Xi'an Jiaotong-Liverpool University
西交利物浦大学

CPT205 Computer Graphics

3D Modelling

Lecture 07
2025-26

Yong Yue and Nan Xiang

Teaching Plan

Week (c/m)	Lecture	Topic	CW	Lecturer
01 (24.09.09)	Lecture 01 Lecture 02	Introduction and hardware/software Mathematics for computer graphics		Yong Yue
02 (24.09.16)	Lecture 03	Geometric primitives		Nan Xiang
03 (24.09.23)	Lecture 04	Geometric transformations	CW1 out	Nan Xiang
04 (24.10.07)	Lecture 05	Viewing and projection		Nan Xiang
05 (24.10.14)	Lecture 06	Parametric curves and surfaces		Yong Yue
06 (24.10.21)	Lecture 07	3D modelling		Yong Yue
07 (24.10.28)		Reading week	CW1 due	
08 (24.11.04)	Lecture 08	Hierarchical modelling	CW2 out	Yong Yue
09 (24.11.11)	Lecture 09	Lighting and materials		Nan Xiang
10 (24.11.18)	Lecture 10	Texture mapping		Nan Xiang
11 (24.11.25)	Lecture 11	Clipping		Yong Yue
12 (24.12.02)	Lecture 12	Hidden surface removal	CW2 due	Yong Yue
13 (24.12.09)	Revision	Summary and highlights of topics covered / Past exam paper		Nan Xiang / Yong Yue

Topics for today

- **3D modelling techniques**
  - Wireframe
  - Surface
  - Solid
- **Constructive solid geometry (CSG)**
  - CSG tree
  - Unambiguous but not unique
- **Boundary representation (B-Rep)**
  - Geometry (shape and size) and topology (connectivity)
  - Types of B-Rep models – manifold and nonmanifold
  - Validity of B-Rep models – Euler’s law
  - Implementation of B-Rep models
  - **Romulus** and **ACIS** modellers

Wireframe modelling (1)

- The oldest and simplest approach

- The model consisting of a finite set of points and curves

- Parametric representation of a space curve

  $x = x(t), \quad y = y(t), \quad z = z(t)$

- Implicit representation of a space curve

  $s1(x,y,z) = 0, \quad s2(x,y,z) = 0$

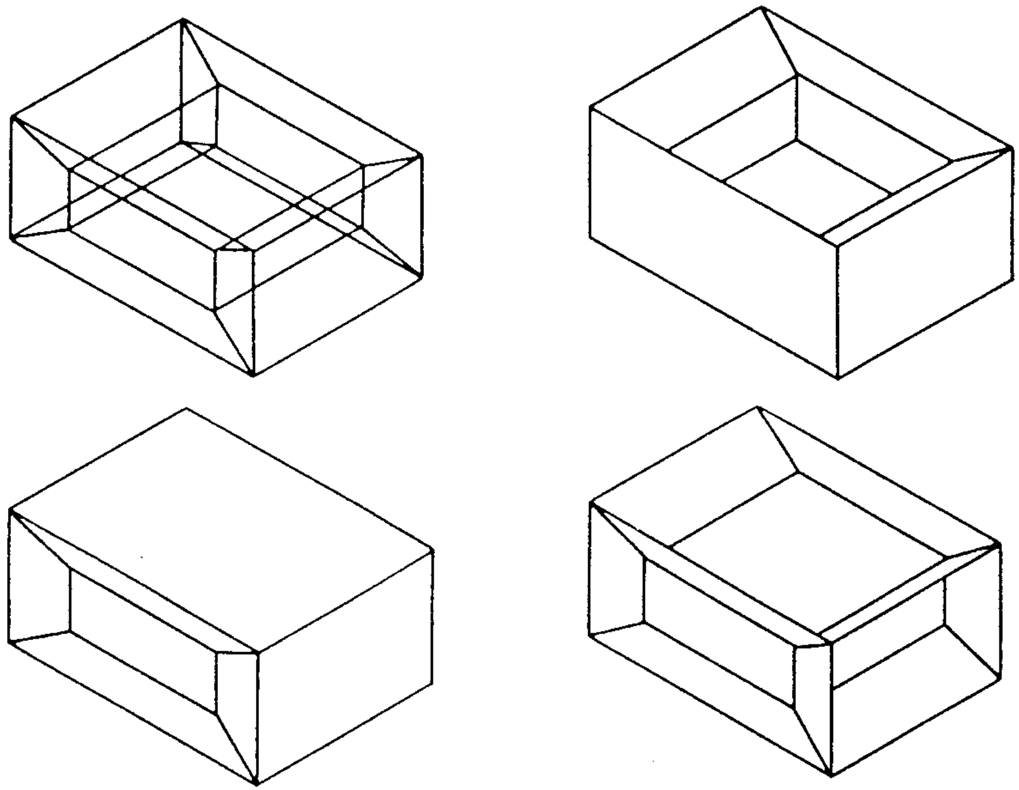
Wireframe modelling (2)

	Parametric representation	Implicit/explicit representation
Straight line	$x = t + 1$ $y = 2t + 1$	$y = 2x - 1$
Circle	$x = r\cos\theta + a$ $y = r\sin\theta + b$	$(x - a)^2 + (y - b)^2 = r^2$

Wireframe modelling (3)

- Combined use of curves can represent 3D objects in the space.
- Its disadvantages are the ambiguity of the model and the severe difficulty in validating the model.
- Furthermore, it does not provide surface and volume-related information.
- However, we should be clear about the difference between the wireframe model and wireframe display of an object.

