机械动力学大作业报告

一、问题背景

两自由度双摆模型如图所示，杆件 DE、GF 均质，质心为杆件中心。杆件 DE 和 GF 长度分别为 0.15m 和 0.1m，质量分别为 0.6kg 和 0.5kg。初始时刻，杆件 DE 与杆件 DF 均与底座 AB 平行，且角速度均为 0。C, D 关节的阻尼系数为 0.05N·m/(rad/s)。以 O 点为原点建立坐标系，C 点坐标为（0，0.5m）。

如图杆 1 为水平杆，杆 2 为竖直杆。

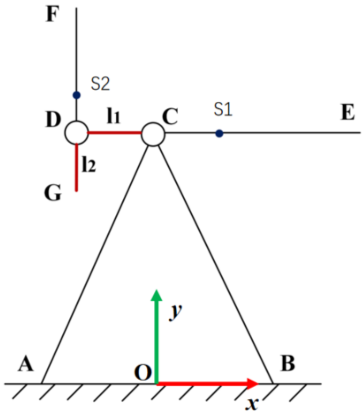


图 1−1 两自由度双摆模型

二、变量设置

杆 1— 长度：L1 质量：m1 质心位置：S1 绕质心转动惯量：J1 与x^+夹角:θ_1

杆 2— 长度：L2 质量：m2 质心位置：S2 绕质心转动惯量：J2 与x^+夹角:θ_2

C 点 阻尼系数：c1 D 点阻尼系数：c2

l1、l2 如图所示

重力加速度:$g = 9.8m/s^2$

将θ_1,θ_2 *设为广义坐标*

三、表达式推导

点 C 坐标：

$$C = [0; 0.5]$$

$$CS1 = \frac{1}{2} * L_1 - l_1$$

点S_1坐标：

$$S = C + CS1 * [\cos(\theta_1)\text{；}\sin(\theta_1)]$$

点 D 坐标：

$$D = C - l_1 * [\cos(\theta_1)\text{；}\sin(\theta_1)]$$

$$DS_2 = \frac{1}{2} * L_2 - l_2$$

点S_2坐标：

$$S_2 = D + DS_2 * [\cos(\theta_2)\text{；}\sin(\theta_2)]$$

点S_1速度：

$$V_1 = \frac{\partial S_1}{\partial \theta_1} * \omega_1 + \frac{\partial S_1}{\partial \theta_2} * \omega_2$$

点S_2速度：

$$V_2 = \frac{\partial S_2}{\partial \theta_1} * \omega_1 + \frac{\partial S_2}{\partial \theta_2} * \omega_2$$

杆角速度：

$$\omega_1 = \frac{\partial \theta_1}{\partial t} \quad \omega_2 = \frac{\partial \theta_2}{\partial t}$$

总动能：

$$E_k = \frac{1}{2} * V_1^T * m_1 * V_1 + \frac{1}{2} * V_2^T * m_2 * V_2 + \frac{1}{2} * J_1^T * \omega_1 * J_1 + \frac{1}{2} * J_2^T * \omega_2 * J_2$$

总势能：

$$E_p = m_1 * S_1(2) * g + m_2 * S_2(2) * g$$

E_k对$\theta_1, \theta_2, \omega_1, \omega_2$的各阶求导:

$$E_{k\_1} = \frac{\partial E_k}{\partial \theta_1} \quad E_{k\_2} = \frac{\partial E_k}{\partial \theta_2} \quad E_{k\_11} = \frac{\partial E_k}{\partial \omega_1} \quad E_{k\_22} = \frac{\partial E_k}{\partial \omega_2}$$

$$E_{k\_1\_1} = \frac{\partial^2 E_k}{\partial \theta_1 \partial \theta_1} \quad E_{k\_1\_2} = \frac{\partial^2 E_k}{\partial \theta_1 \partial \theta_2} \quad E_{k\_1\_11} = \frac{\partial^2 E_k}{\partial \theta_1 \partial \omega_1} \quad E_{k\_1\_22} = \frac{\partial^2 E_k}{\partial \theta_1 \partial \omega_2}$$

$$E_{k\_2\_2} = \frac{\partial^2 E_k}{\partial \theta_2 \partial \theta_2} \quad E_{k\_2\_11} = \frac{\partial^2 E_k}{\partial \theta_2 \partial \omega_1} \quad E_{k\_2\_22} = \frac{\partial^2 E_k}{\partial \theta_2 \partial \omega_2}$$

$$E_{k\_11\_11} = \frac{\partial^2 E_k}{\partial \omega_1 \partial \omega_1} \quad E_{k\_11\_22} = \frac{\partial^2 E_k}{\partial \omega_1 \partial \omega_2} \quad E_{k\_22\_22} = \frac{\partial^2 E_k}{\partial \omega_2 \partial \omega_2}$$

