## Errata for the HoTT Book, first edition

## April 4, 2020

For the benefit of all readers, the available PDF and printed copies of the book are being updated on a rolling basis with minor corrections and clarifications as we receive them. Every copy has a version marker that can be found on the title page and is of the form "first-edition-XX-gYYYYYY", where XX is a natural number and YYYYYYYY is the git commit hash that uniquely identifies the exact version. Higher values of XX indicate more recent copies.

Below is a list of corrections and clarifications that have been made so far (except for trivial formatting and spacing changes), along with the version marker in which they were first made. This list is current as of April 4, 2020 and version marker "first-edition-1250-g3c57f03".

While the page numbering may differ between copies with different version markers (and indeed, already differs between the letter/A4 and printed/ebook copies with the same version marker), we promise that the numbering of chapters, sections, theorems, and equations will remain constant, and no new mathematical content will be added, unless and until there is a second edition.

| Location      | Fixed in Char | nge   |
|---------------|---------------|---|
| §1.1          | 182-gb29ea2f  | Change notation $a \equiv_A b$ to $a \equiv b : A$ , to match that used in  |
|               |               | Appendix A. (Neither are used anywhere else in the book.)   |
| §1.1          | 154-g42698c2  | Clarify that algorithmic decidability of judgmental equality is only meta-theoretic.                                      |
| §1.1          | 154-gac9b226  | Mention notation $a = b = c = d$ to mean " $a = b$ and $b = c$ and  |
|               |               | c = d, hence $a = d''$ , possibly including judgmental equalities.  |
| §1.3          | 42-g4bc5cc2   | Cumulativity means some elements do not have unique types,  |
|               |               | the index $i$ on $U_i$ is not an internal natural number, and typical ambiguity must be justified by reinserting indices. |
| §§1.3 and 1.4 | 42-ga34b313   | Explain that we can't define Fin and fmax yet where we first  |
| 00            | 8             | mention them.   |
| $\S 1.4$      | 165-g0ad2aba  | Add swap as another example of a polymorphic function, and  |
|               |               | discuss the use of subscripts and implicit arguments to depen-  |
|               |               | dent functions.   |
| Remark 1.5.1  | 80-g8f95fa5   | In the discussion of formation rules, the dependent function  |
|               |               | type example should be $\prod_{(x:A)} B(x)$ .   |
| §1.5          | 51-g67e86db   | Better explanation of recursion on product types, why it is jus-  |
|               |               | tified, and how it relates to the uniqueness principle.   |

| Location              | Fixed in Char               | nge   |
|-----------------------|-----------------------------|---|
| §1.6                  | 2-gbe277a8                  | In the types of $g$ and $\operatorname{ind}_{\sum_{(x:A)} B(x)}$ , there is a $\prod_{(a:A)} \prod_{(b:B(x))}$ in which $x$ should be $a$ .   |
| §1.6                  | 27-gd0bfa0d                 | At two places in the definition of ac, $R(a, pr_1(g(x)))$ should be   |
| §1.6                  | 125-g7fdadbf                | $R(x, \operatorname{pr}_1(g(x)))$ . When substituting $\lambda x$ . $\operatorname{pr}_1(g(x))$ for $f$ while verifying that ac is well-typed, the left side of the judgmental equality should be $\prod_{(x:A)} R(x, \operatorname{pr}_1(g(x)))$ , not $\prod_{(x:A)} R(x, \operatorname{pr}_1(f(x)))$ .                 |
| §1.7<br>Theorem 1.8.1 | 30-g264d934<br>391-g1ce619a | In two displayed equations, $f(inl(b))$ should be $f(inr(b))$ .<br>This should not be called a "Theorem", since we have not yet   |
| §1.8                  | 125-g433f87e                | introduced what that means. Instead it should say "We construct an element of". In the definition of binary products in terms of $2$ , the definitions of $\operatorname{pr}_1(p)$ and $\operatorname{pr}_2(p)$ should be switched to match the order of arguments to $\operatorname{rec}_2$ and $\operatorname{ind}_2$ . |
| §1.11                 | 111-g1e868fa                | When translating English to type theory, "unnamed variables" are unnamed in English but must be named in type theory.   |
| §1.12                 | 154-g4ef49f7                | Emphasize that path induction, like all other induction principles, defines a <i>specified</i> function.  |
| §1.12                 | 244-gd58529d                | In proof that path induction implies based path induction, $D(x,y,p)$ should be written $\prod_{(C:\prod_{(z:A)}(x=Az)\to\mathcal{U})}(\cdots)$ so the  |
| Remark 1.12.1         | 563-g3286941                | type of $C$ matches the premise of based path induction. The facts that any $(x,y,p): \sum_{(x,y:A)}(x=y)$ is equal to $(x,x,refl_x)$ , and that any $(y,p): \sum_{(y:A)}(a=_Ay)$ is equal to $(a,refl_a)$ , can be proven by path induction and based path induction respectively.                                       |
| Exercise 1.4          | 78-gcce4dc0                 | The second defining equation of iter should have right-hand side $c_s$ (iter( $C$ , $c_0$ , $c_s$ , $n$ )).   |
| Exercise 1.4          | 293-g4663bfe                | The defining equations of the recursor derived from the iterator only hold propositionally, and require the induction principle to prove.   |
| Exercise 1.6          | 229-ged891f3                | This exercise requires function extensionality (§2.9).  |
| Exercise 1.8          | 450-g7f38c9a                | This exercise requires symmetry and transitivity of equality, Lemmas 2.1.1 and 2.1.2.   |
| Exercise 1.10         | 110-gfe4641b                | To match the usual Ackermann–Péter function, the second displayed equation should be $ack(succ(m), 0) \equiv ack(m, 1)$ .   |
| Chapter 2             | 239-gaf3d682                | In the chapter introduction, clarify that topological homotopies between paths must be endpoint-preserving.   |
| Lemma 2.1.1           | 166-g37b78ef                | Add remarks before and after the proof about how a theorem's statement and proof should be interpreted as exhibiting an element of some type.   |
| Lemma 2.1.2           | 374-g0bc0908                | In the penultimate display in the first proof, $d(x,z,q)$ should be simply $d$ .  |