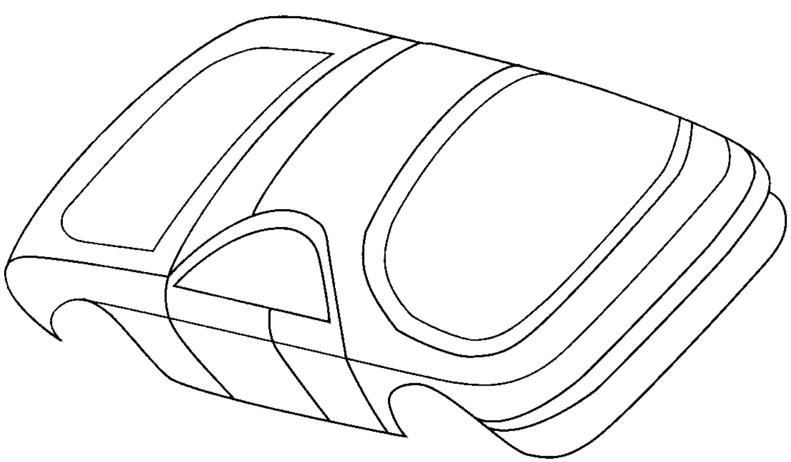
Wireframe modelling (4)



Ambiguous wireframe models

Surface modelling (1)

- It generates objects with a more complete and less ambiguous representation than its wireframe model.
- It is obvious that surface models are suitable for more applications, for example, design and representation of car bodies.

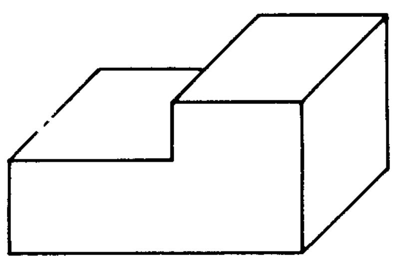


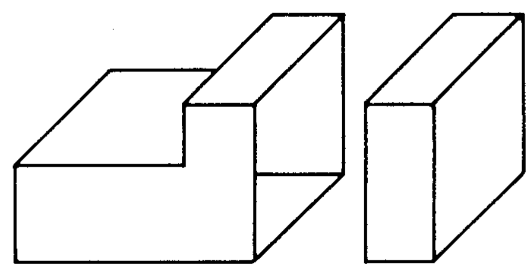
Surface modelling (2)

- Surfaces are built from points and curves.
- Surfaces can be 2D and 3D represented by a closed loop of curves with skin on it, the simplest form being a plane.
- They are very important in modelling objects, and in many situations, used to represent 3D models to a large variety of satisfaction.
- Modelling packages usually provide a range of useful surface creation functions, some of which are similar to those for curves while the geometric characteristics are different.

Surface modelling (3)

- Despite their similar appearance, there are differences between surface and solid models.
- Apart from the lack of volume-related information, surface models normally define the geometry of their corresponding objects.





Surface model - hollow model:
volume, mass, centroid?

Solid modelling

- Solid modelling represents both the geometric properties (e.g., points, curves, surfaces, volume, and centre of shape) and physical properties (e.g., mass, centre of gravity and inertia) of solid objects.
- There is a number of schemes, namely primitive instancing, cell decomposition, constructive solid geometry (CSG) and boundary representation (B-Rep).
- CSG and B-Rep are the most popular.

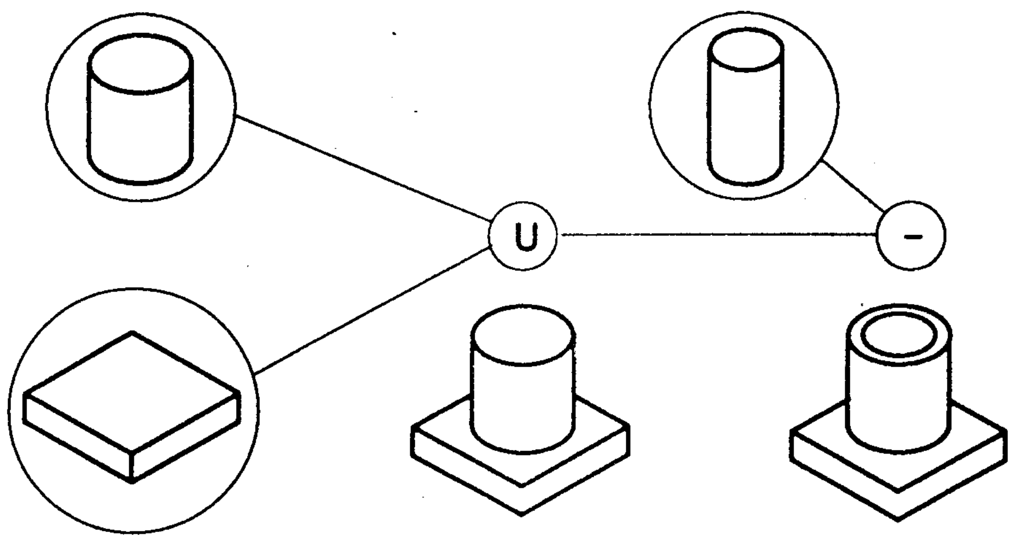
Constructive solid geometry (1)

- The CSG model is an ordered binary tree where the non-terminal nodes represent the operators and the terminal nodes are the primitives or transformation leaves.

