \eta |W| < w. \tag{4}$$

Inequality (4) defines a region of the (w, W) plane for which this policy is optimal. This is graphed in terms of the "real" variables WV_T, wV_T in Figure 3, the values of ε, η, V_T being as presented in the earlier analysis: $\varepsilon = 0.06$, $\eta = 0.33$, $V_T = 20$ m.p.h.

For this case, inequality (4) becomes

$$|WV_T| < 3wV_T - 3.6. \tag{5}$$

The decision rule is simply this: If the terminal point of the vector $V_T(w, W)$ lies in the shaded area of Figure 3, the man should proceed at speed w, if possible. In all other cases, he should run as fast as he can.

Sensitivity analysis. Suppose $\phi < w$, but that the man still runs as fast as he can. The extent to which he gets wet is measured by the wetness function $F(X)$ where X is his top speed (in terms of the terminal velocity of falling raindrops). Define

$$R = \frac{F(X)}{F(w)}. \tag{6}$$

R will now measure the penalty incurred in following the naive rather than the more sophisticated strategy. We consider only the case $X > w$, as otherwise the question of the two strategies does not arise.

Substitution from (1) into (6) gives

$$R = \frac{\phi + X - w}{X} \cdot \frac{w}{\phi}$$

from which it is clear that R becomes larger as ϕ is decreased. The largest possible value of R will thus occur when ϕ achieves its minimum value ε, i.e., when $W = 0$. Under these circumstances

$$R = \frac{w}{X}\left(1 + \frac{X - w}{\varepsilon}\right).$$

R is maximized when

$$X = 2w - \varepsilon$$

and achieves the value

$$R_{\max} = \frac{(X + \varepsilon)^2}{4X\varepsilon}.$$

For the case studied in the earlier paper: $\varepsilon = 0.06$, $X = 1$, this assumes the value 4.68, so that under quite feasible conditions a runner following the naive strategy gets more than four times as wet as a jogger matching his speed to that of the rain.

Conclusion. We have shown that the correction of a technical error in the earlier analysis [1] simplifies the analysis without seriously altering the conclusions. The amended analysis provides a reasonably accessible and plausibly motivated case of a minimum of the $|x|$ type and may be of some use on this account to teachers of elementary college calculus. The simplifications inherent in this paper also suggest the feasibility of more realistic models; e.g., it may be possible to relax the assumptions that the rain is all falling in exactly the same direction or with the same speed. We have not, however, attempted any such generalizations ourselves.

Reference

1. M. A. B. Deakin, Walking in the rain, this MAGAZINE, 45 (1972) 246–253.

AN "ELEPHANTINE" EQUATION

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There are many legendary tales about the wit and wisdom of Birbal, poet, musician, intellectual and the favorite courtier of the Indian Emperor Akbar (1556–1605). Once, three men came to Akbar's court with the problem of sharing the wealth left by their deceased father. The old man had willed that the eldest son should get 1/2, the second son 1/3, and the youngest son 1/9 of his entire property.

The entire property consisted of 17 elephants. It was even suggested that some of the elephants might be slaughtered, since — as the saying goes— "an elephant is worth a thousand gold coins, alive or dead." Birbal's solution was neat and clean. He ordered the Royal Elephant to be lined up with the 17 elephants. The men took their shares of 9, 6, and 2 elephants, leaving behind the Royal Elephant!

Birbal's solution exploited the fact that $1/2 + 1/3 + 1/9 = 17/18$ (not 1). Are there any triplets x_1, x_2, x_3 besides 2, 3, 9 which have a similar property? In other words, what are the solutions of the Diophantine equation

$$\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} = \frac{E}{E+1} \tag{1}$$

where

(a) x_1, x_2, and x_3 are unequal, and

(b) $E + 1$ is the least common multiple of x_1, x_2, x_3?

It turns out that there are 7 solutions. One of the x's must be 2. Otherwise, the maximum value of the left-hand side of (1) is $1/3 + 1/4 + 1/5 = 47/60$; since the least common multiple of the x's must be at least 12, the right-hand side is at least $11/12$. But $47/60$ is already less than $11/12$. Therefore there is no solution without 2 as one of the x's. Arguing similarly, it can be shown that another of the x's must be 3 or 4. Taking $x_1 = 2$ and $x_2 = 3$, x_3 may be relatively prime to 2 and 3 or of the form $2m$, $3m$, or $6m$. Substituting in (1), the corresponding solutions for x_3 are 7, 8, 9, and 12. Taking $x_1 = 2$ and $x_2 = 4$, x_3 may be relatively prime to 2 and 4 or of the form