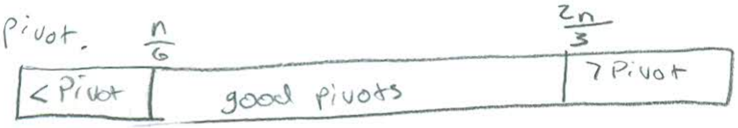
Assignment 2 part 4

- Median of 3 is it more, less, or equally likely to find a better pivot than talking the first element?

Firstly, almost any form of picking a pivot will be better than picking the first element. This is because when you pick a stationary pivot like the first element you have a higher chance of getting a runtime of $O(n^2)$ as the worst case.
With a probability $\frac{1}{3}$ of picking a good pivot we can expect that every three tries will pick a good pivot.

for any $i = 1, 2, 3, 4 \ldots n$ our chosen pivot is equally likely to be $1, \frac{n}{2}, n-1$ with a probability $\frac{1}{3}$, our pivot will be from the middle $\frac{n}{2}$ elements - a good pivot.



any good pivot will create two partitions of size at most $\frac{2n}{3}$, and we can expect to pick a good pivot every three tries.
if we split the array between the bottom sixth and top two thirds we increase our chances of picking a mid pivot, or a good pivot.
if we continue to pick pivots between $\frac{n}{6}$ and $\frac{2n}{3}$ at most we would have to pivot at most $\log_{3/2} n$ times down the longest array created from our pivot. run through this n times and we get an outer loop running n, with an inner loop running $\log n$ thus giving us $\Theta(n \cdot \log n)$.

While we found the same $\Theta(n \cdot \log n)$ for median-of-three as we did for just picking leftmost element the original question was whether or not I thought median-of-three had higher probability of picking a good pivot.

As previously stated, increasing our range of good pivot options from $50\% \left(\frac{3n}{4} - \frac{n}{4}\right)$ with leftmost, to $65\% \left(\frac{2n}{3} - \frac{n}{6}\right)$ with median_of-three increases our probability of picking a good pivot. Therefore, median-of-three is more likely to pick a good pivot than simply picking the leftmost element.