E_p对θ_1, θ_2求导：

$$E_{p\_1} = \frac{\partial E_p}{\partial \theta_1} \quad E_{p\_2} = \frac{\partial E_p}{\partial \theta_2}$$

点S_1对θ_1, θ_2求导：

$$S_1 y = S_1(2)$$

$$S_2 y = S_2(2)$$

$$S_1 y\_1 = \frac{\partial S_1 y}{\partial \theta_1}$$

$$S_1 y\_2 = \frac{\partial S_1 y}{\partial \theta_2}$$

$$S_2 y\_1 = \frac{\partial S_2 y}{\partial \theta_1}$$

$$S_2y\_2 = \frac{\partial S_2 y}{\partial \theta_2}$$

广义力:

$$F_1 = -c_1 * \omega_1 - c_2 * (\omega_1 - \omega_2) - m_1 * g * S_1y\_1 - m_2 * g * S_2y\_1$$
$$F_2 = -c_2 * (\omega_2 - \omega_1) - m_1 * g * S_1y\_2 - m_2 * g * S_1y\_2$$

由拉格朗日方程有：

$$\frac{d}{dt}\left(\frac{\partial E_k}{\partial \dot{q}_i}\right) - \frac{\partial E_k}{\partial \dot{q}} + \frac{\partial E_p}{\partial q_i} = F_i$$

带入两个广义坐标得到：

$$\begin{cases} \frac{\partial^2 E_k}{\partial \theta_1 \partial \omega_1} * \omega_1 + \frac{\partial^2 E_k}{\partial \theta_2 \partial \omega_1} * \omega_2 + \frac{\partial^2 E_k}{\partial \omega_1 \partial \omega_1} * \alpha_1 + \frac{\partial^2 E_k}{\partial \omega_1 \partial \omega_2} \alpha_2 - \frac{\partial E_k}{\partial \theta_1} + \frac{\partial E_p}{\partial \theta_1} = F_1 \\ \frac{\partial^2 E_k}{\partial \theta_1 \partial \omega_2} * \omega_1 + \frac{\partial^2 E_k}{\partial \theta_2 \partial \omega_2} * \omega_2 + \frac{\partial^2 E_k}{\partial \omega_2 \partial \omega_1} * \alpha_1 + \frac{\partial^2 E_k}{\partial \omega_2 \partial \omega_2} \alpha_2 - \frac{\partial E_k}{\partial \theta_2} + \frac{\partial E_p}{\partial \theta_2} = F_2 \end{cases}$$

由此得：

$$A = [\frac{\partial^2 E_k}{\partial \omega_1 \partial \omega_1}, \frac{\partial^2 E_k}{\partial \omega_1 \partial \omega_2}; \frac{\partial^2 E_k}{\partial \omega_1 \partial \omega_2}, \frac{\partial^2 E_k}{\partial \omega_2 \partial \omega_2}]$$

$$B = [F_1 + \frac{\partial E_k}{\partial \theta_1} - \frac{\partial E_p}{\partial \theta_1} - \frac{\partial^2 E_k}{\partial \theta_1 \partial \omega_1} * \omega_1 - \frac{\partial^2 E_k}{\partial \theta_2 \partial \omega_1} * \omega_2;$$
$$F_2 + \frac{\partial E_k}{\partial \theta_2} - \frac{\partial E_p}{\partial \theta_2} - \frac{\partial^2 E_k}{\partial \theta_1 \partial \omega_2} * \omega_1 - \frac{\partial^2 E_k}{\partial \theta_2 \partial \omega_2} * \omega_2]$$

$$\alpha = A \backslash B$$

$$\alpha_1 = \alpha(1) = fun(\theta_1, \theta_2, \omega_1, \omega_2); \alpha_2 = \alpha(2) = fun(\theta_1, \theta_2, \omega_1, \omega_2)$$

可直接带入 ode45 求解。

最小机械能为：

$$E_{min} = \frac{539}{100} - \frac{49\left|\frac{11\, l_1}{10} - \frac{9}{200}\right|}{5} - \frac{49\left|\frac{l_2}{2} - \frac{1}{40}\right|}{5}$$

