1) $\displaystyle\int \frac{x-5}{x^2-2x-2}\,dx$

METODO DEI FRATTI SEMPLICI: $x^2-2x-2 = \left(x-(1+\sqrt{3})\right)\left(x-(1-\sqrt{3})\right)$

$$\frac{x-5}{x^2-2x-2} = \frac{A}{x-(1+\sqrt{3})} + \frac{B}{x-(1-\sqrt{3})} = \frac{Ax - A(1-\sqrt{3}) + Bx - B(1+\sqrt{3})}{(x-(1+\sqrt{3}))(x-(1-\sqrt{3}))} =$$

$$= \frac{(A+B)x - A(1-\sqrt{3}) - B(1+\sqrt{3})}{x^2-2x-2}$$

$$\Rightarrow \begin{cases} A+B=1 \\ -A(1-\sqrt{3})-B(1+\sqrt{3})=-5 \end{cases} \quad \begin{cases} '' \\ -A+\sqrt{3}A-B-\sqrt{3}B=-5 \end{cases} \quad \begin{cases} A=1-B \\ -1+\cancel{B}+\sqrt{3}-\sqrt{3}B-\cancel{B}-\sqrt{3}B=-5 \end{cases}$$

$$\begin{cases} '' \\ -2\sqrt{3}B-1+\sqrt{3}=-5 \end{cases} \quad \begin{cases} '' \\ -2\sqrt{3}B=-4-\sqrt{3} \end{cases} \quad \begin{cases} '' \\ B=\dfrac{4+\sqrt{3}}{2\sqrt{3}} \underset{\substack{\uparrow \\ \text{razionalizzo}}}{=} \dfrac{4\sqrt{3}+3}{6} \end{cases} \quad \begin{cases} A=1-\dfrac{4\sqrt{3}+3}{6} \\ B=\dfrac{4\sqrt{3}+3}{6} \end{cases}$$

$$\begin{cases} A=\dfrac{3-4\sqrt{3}}{6} \\ B=\dfrac{3+4\sqrt{3}}{6} \end{cases}$$

Quindi $\displaystyle\frac{x-5}{x^2-2x-2} = \left(\frac{3-4\sqrt{3}}{6}\right)\left(\frac{1}{x-(1+\sqrt{3})}\right) + \left(\frac{3+4\sqrt{3}}{6}\right)\left(\frac{1}{x-(1-\sqrt{3})}\right)$

$$\int \frac{x-5}{x^2-2x-2}\,dx = \frac{3-4\sqrt{3}}{6}\int \frac{1}{x-(1+\sqrt{3})}\,dx + \frac{3+4\sqrt{3}}{6}\int \frac{1}{x-(1-\sqrt{3})}\,dx =$$

$$= \frac{3-4\sqrt{3}}{6}\ln\left(|x-(1+\sqrt{3})|\right) + \frac{3+4\sqrt{3}}{6}\ln\left(|x-(1-\sqrt{3})|\right) + C$$

2) $$\int \frac{1+x}{\sqrt{3-x^2}}\,dx = \underbrace{\int \frac{1}{\sqrt{3-x^2}}\,dx}_{A} + \underbrace{\int \frac{x}{\sqrt{3-x^2}}\,dx}_{B}$$

Risolviamo separatamente i due integrali:

(A) $$\int \frac{1}{\sqrt{3-x^2}}\,dx = \int \frac{1}{\sqrt{3\left(1-\frac{x^2}{3}\right)}}\,dx = \int \frac{1}{\sqrt{3}}\cdot\frac{1}{\sqrt{1-\frac{x^2}{3}}}\,dx$$

$\left[\text{OSS: } \dfrac{d}{dx}(\arcsin x) = \dfrac{1}{\sqrt{1-x^2}}\right]$

$$= \frac{1}{\sqrt{3}}\int \frac{1}{\sqrt{1-\frac{x^2}{3}}}\,dx$$

SOSTITUZIONE: $\boxed{t=\dfrac{x}{\sqrt{3}} \Rightarrow x=\sqrt{3}t \quad dx=\sqrt{3}\,dt}$

$$= \frac{1}{\sqrt{3}} \int \frac{1}{\sqrt{1-t^2}} \sqrt{3}\, dt = \frac{\cancel{\sqrt{3}}}{\cancel{\sqrt{3}}} \int \frac{1}{\sqrt{1-t^2}}\, dt = \arcsin t + c_1$$

quindi Ⓐ $= \arcsin\left(\frac{x}{\sqrt{3}}\right) + c_1$

Ⓑ $\displaystyle\int \frac{x}{\sqrt{3-x^2}}\, dx$ $\quad$ OSS: $\frac{d}{dx}\left(\sqrt{3-x^2}\right) = (-2x)\frac{1}{2}\frac{1}{\sqrt{3-x^2}} = -\frac{x}{\sqrt{3-x^2}}$

quindi $\displaystyle\int \frac{x}{\sqrt{3-x^2}}\, dx = -\left(\sqrt{3-x^2}\right) + c_2$

