INT201 Decision, Computation and Language

Tutorial 1

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Constructive proof

A method of proof that demonstrates the existence of a mathematical object (anything that has been formally defined) by creating or providing a method for creating the object.

Example

There exists an object with property $\mathcal{P}$.

Proof. Here is the object: [...]

And here is the proof that the object satisfies property $\mathcal{P}$: [...]

Constructive proof

Example

Theorem 1.3.5 *For every even integer $n \geq 4$, there exists a 3-regular graph with n vertices.*

Proof. Define

$$V = \{0, 1, 2, \ldots, n-1\},$$

and

$$E = \{\{i, i+1\} : 0 \leq i \leq n-2\} \cup \{\{n-1, 0\}\} \cup \{\{i, i+n/2\} : 0 \leq i \leq n/2-1\}.$$

Then the graph $G = (V, E)$ is 3-regular.

Convince yourself that this graph is indeed 3-regular. It may help to draw the graph for, say, $n = 8$.

Nonconstructive proof

In a nonconstructive proof, we show that a certain object exists, without actually creating it.

Example

Theorem 1.3.6 *There exist irrational numbers x and y such that x^y is rational.*

Proof. There are two possible cases.

Case 1: $\sqrt{2}^{\sqrt{2}} \in \mathbb{Q}$.

In this case, we take $x = y = \sqrt{2}$. In Theorem 1.3.9 below, we will prove that $\sqrt{2}$ is irrational.

Case 2: $\sqrt{2}^{\sqrt{2}} \notin \mathbb{Q}$.

In this case, we take $x = \sqrt{2}^{\sqrt{2}}$ and $y = \sqrt{2}$. Since

$$x^y = \left(\sqrt{2}^{\sqrt{2}}\right)^{\sqrt{2}} = \sqrt{2}^2 = 2,$$

the claim in the theorem follows.

Proof by Contradiction

A form of proof that establishes the truth or the validity of a proposition, by showing that assuming the proposition to be false leads to a contradiction

Example

Theorem 1.3.7 *Statement $\mathcal{S}$ is true.*

Proof. Assume that statement $\mathcal{S}$ is false. Then, derive a contradiction (such as $1 + 1 = 3$).

Example

Theorem 1.3.8 *Let n be a positive integer. If n^2 is even, then n is even.*

Proof. We will prove the theorem by contradiction. So we assume that n^2 is even, but n is odd. Since n is odd, we know from Theorem 1.3.1 that n^2 is odd. This is a contradiction, because we assumed that n^2 is even. ∎

Proof by Induction

A mathematical proof technique. It is essentially used to prove that a statement P(n) holds for every natural number n = 0, 1, 2, 3, ... ;

Example

For all positive integers n, we have

$$1 + 2 + 3 + \ldots + n = \frac{n(n+1)}{2}.$$

Proof by Induction

Base step: Choose $n = 1$. Then L.H.S =1. and R.H.S $= \frac{(1)(1+1)}{2} = 1$

Induction Assumption: Assume that $1 + 2 + \ldots + k = \dfrac{k(k+1)}{2}$, for $k \in \mathbb{Z}$.

We shall show that $1 + 2 + \ldots + k + (k+1) = \dfrac{(k+1)[(k+1)+1]}{2} = \dfrac{(k+1)(k+2)}{2}$

Consider $1 + 2 + \ldots + k + (k+1)$

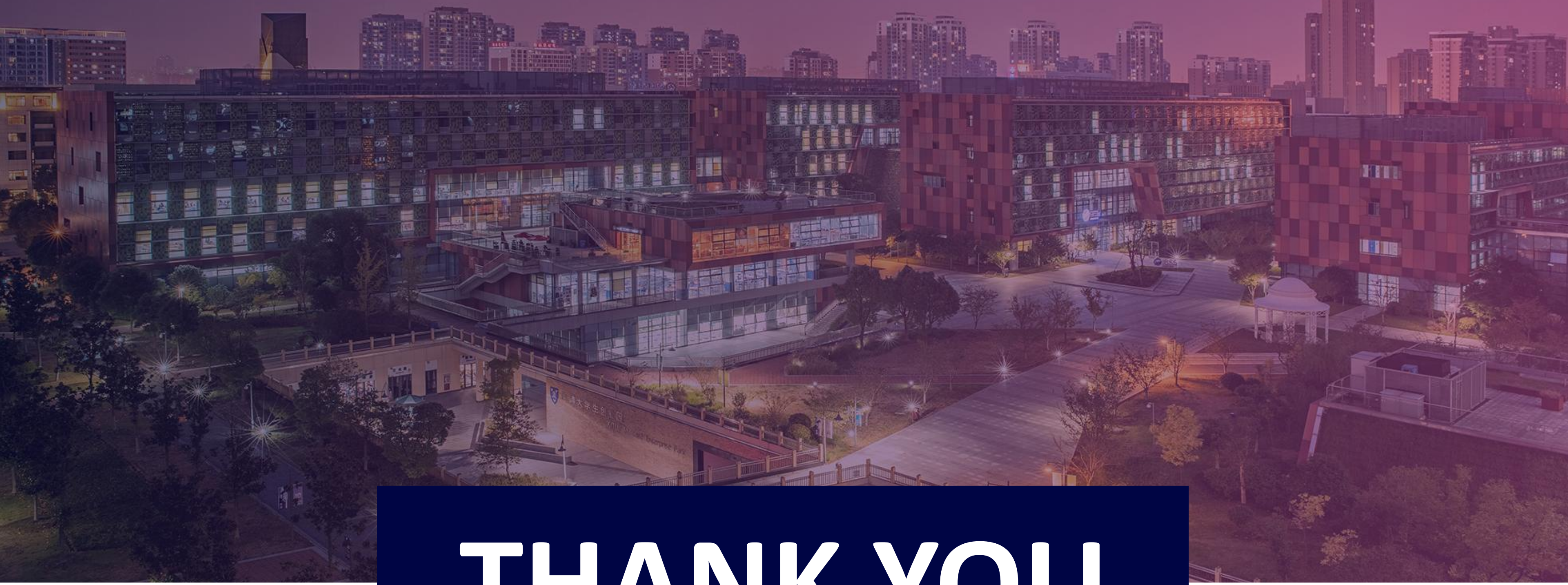
$$= \frac{k(k+1)}{2} + (k+1)$$

$$= (k+1)\left(\frac{k}{2} + \frac{1}{1}\right)$$

$$= (k+1)\left(\frac{k+2}{2}\right)$$

$$= \frac{(k+1)(k+2)}{2}.$$

Thus, by induction we have $1 + 2 + \ldots + n = \dfrac{n(n+1)}{2}$, $\forall n \in \mathbb{Z}$.



THANK YOU

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