- The operators may be rigid motions or regular Boolean operations.

- The primitive leaf is a primitive solid in the modelling space while the transformation leaf defines the arguments of rigid motion.

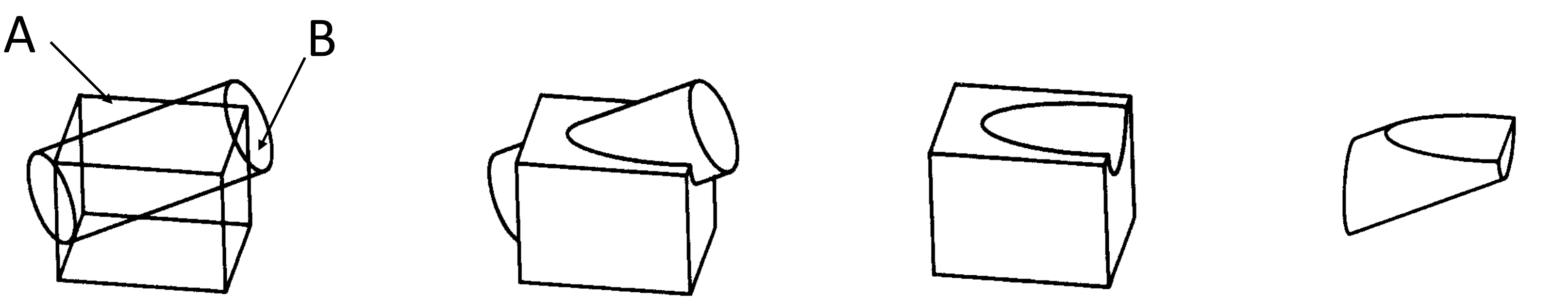
Constructive solid geometry (2)



CSG tree

Constructive solid geometry (3)

- Boolean operations include Boolean Union, Boolean Difference and Boolean Intersection.
- It should be noticed that the resultant solid of a Boolean operation depends not only on the solids but also on their location and orientation.



Objects | Union $A \cup B$ | Subtraction $A - B$ | Intersection $A \cap B$

Constructive solid geometry (4)

- Each solid usually has its <u>local</u> co-ordinate frame specified relative to the <u>world</u> co-ordinate frame.

- Before a Boolean operation is performed, it may be necessary to translate and/or rotate the solids in order to obtain the required relative location and orientation relationship between them.

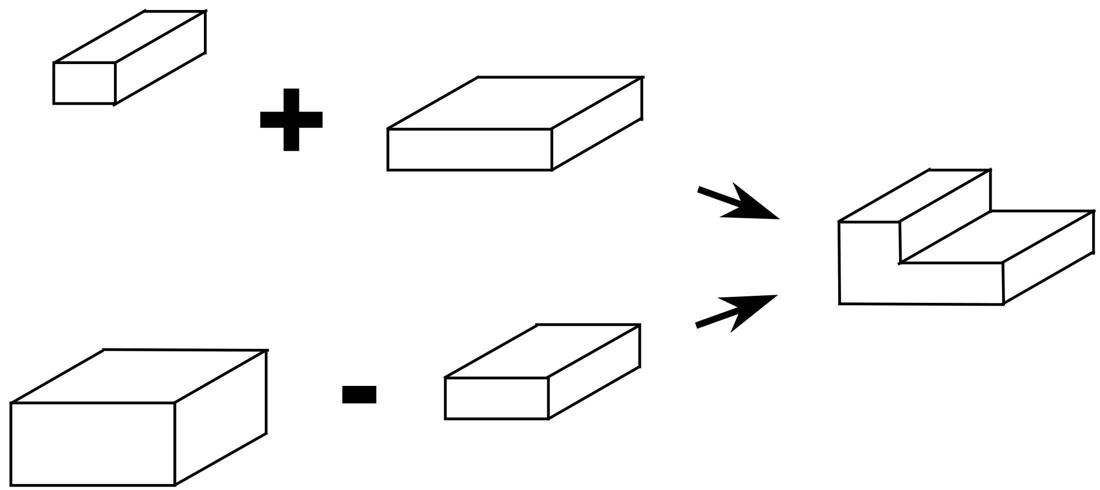
Constructive solid geometry (5)

- If an object can be represented by a unique set of data, the representation is said to be unique.

- The representation scheme for some applications (e.g., geometric reasoning) should ideally be both unambiguous and unique.

- Solid representations are usually unambiguous but few of them can be unique, and it is not feasible to make CSG representation unique.