Conclusione: $\displaystyle\int \frac{1+x}{\sqrt{3-x^2}}\, dx = \arcsin\left(\frac{x}{\sqrt{3}}\right) - \sqrt{3-x^2} + c$ $\quad$ (dove $c = c_1 + c_2$)

3) $\displaystyle\int \frac{\ln x}{\sqrt{x}}\, dx$ $\quad$ SOSTITUZIONE: $\boxed{\begin{array}{l}\sqrt{x} = t \\ x = t^2 \quad dx = 2t\,dt\end{array}}$

$$= \int \frac{\ln(t^2)}{\cancel{t}} 2\cancel{t}\, dt = 2 \int \ln(t^2)\, dt \longrightarrow \text{Integrazione per parti}$$

$$= 2 \int 1 \cdot \ln(t^2)\, dt = 2\left(t \ln(t^2) - \int \cancel{t}\frac{2}{\cancel{t}}\, dt\right) = 2\left(t \ln(t^2) - 2t\right) + c$$

$$\left(\begin{array}{ll} f(t) = t & f'(t) = 1 \\ g(t) = \ln(t^2) & g'(t) = \frac{2t}{t^2} = \frac{2}{t}\end{array}\right)$$

Quindi $\displaystyle\int \frac{\ln x}{\sqrt{x}}\, dx = 2\left(\sqrt{x} \ln(x) - 2\sqrt{x}\right) + c$

$$= 2\sqrt{x}\left(\ln(x) - 2\right) + c$$

4) $\displaystyle\int \cos x \ln\left(\frac{\sin x}{4} + 4\right) dx = \int \cos x \ln\left(\frac{\sin x + 16}{4}\right) dx =$

$$= \int \cos x \left(\ln(\sin x + 16) - \ln 4\right) dx = \int \cos x \ln(\sin x + 16)\, dx - \int \cos x \ln 4\, dx =$$

$$= \int \cos x \ln(\sin x + 16)\, dx - \ln 4 \int \cos x\, dx = \underline{\int \cos x \ln(\sin x + 16)\, dx} - \ln(4) \sin x + c_1$$

Risolviamo questo integrale : SOSTITUZIONE $\boxed{\begin{array}{l} t = \sin x \\ dt = \cos x\, dx\end{array}}$

$\rightarrow \int \ln(t+16)\,dt$ SOSTITUZIONE $\boxed{\begin{array}{l} u=t+16 \\ du=dt \end{array}}$

$= \int \ln u \, du$ $\rightarrow$ integrazione per parti $\begin{pmatrix} f(u)=u & f'(u)=1 \\ g(u)=\ln u & g'(u)=\frac{1}{u} \end{pmatrix}$ $= u\ln(u) - \int \frac{\cancel{u}}{\cancel{u}}\,du = u \ln(u) - u + C_2$

Allora $\int \ln(t+16)\,dt = (t+16)\ln(t+16) - (t+16) + C_2 = (t+16)\left(\ln(t+16)-1\right)+C_2$

e $\int \cos x \ln(\sin x+16)\,dx = (\sin x+16)\left(\ln(\sin x+16)-1\right)+C_2$

Conclusione: $\int \cos x \ln\left(\frac{\sin x}{4}+4\right)dx = (\sin x+16)\left(\ln(\sin x+16)-1\right)-\ln(4)\sin x + C$

$= (\sin x+16)\ln(\sin x+16) - \sin x \underbrace{-16 - \ln(4)\sin x + C}$ $= C'$ (nuova costante arbitraria)

$= \sin x \ln(\sin x+16) + 16\ln(\sin x+16) - \sin x - \ln(4)\sin x + C'$

$= \sin x\left(\ln(\sin x+16)-\ln(4)\right) + 16\left(\ln(\sin x+16)-\ln(4)\right) - \sin x \underbrace{+16\ln(4)+C'}_{=C''}$

