left

$$\exp\left[-\frac{1}{2}\left(\sum_{i=1}^{n}\phi(x_i-\theta)^2+\tau(\theta-\theta_0)^2\right)\right]$$

$$=\exp\left[-\frac{1}{2}\left(\sum_{i=1}^{n}\phi(x_i^2-2\theta x_i+\theta^2)+\tau\theta^2+\tau\theta_0^2-2\theta\theta_0\tau\right)\right]$$

$$=\exp\left[-\frac{1}{2}\left(\phi\Sigma x_i^2-2x_i\phi\theta+n\phi\theta^2+\tau\theta^2+\tau\theta_0^2-2\theta_0\tau\theta\right)\right]$$

$$=\exp\left[-\frac{1}{2}\left((n\phi+\tau)\theta^2+(2\phi\Sigma x_i+2\theta_0\tau)\theta+\phi\Sigma x_i^2+\theta_0^2\tau\right)\right]$$

$$=\exp\left[-\frac{1}{2}\left((n\phi+\tau)\theta^2-(2\phi\Sigma x_i+2\theta_0\tau)\theta\right)\right]\times\exp\left[-\frac{1}{2}(\phi\Sigma x_i^2+\theta_0^2\tau)\right]$$

∵ as ϕ、x_i、θ_0、τ are all given.

∴ $\exp\left[-\frac{1}{2}(\phi\Sigma x_i^2+\tau\theta_0^2)\right]$ is a constant

∴ left $\propto \exp\left[-\frac{1}{2}\left((n\phi+\tau)\theta^2-(2\phi\Sigma x_i+2\theta_0\tau)\theta\right)\right]$

$$=\exp\left[-\frac{1}{2}(n\phi+\tau)\left(\theta^2-\frac{2(\phi\Sigma x_i+\theta_0\tau)}{n\phi+\tau}\theta\right)\right]$$

$$\propto\exp\left[-\frac{1}{2}(n\phi+\tau)\left(\theta^2-2\frac{(\phi\Sigma x_i+\tau\theta_0)}{n\phi+\tau}\theta+\left(\frac{\phi\Sigma x_i+\theta_0\tau}{n\phi+\tau}\right)^2\right)\right]$$

(multiply a constant)

$$=\exp\left[-\frac{1}{2}(n\phi+\tau)\left(\theta-\frac{\phi\sum_{i=1}^{n}x_i+\tau\theta_0}{n\phi+\tau}\right)^2\right]$$ ~~Q.E.D~~ ⇒ right

∴ left $\propto$ right Q.E.D