Constructive solid geometry (6)



Nonuniqueness of CSG model

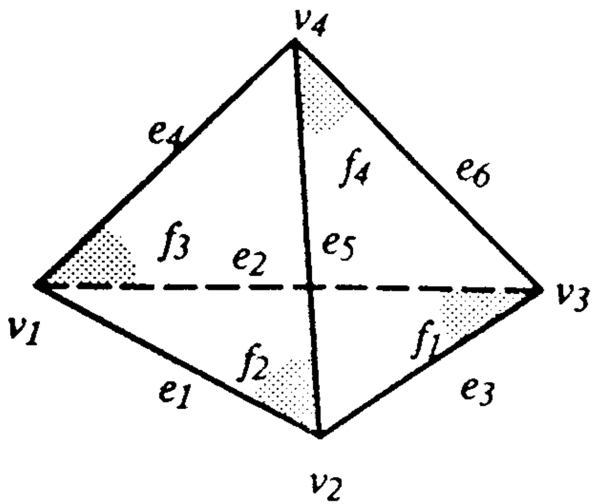
Boundary representation (1)

- The boundary representation (B-Rep) model represents a solid by segmenting its boundary into a finite number of bounded subsets (about the geometry and topology).

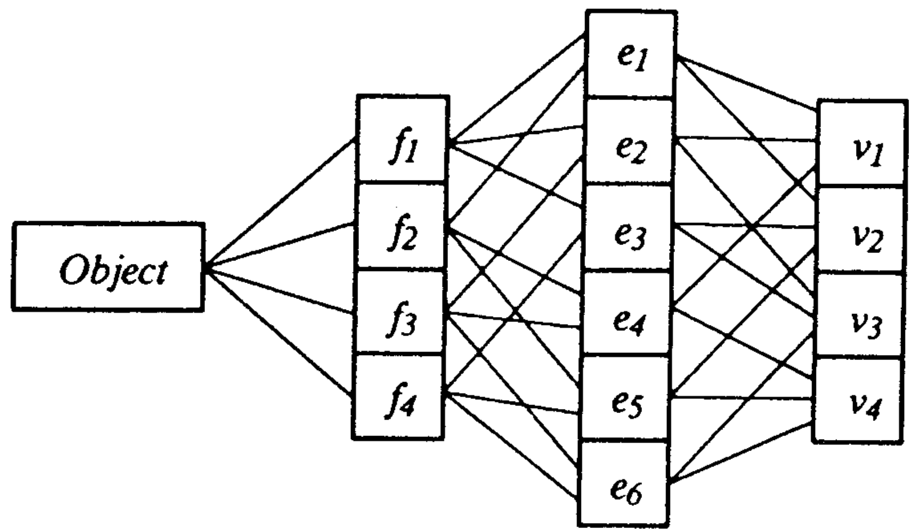
- The **geometry** is about the <u>shape and size</u> of the boundary entities called *points*, *curves* and *surfaces* while the **topology** keeps the <u>connectivity</u> of the boundary entities referred as *vertices*, *edges* and *faces* (corresponding to points, curves and surfaces).

- B-Rep is basically a <u>topologically explicit</u> representation where both geometric and topological information is stored in the data structure.

Boundary representation (2)



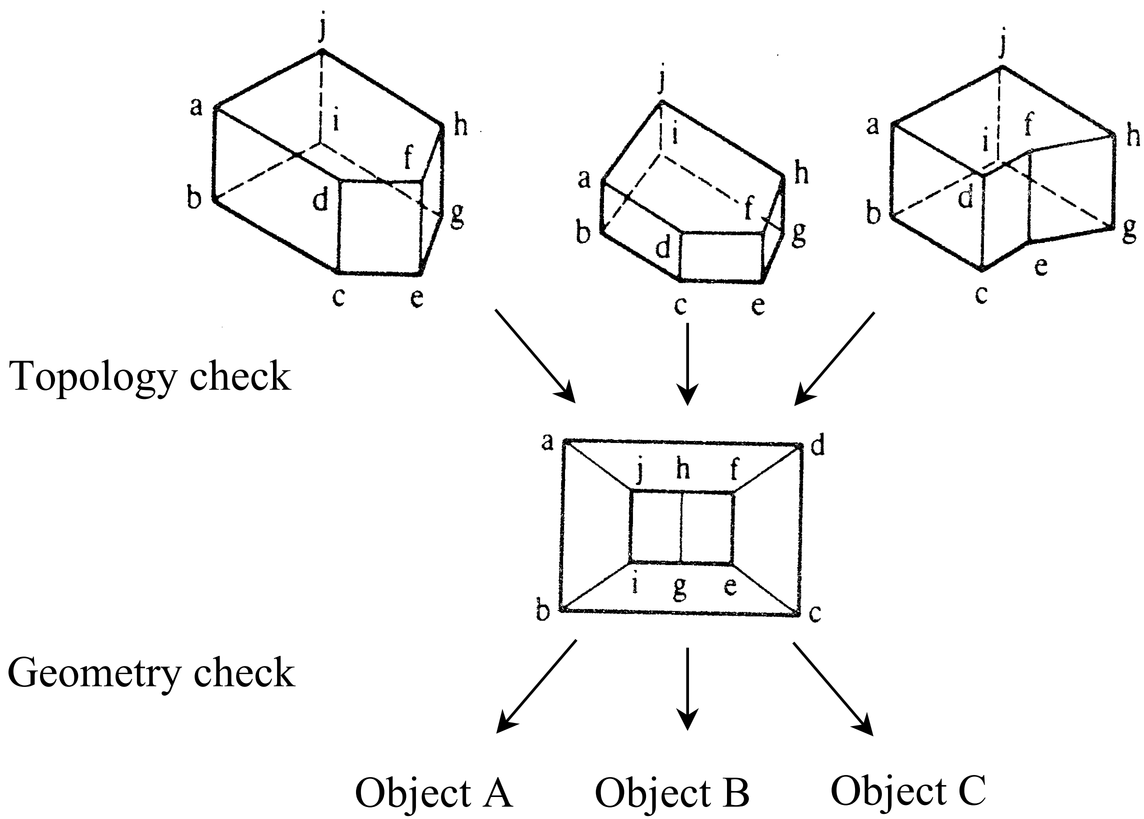
Tetrahedron



Topology of tetrahedron

Boundary representation (3)

