

# INT201 Decision, Computation and Language

Tutorial 5

Dr Yushi Li



Xi'an Jiaotong-Liverpool University

西交利物浦大學

1. Prove the following language is not regular

$$L = \{w \in \{a, b\}^* \mid w = w^R\}$$

2. Prove that if we add a finite set of strings to a regular language, the result is a regular language.

3. Convert the regular expression  $(00)^*(11)$  into an NFA.



## Solution

1.

**Answer:** Suppose that  $A_2$  is a regular language. Let  $p$  be the “pumping length” of the Pumping Lemma. Consider the string  $s = a^pba^p$ . Note that  $s \in A_2$  since  $s = s^R$ , and  $|s| = 2p + 1 \geq p$ , so the Pumping Lemma will hold. Thus, we can split the string  $s$  into 3 parts  $s = xyz$  satisfying the conditions

- i.  $xy^i z \in A_2$  for each  $i \geq 0$ ,
- ii.  $|y| > 0$ ,
- iii.  $|xy| \leq p$ .

Since the first  $p$  symbols of  $s$  are all  $a$ 's, the third condition implies that  $x$  and  $y$  consist only of  $a$ 's. So  $z$  will be the rest of the first set of  $a$ 's, followed by  $ba^p$ . The second condition states that  $|y| > 0$ , so  $y$  has at least one  $a$ . More precisely, we can then say that

$$\begin{aligned}x &= a^j \text{ for some } j \geq 0, \\y &= a^k \text{ for some } k \geq 1, \\z &= a^m ba^p \text{ for some } m \geq 0.\end{aligned}$$

Since  $a^pba^p = s = xyz = a^j a^k a^m ba^p = a^{j+k+m} ba^p$ , we must have that  $j + k + m = p$ . The first condition implies that  $xy^2z \in A_2$ , but

$$\begin{aligned}xy^2z &= a^j a^k a^k a^m ba^p \\&= a^{p+k} ba^p\end{aligned}$$

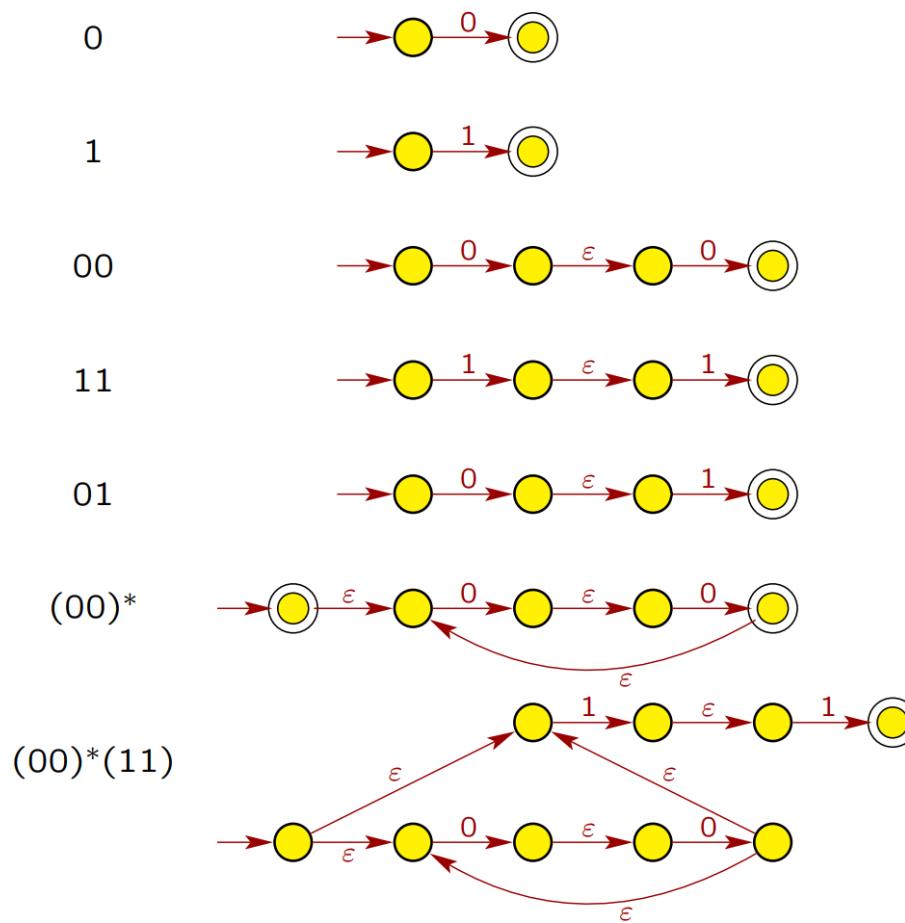
since  $j + k + m = p$ . Hence,  $xy^2z \notin A_2$  because  $(a^{p+k} ba^p)^R = a^p ba^{p+k} \neq a^{p+k} ba^p$  since  $k \geq 1$ , and we get a contradiction. Therefore,  $A_2$  is a nonregular language.

