Investigation of factors affecting the precession of a gyroscope

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Abstract

We propose to investigate factors that affect the dynamics of a precessing gyroscope. We believe that the gyroscopic motion is generally ill understood by current students. As a paradigm for demonstrating the relationship between torque and rate of change of angular momentum, it has almost no competitor. With a detailed study of this complex motion, we plan to bring out various aspect of this complex instrument that carry substantial pedagogical value and beg to be introduced into the curriculum. We will present this work at the National meeting of the Am. Assoc. of Physics Teachers during the academic year 2009-10.

A gyroscope consists of a spinning rotor, typically a disk or wheel with large moment of inertia that is free to rotate about a static or moving axis. Large angular momentum associated with this spinning mass provides resistance to any change in direction of the axis of rotation.

Gyroscopic motion, due to its complexity, is generally not covered in an undergraduate physics program and hence remains an enigma to all students. Study of a gyroscope, or at least its demonstration, used to be quite common in American universities in the beginning of the twentieth century.

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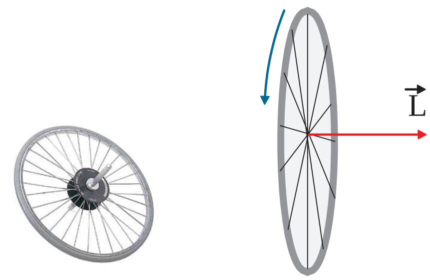
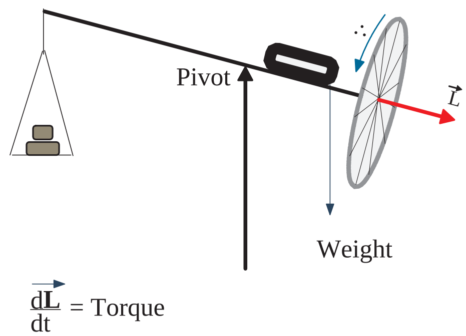


(a) (b)



(c)

Figure 1:

One of the very popular demonstration of gyroscopic motion in any introductory physics class is that of a bicycle wheel. If the bicycle wheel like the the one shown in Fig. (1-a) is spun and is hung from one end of the handle by a string, the plane of the wheel precesses around the string. We will like to study the various factors that affect this precession.

As shown in Fig. (1-b), we will build a gyroscope by suitably choosing a bar to form chases of our gyroscope. At one end of the bar we will attach a fly wheel that will be connected to a motor and a power supply. This bar will be pivoted on a very sharp edge to minimize friction. On the other end of the bar, we will attach a hanger to carry weight. The weight thus hung will be one of the variables whose effect we plan to study. We will also vary the pivot point and se how changing the point of suspension affects the rate of precession. Here we describe some of the basic formalism that we plan to use in this project. We expect some of this formalism to get refined as we progress in this project.

Theroetical Formalism:

The wheel attached, once spun, would represent the major part of the angular momentum $\vec{L}$ of the system. The strength of the motor will determine the angular speed Ω_w of the rotating wheel, and once combined with the moment if inertia I its angular momentum is given by $I\,\Omega_w$. Another variable that would affect the angular momentum is nature of the rotating object. We will work with objects made of different material and of varying shapes to gather data on their effect and to finally analyze it. The mass of the bar, especially if the pivot is off centered, and the mass of the wheel causes a torque on the system about the pivot point. In Fig. (1-c), the net weight of the system acts along the arrow denoted by the weight vector. This vector passes through the center of mass of the bar, motor wheel system.

According to an extension Newton's law of motion $\left(\overrightarrow{F} = m\vec{a}\right)$ for rotating objects, one has

$$\overrightarrow{\tau} = \frac{d\,\vec{L}}{dt} \ .$$

The torque caused by the weight is given by the product of the weight Mg, and its distance from the point of suspension. Thus,

$$Mg\ d\ \hat{e} = \frac{d\,\vec{L}}{dt} \ ,$$

where d is the distance of the weight vector from the pivot point and $\hat{e}$ is a unit vector that points in a direction perpendicular to the bar as well as the angular momentum $\vec{L}$ of the wheel depicted in Fig. (1-c).

Let us assume that the bar precesses with an angular frequency $\overrightarrow{\Omega}_P$. Thus, the angular velocity vector $\vec{L}$ also precesses with the same frequency $\overrightarrow{\Omega}_P$. In such case, the rate of change of the angular momentum is given by

$$\frac{d\,\overrightarrow{L}}{dt} = \overrightarrow{\Omega}_P \times \overrightarrow{L} \ .$$

This leads to

$$Mg\ d\ \hat{e} = \overrightarrow{\Omega}_P \times \overrightarrow{L} = I\,\overrightarrow{\Omega}_P \times \overrightarrow{\Omega}_w \ .$$

Extracting the magnitudes of both sides of the above equation, we get

$$Mg\ d = I\,\Omega_P \Omega_w \ ,$$

or

$$\Omega_P = \frac{Mg\ d}{I\,\Omega_w} \ .$$

Thus, the rate of precession of the bar is directly proportional to the distance d of the pivot point from the center of mass and inversely proportional to the angular speed of the wheel. We will verify this relationship in the laboratory by building a gyroscope of the type shown in (1-c).

In summary, gyroscopic motion used to be an integral part of education in early part of the twentieth century as is evident from models available at museums of universities around the world. We plan to carry out the above research and show the utility of the phenomenon of precession of a gyroscope in the understanding of the Newton's second law of motion for rotating objects, i.e., $\overrightarrow{\tau} = \frac{d\vec{L}}{dt}$.

Roles of faculty mentors:

1. To teach the students part of the mechanics pertaining to rotation of rigid bodies.

2. To help them procure material to build a gyroscope.

3. To guide them as they carry out scientific investigation of factors that affect the precession of a gyroscope.

Roles of the students: With the supervision of the faculty mentors to learn the mechanics of a complex rotating object. Build a gyroscope such that its position relative to the pivot can be varied and the torque on the bar carrying the gyroscope can be controlled. Study the change in precession as these two parameters (distance from the pivot and torque on the bar) are varied. Present this work and its pedagogical importance at a national meeting of AAPT and prepare a manuscript for publication.

Budget:

We expect to present our work at the Undergraduate Research Symposium at the national meeting of the American Association of Physics Teachers(AAPT).

1)	Purchase of equipment		
	Motors	$4\times$ $50	$200
	Material	Metal, Plexiglass	$200
	Registration and travel expenses	Cost per person	Total cost
2)	National AAPT meeting	$800	$1600
	Total budget	$1000	$2000