| Location              | Fixed in Chang               | ge   |
|-----------------------|------------------------------|--|
| Lemma 2.1.4           | 750-g91b7348                 | In the first proofs of (i)–(iii), $ind_{A}(D,d,p)$ should be $ind_{A}(D,d,x,y,p)$ .  |
| §2.1                  | 435-gee0b28a                 | In the third paragraph after Lemma 2.1.2, $p \cdot refl_x \equiv p$ should   |
| 0                     | 9                            | be $p \cdot refl_{y} \equiv p$ .   |
| § <b>2.</b> 1         | 165-g18642ca                 | Mention that the notation $a = b = c = d$ , and its displayed  |
| -                     | G                            | variant, indicate concatenation of paths.  |
| § <b>2.</b> 1         | 253-gdd47c75                 | Lemma 2.1.4(iv) justifies writing $p \cdot q \cdot r$ and so on.   |
| Theorem 2.1.6         | 253-gdd47c75                 | The induction defining $\alpha \cdot_{\mathbf{r}} r$ has defining equation $\alpha \cdot_{\mathbf{r}} \operatorname{refl}_b \equiv$  |
|                       |                              | $\operatorname{ru}_p^{-1} \cdot \alpha \cdot \operatorname{ru}_q$ , with $\operatorname{ru}_p$ the right unit law. For $\alpha \star \beta = \alpha \cdot \beta$ to        |
|                       |                              | be well-typed, we assume $p \equiv q \equiv r \equiv s \equiv refl_a$ and use  |
|                       |                              | $ru_{refl_a} = refl_{refl_a}$ and its dual. Proving $\alpha \star \beta = \alpha \star' \beta$ requires  |
|                       |                              | induction not only on $\alpha$ and $\beta$ but then on the two remaining   |
|                       |                              | 1-paths. After the proof, remark that we trust the reader to   |
| Definition 2.1.9      | 222 ~25b777                  | construct such operations from now on.   |
| Definition 2.1.8 §2.2 | 233-gc3fb777<br>336-g8ff8a7f | The three displays should be $:\equiv$ 's rather than $=$ 's. In the type of $ap_f$ towards the end of the first proof of  |
| 32.2                  | 550 g0110a71                 | Lemma 2.2.1, $g(x)$ should be $f(y)$ .   |
| §2.3                  | 154-g4ef49f7                 | Emphasize that unlike fibrations in classical homotopy theory,   |
| o .                   | O                            | type families come with a <i>specified</i> path-lifting function.  |
| § <b>2.</b> 3         | 343-g6efd724                 | The functions Eq. (2.3.6) and Eq. (2.3.7) are obtained by con-   |
|                       |                              | catenating with transportconst $_p^B(f(x))$ and its inverse, respec-   |
|                       |                              | tively.  |
| Corollary 2.4.4       | 253-gdd47c75                 | Canceling $H(x)$ may be done by whiskering with $(H(x))^{-1}$ .  |
| §2.4                  | 1171-gab3c0aa                | In the proof that $\operatorname{isequiv}(f) \to \operatorname{qinv}(f)$ , the definition of $\gamma$  |
| 00.6                  | <b>7.</b> 0007 <b>20</b>     | should be $\gamma(x) := \beta(g(x))^{-1} \cdot h(\alpha(x))$ .   |
| §2.6                  | 74-g9896e32                  | In the type of pair (just after the proof of Theorem 2.6.2), the   |
| 92.6                  | 005 ~07 41-004               | second factor in the domain should be $pr_2(x) = pr_2(y)$ .  |
| §2.6                  | 895-g96db894                 | In the displayed equation just before Theorem 2.6.4, pair $(p \cdot q, r, p' \cdot q', r)$ should be pair $(p \cdot q, r, p' \cdot q', r')$ and pair $(p, q \cdot q', r')$ |
|                       |                              | $r, p', q' \cdot r)$ should be pair $(p, q \cdot r, p', q' \cdot r')$ (two primes on $rs$  |
|                       |                              | are missing).  |
| Theorem 2.6.4         | 349-gc7fd9d8                 | The path is in $A(w) \times B(w)$ , not $A(y) \times B(y)$ .   |
| Theorem 2.6.4         | 76-ga42354c                  | The third displayed judgmental equality in the proof should  |
|                       |                              | be transport <sup>B</sup> $(p, \operatorname{pr}_2 x) \equiv \operatorname{pr}_2 x$ .  |
| Theorem 2.7.2         | 507-g8f10eda                 | In the proof, the equation $f(g(refl, refl)) = refl should be$   |
|                       |                              | $f(g(refl_{w_1}, refl_{w_2})) = (refl_{w_1}, refl_{w_2}).$   |
| §2.9                  | 269-g3880fe2                 | The paragraph preceding the definition of transport $I_{A}(B)(p, f)$   |
| Aviam 2 10 2          | 000 ~~4-5214                 | (before Eq. $(2.9.5)$ ) misstated the (already given) type of $p$ .  |
| Axiom 2.10.3          | 992-gc4a5314                 | The axiom should read "For any $A$ , $B$ : $\mathcal{U}$ , the function (2.10.2) is an equivalence. The display $(A =_{\mathcal{U}} B) \simeq (A \simeq B)$ should         |
|                       |                              | be deduced afterwards, outside the axiom statement.  |
|                       |                              | be deduced after wards, butside the axion statement.   |

