Assignment 2 Part 2

- Prove that if two graphs A and B have the same number of nodes and are Completely connected, they must be isomorphic.
  - Isomorphic: Same structure $G_1 = (V_1, E_1)$ is isomorphic to $G_2 = (V_2, E_2)$ if there is a one-to-one and onto fuction $f: V_1 \to V_2$ such that $(u,v) \in E_1$ iff $(f(u), f(v)) \in E_2$
    - $f: V_1 \to V_2$ $V_1 = V_2$ and $E_1 = E_2$
  - Completely connected $V = n$ $|E| = \frac{n(n-1)}{2}$
  - Degree $- \sum_{v \in V} deg(v) = 2|E|$

- (Direct proof) show that if $A = (V_1, E_1)$ and $B = (V_2, E_2)$ then $V_1 = V_2$ and $E_1 = E_2$ with $deg(V_1) = deg(V_2)$ and thus satisfying $f: V_1 \to V_2$ such that $(u,v) \in E_1$ iff $(f(u), f(v)) \in E_2$

Let $A = (V_1, E_1)$ and $V_1 = \{1, 2, 3, 4\}$ and $E_1 = \{(1,2), (1,3), (1,4), (2,4), (2,3), (3,4)\}$

Let $B = (V_2, E_2)$ and $V_2 = \{A, B, C, D\}$ and $E_2 = \{(A,B), (A,C), (A,D), (B,D), (B,C), (C,D)\}$

so, $V_1 = V_2 = n$ where $n = 4$ with $|E_1| = 6$ and $|E_2| = 6$

$\therefore$ thus $\sum_{v \in V_1} deg(v) = 2|E_1| = \sum_{v \in V_2} deg(v) = 2|E_2|$

$$2|E_1| = 2|E_2|$$
$$2(6) = 2(6)$$
$$12 = 12$$

So, the degree of both E_1 and E_2 is 12 and we know that for a complete graph $\sum_{v \in V} deg(v) = 2|E|$ where $E = \frac{n(n-1)}{2}$

for $n=4$ $|E| = \frac{4(4-1)}{2} = 6$

so, $\sum_{v \in V} deg(v) = 2|E| = 2(6) = 12$

therefore, with $V_1 = V_2$ and $E_1 = E_2$ the statement $f: V_1 \to V_2$ such that $(u,v) \in E_1$ iff $(f(u), f(v)) \in E_2$, must be true. This shows that A and B are isomorphic while the statement $\sum_{v \in V_1} deg(v) = 12$ and $\sum_{v \in V_2} deg(v) = 12$ shows, A and B have the same degree as a complete graph, given $n=4$. Thus A and B are isomorphic and complete.