

INT201 Decision, Computation and Language

Tutorial 5

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1. Prove the following language is not regular

$$L = \{w \in \{a, b\}^* \mid w = w^R\}$$

2. Prove that if we add a finite set of strings to a regular language, the result is a regular language.

3. Convert the regular expression $(00)^*(11)$ into an NFA.



Solution

1.

Answer: Suppose that A_2 is a regular language. Let p be the “pumping length” of the Pumping Lemma. Consider the string $s = a^pba^p$. Note that $s \in A_2$ since $s = s^{\mathcal{R}}$, and $|s| = 2p + 1 \geq p$, so the Pumping Lemma will hold. Thus, we can split the string s into 3 parts $s = xyz$ satisfying the conditions

- i. $xy^iz \in A_2$ for each $i \geq 0$,
- ii. $|y| > 0$,
- iii. $|xy| \leq p$.

Since the first p symbols of s are all a 's, the third condition implies that x and y consist only of a 's. So z will be the rest of the first set of a 's, followed by ba^p . The second condition states that $|y| > 0$, so y has at least one a . More precisely, we can then say that

$$\begin{aligned}x &= a^j \text{ for some } j \geq 0, \\y &= a^k \text{ for some } k \geq 1, \\z &= a^m ba^p \text{ for some } m \geq 0.\end{aligned}$$

Since $a^pba^p = s = xyz = a^ja^ka^mba^p = a^{j+k+m}ba^p$, we must have that $j + k + m = p$. The first condition implies that $xy^2z \in A_2$, but

$$\begin{aligned}xy^2z &= a^ja^ka^ka^mba^p \\&= a^{p+k}ba^p\end{aligned}$$

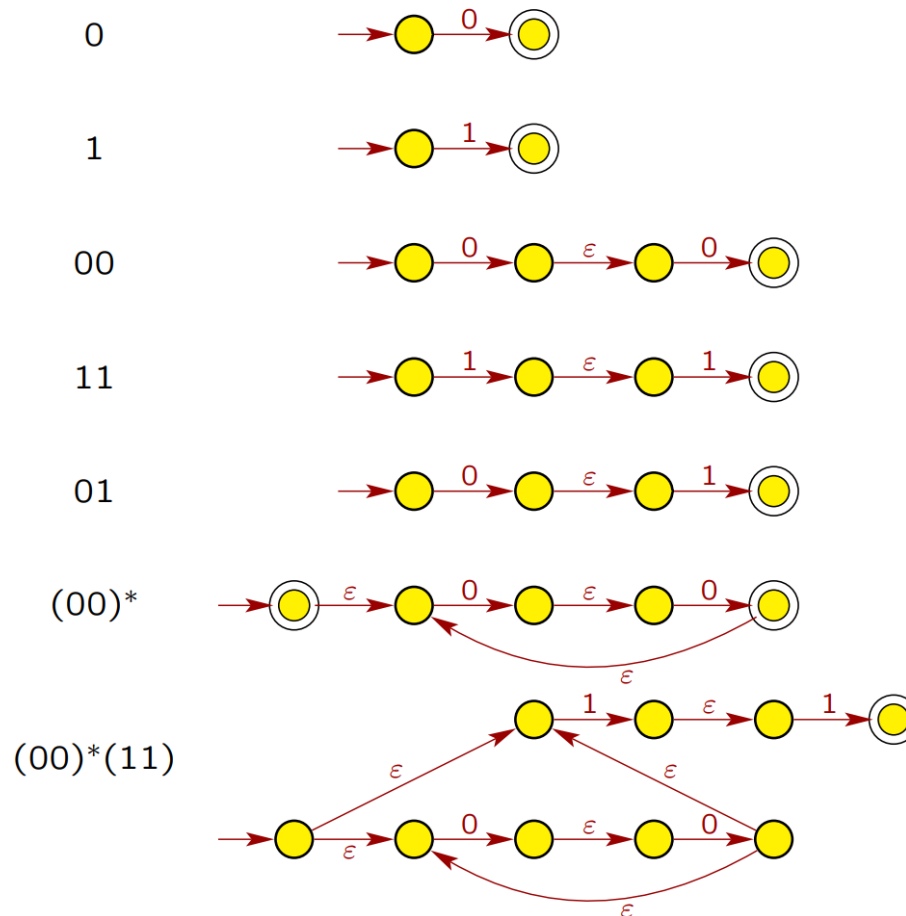
since $j + k + m = p$. Hence, $xy^2z \notin A_2$ because $(a^{p+k}ba^p)^{\mathcal{R}} = a^pba^{p+k} \neq a^{p+k}ba^p$ since $k \geq 1$, and we get a contradiction. Therefore, A_2 is a nonregular language.