| Location F          | ixed in Change               | 2  |
|---------------------|------------------------------|--|
| Theorem 2.11.1      | 310-gd5fa240                 | The second half of the proof is more involved than the first. It follows abstractly using the 2-out-of-6 property (Exercise 4.5), or more concretely by concatenating with $\alpha_{f(a)}^{-1} \cdot \alpha_{f(a)}$ on each side and then repeatedly using naturality and functoriality. |
| §2.11               | 236-g32be999                 | The second display after the proof of Theorem 2.11.1 should be $\prod_{(x:A)} (happly(p)(x) =_{f(x)=g(x)} happly(q)(x))$ .   |
| Theorem 2.11.3      | 628-g1bd8602                 | The sentence preceding the theorem suggests that it follows from Lemmas 2.3.10 and 2.11.2, but actually it requires a separate path induction.   |
| Theorem 2.11.3      | 704-g70c069e                 | The sentence after the theorem should say that $ap_{(x\mapsto c)}$ is $p\mapsto refl_c$ , not $refl_c$ .   |
| Theorem 2.11.4      | 364-g3c47534                 | The right-hand side of the displayed equality should be $(apd_f(p))^{-1} \cdot ap_{(transport^B p)}(q) \cdot apd_g(p).$  |
| §2.12               | 101-g645f763                 | In Theorem 2.12.5 and the preceding paragraph, in the equivalence $(inl(a) = x) \simeq code(x)$ , the variable $a$ should be $a_0$ .   |
| §2.12               | 370-g114db82                 | In the two displays after the proof of Theorem 2.12.5, the terms should be $encode(inl(a), -)$ and $encode(inr(b), -)$ .   |
| §2.14.2             | 261-g4ccda0a                 | In the first displayed pair of equations, the type of $p_2$ should be transport $(p_1, (m, a)) = (m', a')$ .   |
| §2.14.2             | 402-g2297ecb                 | The right hand side of the last displayed equation should be $m'(e(x_1), e(x_2))$ .  |
| §2.15               | 305-g64685f1                 | In the discussion of universal properties for product types and $\Sigma$ -types surrounding Eq. (2.15.9), the phrases "left-to-right" and "right-to-left" should be switched.  |
| Chapter 2 Notes     | 379-ga57eab2                 | It should be mentioned that Hofmann and Streicher (1998) proposed an axiom similar to univalence, which is correct (and equivalent to univalence) for a universe of 1-types.   |
| Eq. (3.2.1)<br>§3.5 | 1193-g54b20e3<br>86-g39feab1 | The domain of $g: \prod_{(x:A)} A(x)$ should be $X$ .<br>The definition of subset containment should say $\prod_{(x:A)} (P(x) \to Q(x))$ , not $\forall (x:A). (P(x) \Rightarrow Q(x))$ , as the latter notation has not been introduced yet.  |
| Lemma 3.11.7        | 95-gce0131f                  | In the proof, $p$ should be $r$ to match the preceding definition of retraction.   |
| Exercise 3.14       | 1162-ga97cb70                | Should be to show that $\neg \neg A$ satisfies the recursion principle of $  A  $ but with only a propositional computation rule.  |
| Lemma 4.1.1         | 87-g693e9b9                  | At the end of the proof, Lemma 3.11.8 should be cited as the reason why $\sum_{(g:A\to A)} (g = id_A)$ is contractible.  |
| Theorem 4.2.3       | 275-g8ea9f71                 | In the proof, the path concatenations in the definitions of $\epsilon'$ and $\tau$ were written in reverse order.  |
| Theorem 4.2.3       | 1043-gcfce4d7                | In the proof, the type of $\tau(a)$ should be $f(\eta(a)) = \epsilon(f(g(f(a))))^{-1} \cdot (f(\eta(g(f(a)))) \cdot \epsilon(f(a)))$ , instead of $\epsilon(f(g(f(a))))^{-1} \cdot (f(\eta(g(f(a)))) \cdot \epsilon(f(a))) = f(\eta(a))$ .   |