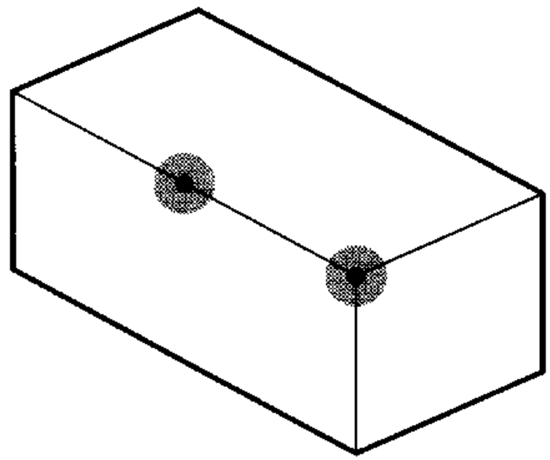
The same topology may represent different geometric shapes/sizes and therefore both topological and geometric data is necessary to fully and uniquely define an object.



Types of B-Rep

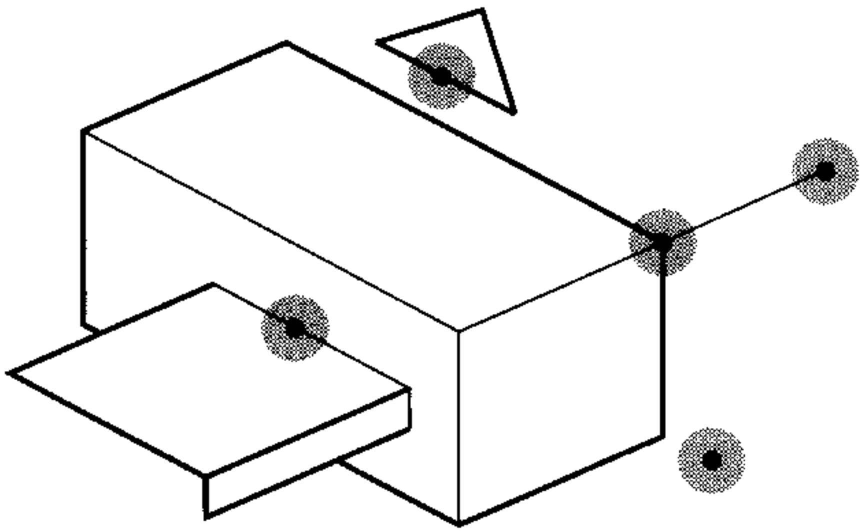
- B-Rep models can be divided into two types: *manifold* and *nonmanifold*.
- In a manifold model, an edge connects exactly two faces and a vertex connects at least three edges.
- A nonmanifold model may have dangling faces, edges and vertices, and therefore represent a non-realistic / non-physical object.

Manifold B-Rep model



Two faces meet exactly at one edge,
and at least three edges meet at a vertex

Nonmanifold B-Rep model



Dangling faces, edge and vertices

Euler's law for manifold B-Rep (1)

To ensure the topological validity for a solid (i.e., manifold model), a manifold model must satisfy the following Euler (Leonhard Euler, 1707-1783) formula,

$$V - E + F - R + 2H - 2S = 0$$

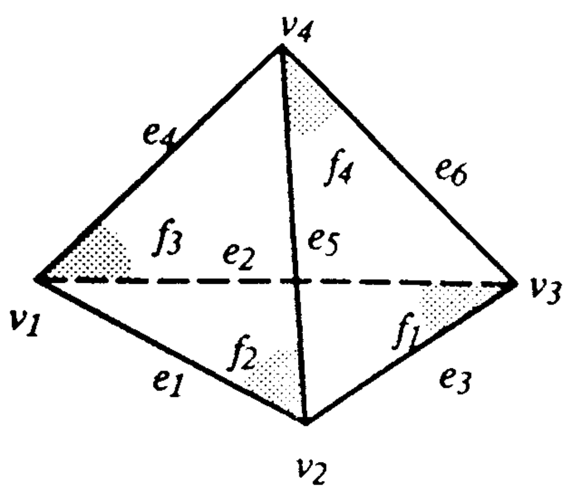
where V, E, F, R, H and S are the numbers of vertices, edges, faces, rings (inner loops on faces), passages/through-holes (genus) and shells (disjoint bodies), respectively.