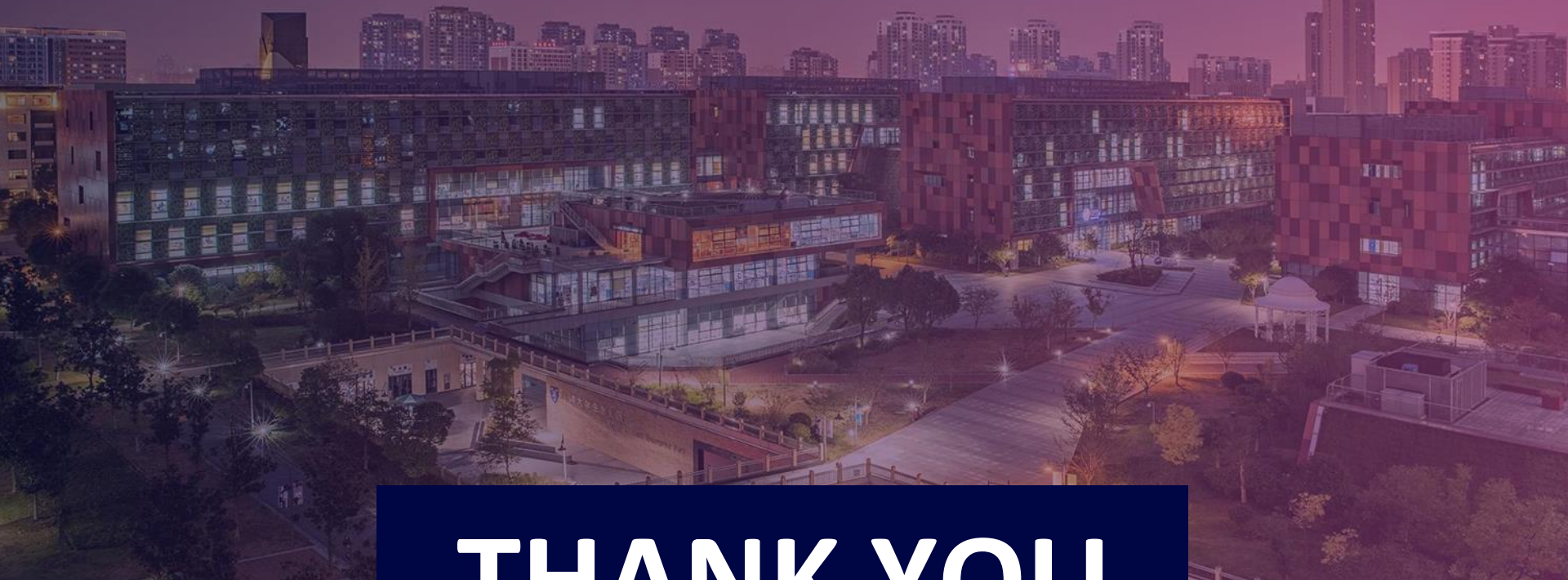


Solution

2. Let A be a regular language, and let B be a finite set of strings. We know from class that finite languages are regular, so B is regular. Thus, $A \cup B$ is regular since the class of regular languages is closed under union.

3.





THANK YOU



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