- $P(E\cup F) = P(E) + P(F) - P(E\cap F)$
- $P(A|B) = \dfrac{P(A\cap B)}{P(B)}$
- $P(A\cap B) = P(A|B)\cdot P(B)$
- F. BAYES $\rightsquigarrow$ $P(A|B) = \dfrac{P(B|A)\cdot P(A)}{P(B)}$
- $P(A\cap B\cap C) = P(A|B\cap C)\cdot P(B|C)\cdot P(C)$

  $\hookrightarrow$ RIBALTABILE CON BAYES.

- F. PROB. TOT. SIA E_i = PARTIZ. DI EVENTI

$$P(A) = \sum_{i=1}^{n} P(A|E_i)\cdot P(E_i)$$

- INDIP. FRA EVENTI $\rightsquigarrow$ $P(A\cap B) = P(A)\cdot P(B)$.
- $\#D_{n,k}$ = # {SEQ. ORDINATE DI EL. $\{1,\dots,n\}$ DI LEN K}

$$\#D_{n,k} = \frac{n!}{(n-k)!}$$

- $\#C_{n,k}$ $\rightsquigarrow$ COME D MA SONO SOTTOINSIEMI

$$\#C_{n,k} = \binom{n}{k} = \frac{n!}{k!\,(n-k)!}$$

- ESTRA. IN BLOCCO.

× 1 TIPO OGG. k $\quad n_1 + n_2 = n$

$$P_k = \frac{\binom{n_1}{k}\binom{n_2}{n-k}}{\binom{n_1+n_2}{n}}$$

× + OGG. $k_1, k_2, .., k_n$

$$\frac{\binom{n_1}{k_1}\binom{n_2}{k_2}\cdots\binom{n_n}{k_n}}{\binom{n_1+n_2+\dots+n_n}{n}}$$

- DENSITÀ DISCRETA GENERALE

$$P_x(x) = P(X=x)$$

- DISTRIB. BERNOULLIANA $\rightsquigarrow$ CASO SI/NO

$$P(X=1) = P(B)$$
$$P(X=0) = P(B^c)$$

- $\forall\, k \in S_x \rightarrow$ RISULTATI DI X

\# SEQ. ω DOVE $X(\omega)=k$

$$P_x(k) = \mu_{n,k} \cdot q_k$$

$\hookrightarrow = \binom{n}{k}$

✗ CASO DISTR. BINOMIALE

· # SUCCESS. DI n PROVE INDIPEND. (= PROB. p)

$\forall k \in S_X \in \{0, 1, \dots, n\}$

$$P_X(k) = \binom{n}{k} p^k \cdot (1-p)^{n-k}$$

✗ DISTR. IPERGEOMETRICA

EXTR. CON PROVE ~~NON~~ INDIP. e EXTR. K OGG. DI UN TIPO

$$P_X(k) = \frac{\binom{n_1}{k}\binom{n_2}{n-k}}{\binom{n_1+n_2}{n}}$$

✗ DISTR. MULTINOMIALE

r RISULTATI POSSIB.

$$P\begin{pmatrix} k_1 \text{ VOLTE NUMERO } 1 \\ \vdots \\ k_n \text{ VOLTE NUMERO } n \end{pmatrix} = \frac{n!}{k_1! \cdots k_r!}\left(\frac{1}{r}\right)^n$$

✗ SIA $E = \{x_1, x_2, \dots, x_n\} \subset \mathbb{R}$

$$P(X \in B) = \frac{\#(B \cap E)}{\# E} = \frac{\#(B \cap E)}{n}$$

DISTR. POISSON

$$P_K(x) = \frac{\lambda^k}{k!} e^{-k}$$

CON n MOLTO GRANDI $n \to \infty$

U $X = X_n \sim BIN(n, p_n = \frac{\lambda}{n})$

$Z \sim POISSON(\lambda)$ $\lambda = n p_n$

$$\lim_{n\to\infty} P_{X_n}(k) = P_Z(k)$$

DISTR. GEOM. (GEOM. TRASL.) $\nearrow$ PROVE INDIP. U P UGUALE

$X \sim GEO(P)$ = # FAIL BEFORE 1° SUCC.

$Y \sim GEOTRASL(P)$ = # PROVE BEF. 1° SUCC.

- PER $k \geq 0$ $P_X(k) = (1-P)^k P$
- PER $h \geq 1$ $P_Y(h) = (1-P)^{h-1} P \rightsquigarrow P_X(X = h-1)$
- SERIE GEOMETRICA

$$h \geq 0 \quad \sum_{k=h}^{\infty} r^k = \frac{r^h}{1-r}$$

CASI CON CALCOLI VERSO ∞

PER $J \geq 0$

$$P_X(X \geq J) = \sum_{k=J}^{\infty} P_X(k) = p \sum_{k=J}^{\infty} (1-p)^k = (1-p)^J$$

$$P_Y(Y \geq J) = (1-p)^{J-1}$$

* PROP. "MANCANZA MEM."

$$\forall k, h \geq 0 \quad P(X = k+h \mid X \geq h) = P(X = k)$$

* DISTR. BIN. NEGAT. (NEG. TRASLATA)

$X = \#$ FAIL BEF. k SUCC. $= Y - k \sim \text{BIN-NEG}(k, p)$

$Y = \#$ PROVE " " " $= X = Y + k \sim \text{BIN-N-T}(k, p)$

$$P_X(k) = b_{r,k}\, p^r (1-p)^k = \binom{k+r-1}{k} p^r (1-p)^k$$

$b_{r,k}$ ↓ # SEQ. CON k VOLTE "F" (↑ FAIL), r VOLTE "S", CHE FINISCE IN "S"

$$P_Y(h) = \binom{h-1}{h-r} p^r (1-p)^{h-r} \quad \forall\, h \geq r \text{ INTERO}$$

