

# Errata for the HoTT Book, first edition

February 4, 2021

For the benefit of all readers, the available PDF and printed copies of the book are being updated on a rolling basis with minor corrections and clarifications as we receive them. Every copy has a version marker that can be found on the title page and is of the form “first-edition-XX-gYYYYYYY”, where XX is a natural number and YYYYYYY is the git commit hash that uniquely identifies the exact version. Higher values of XX indicate more recent copies.

Below is a list of corrections and clarifications that have been made so far (except for trivial formatting and spacing changes), along with the version marker in which they were first made. This list is current as of February 4, 2021 and version marker “first-edition-1278-gb993f64”.

While the page numbering may differ between copies with different version markers (and indeed, already differs between the letter/A4 and printed/ebook copies with the same version marker), we promise that the numbering of chapters, sections, theorems, and equations will remain constant, and no new mathematical content will be added, unless and until there is a second edition.

Location	Fixed in	Change
§1.1	182-gb29ea2f	Change notation $a \equiv_A b$ to $a \equiv b : A$ , to match that used in Appendix A. (Neither are used anywhere else in the book.)
§1.1	154-g42698c2	Clarify that algorithmic decidability of judgmental equality is only meta-theoretic.
§1.1	154-gac9b226	Mention notation $a = b = c = d$ to mean “ $a = b$ and $b = c$ and $c = d$ , hence $a = d$ ”, possibly including judgmental equalities.
§1.3	42-g4bc5cc2	Cumulativity means some elements do not have unique types, the index $i$ on $\mathcal{U}_i$ is not an internal natural number, and typical ambiguity must be justified by reinserting indices.
§§1.3 and 1.4	42-ga34b313	Explain that we can’t define $\text{Fin}$ and $\text{fmax}$ yet where we first mention them.
§1.4	165-g0ad2aba	Add swap as another example of a polymorphic function, and discuss the use of subscripts and implicit arguments to dependent functions.
Remark 1.5.1	80-g8f95fa5	In the discussion of formation rules, the dependent function type example should be $\prod_{(x:A)} B(x)$ .
§1.5	51-g67e86db	Better explanation of recursion on product types, why it is justified, and how it relates to the uniqueness principle.

Location	Fixed in	Change
§1.6	2-gbe277a8	In the types of $g$ and $\text{ind}_{\sum_{(x:A)} B(x)}$ , there is a $\prod_{(a:A)} \prod_{(b:B(x))}$ in which $x$ should be $a$ .
§1.6	27-gd0bfa0d	At two places in the definition of $\text{ac}$ , $R(a, \text{pr}_1(g(x)))$ should be $R(x, \text{pr}_1(g(x)))$ .
§1.6	125-g7fdadbf	When substituting $\lambda x. \text{pr}_1(g(x))$ for $f$ while verifying that $\text{ac}$ is well-typed, the left side of the judgmental equality should be $\prod_{(x:A)} R(x, \text{pr}_1(g(x)))$ , not $\prod_{(x:A)} R(x, \text{pr}_1(f(x)))$ .
§1.7	30-g264d934	In two displayed equations, $f(\text{inl}(b))$ should be $f(\text{inr}(b))$ .
Theorem 1.8.1	391-g1ce619a	This should not be called a “Theorem”, since we have not yet introduced what that means. Instead it should say “We construct an element of...”.
§1.8	125-g433f87e	In the definition of binary products in terms of <b>2</b> , the definitions of $\text{pr}_1(p)$ and $\text{pr}_2(p)$ should be switched to match the order of arguments to $\text{rec}_2$ and $\text{ind}_2$ .
§1.11	111-g1e868fa	When translating English to type theory, “unnamed variables” are unnamed in English but must be named in type theory.
§1.12	154-g4ef49f7	Emphasize that path induction, like all other induction principles, defines a <i>specified</i> function.
§1.12	244-gd58529d	In proof that path induction implies based path induction, $D(x, y, p)$ should be written $\prod_{(C: \prod_{(z:A)} (x=Az) \rightarrow \mathcal{U})} (\dots)$ so the type of $C$ matches the premise of based path induction.
Remark 1.12.1	563-g3286941	The facts that any $(x, y, p) : \sum_{(x,y:A)} (x = y)$ is equal to $(x, x, \text{refl}_x)$ , and that any $(y, p) : \sum_{(y:A)} (a =_A y)$ is equal to $(a, \text{refl}_a)$ , can be proven by path induction and based path induction respectively.
Exercise 1.4	78-gcce4dc0	The second defining equation of $\text{iter}$ should have right-hand side $c_s(\text{iter}(C, c_0, c_s, n))$ .
Exercise 1.4	293-g4663bfe	The defining equations of the recursor derived from the iterator only hold propositionally, and require the induction principle to prove.
Exercise 1.6	229-ged891f3	This exercise requires function extensionality (§2.9).
Exercise 1.8	450-g7f38c9a	This exercise requires symmetry and transitivity of equality, Lemmas 2.1.1 and 2.1.2.
Exercise 1.10	110-gfe4641b	To match the usual Ackermann–Péter function, the second displayed equation should be $\text{ack}(\text{succ}(m), 0) \equiv \text{ack}(m, 1)$ .
Chapter 2	239-gaf3d682	In the chapter introduction, clarify that topological homotopies between paths must be endpoint-preserving.
Lemma 2.1.1	166-g37b78ef	Add remarks before and after the proof about how a theorem’s statement and proof should be interpreted as exhibiting an element of some type.
Lemma 2.1.2	374-g0bc0908	In the penultimate display in the first proof, $d(x, z, q)$ should be simply $d$ .