$= (\sin x+16)\ln\left(\frac{\sin x+16}{4}\right) - \sin x + C''$

$= (\sin x+16)\ln\left(\frac{\sin x}{4}+4\right) - \sin x + C''$

5) $\int e^{8x}\sin(x+1)\,dx$ Integrazione per parti

$$\begin{pmatrix} f(x)=\sin(x+1) & f'(x)=\cos(x+1) \\ g(x)=\frac{e^{8x}}{8} & g'(x)=e^{8x} \end{pmatrix}$$

$= \frac{e^{8x}}{8}\sin(x+1) - \frac{1}{8}\underbrace{\int e^{8x}\cos(x+1)\,dx}$ $\rightarrow$ Integrazione per parti : $\begin{pmatrix} f(x)=\cos(x+1) & f'(x)=-\sin(x+1) \\ g(x)=\frac{e^{8x}}{8} & g'(x)=e^{8x} \end{pmatrix}$

$= \frac{e^{8x}}{8}\sin(x+1) - \frac{1}{8}\left(\frac{e^{8x}}{8}\cos(x+1) + \frac{1}{8}\int e^{8x}\sin(x+1)\,dx\right)$

Quindi:

$\int e^{8x}\sin(x+1)\,dx = \frac{e^{8x}}{8}\sin(x+1) - \frac{e^{8x}\cos(x+1)}{64} - \frac{1}{64}\underbrace{\int e^{8x}\sin(x+1)\,dx}$

porto questo a sinistra di "="

$\int e^{8x}\sin(x+1)\,dx + \frac{1}{64}\int e^{8x}\sin(x+1)\,dx = e^{8x}\left(\frac{\sin(x+1)}{8} - \frac{\cos(x+1)}{64}\right) \longrightarrow$

$$\int e^{8x}\sin(x+1)\,dx\left(1+\frac{1}{64}\right)=e^{8x}\left(\frac{\sin(x+1)}{8}-\frac{\cos(x+1)}{64}\right)$$

$$\frac{64+1}{64}=\frac{65}{64}$$

$$\Rightarrow \int e^{8x}\sin(x+1)\,dx=\frac{64}{65}\left(e^{8x}\left[\left(\frac{\sin(x+1)}{8}\right)-\left(\frac{\cos(x+1)}{64}\right)\right]\right)+c$$

$$=\frac{e^{8x}}{65}\left(8\sin(x+1)-\cos(x+1)\right)+c$$

6) $\int (x^2+5x+6)\cos^2 x\,dx$ → integrazione per parti $\left[\begin{array}{ll} f(x)=\frac{x^3}{3}+\frac{5}{2}x^2+6x & f'(x)=x^2+5x+6 \\ g(x)=\cos^2 x & g'(x)=-2\cos x\sin x \\ & \quad =-\sin(2x)\end{array}\right]$

$$=\left(\frac{x^3}{3}+\frac{5}{2}x^2+6x\right)\cos^2 x+\underbrace{\int\left(\frac{x^3}{3}+\frac{5}{2}x^2+6x\right)\sin(2x)\,dx}_{A}$$

Ⓐ: Integrazione per parti $\left[\begin{array}{ll} f(x)=-\frac{\cos 2x}{2} & f'(x)=\sin 2x \\ g(x)=\frac{x^3}{3}+\frac{5}{2}x^2+6x & g'(x)=x^2+5x+6\end{array}\right]$

$$A=\left(\frac{x^3}{3}+\frac{5}{2}x^2+6x\right)\left(-\frac{\cos 2x}{2}\right)+\frac{1}{2}\underbrace{\int (x^2+5x+6)\cos 2x\,dx}_{B}$$

Ⓑ: Integrazione per parti $\left[\begin{array}{ll} f(x)=\frac{\sin 2x}{2} & f'(x)=\cos 2x \\ g(x)=x^2+5x+6 & g'(x)=2x+5\end{array}\right]$

$$B=(x^2+5x+6)\left(\frac{\sin 2x}{2}\right)-\frac{1}{2}\underbrace{\int (2x+5)\sin 2x\,dx}_{C}$$

Ⓒ: Integrazione per parti $\left[\begin{array}{ll} f(x)=-\frac{\cos 2x}{2} & f'(x)=\sin 2x \\ g(x)=2x+5 & g'(x)=2\end{array}\right]$

$$C=(2x+5)\left(-\frac{\cos 2x}{2}\right)+\frac{\cancel{2}}{\cancel{2}}\int\cos 2x\,dx=(2x+5)\left(-\frac{\cos 2x}{2}\right)+\frac{\sin 2x}{2}+c$$