设置终止条件：

$$E_p + E_k - 1.001E_{min} < 0$$

四、第一问

将$l_1 = 0.05$ $l_2 = 0.025$ $c_1 = 0.05$ $c_2 = 0.025$带入方程

仿真视频见附件视频文件。

得到 E、F 两点位置、速度、加速度随时间变化图像如下：

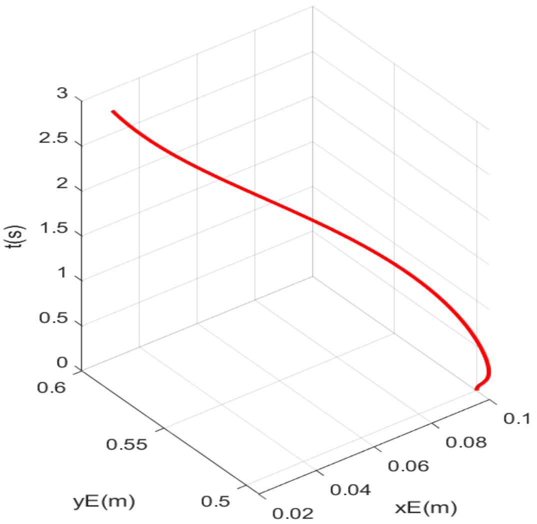


图 4-1 E 点位置随时间变化

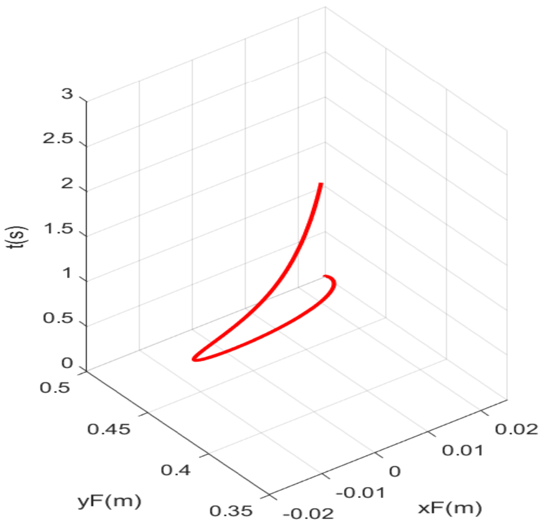


图 4-2 F 点位置随时间变化

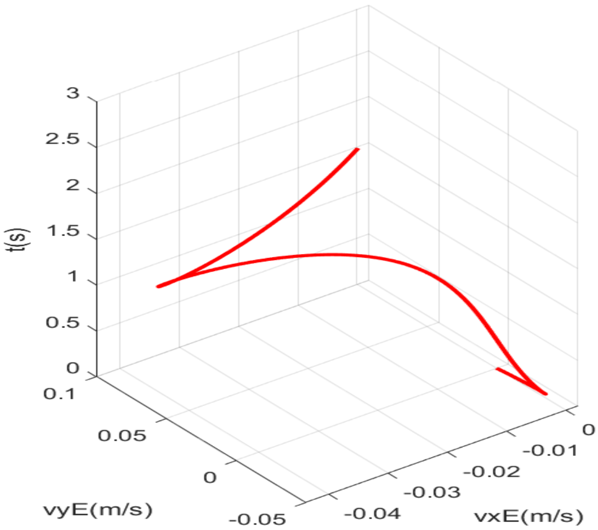


图 4-1 E 点速度随时间变化

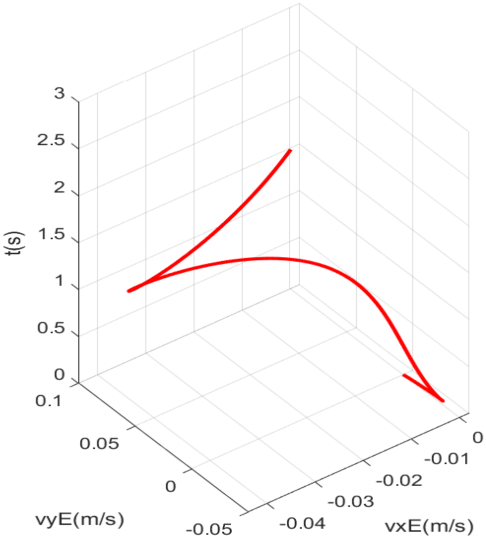