Location	Fixed in	Change
Lemma 2.1.4	750-g91b7348	In the first proofs of (i)–(iii), $\text{ind}_{=A}(D, d, p)$ should be $\text{ind}_{=A}(D, d, x, y, p)$ .
§2.1	435-gee0b28a	In the third paragraph after Lemma 2.1.2, $p \star \text{refl}_x \equiv p$ should be $p \star \text{refl}_y \equiv p$ .
§2.1	165-g18642ca	Mention that the notation $a = b = c = d$ , and its displayed variant, indicate concatenation of paths.
§2.1	253-gdd47c75	Lemma 2.1.4(iv) justifies writing $p \star q \star r$ and so on.
Theorem 2.1.6	253-gdd47c75	The induction defining $\alpha \star_r r$ has defining equation $\alpha \star_r \text{refl}_b \equiv \text{ru}_p^{-1} \star \alpha \star \text{ru}_q$ , with $\text{ru}_p$ the right unit law. For $\alpha \star \beta = \alpha \star \beta$ to be well-typed, we assume $p \equiv q \equiv r \equiv s \equiv \text{refl}_a$ and use $\text{ru}_{\text{refl}_a} = \text{refl}_{\text{refl}_a}$ and its dual. Proving $\alpha \star \beta = \alpha \star' \beta$ requires induction not only on $\alpha$ and $\beta$ but then on the two remaining 1-paths. After the proof, remark that we trust the reader to construct such operations from now on.
Definition 2.1.8	233-gc3fb777	The three displays should be $:\equiv$ 's rather than $=$ 's.
§2.2	336-g8ff8a7f	In the type of $\text{ap}_f$ towards the end of the first proof of Lemma 2.2.1, $g(x)$ should be $f(y)$ .
§2.3	154-g4ef49f7	Emphasize that unlike fibrations in classical homotopy theory, type families come with a <i>specified</i> path-lifting function.
§2.3	343-g6efd724	The functions Eq. (2.3.6) and Eq. (2.3.7) are obtained by concatenating with $\text{transportconst}_p^B(f(x))$ and its inverse, respectively.
Corollary 2.4.4	253-gdd47c75	Canceling $H(x)$ may be done by whiskering with $(H(x))^{-1}$ .
§2.4	1171-gab3c0aa	In the proof that $\text{isequiv}(f) \rightarrow \text{qinv}(f)$ , the definition of $\gamma$ should be $\gamma(x) := \beta(g(x))^{-1} \star h(\alpha(x))$ .
§2.6	74-g9896e32	In the type of $\text{pair}^=$ (just after the proof of Theorem 2.6.2), the second factor in the domain should be $\text{pr}_2(x) = \text{pr}_2(y)$ .
§2.6	895-g96db894	In the displayed equation just before Theorem 2.6.4, $\text{pair}^*(p \star q, r, p' \star q', r)$ should be $\text{pair}^*(p \star q, r, p' \star q', r')$ and $\text{pair}^*(p, q \star r, p', q' \star r)$ should be $\text{pair}^*(p, q \star r, p', q' \star r')$ (two primes on $rs$ are missing).
Theorem 2.6.4	349-gc7fd9d8	The path is in $A(w) \times B(w)$ , not $A(y) \times B(y)$ .
Theorem 2.6.4	76-ga42354c	The third displayed judgmental equality in the proof should be $\text{transport}^B(p, \text{pr}_2 x) \equiv \text{pr}_2 x$ .
Theorem 2.7.2	507-g8f10eda	In the proof, the equation $f(g(\text{refl}, \text{refl})) = \text{refl}$ should be $f(g(\text{refl}_{w_1}, \text{refl}_{w_2})) = (\text{refl}_{w_1}, \text{refl}_{w_2})$ .
§2.9	269-g3880fe2	The paragraph preceding the definition of $\text{transport}^{\Pi_A(B)}(p, f)$ (before Eq. (2.9.5)) misstated the (already given) type of $p$ .
Axiom 2.10.3	992-gc4a5314	The axiom should read “For any $A, B : \mathcal{U}$ , the function (2.10.2) is an equivalence. The display $(A =_{\mathcal{U}} B) \simeq (A \simeq B)$ should be deduced afterwards, outside the axiom statement.

