

## Lab 06 - Dijkstra's algorithm

- find the  $\Theta$  complexity for your algorithm
- $f(n)$  is  $O(g(n))$  iff there exist positive constants  $c, k$  such that  $f(n) \leq c \cdot g(n)$  for all  $n \geq k$
- $f(n)$  is  $\Theta(g(n))$  iff there exists positive constants  $c, d, k$  such that  $c \cdot g(n) \leq f(n) \leq d \cdot g(n)$  for all  $n \geq k$  (both  $O(g(n))$  and  $\Omega(g(n))$ )
- $f(n)$  is  $\Omega(g(n))$  iff there exist positive constants  $d, k$  such that  $d \cdot g(n) \leq f(n)$  for all  $n \geq k$

$$\text{Let } f(n) = 32n^2 + 3n$$

Prove  $f(n)$  is  $O(n^2)$

$$32n^2 + 3n \leq c \cdot n^2$$

$$\text{let } c = 32$$

$$32n^2 + 3n \leq 32n^2$$

$$3n \leq 0$$

$$n \geq 0$$

This relationship holds true for  $c=32$  and any positive  $k$  so  $f(n)$  is  $O(n^2)$ .

Prove  $f(n)$  is  $\Omega(n^2)$

$$\text{let } d = 16$$

$$32n^2 + 3n \geq 16n^2$$

$$16n^2 + 3n \geq 0$$

$$16n^2 \geq -3n \quad n \text{ has to be positive}$$

$$16n^2 \geq 3n \quad \text{so we can divide}$$

$$16n \geq 3$$

$$n \geq \frac{3}{16}$$

This relationship holds true for  $d=16$  and  $k \geq \frac{3}{16}$  so  $f(n)$  is  $\Omega(n^2)$

Therefore, because  $f(n)$  is both  $O(n^2)$  and  $\Omega(n^2)$  it is also  $\Theta(n^2)$ .