Rimettiamo insieme tutti i pezzi:

$$\int (x^2+5x+6)\cos^2 x\,dx=\left(\frac{x^3}{3}+\frac{5}{2}x^2+6x\right)\cos^2 x+\left(\frac{x^3}{3}+\frac{5}{2}x^2+6x\right)\left(-\frac{\cos 2x}{2}\right)+\frac{1}{2}\left(x^2+5x+6\right)\left(\frac{\sin 2x}{2}\right)-$$

$$-\frac{1}{4}(2x+5)\left(-\frac{\cos 2x}{2}\right)-\frac{1}{8}\sin 2x+c$$

<u>Ricorda</u>: $\cos 2x=\cos^2 x-\sin^2 x$

$$=\left(\frac{x^3}{3}+\frac{5}{2}x^2+6x\right)\cos^2 x-\frac{1}{2}\left(\frac{x^3}{3}+\frac{5}{2}x^2+6x\right)\left(\cos^2 x-\sin^2 x\right)+\frac{1}{4}\left(x^2+5x+6\right)\sin 2x+\frac{1}{8}\left(2x+5\right)\cos 2x$$

$$-\frac{1}{8}\sin 2x+c$$

$$= \left(\frac{x^3}{3}+\frac{5}{2}x^2+6x\right)\left(\cos^2 x-\frac{1}{2}\cos^2 x+\frac{1}{2}\sin^2 x\right)+\left(\frac{1}{4}\left(x^2+5x+6\right)-\frac{1}{8}\right)\sin 2x+\frac{1}{8}\left(2x+5\right)\cos 2x+c$$ (5)

$$= \left(\frac{x^3}{3}+\frac{5}{2}x^2+6x\right)\Big(\frac{1}{2}\big(\underbrace{\cos^2 x+\sin^2 x}_{1}\big)\Big)+\left(\frac{1}{4}\left(x^2+5x+6\right)-\frac{1}{8}\right)\sin 2x+\frac{1}{8}\left(2x+5\right)\cos 2x+c$$

$$= \frac{1}{24}\left[\left(4x^3+30x^2+72x\right)+\left(6x^2+30x+33\right)\sin 2x+\left(6x+15\right)\cos 2x\right]+c$$

7) $\int \cos^3 x\,dx = \int \cos x\cdot\cos^2 x\,dx$ → Integrazione per parti $\left[\begin{array}{ll} f(x)=\sin x & f'(x)=\cos x \\ g(x)=\cos^2 x & g'(x)=-2\cos x\sin x \\ & =-\sin 2x \end{array}\right]$

$= \sin x\cos^2 x+2\int \cos x\sin^2 x\,dx$

$= \sin x\cos^2 x+2\int \cos x(1-\cos^2 x)dx = \sin x\cos^2 x+2\int\cos x\,dx-2\int\cos^3 x\,dx =$

$= \sin x\cos^2 x+2\sin x-2\int\cos^3 x\,dx$

Conclusione: $\int\cos^3 x\,dx = \sin x\cos^2 x+2\sin x\underbrace{-2\int\cos^3 x\,dx}$ → porto questo a sinistra

$3\int\cos^3 x\,dx = \sin x\cos^2 x+2\sin x$

$\Rightarrow \int\cos^3 x\,dx = \frac{\sin x\cos^2 x+2\sin x}{3}+c$

$= \frac{\sin x(1-\sin^2 x)+2\sin x}{3}+c$

$= \frac{3\sin x-\sin^3 x}{3}+c = \sin x-\frac{\sin^3 x}{3}+c$

8) $\int e^{\sqrt{x}}dx$ SOSTITUZIONE : $\boxed{\begin{array}{l}\sqrt{x}=t \\ x=t^2 \quad dx=2t\,dt\end{array}}$

$\int e^t 2t\,dt = 2\int t e^t dt$ Integrazione per parti $\left(\begin{array}{ll} f(t)=t & f'(t)=1 \\ g(t)=e^t & g'(t)=e^t\end{array}\right)$