Location	Fixed in	Change
Theorem 2.11.1	310-gd5fa240	The second half of the proof is more involved than the first. It follows abstractly using the 2-out-of-6 property (Exercise 4.5), or more concretely by concatenating with $\alpha_{f(a)}^{-1} \cdot \alpha_{f(a)}$ on each side and then repeatedly using naturality and functoriality.
§2.11	236-g32be999	The second display after the proof of Theorem 2.11.1 should be $\prod_{(x:A)} (\text{happly}(p)(x) =_{f(x)=g(x)} \text{happly}(q)(x))$ .
Theorem 2.11.3	628-g1bd8602	The sentence preceding the theorem suggests that it follows from Lemmas 2.3.10 and 2.11.2, but actually it requires a separate path induction.
Theorem 2.11.3	704-g70c069e	The sentence after the theorem should say that $\text{ap}_{(x \mapsto c)}$ is $p \mapsto \text{refl}_c$ , not $\text{refl}_c$ .
Theorem 2.11.4	364-g3c47534	The right-hand side of the displayed equality should be $(\text{apd}_f(p))^{-1} \cdot \text{ap}_{(\text{transport}^{B_p})}(q) \cdot \text{apd}_g(p)$ .
§2.12	101-g645f763	In Theorem 2.12.5 and the preceding paragraph, in the equivalence $(\text{inl}(a) = x) \simeq \text{code}(x)$ , the variable $a$ should be $a_0$ .
§2.12	370-g114db82	In the two displays after the proof of Theorem 2.12.5, the terms should be $\text{encode}(\text{inl}(a), -)$ and $\text{encode}(\text{inr}(b), -)$ .
§2.14.2	261-g4ccda0a	In the first displayed pair of equations, the type of $p_2$ should be $\text{transport}^{\text{SemigroupStr}}(p_1, (m, a)) = (m', a')$ .
§2.14.2	402-g2297ecb	The right hand side of the last displayed equation should be $m'(e(x_1), e(x_2))$ .
§2.15	305-g64685f1	In the discussion of universal properties for product types and $\Sigma$ -types surrounding Eq. (2.15.9), the phrases “left-to-right” and “right-to-left” should be switched.
Chapter 2 Notes	379-ga57eab2	It should be mentioned that Hofmann and Streicher (1998) proposed an axiom similar to univalence, which is correct (and equivalent to univalence) for a universe of 1-types.
Eq. (3.2.1)	1193-g54b20e3	The domain of $g : \prod_{(x:A)} A(x)$ should be $X$ .
§3.5	86-g39feab1	The definition of subset containment should say $\prod_{(x:A)} (P(x) \rightarrow Q(x))$ , not $\forall (x : A). (P(x) \Rightarrow Q(x))$ , as the latter notation has not been introduced yet.
Lemma 3.11.7	95-gce0131f	In the proof, $p$ should be $r$ to match the preceding definition of retraction.
Exercise 3.14	1162-ga97cb70	Should be to show that $\neg\neg A$ satisfies the recursion principle of $\ A\ $ but with only a propositional computation rule.
Lemma 4.1.1	87-g693e9b9	At the end of the proof, Lemma 3.11.8 should be cited as the reason why $\sum_{(g:A \rightarrow A)} (g = \text{id}_A)$ is contractible.
Theorem 4.2.3	275-g8ea9f71	In the proof, the path concatenations in the definitions of $\epsilon'$ and $\tau$ were written in reverse order.
Theorem 4.2.3	1043-gcfce4d7	In the proof, the type of $\tau(a)$ should be $f(\eta(a)) = \epsilon(f(g(f(a))))^{-1} \cdot (f(\eta(g(f(a)))) \cdot \epsilon(f(a)))$ , instead of $\epsilon(f(g(f(a))))^{-1} \cdot (f(\eta(g(f(a)))) \cdot \epsilon(f(a))) = f(\eta(a))$ .