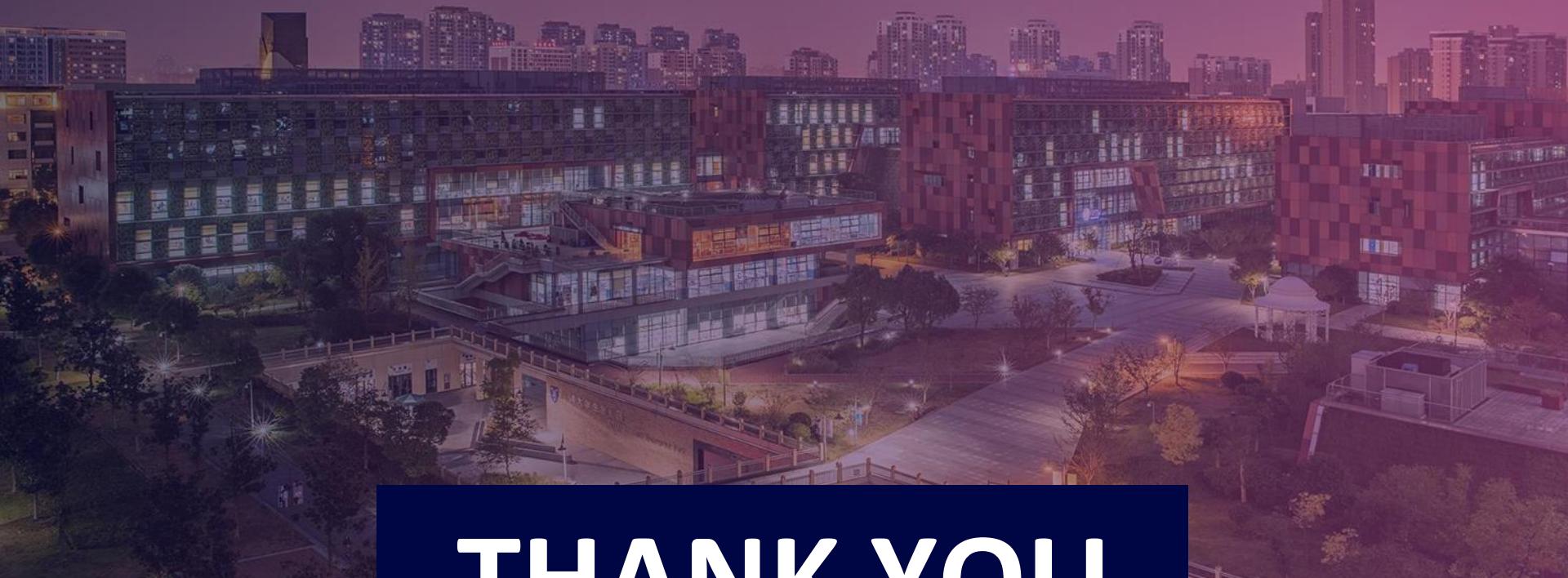


## Solution

2. Let A be a regular language, and let B be a finite set of strings. We know from class that finite languages are regular, so B is regular. Thus,  $A \cup B$  is regular since the class of regular languages is closed under union.

3.





# THANK YOU