№2

1. $y = x^2z^4 - x^3z^3 + x^4z^2$

$\partial y = \frac{\partial y}{\partial x}dx + \frac{\partial y}{\partial z}dz$

$\frac{\partial y}{\partial x} = 2xz^4 - 3x^2z^3 + 4x^3z^2$

$\frac{\partial y}{\partial z} = 4x^2z^3 + 3x^3z^2 + 2x^4z$

$dy = (2xz^4 - 3x^2z^3 + 4x^3z^2)dx + (4x^2z^3 - 3x^3z^2 + 2x^4z)dz$

2. $u = \frac{y+z}{z-y}$

$\frac{\partial u}{\partial y} = \frac{1\cdot(z-y)-(y+z)(-1)}{(z-y)^2} = \frac{z-y+y+z}{(z-y)^2}$

$= \frac{2z}{(z-y)^2}$

$\frac{\partial u}{\partial z} = \frac{1\cdot(x-y)-(y+z)(1)}{(z-y)^2} = \frac{z-y-y-z}{(z-y)^2} =$

$\frac{-2y}{(x-y)^2}$; $du = \frac{2z}{(z-y)^2}dy - \frac{2y}{(z-y)^2}dz$

N3

1. $z = \frac{1}{1-x-y+xy}$

$1-x-y+xy = (1-x)(1-y) \Rightarrow z = \frac{1}{(1-x)(1-y)}$

$\frac{dz}{dx} = \frac{1}{(1-y)} \cdot \frac{1}{(1-x)^2} = \frac{1}{(1-x)^2(1-y)}$

$\frac{d^2z}{dx^2} = \frac{2}{(1-x)^3(1-y)}$

2. $v = \sin(t^2+s^2)$

$\frac{\partial v}{\partial s} = \cos(t^2+s^2) \cdot 2s$

$\frac{\partial^2 v}{\partial s^2} = -\sin(t^2+s^2) \cdot (2s)^2 + \cos(t^2+s^2) \cdot 2$

$= -4s^2 \sin(t^2+s^2) + 2\cos(t^2+s^2)$

3. $f = \ln(1-z)\ln(1-y)$

$\frac{\partial f}{\partial z} = -\frac{\ln(1-y)}{1-z}$

Домашная работа №1

1. $z = x - y$

$\frac{\partial z}{\partial x} = 1, \ \frac{\partial z}{\partial y} = -1$

2. $z = \frac{u}{v} + \frac{v}{u}$

$\frac{\partial z}{\partial u} = \frac{1}{v} - \frac{v}{u^2}, \ \frac{\partial z}{\partial v} = -\frac{u}{v^2} + \frac{1}{u}$

3. $z = xyt$

$\frac{\partial z}{\partial x} = yt, \ \frac{\partial z}{\partial y} = xt, \ \frac{\partial z}{\partial t} = xy$

4. $z = \ln(x^2 + y^2)$

$\frac{\partial z}{\partial x} = \frac{2x}{x^2 + y^2}, \ \frac{\partial z}{\partial y} = \frac{2y}{x^2 + y^2}$

5. $z = \sqrt{x^2 + y^2 + t^2}$

$\frac{\partial z}{\partial x} = \frac{x}{\sqrt{x^2 + y^2 + t^2}}, \ \frac{\partial z}{\partial y} = \frac{y}{\sqrt{x^2 + y^2 + t^2}}, \ \frac{\partial z}{\partial t} =$

$= \frac{t}{\sqrt{x^2 + y^2 + t^2}}$

$$d^2z = \frac{\partial^2 z}{\partial x^2}dx^2 + 2\frac{\partial^2 z}{\partial x \partial y}dx\,dy + \frac{\partial^2 z}{\partial y^2}dy^2$$

$$\frac{\partial^2 z}{\partial x^2} = 6x, \quad \frac{\partial^2 z}{\partial y^2} = 12y, \quad \frac{\partial^2 z}{\partial x \partial y} = -1,$$

$$d^2z = 6x\,dx^2 - 2dx\,dy$$

№4

1. $z = x^3 + 2y^3 - xy$

$$dz = (3x^2 - y)\,dx + (6y^2 - x)\,dy$$

$$d^2z = \frac{\partial^2 z}{\partial x^2}dx^2 + 2\frac{\partial^2 z}{\partial x \partial y}dx\,dy + \frac{\partial^2 z}{\partial y^2}dy^2$$

$$\frac{\partial^2 z}{\partial x^2} = 6x, \quad \frac{\partial^2 z}{\partial y^2} = 12y, \quad \frac{\partial^2 z}{\partial x \partial y} = -1$$