Location	Fixed in	Change
Lemma 4.2.12	296-ge3dc076	In the proof, $(fgx, \epsilon(fx)) =_{\text{fib}_f(fx)} (x, \text{refl}_{fx})$ should be $(gfx, \epsilon(fx)) =_{\text{fib}_f(fx)} (x, \text{refl}_{fx})$ .
Corollary 4.3.3	272-gfd47093	At the end of the proof, the equivalence follows from the fact that $\text{ishae}(f)$ , not $\text{isContr}(f)$ , is a mere proposition.
Theorem 4.4.3	299-g85b729b	In the proof, $\text{lcoh}_f(g, \epsilon)$ should be $\text{rcoh}_f(g, \epsilon)$ , and the final displayed equation should have $\text{pr}_2$ applied to both occurrences of $P(fx)$ .
Lemma 4.7.3	265-g64000fb	The path concatenations in the definitions of $\varphi_b$ and $\psi_b$ (and subsequent equations) are reversed, and each $f(a)$ in the next two displayed equations should be $g(a)$ .
Theorem 4.7.6	275-g84ab032	The first equivalence in the proof is not by (2.15.9) but by Exercise 2.10.
Theorem 4.7.6	202-g775a3f0	The last equivalence in the proof is not by (2.15.10) but by Lemmas 3.11.8 and 3.11.9 and Exercise 2.10.
Theorem 4.8.3	205-gf9fe386	In the proof, $e \cdot \text{pr}_1$ should be $(\text{ua}(e))_*(\text{pr}_1)$ . Also, explain its computation better.
§4.9	114-gaba76c8	The point of Lemma 4.9.2 is that it follows from univalence without assuming function extensionality separately.
Corollary 4.9.3	484-g2ce1249	In the statement, “precomposition” should be “postcomposition”.
Theorem 4.9.4	746-g4d540d6	In the definition of $\psi$ in the proof, transport has to be along $\text{happly}(p, x)$ instead of along $p$ .
Exercise 4.2	358-g9543064	The text should be “Show that for any $A, B : \mathcal{U}$ , the following type is equivalent to $A \simeq B$ . Can you extract from this a definition of a type satisfying the three desiderata of $\text{isequiv}(f)$ ?”
§5.2	706-ged2c765	In the proof that $\mathbb{N} \simeq \mathbb{N}'$ , the definitions of $f$ and $g$ should be $\text{rec}_{\mathbb{N}}(\mathbb{N}', 0', \lambda n. \text{succ}')$ and $\text{rec}_{\mathbb{N}'}(\mathbb{N}, 0, \lambda n. \text{succ})$ respectively.
§5.3	125-g433f87e	In the definition of $\mathbf{N}^w$ , use $0_2$ for 0 and $1_2$ for $\text{succ}$ , to match the ordering of $0_2$ and $1_2$ in §1.8.
§5.3	551-g82b74bf	The definitions of $\mathbf{N}^w$ and $\text{List}(A)$ as W-types should be $W_{(b:2)} \text{rec}_2(\mathcal{U}, \mathbf{0}, \mathbf{1}, b)$ and $W_{(x:1+A)} \text{rec}_{1+A}(\mathcal{U}, \mathbf{0}, \lambda a. \mathbf{1}, x)$ .
§5.3	218-g42219cb	In the description of the constructor $\text{sup}$ , its second argument is more clearly written as $f : B(a) \rightarrow W_{(x:A)} B(x)$ .
§5.3	525-gb1957b8	In the computation rule, the recursive call to $\text{rec}$ is missing an argument. It should read $\text{rec}_{W_{(x:A)} B(x)}(E, e, \text{sup}(a, f)) \equiv e(a, f, (\lambda b. \text{rec}_{W_{(x:A)} B(x)}(E, e, f(b))))$ .
§5.3	570-g6ec04c3	In the verification that double computes as expected, $e_t$ should be $e_0$ and $e_f$ should be $e_1$ .
§5.4	554-g9b2a34b	The definition of the type of W-homomorphisms (just before Theorem 5.4.7) should read $W\text{Hom}_{A,B}((C, s_C), (D, s_D)) := \sum_{(f:C \rightarrow D)} \prod_{(a:A)} \prod_{(h:B(a) \rightarrow C)} f(s_C(a, h)) = s_D(a, f \circ h)$ .