图 4-2 F 点速度随时间变化

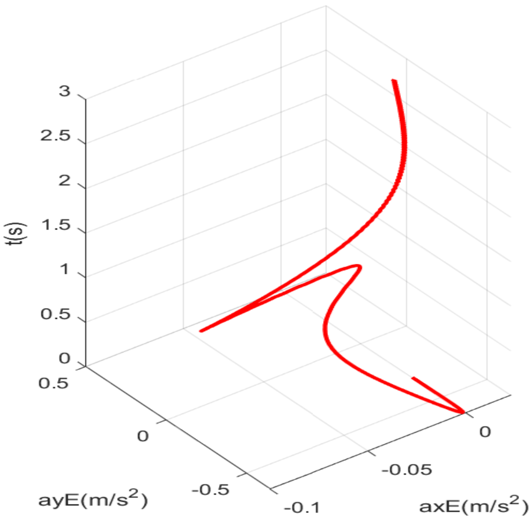


图 4-1 E 点加速度随时间变化

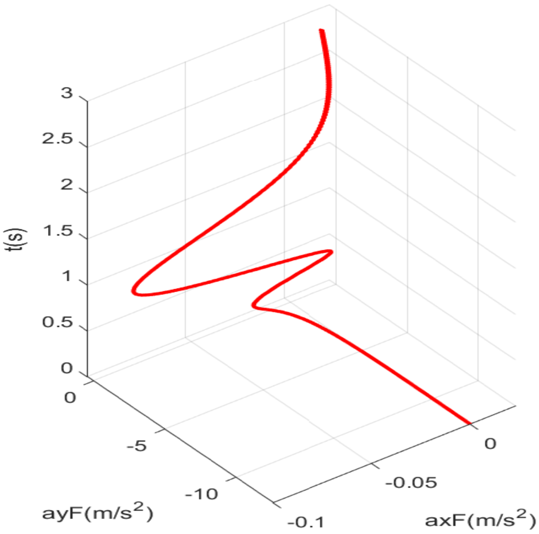


图 4-2 F 点加速度随时间变化

五、第二问

5.1

改变l_1、l_2的长度，得到双摆摆动时间随l_1、l_2的长度变化的改变图像如下：

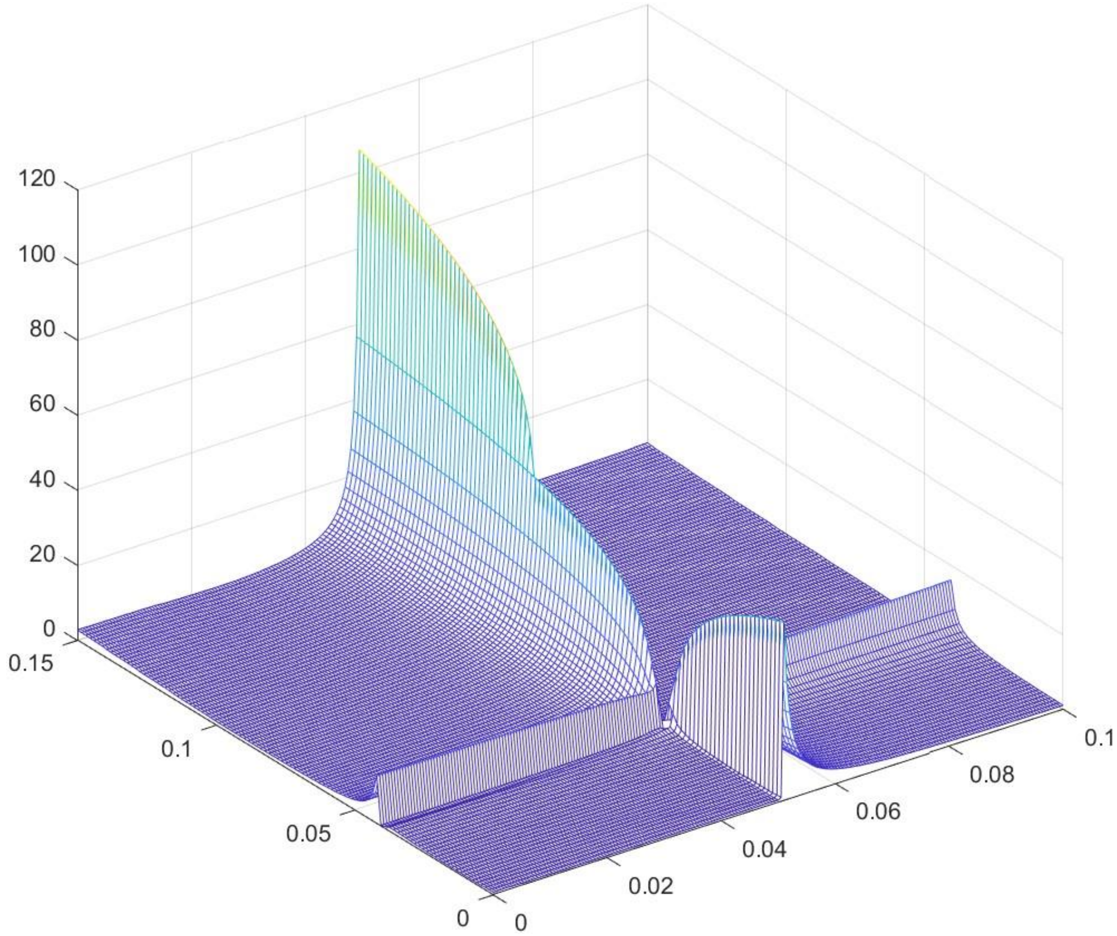


图 5-1、双摆摆动时间随l_1、l_2的长度变化改变图像

其中$l_1 = linspace(0, L_1, 101)$ $l_2 = linspace(0, L_2, 201)$