Euler's law for manifold B-Rep (2)

The Euler's law in its simplest form is

$$V - E + F - 2 = 0$$

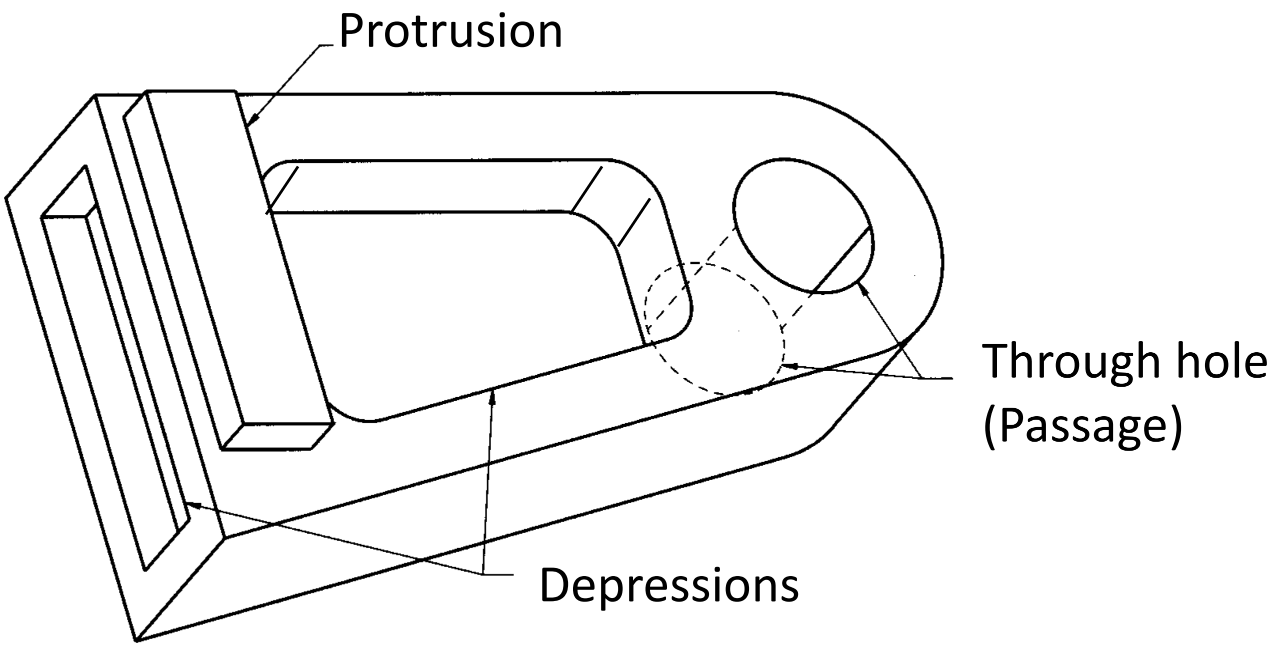
which can be applied to simple polyhedra (i.e., objects without inner loops of edges and passages).



For the tetrahedron

$4 - 6 + 4 - 2 = 0$

Euler's law for manifold B-Rep (3)



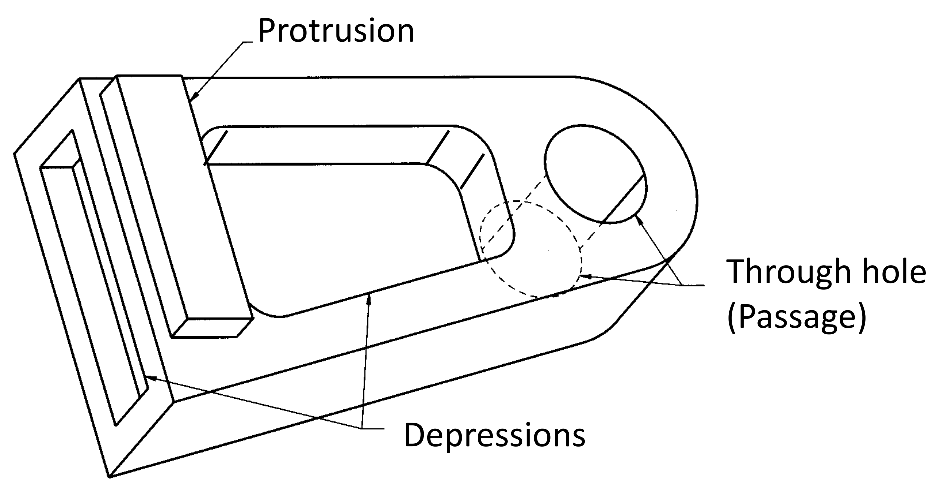
Verify if this is a manifold object

Euler's law for manifold B-Rep (4)

	V	E	F	R	H	S
Basic shape	8	12	6			1
Protrusion	8	12	5	1		
Sharp corner depression	8	12	5	1		
Round corner depression	16	24	9	1		
Hole	4	6	2	2	1	
Total	44	66	27	5	1	1

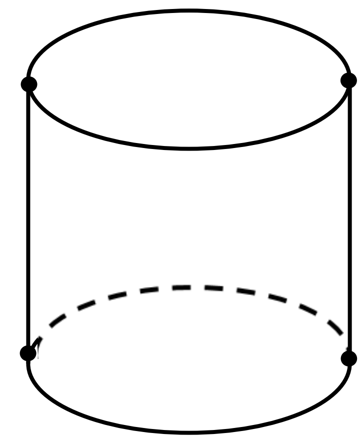
$V - E + F - R + 2H - 2S = 0$

44 - 66 + 27 - 5 + 2x1 - 2x1 = 0



Euler's law for manifold B-Rep (5)

