Trigonometrische Gleichungen

$$-8\sin^2\left(3x - \frac{\pi}{2}\right) + 2\sin\left(3x - \frac{\pi}{2}\right) = -3 \quad \Leftrightarrow \quad -8\sin^2\left(3x - \frac{\pi}{2}\right) + 2\sin\left(3x - \frac{\pi}{2}\right) + 3 = 0$$

$$Subs.: \sin\left(3x - \frac{\pi}{2}\right) = u; \quad \Rightarrow \quad -8u^2 + 2u + 3 = 0 \quad \Leftrightarrow \quad u^2 - \frac{1}{4}u - \frac{3}{8} = 0$$

$$u_{1/2} = \frac{1}{8} \pm \sqrt{\frac{1}{64} + \frac{24}{64}} = \frac{1}{8} \pm \sqrt{\frac{25}{64}} = \frac{1}{8} \pm \frac{5}{8}; \quad \Rightarrow \quad u_1 = \frac{1}{8} + \frac{5}{8} = \frac{6}{8} = \frac{3}{4}; \quad \& \quad u_2 = \frac{1}{8} - \frac{5}{8} = -\frac{4}{8} = -\frac{1}{2}$$

$$Rücksubs.: \sin\left(3x - \frac{\pi}{2}\right) = u \quad \Rightarrow \quad 3x - \frac{\pi}{2} = \sin^{-1}(u)$$

$u_1 = \dfrac{3}{4}$		$u_2 = -\dfrac{1}{2}$	
$\sin(x) = \sin(\pi - x)$		$\sin(x) = \sin(\pi - x)$	
$3x - \dfrac{\pi}{2} = \sin^{-1}\left(\dfrac{3}{4}\right)$ $3x = \sin^{-1}\left(\dfrac{3}{4}\right) + \dfrac{\pi}{2}$ $x = \dfrac{1}{3}\sin^{-1}\left(\dfrac{3}{4}\right) + \dfrac{\pi}{6}$	$\pi - \left(3x - \dfrac{\pi}{2}\right) = \sin^{-1}\left(\dfrac{3}{4}\right)$ $\dfrac{3}{2}\pi - 3x = \sin^{-1}\left(\dfrac{3}{4}\right)$ $\dfrac{3}{2}\pi - \sin^{-1}\left(\dfrac{3}{4}\right) = 3x$ $x = \dfrac{1}{2}\pi - \dfrac{1}{3}\sin^{-1}\left(\dfrac{3}{4}\right)$	$3x - \dfrac{\pi}{2} = \sin^{-1}\left(-\dfrac{1}{2}\right)$ $3x = \sin^{-1}\left(-\dfrac{1}{2}\right) + \dfrac{\pi}{2}$ $x^1 = \dfrac{1}{3}*\left(-\dfrac{1}{6}\pi\right) + \dfrac{\pi}{6}$ $x = \dfrac{1}{9}\pi$	$\pi - \left(3x - \dfrac{\pi}{2}\right) = \sin^{-1}\left(-\dfrac{1}{2}\right)$ $\dfrac{3}{2}\pi - 3x = -\dfrac{1}{6}\pi$ $\dfrac{5}{3}\pi = 3x$ $\dfrac{5}{3}\pi = x$
Hinzufügen der periodischen Lösungen: $\pm k * Periode; k \in \mathbb{N}; Periode\ p = \dfrac{2\pi}{b}$			
$x = \dfrac{1}{3}\sin^{-1}\left(\dfrac{3}{4}\right) + \dfrac{\pi}{6} \pm k * \dfrac{2}{3}\pi$	$x = \dfrac{1}{2}\pi - \dfrac{1}{3}\sin^{-1}\left(\dfrac{3}{4}\right) \pm k * \dfrac{2}{3}\pi$	$x = \dfrac{1}{9}\pi \pm k * \dfrac{2}{3}\pi$	$\dfrac{5}{3}\pi = x \pm k * \dfrac{2}{3}\pi$

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[1] $\sin^{-1}\left(-\dfrac{1}{2}\right) = -\dfrac{1}{6}\pi$