| Location 1      | Fixed in Change | 2  |
|-----------------|-----------------|--|
| Lemma 4.2.12    | 296-ge3dc076    | In the proof, $(fgx, \epsilon(fx)) =_{fib_f(fx)} (x, refl_{fx})$ should be $(gfx, \epsilon(fx)) =_{fib_f(fx)} (x, refl_{fx}).$   |
| Corollary 4.3.3 | 272-gfd47093    | At the end of the proof, the equivalence follows from the fact that $ishae(f)$ , not $isContr(f)$ , is a mere proposition.   |
| Theorem 4.4.3   | 299-g85b729b    | In the proof, $lcoh_f(g, \epsilon)$ should be $rcoh_f(g, \epsilon)$ , and the final displayed equation should have $pr_2$ applied to both occurrences of $P(fx)$ .   |
| Lemma 4.7.3     | 265-g64000fb    | The path concatenations in the definitions of $\varphi_b$ and $\psi_b$ (and subsequent equations) are reversed, and each $f(a)$ in the next two displayed equations should be $g(a)$ .   |
| Theorem 4.7.6   | 275-g84ab032    | The first equivalence in the proof is not by (2.15.9) but by Exercise 2.10.  |
| Theorem 4.7.6   | 202-g775a3f0    | The last equivalence in the proof is not by (2.15.10) but by Lemmas 3.11.8 and 3.11.9 and Exercise 2.10.   |
| Theorem 4.8.3   | 205-gf9fe386    | In the proof, $e \cdot \operatorname{pr}_1$ should be $(\operatorname{ua}(e))_*(\operatorname{pr}_1)$ . Also, explain its computation better.  |
| §4.9            | 114-gaba76c8    | The point of Lemma 4.9.2 is that it follows from univalence without assuming function extensionality separately.   |
| Corollary 4.9.3 | 484-g2ce1249    | In the statement, "precomposition" should be "post-composition".   |
| Theorem 4.9.4   | 746-g4d540d6    | In the definition of $\psi$ in the proof, transport has to be along happly( $p$ , $x$ ) instead of along $p$ .   |
| Exercise 4.2    | 358-g9543064    | The text should be "Show that for any $A$ , $B$ : $\mathcal{U}$ , the following type is equivalent to $A \simeq B$ . Can you extract from this a definition of a type satisfying the three desiderata of isequiv( $f$ )?"  |
| §5.2            | 706-ged2c765    | In the proof that $\mathbb{N} \simeq \mathbb{N}'$ , the definitions of $f$ and $g$ should be $\operatorname{rec}_{\mathbb{N}}(\mathbb{N}', 0', \lambda n.\operatorname{succ}')$ and $\operatorname{rec}_{\mathbb{N}'}(\mathbb{N}, 0, \lambda n.\operatorname{succ})$ respectively. |
| §5.3            | 125-g433f87e    | In the definition of $N^w$ , use $0_2$ for 0 and $1_2$ for succ, to match the ordering of $0_2$ and $1_2$ in §1.8.   |
| §5.3            | 551-g82b74bf    | The definitions of $\mathbf{N}^{\mathbf{w}}$ and $\operatorname{List}(A)$ as W-types should be $W_{(b:2)}\operatorname{rec}_{2}(\mathcal{U},0,1,b)$ and $W_{(x:1+A)}\operatorname{rec}_{1+A}(\mathcal{U},0,\lambda a.1,x)$ .   |
| §5.3            | 218-g42219cb    | In the description of the constructor sup, its second argument is more clearly written as $f: B(a) \to W_{(x:A)}B(x)$ .  |
| §5.3            | 525-gb1957b8    | In the computation rule, the recursive call to rec is missing an argument. It should read $\operatorname{rec}_{W_{(x:A)}B(x)}(E,e,\sup(a,f)) \equiv e(a,f,(\lambda b.\operatorname{rec}_{W_{(x:A)}B(x)}(E,e,f(b))))$ .   |
| §5.3            | 570-g6ec04c3    | In the verification that double computes as expected, $e_t$ should be $e_0$ and $e_f$ should be $e_1$ .  |
| §5.4            | 554-g9b2a34b    | The definition of the type of W-homomorphisms (just before Theorem 5.4.7) should read WHom <sub><math>A,B</math></sub> $((C,s_C),(D,s_D)) :\equiv \sum_{(f:C\to D)} \prod_{(a:A)} \prod_{(h:B(a)\to C)} f(s_C(a,h)) = s_D(a,f\circ h).$  |

