assignment 2 Part 2

- Prove that if two graphs A and B are isomorphic they do not need to be completely connected
  - Isomorphic: Same structure $G_1 = (U_1, E_1)$ is isomorphic to $G_2 = (U_2, E_2)$ if there is a one-to-one and onto function $f: U_1 \rightarrow V_2$ such that $(u, U_1) \in E_1$ iff $(f(u), f(U)) \in E_2$
  - Must have same degree sequence proven with handshaking theorem
    - for any graph G we have:

$$\sum_{v \in V(G)} deg(v) = 2|E(G)|$$

where $|E(G)|$ is edges in G

- Every edge has two endpoints and the sum of any degree sequence is even, where the graph will have an even number of verticies and an odd degree.
- for graphs to be isomorphic they must both be connected or disconnected.

Prove that two graphs A and B are isomorphic and dont need to be completely connected

let $A = (U_1, E_1)$ and $B = (U_2, E_2)$ where $U_1 = U_2 = n$

for A, $\sum_{v \in V(A)} deg(v) = 2|E_1(A)| = \{d_1, d_2, d_3, d_4, \ldots d_{n-1}\} = 2(E_1)$

for B, $\sum_{v \in V_2(B)} deg(v) = 2|E_2(B)| = \{d_1, d_2, d_3, d_4, \ldots d_{n-1}\} = 2(E_2)$

we can show $2|E_1(A)| = 2|E_2(B)|$

$2(E_1) = 2(E_2)$

$E_1 = E_2$

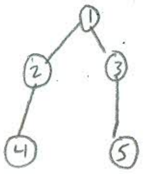
therefore, if $E_1 = E_2$ and $U_1 = U_2 = n$ graphs $A(U_1, E_1)$ and $B(U_2, E_2)$ must be isomorphic

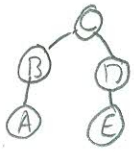
if $U_1 = \{1, 2, 3, 4, 5\}$ and $E_1 = \{(1,2), (1,3), (2,4), (3,5)\}$

and $U_2 = \{A, B, C, D, E\}$ and $E_2 = \{(A,B), (B,C), (C,D), (D,E)\}$

Then $\sum_{v \in U_1(A)} deg(v) = 2|E_1| = \sum_{v \in U_2(B)} deg(v) = 2|E_2|$ where $|E_1| = |E_2| = 4$

and $A(U_1, E_1)$ can be represented as with $B(U_2, E_2)$ being represented as





for graphs A or B to be completely connected $\sum_{v \in V} deg(v) = 2|E_0|$ where $|E_0| = \frac{n(n-1)}{2}$,

with $V_1 = V_2 = n = 5$ we get $deg(V) = 2\left|\frac{n(n-1)}{2}\right| = 2\left|\frac{5(5-1)}{2}\right| = 2(10) = 20$

Therefore because graphs A and be are isomorphic and $E_0 \neq E_1$ and $E_0 \neq E_2$ neither graph A or B is connected, so two graphs do not have to be connected completely to be isomorphic. ■