# INT201 Decision, Computation and Language

**Tutorial 1** 

Dr Yushi Li



## **Constructive proof**

A method of proof that demonstrates the existence of a mathematical object (anything that has been formally defined) by creating or providing a method for creating the object.

#### **Example**

There exists an object with property  $\mathcal{P}$ .

**Proof.** Here is the object: [...] And here is the proof that the object satisfies property  $\mathcal{P}$ : [...]



## **Constructive proof**

## **Example**

**Theorem 1.3.5** For every even integer  $n \geq 4$ , there exists a 3-regular graph with n vertices.



## **Nonconstructive proof**

In a nonconstructive proof, we show that a certain object exists, without actually creating it.

#### **Example**

**Theorem 1.3.6** There exist irrational numbers x and y such that  $x^y$  is rational.



### **Proof by Contradiction**

A form of proof that establishes the truth or the validity of a proposition, by showing that assuming the proposition to be false leads to a contradiction

#### **Example**

Theorem 1.3.7 Statement S is true.

**Proof.** Assume that statement S is false. Then, derive a contradiction (such as 1 + 1 = 3).

#### **Example**

**Theorem 1.3.8** Let n be a positive integer. If  $n^2$  is even, then n is even.



#### **Proof by Induction**

A mathematical proof technique. It is essentially used to prove that a statement P(n) holds for every natural number n = 0, 1, 2, 3, ...;

#### **Example**

For all positive integers n, we have

$$1+2+3+\ldots+n=\frac{n(n+1)}{2}$$
.