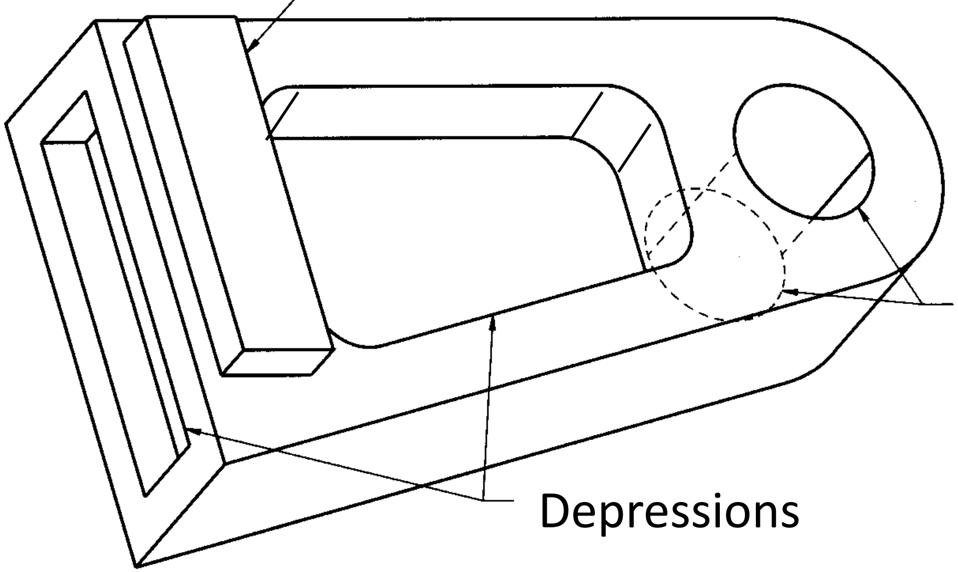
- A cylindrical body can be represented by 4 vertices, 6 edges and 4 faces, satisfying the Euler's law.



- Therefore, the cylindrical hole in the previous slide is represented by two edges along its axis, resulting in 4 vertices, 6 edges and 2 faces in total.

- It is worth noting that the Euler's law also applies to curved objects represented by patches, curve segments and vertices.

Euler’s law for manifold B-Rep (3)



Can you verify if the object is still a manifold object,
when the Round corner depression becomes a through hole (passage)?

Implementation of B-Rep (1)

- B-Rep models can be conveniently implemented on computers by representing the topology as pointers and the geometry as numerical information in the data structure for extraction and manipulation using object-oriented programming (OOP) techniques (e.g., C++).

- The latest B-Rep modellers also provide facilities to tag attributes (such as colour, tolerance and surface finish) on the boundary elements, useful for applications such as computer aided design and manufacturing (CAD/CAM).

Implementation of B-Rep (2)

Faces

Face	Edge
1	1
	2
	3
	4
2	.
	.
	.

Edges

Edge	Vertex
1	1
	2
2	5
	6
3	.
	.
	.

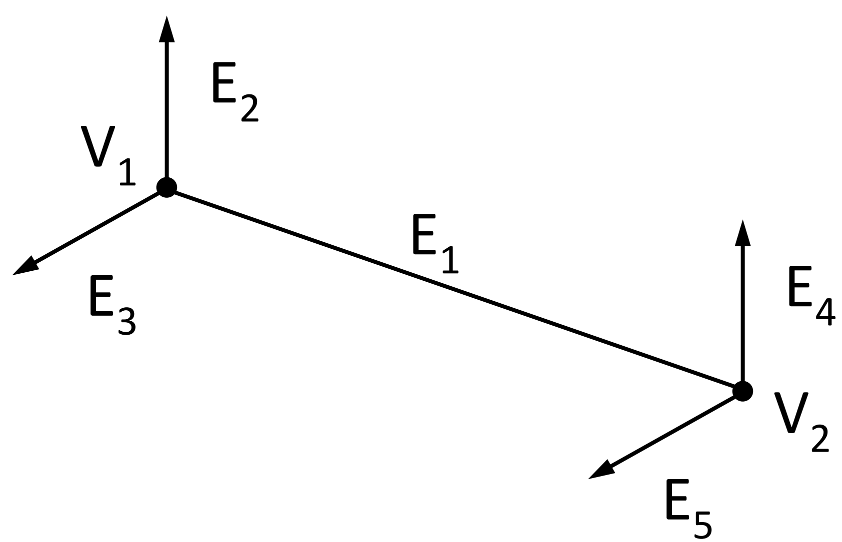
Vertices

Vertex	X	Y	Z
1	2	1	3
2	4	2	5
3	2	3	6
4	.	4	.
5	.	.	.
.	.	.	.
.	.	.	.

The three table-structure

Implementation of B-Rep (3)