$= 2\left(te^t-\int e^t dt\right) = 2\left(te^t-e^t\right)+c$

Quindi ; $\int e^{\sqrt{x}}dx = 2(\sqrt{x}-1)e^{\sqrt{x}}+c$

Integrali definiti

1) $$\int_9^{16} \frac{\sqrt{x}+3}{x-3\sqrt{x}+2}\,dx$$

Soggerimento: utilizzare il metodo dei fratti semplici

$\Rightarrow$ trovare a, A, b, B tali che:

$$\frac{\sqrt{x}+3}{x-3\sqrt{x}+2} = \frac{A}{\sqrt{x}+a} + \frac{B}{\sqrt{x}+b}$$

$$\frac{A}{\sqrt{x}+a} + \frac{B}{\sqrt{x}+b} = \frac{A\sqrt{x}+Ab+B\sqrt{x}+Ba}{(\sqrt{x}+a)(\sqrt{x}+b)} = \frac{\sqrt{x}(A+B)+Ab+Ba}{x+(a+b)\sqrt{x}+ab}$$

Quindi dobbiamo risolvere il sistema:

$$\begin{cases} A+B=1 \\ Ab+Ba=3 \\ a+b=-3 \\ ab=2 \end{cases} \quad \begin{cases} // \\ // \\ // \\ a=\frac{2}{b} \end{cases} \quad \begin{cases} // \\ // \\ \frac{2}{b}+b=-3 \\ // \end{cases} \quad \begin{cases} // \\ // \\ 2+b^2=-3b \\ // \end{cases} \rightarrow b^2+3b+2=0$$

$$\Delta = 9-4(2)=1$$

$$b_{1,2} = \frac{-3\pm 1}{2} \begin{cases} -1 \\ -2 \end{cases}$$

$\boxed{b=-1}$
$$\begin{cases} A+B=1 \\ -A-2B=3 \\ a=-2 \end{cases} \quad \begin{cases} A=1-B \\ -1+B-2B=3 \\ // \end{cases} \quad \begin{cases} // \\ -B=4 \\ // \end{cases} \quad \begin{cases} A=5 \\ B=-4 \\ a=-2 \end{cases}$$

$\boxed{b=-2}$
$$\begin{cases} A+B=1 \\ -2A-B=3 \\ a=-1 \end{cases} \Rightarrow \begin{cases} A=-4 \\ B=5 \\ a=-1 \end{cases}$$

<u>OSS</u>: Sia nel caso : $a=-2,\ b=-1,\ A=5,\ B=-4$

che nel caso : $a=-1,\ b=-2,\ A=-4,\ B=5$

otteniamo che $\dfrac{\sqrt{x}+3}{x-3\sqrt{x}+2} = \dfrac{5}{\sqrt{x}-2} - \dfrac{4}{\sqrt{x}-1}$

Quindi: $$\int \frac{\sqrt{x}+3}{x-3\sqrt{x}+2}\,dx = 5\int \frac{1}{\sqrt{x}-2}\,dx - 4\int \frac{1}{\sqrt{x}-1}\,dx$$

SOSTITUZIONE: $\boxed{\begin{array}{l} t=\sqrt{x} \\ x=t^2 \\ dx=2t\,dt \end{array}}$

$$= 5\int \frac{t}{t-2}\,dt - 4\int \frac{t}{t-1}\,dt = 5\int \frac{t-2+2}{t-2}\,dt - 4\int \frac{t-1+1}{t-1}\,dt =$$ ⑦

$$= 5\int 1 + \frac{2}{t-2}\,dt - 4\int 1 + \frac{1}{t-1}\,dt = 5\int dt + 10\int \frac{1}{t-2}\,dt - 4\int dt - 4\int \frac{1}{t-1}\,dt$$

$$= 5t + 10\ln(|t-2|) - 4t - 4\ln(|t-1|) + c$$

$$= t + 10\ln(|t-2|) - 4\ln(|t-1|) + c$$

Quindi $= \sqrt{x} + 10\ln(|\sqrt{x}-2|) - 4\ln(|\sqrt{x}-1|) + c$ è una primitiva di $\dfrac{\sqrt{x}+3}{x-3\sqrt{x}+2}$

$$\int_9^{16} \frac{\sqrt{x}+3}{x-3\sqrt{x}+2} = \sqrt{16} + 10\ln\left(|\sqrt{16}-2|\right) - 4\ln\left(|\sqrt{16}-1|\right) - \left(\sqrt{9} + 10\ln\left(|\sqrt{9}-2|\right) - 4\ln\left(|\sqrt{9}-1|\right)\right)$$

$$= 4 + 10\ln(2) - 4\ln(3) - 3 - \cancel{10\ln(1)} + 4\ln(2) =$$

$$= 1 + 14\ln(2) - 4\ln(3)$$ ✓

2) $\displaystyle\int_0^1 \frac{e^x}{1+e^{2x}}\,dx$ SOSTITUZIONE $\boxed{\begin{array}{l} t = e^x \\ dt = e^x dx \end{array}}$

$$\int_0^1 \frac{e^x}{1+e^{2x}}\,dx = \int_1^e \frac{dt}{1+t^2} = \operatorname{arctg}(t)\Big|_1^e = \operatorname{arctg}(e) - \operatorname{arctg}(1)$$