| Location      | Fixed in Change |  |
|---------------|-----------------|--|
| §5.5          | 917-gd6960ad    | In the first paragraph, the definition of $N^w$ should be $W_{(b:2)}rec_2(\mathcal{U}, 0, 1, b)$ .   |
| §5.5          | 608-g6af101f    | In the computation rule for homotopy W-types, the left-hand side should be $\operatorname{rec}_{W^h_{(x:A)}B(x)}(E,e,\sup(a,f))$ .                 |
| Eq. (5.6.6)   | 912-g04d3fb6    | In the preceeding sentence, $\delta$ : $d$ should be $\delta$ : $D$ .  |
| §5.7          | 908-g4b2eb10    | The second two constructors of paritynat should be esucc : $paritynat(1_2) \to paritynat(0_2)$ and $osucc$ : $paritynat(0_2) \to paritynat(1_2)$ . |
| Theorem 5.8.2 | 139-gd5c5d01    | In the proof of (iv) $\Rightarrow$ (i), the type of $D'$ should be $(\sum_{(b:A)} R(b)) \rightarrow \mathcal{U}$ .                                 |
| Exercise 5.2  | 622-ga0bd007    | The two functions should satisfy the same recurrence judgmentally.   |
| Exercise 5.3  | 622-ga0bd007    | The function should satisfy both recurrences judgmentally.   |
| §6.2          | 54-gd4a47c2     | Soon after Remark 6.2.1, the phrase "An element $b: P(base)$ in  |
| Lemma 6.2.8   | 400 af760aa1    | the fiber over the constructor base: N" should say base: S <sup>1</sup> .  Theorems 2.11.2 and 2.11.5 are needed to not a in the form              |
| Lemma 6.2.6   | 423-gf763ae1    | Theorems 2.11.3 and 2.11.5 are needed to put $q$ in the form required by the induction principle.  |
| Lemma 6.3.2   | 417-g4aa6a15    | Added Exercise 6.10: the function constructed in Lemma 6.3.2   |
|               | O               | is actually an inverse to happly, so that the full function exten-   |
|               |                 | sionality axiom follows from an interval type.   |
| Lemma 6.4.2   | 625-g950efa9    | In the second paragraph of the proof, the appeal to function extensionality should be omitted.   |
| §6.4          | 327-g7cbe31c    | In the first sentence after the proof of Lemma 6.4.6, " $P: \mathbb{S}^2 \to P$ " should be " $P: \mathbb{S}^2 \to U$ ".                           |
| $\S6.4$       | 1039-g30da4c6   | In the sentence after the proof of Lemma 6.4.6, the type family  |
|               |                 | in which <i>s</i> is a dependent path should be $\lambda p$ . $b =_p^P b$ instead of <i>P</i> .  |
| §6.6          | 289-gdefeb8c    | In the induction principle for the torus, the types of $p'$ and $q'$ should be $b' = p b'$ and $b = p b'$ respectively.                            |
| §6.7          | 289-gdefeb8c    | In the induction principle for the torus, the types of $p'$ and $q'$ should be $b' = p b'$ and $b = p b'$ respectively.                            |
| §6.9          | 468-g5472874    | The induction principle for $  A  $ should conclude $f( a ) \equiv$  |
|               |                 | $g(a)$ , not $f( a ) \equiv a$ . And in the hypotheses of the induc-   |
|               |                 | tion principle for $  A  _0$ and in the proof of Lemma 6.9.1, $v: p = {}^B_{u(x,y,p,q)} q$ should instead be $v: r = {}^B_{u(x,y,p,q)} s$ .        |
| §6.9          | 860-gc7d862c    | In the penultimate paragraph, the "unobjectionable" construc-  |
|               |                 | tor for $  A  _0$ should begin "For every $f: S \to   A  _0$ ", not "For every $f: S \to A$ ".   |
| Lemma 6.10.3  | 961-gde36592    | The first sentence of the second paragraph of the proof should   |
|               |                 | end with $g(x) = \overline{g \circ q}(x)$ .  |

| Location F        | ixed in Change |  |
|-------------------|----------------|--|
| Lemma 6.10.8      | 514-g18ade45   | Instead of "is the set-quotient of $A$ by $\sim$ ", the statement should say "satisfies the universal property of the set-quotient of $A$ by $\sim$ , and hence is equivalent to it". In the proof, the second displayed equation should be $e'(g,s)(x,p) :\equiv g(x)$ . The fourth displayed equation should be $e(e'(g,s)) \equiv e(g \circ \operatorname{pr}_1) \equiv (g \circ \operatorname{pr}_1 \circ q, \_)$ , the fifth should be $g(\operatorname{pr}_1(q(x))) \equiv g(r(x)) = g(x)$ , and the proof should conclude with " $g$ respects $\sim$ by the assumption $s$ ". |
| Lemma 6.10.12     | 535-g0a9abfe   | The "computation rules" satisfied by $f$ are only propositional equalities. Also, the proof requires transport across a few unmentioned equivalences.  |
| Corollary 6.10.13 | 535-g0a9abfe   | The defining clauses should use := rather than := (see the erratum for Lemma 6.10.12). Also, the first clause should say refl <sub>a</sub> rather than refl <sub>base</sub> .  |
| Lemma 6.12.1      | 682-g3af5dbe   | Three occurrences of <i>P</i> in the statement should be <i>B</i> .  |
| Lemma 6.12.3      | 457-g411ec6d   | The right-hand side of the displayed equation in the proof should be $(c(g(b)), D(b)(y))$ .  |
| Lemma 6.12.3      | 961-gde36592   | After the display we should have $p(b) : c(f(b)) = c(g(b))$ .  |
| §6.12             | 519-gc99a54c   | $f$ denotes a map $B \to A$ in this section and should not be reused for functions defined by induction on $\sum_{(w:W)} P(w)$ ; we may use $k$ instead. Thus $f$ should be $k$ in the last sentence of Lemma 6.12.4; the first sentence of its proof; the second and third sentences of the paragraph after its proof; the last sentence of Lemma 6.12.5; the first, second, and last sentences of its proof; throughout the statement and proof of Lemma 6.12.7; the statement of Lemma 6.12.8; and the second sentence of its proof.  |
| Lemma 6.12.4      | 537-gdf3b51d   | In the display after the definition of $q$ , the transport in the first line should be with respect to $x \mapsto Q(\widetilde{c}'(g(b), x))$ , and in the second line the subscript of ap should be $x \mapsto \widetilde{c}'(g(b), x)$ .   |
| Lemma 6.12.4      | 961-gde36592   | The subscript of ap should also be $x \mapsto \widetilde{c}'(g(b), x)$ in the third, fourth, and fifth displays. In the fourth and fifth displays, the path-concatenations should be in the other order. And in the fifth display, $\operatorname{refl}_{g(b)}$ should be $\operatorname{refl}_{c(g(b))}$ .  |
| Lemma 6.12.7      | 501-ge895f81   | Both occurrences of $P$ in the statement should be $Y$ , and both occurrences of $Q$ in the proof should be $Z$ .  |
| Theorem 7.1.4     | 180-gb672a4d   | In the last displayed equation of the proof, $q$ should be $r$ .   |
| Theorem 7.1.10    | 101-g713f48c   | The base case in the proof is just Lemma 3.11.4.   |
| §7.3              | 480-gdc84050   | The third paragraph is wrong: in contrast to Remark 6.7.1, it <i>would</i> actually work to define $  A  _n$ omitting the hub point.   |
| Theorem 7.2.2     | 1131-gc1748fa  | In the second paragraph of the first proof, the codomain of the function $f(x, x)$ should be $x =_X x$ , not $x =_X y$ .   |