Location	Fixed in	Change
§5.5	917-gd6960ad	In the first paragraph, the definition of $\mathbf{N}^w$ should be $W_{(b:2)}\text{rec}_2(\mathcal{U}, \mathbf{0}, \mathbf{1}, b)$ .
§5.5	608-g6af101f	In the computation rule for homotopy $W$ -types, the left-hand side should be $\text{rec}_{W_{(x:A)}^h B(x)}(E, e, \text{sup}(a, f))$ .
§5.5	1261-g4cdab82	In the commutative diagram preceding the definition of $W_s(A, B)$ , all occurrences of $x$ should be replaced with $a$ .
§5.5	1261-g4cdab82	In the definition of $W_s(A, B)$ , $\alpha(\text{sup}(x, f))$ should be $\alpha(\text{sup}(a, f))$ , and $\prod_{(a,f)}$ should be inserted after $\sum_{(\alpha)}$ .
Eq. (5.6.6)	912-g04d3fb6	In the preceding sentence, $\delta : d$ should be $\delta : D$ .
§5.7	908-g4b2eb10	The second two constructors of $\text{paritynat}$ should be $\text{esucc} : \text{paritynat}(1_2) \rightarrow \text{paritynat}(0_2)$ and $\text{osucc} : \text{paritynat}(0_2) \rightarrow \text{paritynat}(1_2)$ .
Theorem 5.8.2	139-gd5c5d01	In the proof of (iv) $\Rightarrow$ (i), the type of $D'$ should be $(\sum_{(b:A)} R(b)) \rightarrow \mathcal{U}$ .
Exercise 5.2	622-ga0bd007	The two functions should satisfy the same recurrence judgmentally.
Exercise 5.3	622-ga0bd007	The function should satisfy both recurrences judgmentally.
§5.8	171-gdc4966e	The subscript of $\text{refl}_A : a =_A a$ should be $a$ , i.e. $\text{refl}_a$ .
§6.2	54-gd4a47c2	Soon after Remark 6.2.1, the phrase “An element $b : P(\text{base})$ in the fiber over the constructor $\text{base} : \mathbb{N}$ ” should say $\text{base} : \mathbb{S}^1$ .
Lemma 6.2.8	423-gf763ae1	Theorems 2.11.3 and 2.11.5 are needed to put $q$ in the form required by the induction principle.
Lemma 6.3.2	417-g4aa6a15	Added Exercise 6.10: the function constructed in Lemma 6.3.2 is actually an inverse to $\text{happly}$ , so that the full function extensionality axiom follows from an interval type.
Lemma 6.4.2	625-g950efa9	In the second paragraph of the proof, the appeal to function extensionality should be omitted.
§6.4	327-g7cbe31c	In the first sentence after the proof of Lemma 6.4.6, “ $P : \mathbb{S}^2 \rightarrow P$ ” should be “ $P : \mathbb{S}^2 \rightarrow \mathcal{U}$ ”.
§6.4	1039-g30da4c6	In the sentence after the proof of Lemma 6.4.6, the type family in which $s$ is a dependent path should be $\lambda p. b =_p^P b$ instead of $P$ .
§6.6	289-gdefeb8c	In the induction principle for the torus, the types of $p'$ and $q'$ should be $b' =_p^P b'$ and $b =_q^P b$ respectively.
§6.7	289-gdefeb8c	In the induction principle for the torus, the types of $p'$ and $q'$ should be $b' =_p^P b'$ and $b =_q^P b$ respectively.
§6.9	468-g5472874	The induction principle for $\ A\ $ should conclude $f( a ) \equiv g(a)$ , not $f( a ) \equiv a$ . And in the hypotheses of the induction principle for $\ A\ _0$ and in the proof of Lemma 6.9.1, $v : p =_{u(x,y,p,q)}^B q$ should instead be $v : r =_{u(x,y,p,q)}^B s$ .