3) $\displaystyle\int_e^{e^2} \frac{dx}{x\ln x}$ SOSTITUZIONE $\boxed{\begin{array}{l} t = \ln(x) \\ dt = \frac{1}{x}\,dx \end{array}}$

$$\int_e^{e^2} \frac{dx}{x\ln x} = \int_1^2 \frac{dt}{t} = \ln(t)\Big|_1^2 = \ln(2) - \underset{0}{\underset{\parallel}{\ln(1)}} = \ln(2)$$

4) $\displaystyle\int_0^{\sqrt{2}/2} \frac{dx}{\sqrt{1-x^2}} \underset{\uparrow}{=} \arcsin x\Big|_0^{\sqrt{2}/2} = \arcsin\left(\frac{\sqrt{2}}{2}\right) - \arcsin(0) = \frac{\pi}{4} - 0 = \frac{\pi}{4}$

$$\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$$

5) $\displaystyle\int_1^e \frac{\sin(\ln x)}{x}\,dx$ SOSTITUZIONE: $\boxed{\begin{array}{l} t = \ln x \\ dt = \frac{1}{x}dx \end{array}}$ (ATTENZIONE: Errore nel testo dell'esercizio!! $\displaystyle\int_0^e \frac{\sin(\ln x)}{x}dx$ non esiste)

$$\int_1^e \frac{\sin(\ln x)}{x}\,dx = \int_0^1 \sin(t)\,dt = -\cos t\Big|_0^1 = -\cos(1) + \cos(0) = 1 - \cos(1)$$

6) $$\int_0^{-3} \frac{dx}{\sqrt{25+3x}} = \frac{2}{3}\sqrt{25+3x}\Big|_0^{-3} = -\left(\frac{2}{3}\sqrt{25+3x}\Big|_{-3}^{0}\right) = -\left(\frac{2}{3}\sqrt{25} - \frac{2}{3}\sqrt{25-9}\right) =$$

$$= -\left(\frac{2}{3}\cdot 5 - \frac{2}{3}\cdot 4\right) = -\frac{2}{3}$$

- Determinare la primitiva di $f(x) = \frac{1}{x}$ che vale 1 per $x = -1$

Calcoliamo $\int \frac{1}{x}dx \Rightarrow \int \frac{1}{x}dx = \ln(|x|) + c \Rightarrow$ al variare di $c \in \mathbb{R}$ ottengo una famiglia di primitive

Tra tutte queste primitive, voglio trovare l'unica primitiva $F(x)$ tale che $F(-1) = 1$.

Dobbiamo perciò trovare la c che soddisfa l'equazione:

$$\ln(|-1|) + c = 1 \iff \ln(1) + c = 1 \iff \boxed{c = 1}$$

quindi $F(x) = \ln(|x|) + 1$ ✓

- Sia $F(x)$ la primitiva di $f(x) = \ln(x^2)$ passante per il punto $P = (-1, 1)$

Calcolare quanto valgono $F(-\frac{1}{e})$, $F(-e^2)$, $F(-e)$

Svolgimento: $\int \ln(x^2)dx = \int 2\ln(x)dx = 2\int \ln(x)dx = 2(x\ln(x) - x) = x(\ln(x^2) - 2) + c$

Troviamo c imponendo il passaggio per $P = (-1, 1)$ $\Rightarrow -1(\ln(1) - 2) + c = 1 \iff 2 + c = 1 \iff \boxed{c = -1}$

Quindi $F(x) = x(\ln(x^2) - 2) - 1$

- $F(-\frac{1}{e}) = -\frac{1}{e}(\ln(1/e^2) - 2) - 1 = -\frac{1}{e}(-2 - 2) - 1 = \frac{4}{e} - 1$
- $F(-e^2) = -e^2(\ln(e^4) - 2) - 1 = -e^2(4 - 2) - 1 = -2e^2 - 1$
- $F(-e) = -e(\ln(e^2) - 2) - 1 = -e(2 - 2) - 1 = -1$