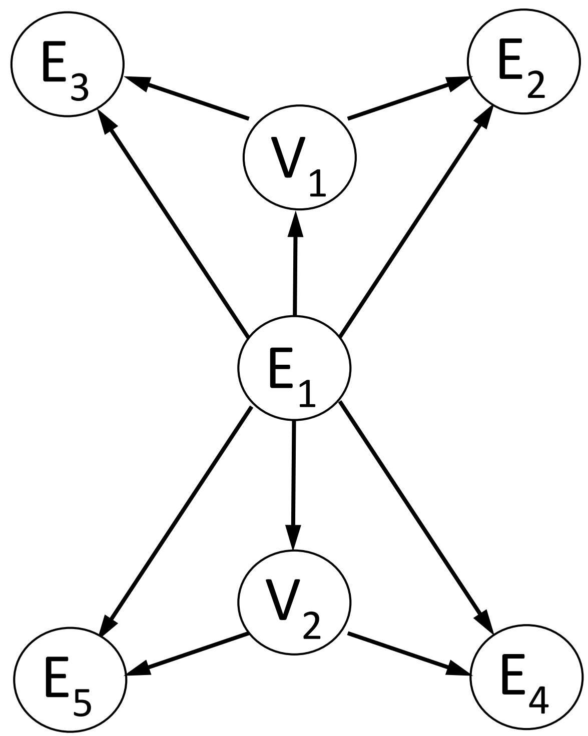
- The figure shows a single edge, ending in two vertices, which then each has two other edges leading off from them.
- The edge might, for example, be an edge of a cuboid.
- This is how the edge might be represented in the computer.



Implementation of B-Rep (4)

- The edge has *pointers* to the vertices at its ends, and to the next edges. A pointer is essentially the address in the computer's memory where something (data) is stored.
- The vertices have pointers to their coordinates (*x, y, z*) and so on.
- This structure is called *Baugmart's winged edge* data structure, named after its inventor (Brace Guenther Baugmart). The 'winged edge' phrase refers to the edge and its adjoining faces, which look like a dihedral wing.

Implementation of B-Rep (5)



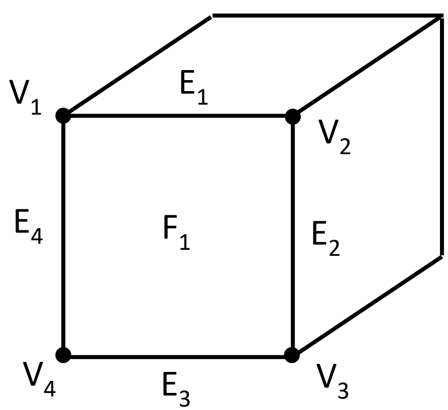
Baugmart's winged edge data structure

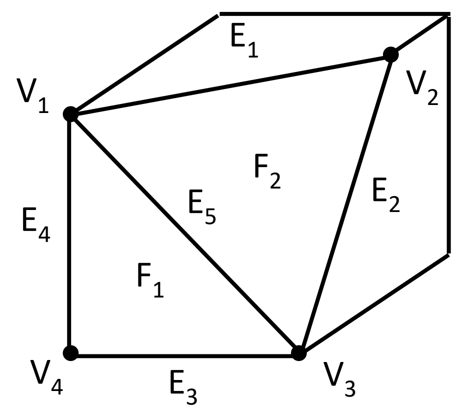
Implementation of B-Rep (6)

- Many B-Rep modelling systems have procedures called Euler-operators.
- These operators modify the <u>face-edge-vertex</u> pointer structure in such a way that the Euler formula is always kept true.

Implementation of B-Rep (7)

- An example: *make_edge_and_face(f1, v1, v3)*.
  - This would take the cuboid on the left, and split the face into two along its diagonal, making a new object.
  - Note that in the original cuboid, v1, v2, v3 and v4 must all lie in the same plane, but in the new object v2 is free to move along the top-right edge.





B-Rep Geometric Modellers (1)

- **Romulus** (modeling kernel), known as Romulus (b-rep solid modeler), Romulus (disambiguation), Romulus b-rep
- The Romulus b-rep solid modeler (or simply Romulus) was released in 1978 by **Ian Braid**, **Charles Lang**, **Alan Grayer**, and the Shape Data team in Cambridge, England.
- It was the first commercial solid modeling kernel designed for straightforward integration into CAD software.

https://www.semanticscholar.org/topic/Romulus-(modelling-kernel)/589274

B-Rep Geometric Modellers (2)

- **Romulus** incorporated the CAM-I AIS (Computer Aided Manufacturers International's Application Interface Specification) and was the only solid modeler (other than its successors Parasolid and ACIS) ever to offer a third-party standard API to facilitate high-level integration into a host CAD software program.
- It was quickly licensed by Siemens, HP and several other CAD software vendors.

https://www.semanticscholar.org/topic/Romulus-(modelling-kernel)/589274

B-Rep Geometric Modellers (3)

- **ACIS**, known as .sab, **A**lan, **C**harles and **I**an's **S**ystem (disambiguation)

- The 3D ACIS Modeler (ACIS) is a geometric modeling kernel developed by Spatial Corporation (formerly Spatial Technology), part of Dassault Systems.

SPATIAL (https://www.spatial.com/)

https://www.semanticscholar.org/topic/3D-ACIS-Modeler/3712130

B-Rep Geometric Modellers (4)

- **ACIS** is used by many software developers in industries such as computer-aided design (CAD), computer-aided manufacturing (CAM), computer-aided engineering (CAE), architecture, engineering and construction (AEC), coordinate-measuring machine (CMM), 3D animation and shipbuilding.

- ACIS provides software developers and manufacturers with the underlying 3D modeling functionality.

https://www.semanticscholar.org/topic/3D-ACIS-Modeler/3712130

Summary

- **3D modelling techniques**
  - Wireframe
  - Surface
  - Solid
- **Constructive solid geometry (CSG)**
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- **Boundary representation (B-Rep)**
  - Geometry (shape and size) and topology (connectivity)
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  - Implementation of B-Rep models
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