$$d^2z = 6x\,dx^2 - 2dx\,dy + 12y\,dy^2$$

3. $z = y \ln x$

$$\frac{\partial z}{\partial x} = \frac{y}{x}, \quad \frac{\partial z}{\partial y} = \ln x$$

$$\frac{\partial z}{\partial x^2} = -\frac{y}{x^2}, \quad \frac{\partial^2 z}{\partial y^2} = 0, \quad \frac{\partial^2 z}{\partial x \partial y} = \frac{1}{x}$$

$$d^2 z = -\frac{y}{x^2} dx^2 + \frac{2}{x} dx\, dy$$

2. $z = \sin(xy)$

$$\frac{\partial z}{\partial x} = y \cos(xy), \quad \frac{\partial z}{\partial y} = x \cos(xy)$$

$$\frac{\partial^2 z}{\partial x^2} = -y^2 \sin(xy), \quad \frac{\partial^2 z}{\partial y^2} = -x^2 \sin(xy),$$

$$\frac{\partial^2 z}{\partial x \partial y} = \cos(xy) - xy \sin(xy)$$

$$d^2 z = -y^2 \sin(xy) dx^2 + 2 [\cos(xy) -$$
$$- xy \sin(xy)] dx\, dy - x^2 \sin(xy) dy^2$$

№5

$$\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$$

1. $f = x^6 + xy^2 - 5xy + y^5$

$f_x = 6x^5 + y^2 - 5y, \quad f_y = 2xy - 5x + 5y^4$

$f_{xy} = 2y - 5, \quad f_{yx} = 2y - 5$ равны

2. $f = \frac{2y + 3x}{x - 5y}$

$f_x = \frac{3(x - 5y) - (2y + 3x)(1)}{(x - 5y)^2} = \frac{3x - 15y - 2y - 3x}{(x - 5y)^2}$

$= \frac{-17y}{(x - 5y)^2}$

$f_y = \frac{2(x - 5y) - (2y + 3x)(-5)}{(x - 5y)^2} = \frac{2x - 10y + 10y + 15x}{(x - 5y)^2}$

$= \frac{17x}{(x - 5y)^2}$

$f_{xy} = \frac{\partial}{\partial y}\left(\frac{-17}{(x - 5y)^2}\right)$ производная частно

$= \frac{-17(x - 5y)^2 - (-17y) \cdot 2(x - 5y)(-5)}{(x - 5y)^4}$

$= \frac{-17(x - 5y)^2 - 170y(x - 5y)}{(x - 5y)^4} = \frac{-17x + 85y - 170y}{(x - 5y)^3}$

$$\frac{\partial^2 f}{\partial z \partial y} = -\frac{1}{1-z} \cdot \left(-\frac{1}{1-y}\right) = \frac{1}{(1-z)(1-y)}$$

4. $h = e^u \cos(uv)$

$$\frac{\partial h}{\partial v} = e^u \cdot (-\sin(uv)) \cdot u = -ue^u \sin(uv)$$

$$\frac{\partial^2 h}{\partial v^2} = -ue^u \cdot \cos(uv) \cdot u = -u^2 e^u \cos(uv)$$

$$\frac{\partial^3 h}{\partial^2 v \partial u} = \frac{\partial}{\partial u}\left(-u^2 e^u \cos(uv)\right)$$

производная по u:

$$= -\left[\left(2ue^u + u^2 e^u\right)\cos(uv) + \left(-u^2 e^u \cdot (-v \sin(uv))\right)\right]$$

$$= -\left[(2u + u^2) e^u \cos(uv) + u^2 v e^u \sin(uv)\right]$$

$$= -e^u \left[(2u + u^2)\cos(uv) + u^2 v e^u \sin(uv)\right]$$

№4

1. $z = x^3 + 2y^3 - xy$

$$dz = (3x^2 - y)\,dx + (6y^2 - x)\,dy$$

$$f_{yx} = \frac{\partial}{\partial x}\left(\frac{17x}{(x-5y)^2}\right) = \frac{17(x-5y)^2 - 17x\cdot 2(x-5y)}{(x-5y)^4}$$

$$= \frac{17(x-5y) - 34x}{(x-5y)^3} = \frac{17x - 85y - 34x}{(x-5y)^3} = \frac{-17x - 85y}{(x-5y)^3}$$

3. $f = e^{xy^2}$

$$f_x = y^2 e^{xy^2}, \quad f_y = 2xy e^{xy^2}$$

$$f_{xy} = 2ye^{xy^2} + y^2\cdot 2xye^{xy^2} = 2ye^{xy^2}(1+xy^2)$$

$$f_{yx} = 2ye^{xy^2} + 2xy\cdot y^2e^{xy^2} = 2ye^{xy^2}(1+xy^2)$$