| Location Fi      | ixed in Change |   |
|------------------|----------------|---|
| Lemma 7.2.4      | 644-g627c0a8   | In the proof of the lemma, "If $x$ is $inr(f)$ " should be "If $x$ is $inr(t)$ ".   |
| Theorem 7.3.12   | 412-gb9582fc   | In the proof, encode and decode should be switched.   |
| Lemma 7.5.12     | 801-g01922a8   | The converse direction is false unless Q is fiberwise merely  |
|                  | J              | inhabited. Also, the occurrences of $f(p)$ and $f(pr_2w)$ in the proof should be just $p$ and $pr_2w$ , respectively.   |
| Lemma 7.5.14     | 367-g1c8c07e   | In the proof that the first composite is the identity, all occurrences of $y$ should be $f(x)$ .  |
| Theorem 7.7.4    | 658-g016f3a4   | In the second paragraph of the proof, the first two occurrences of pr <sub>2</sub> (but not the third) should be pr <sub>1</sub> .  |
| Exercise 7.2     | 101-ga366be2   | "entires" should be "entirely".   |
| Exercise 7.2     | 683-g8941e50   | This exercise needs more precise definitions of "diagram" and "colimit".  |
| Exercise 7.8     | 1074-gcd42187  | $AC_{\infty,\infty}$ is not Theorem 2.15.7, but the identity function.  |
| Exercise 7.8     | 603-ge113e08   | The penultimate sentence should ask "Is $AC_{n,m}$ consistent with univalence for any $m \ge 0$ and any $n$ ?".   |
| Lemma 8.1.8      | 535-g0a9abfe   | The proof by induction on $n : \mathbb{Z}$ is justified by Lemma 6.10.12, not Corollary 6.10.13.  |
| Lemma 8.1.12     | 535-g0a9abfe   | The clauses defining $q_z$ should use := rather than := (see the erratum for Lemma 6.10.12).  |
| Theorem 8.2.1    | 1062-gf3bfeae  | In the proof, $E$ is not $(n + 1)$ -connected but $(n + 1)$ -truncated.   |
| Lemma 8.4.4      | 1181-g3e51973  | In the proof, $(x : A)$ should be $(x : X)$ .   |
| Corollary 8.4.8  | 1023-gf188aeb  | The proof requires a separate argument for $k = 0$ .  |
| Theorem 8.5.1    | 256-g9e6fcb8   | The phrase "whose fibers are $S^1$ " should be "whose fiber over the basepoint is $S^1$ ". The same change should be made in Exercises 8.8 and 8.9.   |
| Lemma 8.5.3      | 1062-gf3bfeae  | In the definition of $E^{\text{tot}'}$ in the proof, $e_C$ should be $e_X$ .  |
| Lemma 8.6.1      | 396-g868335b   | In the proof, the function $k$ should have type $\prod_{(a:A)} P(f(a))$ . It should also be named $\ell$ , to avoid confusion with the integer $k$ .  |
| Definition 8.6.5 | 87-g3f977b2    | In the second displayed equation in the proof, $\operatorname{merid}(x_1)$ should be $\operatorname{merid}(x_1)^{-1}$ .   |
| Lemma 8.6.2      | 1203-g7464bf1  | The type family $P$ defined in the proof should instead be called $Q$ , to avoid clashes with the type family $P$ assumed in the statement.   |
| Lemma 8.6.2      | 399-g8897c94   | In the last sentence of the proof, " $(n-1)$ -connected" should be " $(n-1)$ -truncated".   |
| Lemma 8.6.10     | 88-g0c0be67    | The type of $m$ should be $a_1 = a_2$ , the second display should begin with $C(a_1, \operatorname{transport}^B(m^{-1}, b))$ , and the proof should say "we may assume $a_2$ is $a_1$ and $m$ is $\operatorname{refl}_{a_1}$ ". |