通过图像可以发现$l_1 = 0.15m; l_2 \in (0.045, 0.052)$时时间取到最大值。

令$l_1 = 0.15$，$l_2 = linspace(0.045, 0.052, 10000)$

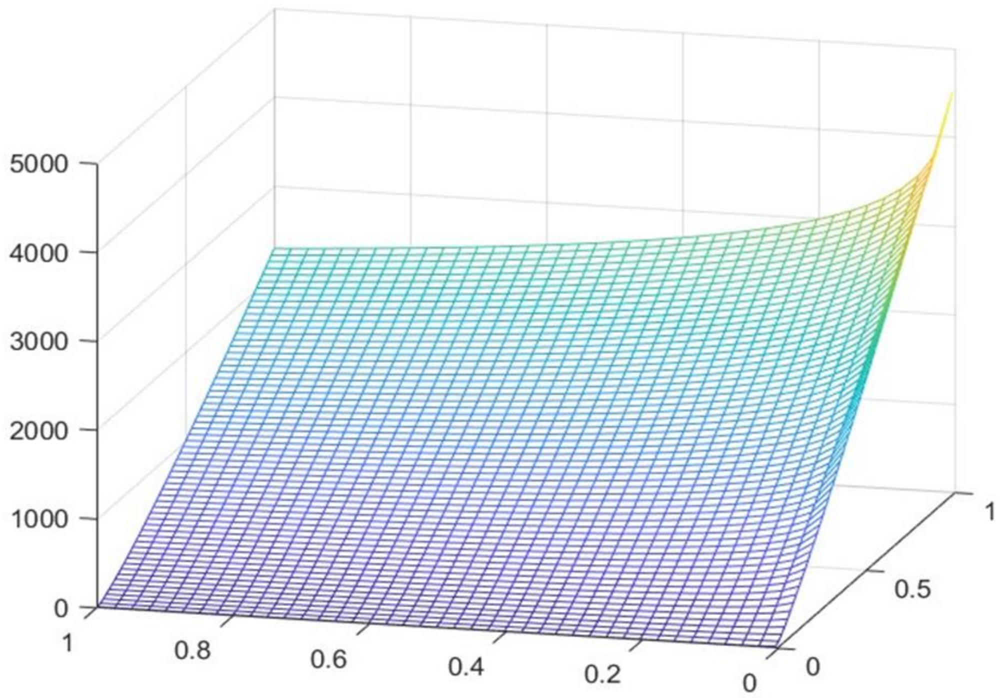
找到：$l_1 = 0.15$ $l_2 = 0.0495309$ 时取最大值。

5.2

固定$l_1 = 0.15$ $l_2 = 0.0495309$，改变c_1、c_2值，观察不同阻尼系数对摆动时间的影响规律。

令$c_1 = linspace(0.005, 1, 41)$ $c_2 = linspace(0.005, 1, 61)$

获得图像如下



发现随c_2提升，停止时间大幅上升，而随c_1提升，停止时间会出现下降的情况。