V.a MULTIDIMENSIONALE

$$\underline{X}(X_1,\dots,X_n) : \Omega \to \mathbb{R}^n$$

$$\underline{X}(\omega) = \left(X_1(\omega), X_2(\omega),\dots, X_n(\omega)\right)$$

$$P_{\underline{X}}(\underline{x}) = P(\underline{X} = \underline{x}) \qquad \underline{x} = (x_1, x_2, \dots, x_n) \in \mathbb{R}^n$$

$P_{\underline{X}}$ → DENSITÀ CONGIUNTA

$P_{X_1}, P_{X_2}, \dots, P_{X_n}$ → DENSITÀ MARGINALE

$$\forall i \in \{1, \dots, n\} \qquad \underline{P_{X_i}(x_i) = \sum_{x \in S_x} P_{\underline{X}}(\underline{x})}$$

$X_1, X_2, \dots, X_n$ INDIP. $\iff$

$$P_{\underline{X}}(x_1 \cdot x_2 \cdots x_n) = P_{X_1}(x_1) \cdot P_{X_2}(x_2) \cdots P_{x_3}(x_3)$$

$$\forall (x_1, \dots, x_n) \in \mathbb{R}^n$$

D. DISCRETE CON F. COMPOSTE

$$\underline{X} : \Omega \to \mathbb{R}^n$$

$$f : A \subset \mathbb{R}^n \to \mathbb{R}^n$$

$$\underline{Y} : \underline{X} \circ f : \Omega \to \mathbb{R}^n$$

$$\Rightarrow \boxed{P_{\underline{Y}}(y) = \sum_{\substack{x \in S_x : \\ f(x) = y}} P_{\underline{X}}(x)}$$

V.A. O MIN, MAX

$$Y = \max\{X_1, X_2\} \rightarrow P(Y) = P(Y \leq y) - P(Y \leq y-1)$$

$$W = \min\{X_1, X_2\} \rightarrow P(X) = P(W \geq w) - P(W \geq w+1).$$

VALORE ATTESO

$$E[X] = \sum_{x_i \in S_X} |x_i| \, P_X(x_i)$$

PROPRIETÀ

- $E[a_1 X_1 + a_2 X_2 + \dots + a_n X_n] = a_1 E[X_1] + \dots + a_n E[X_n]$.

- $X_1, X_2, \dots, X_n$ INDIP. SU = S. PROB.

$$E[X_1 \cdot X_2 \cdot \dots \cdot X_n] = E[X_1] \cdot E[X_2] \cdot \dots \cdot E[X_n]$$

- $\underline{X}$, $Y = f(\underline{X})$

$$E[Y] = \sum_{\underline{x}_i \in S_{\underline{X}}} f(\underline{x}_i) \, P_{\underline{X}}(\underline{x}_i)$$

E[X] SU DIST. NOTE

- $X \sim B(p) \longrightarrow E[X] = p$

- $X \sim BIN(n, p) \longrightarrow E[X] = np$

- $X \sim$ IPERG $\longrightarrow$ $E[X] = n \dfrac{m_1}{m_1+m_2}$

- $X \sim$ POISSON(λ) $\rightarrow$ $E[X] = \lambda$
- $X \sim$ GEOM(p) $\longrightarrow$ $E[X] = \dfrac{1}{p} - 1$
- $X \sim$ GEOM TRASLATA $\longrightarrow$ $E[X] = \dfrac{1}{p}$
- $X \sim$ BIN-NEG $\longrightarrow$ $E[X] = n\left(\dfrac{1}{p} - 1\right)$
- $Y \sim$ BIN-N.-TRASL $\longrightarrow$ $E[X] = n/p$

☆ • MOMENTO $= E[X^k]$ $\rightarrow$ SU x_i METTO x_i^k

- MOMENTO CENTRATO $= E\left[(X - E[X])^k\right]$
- $VAR[X] = E\left[(X - E[X])^2\right]$
- SCARTO QUAD. MEDIO $= \sqrt{VAR[X]}$

☆ FORM. ALTERNATIVE X VARIANZA

- $VAR[X] = E[X^2] - (E[X])^2$

- $a \in \mathbb{R} \rightarrow VAR[aX] = a\, VAR[X]$
- $a \in \mathbb{R} \rightarrow VAR[a+X] = VAR[X]$

- $VAR[X_1+X_2] = VAR[X_1] + VAR[X_2] - 2\, COV[X_1, X_2]$

☆ $COV[X_1, X_2] = E[X_1 X_2] - E[X_1] \cdot E[X_2]$

- $COV(X_1, X_2) = COV(X_2, X_1)$
- $COV(X, X) = VAR[X]$

✗ VAR DI DIST. NOTE

- B → $VAR[X] = p(1-p)$
- BIN → // $= n p (1-p)$
- IP. GEOM → // $= np(1-p)\frac{n_1+n_2-n}{n_1+n_2-1}$
- POISSON → // $= \lambda$
- GEOM → // $= \frac{1-p}{p^2}$
- GEOM. TRASLATA → // $= \frac{1-p}{p^2}$
- BIN-NEG → // $= n \frac{1-p}{p^2}$
- BIN-NEG-TR → // $=$?