| Location F       | ixed in Change |  |
|------------------|----------------|--|
| §8.6             | 165-gd5584c6   | In (8.6.11), $r''$ should be $r'$ , the end point of $r$ should be transport $^B(\text{merid}(x_0)^{-1}, q)$ , and obtaining $r'$ requires also identifying this with $q \cdot \text{merid}(x_0)^{-1}$ . Similarly, in (8.6.12), the end point of $r$ should be transport $^B(\text{merid}(x_1)^{-1}, q)$ .  |
| §8.6             | 474-g5289470   | $\pi_3(S^2) = \mathbb{Z}$ should be stated as Corollary 8.6.19, following from Corollary 8.5.2 and Theorem 8.6.17.   |
| Theorem 8.8.3    | 1092-ge3b8b71  | After applying the induction hypothesis, it additionally needs to be checked that for every path $p: a = a$ the map $\pi_k(ap_f): \pi_k(x = x, p) \to \pi_k(f(x) = f(x), ap_f(p))$ is a bijection.   |
| §8.9             | 1154-g301662b  | In the strengthening of condition (iii) from Lemma 8.9.1, the right side should read just "c" instead of "c.a".  |
| Definition 9.2.1 | 807-gebec78b   | In Item (iv), it should read "hom <sub><math>A</math></sub> ( $b$ , $c$ )" instead of "hom <sub><math>B</math></sub> ( $b$ , $c$ )".   |
| §9.4             | 1218-gcb6ba30  | Just before Definition 9.4.6, it should say "However, if <i>A</i> is not a category" instead of "However, if <i>B</i> is not a category".  |
| Theorem 9.5.4    | 971-g6096085   | The sequence of equations at the end of the proof should begin with $\alpha_{a'}(f) = \alpha_{a'}(\mathbf{y}a_{a,a'}(f)(1_a))$ , and thereafter the subscripts should remain $a, a'$ rather than $a', a$ .   |
| Definition 9.8.1 | 897-g94fb722   | In (iv), "if $f$ : hom $_X(x,y)$ " should be "if $f$ : hom $_X(x,y)$ and $g$ : hom $_X(y,z)$ ".  |
| §9.8             | 1111-g3332a31  | The type of objects $A_0$ of the precategory $A$ of $(P, H)$ - structures should be defined as $\sum_{(x:X_0)} Px$ , not $\sum_{(x:X)} Px$ .   |
| Chapter 9        | 966-g04374f5   | The first sentence after Theorem 9.9.4 should begin "Therefore, if a precategory $A$ admits a weak equivalence functor $A \to \widehat{A}$ into a category".   |
| Theorem 9.9.5    | 313-g8ee79db   | In the second proof, the third constructor of $\widehat{A}_0$ is unneeded; it follows from the fourth constructor and path induction. In the fifth constructor, $j(g) \cdot j(f)$ should be $j(f) \cdot j(g)$ , and similarly throughout the proof. Finally, for consistency, the 1-truncation constructor should be included explicitly (this was intended to be implied by "higher inductive 1-type"). |
| Chapter 9 Notes  | 379-ga57eab2   | It should be mentioned that Hofmann and Streicher (1998) also considered this definition of category.  |
| Theorem 10.3.20  | 140-g55de417   | The second sentence of the proof should say "By well-founded induction on $A$ , suppose $A_{/b}$ is accessible for all $b < a$ ".  |
| Lemma 10.3.22    | 140-gd7f8960   | The statement should say $X : \mathcal{U}$ rather than $X : \mathcal{U}_{\mathcal{U}}$ .   |
| Theorem 10.4.3   | 140-gcca0bcf   | The penultimate sentence of the proof should say "if $a < b$ and $b < c$ " rather than "if $a < b$ and $a < c$ ".  |
| Theorem 10.4.4   | 871-g85bcd11   | The statement of (i) should end with $Y : \mathcal{P}_+(X)$ , not $Y : \mathcal{P}(X)$ .   |

| Location Fi       | xed in Change |  |
|-------------------|---------------|--|
| §10. <del>5</del> | 753-gc87ce23  | The second clause in the induction principle for $V$ should say "Verify that if $f: A \to V$ and $g: B \to V$ satisfy (10.5.2), then $h(\operatorname{set}(A,f)) = {}^p_q h(\operatorname{set}(B,g))$ , where $q$ is the path arising from the second constructor of $V$ and (10.5.2), assuming inductively that $h(f(a)) = {}^p_p h(g(b))$ whenever $p: f(a) = g(b)$ ." |
| §10.5             | 706-ged2c765  | The proof that membership is well-defined should end with "hence $x = g(b)$ and $x \in set(B, g)$ ."   |
| §10.5             | 1056-g4060c2b | In the definition of $V$ -set, the notation $v \in V$ should be $v : V$ .  |
| Theorem 10.5.8    | 708-g6f53189  | In the pairing axiom, the pair class should be denoted $\{u,v\}$ , not $u \cup v$ .  |
| Theorem 10.5.8    | 723-g9cf5b44  | The replacement axiom should be given $x:V$ (not $a:V$ ) and the displayed class should be $\{y\mid \exists (z:V).z\in x \land y=r(z)\}$ . Its proof should begin "let $C$ denote the class in question."  |
| Theorem 10.5.8    | 706-ged2c765  | In the proof of the function set axiom, "the types of elements $[u] \rightarrow V$ and $[u] \rightarrow V$ " should be "the types of members $[u] \rightarrow V$ and $[v] \rightarrow V$ ."  |
| Exercise 10.12    | 1053-ge13dd65 | Extra parentheses around $\forall (x \in v)$ . $\exists (y)$ . $R(x,y)$ are needed to make the formula unambiguous.  |
| Exercise 10.13    | 1053-ge13dd65 | Extra parentheses around $\forall (y \in x). \exists (z \in V). z \in y$ are needed to make the formula unambiguous.   |
| Exercise 10.13    | 1056-g4060c2b | The notation $\in V$ should be : $V$ .   |
| Lemma 11.2.2      | 165-gb002a64  | The statement should say "For all $x : \mathbb{R}_d$ and $q : \mathbb{Q}$ , $L_x(q) \Leftrightarrow (q < x)$ and $U_x(q) \Leftrightarrow (x < q)$ ".   |
| Theorem 11.2.4    | 165-g179b359  | In the proof, the sentence beginning "From $0 < ac$ it follows" should be replaced by "From $0 < ac$ and $0 < bc$ it follows that $a$ , $b$ , and $c$ are either all positive or all negative. Hence either $0 < a < x$ or $x < b < 0$ , so that $x \# 0$ ".   |
| §11.2.2           | 832-g0cb658e  | In the second paragraph, at "From this we get", the universal quantification should be over $\delta$ as well.  |
| §11.3.2           | 1209-g3e5ad94 | In the statement of $(\mathbb{R}_c, \sim)$ -recursion, " $f(x) : A$ " should be " $f(\lim(x)) : A$ ".  |
| Theorem 11.3.16   | 1069-g3b333d5 | In the description of openness of $\approx$ , " $\exists (\varepsilon : \mathbb{Q}_+)$ ." should be " $\exists (\delta : \mathbb{Q}_+)$ .".  |
| Lemma 11.4.1      | 87-g82b27c3   | (11.4.2) should be $c: \prod_{(q,r:\mathbb{Q})} (q < r) \to (q < x) + (x < r)$ , and therefore the use of $c$ in the proof should be $c(s,t)$ rather than $c(x,s,t)$ .   |
| §11.6             | 1189-ga9c35f0 | The inductive case of $\iota_{\mathbb{Q}_D}$ should be defined as $\iota_{\mathbb{Q}_D}(a/2^n) := \{ \iota_{\mathbb{Q}_D}(a/2^n - 1/2^n) \mid \iota_{\mathbb{Q}_D}(a/2^n + 1/2^n) \}.$   |
| Example 11.6.18   | 636-g827e7ea  | In the first bullet point, to prove $x^L + z < x + z$ requires a No-<br>induction on $z$ , since only when $z$ is defined by a cut can we<br>say that $x^L + z$ is a left option of $x + z$ .  |

| Location Fi     | xed in Change |  |
|-----------------|---------------|--|
| Exercise 11.13  | 222-g3453cf1  | This is the intermediate value theorem, not the mean value   |
|                 |               | theorem.   |
| Example 11.6.18 | 980-ge9d0398  | For the codomain of the outer recursion, the conditions should be $(x < y) \rightarrow (g(x) < g(y))$ and $(x \le y) \rightarrow (g(x) \le g(y))$ . In the first bullet of the verification that inequalities are preserved, the outer inductive hypotheses give non-strict inequalities $x^L + y \le x^L + z$ and $x^R + y \le x^R + z$ , and no additional No-induction on $z$ is required (it is already known to be defined by a cut). |
| Example 11.6.18 | 980-ge9d0398  | The verification that Conway's definition of $x + y$ is a surreal number (i.e. all its left options are $<$ all its right options) was omitted. This requires turning the inner recursion into an inner induction with codomain a varying subset of No, as in Theorem 11.6.7.  |
| Appendix A      | 165-g76db618  | After the introduction of the judgment " $\Gamma$ ctx" in the Preliminaries, the sentence beginning "Therefore, if $\Gamma \vdash a : A,$ " should say instead "In particular, therefore, if $\Gamma \vdash a : A,$ ".   |
| Appendix A.2.1  | 64-g7c2312e   | Clarify the distinction between typing judgments and context well-formedness judgments, and remove the $\vdash$ from the notation for the latter.  |
| Appendix A.2.5  | 26-gcd691e8   | In $\Sigma$ -COMP and the following paragraph, $y.C$ should be $z.C$ , and "we bind $y$ in $C$ " should likewise say $z$ .   |
| Appendix A.2.8  | 338-g4e1c688  | The $c$ argument in the eliminator for 1 (in the 1-ELIM and 1-COMP rules) should not bind a variable of type 1.  |
| Appendix A.2.10 | 578-ga4b94a5  | The unbased eliminator for the identity type should be named $\operatorname{ind}_{=_A}$ , not $\operatorname{ind}'_{=_A}$ .  |