Location	Fixed in	Change
§6.9	860-gc7d862c	In the penultimate paragraph, the “unobjectionable” constructor for $\ A\ _0$ should begin “For every $f : S \rightarrow \ A\ _0$ ”, not “For every $f : S \rightarrow A$ ”.
Lemma 6.10.3	961-gde36592	The first sentence of the second paragraph of the proof should end with $g(x) = \overline{g \circ q}(x)$ .
Lemma 6.10.8	514-g18ade45	Instead of “is the set-quotient of $A$ by $\sim$ ”, the statement should say “satisfies the universal property of the set-quotient of $A$ by $\sim$ , and hence is equivalent to it”. In the proof, the second displayed equation should be $e'(g, s)(x, p) \equiv g(x)$ . The fourth displayed equation should be $e(e'(g, s)) \equiv e(g \circ \text{pr}_1) \equiv (g \circ \text{pr}_1 \circ q, \_)$ , the fifth should be $g(\text{pr}_1(q(x))) \equiv g(r(x)) = g(x)$ , and the proof should conclude with “ $g$ respects $\sim$ by the assumption $s$ ”.
Lemma 6.10.12	535-g0a9abfe	The “computation rules” satisfied by $f$ are only propositional equalities. Also, the proof requires transport across a few unmentioned equivalences.
Corollary 6.10.13	535-g0a9abfe	The defining clauses should use $:=$ rather than $\equiv$ (see the erratum for Lemma 6.10.12). Also, the first clause should say $\text{refl}_a$ rather than $\text{refl}_{\text{base}}$ .
Lemma 6.12.1	682-g3af5dbe	Three occurrences of $P$ in the statement should be $B$ .
Lemma 6.12.3	457-g411ec6d	The right-hand side of the displayed equation in the proof should be $(c(g(b)), D(b)(y))$ .
Lemma 6.12.3	961-gde36592	After the display we should have $p(b) : c(f(b)) = c(g(b))$ .
§6.12	519-gc99a54c	$f$ denotes a map $B \rightarrow A$ in this section and should not be re-used for functions defined by induction on $\sum_{(w:W)} P(w)$ ; we may use $k$ instead. Thus $f$ should be $k$ in the last sentence of Lemma 6.12.4; the first sentence of its proof; the second and third sentences of the paragraph after its proof; the last sentence of Lemma 6.12.5; the first, second, and last sentences of its proof; throughout the statement and proof of Lemma 6.12.7; the statement of Lemma 6.12.8; and the second sentence of its proof.
Lemma 6.12.4	537-gdf3b51d	In the display after the definition of $q$ , the transport in the first line should be with respect to $x \mapsto Q(\tilde{c}'(g(b), x))$ , and in the second line the subscript of $\text{ap}$ should be $x \mapsto \tilde{c}'(g(b), x)$ .
Lemma 6.12.4	961-gde36592	The subscript of $\text{ap}$ should also be $x \mapsto \tilde{c}'(g(b), x)$ in the third, fourth, and fifth displays. In the fourth and fifth displays, the path-concatenations should be in the other order. And in the fifth display, $\text{refl}_{g(b)}$ should be $\text{refl}_{c(g(b))}$ .
Lemma 6.12.7	501-ge895f81	Both occurrences of $P$ in the statement should be $Y$ , and both occurrences of $Q$ in the proof should be $Z$ .
Theorem 7.1.4	180-gb672a4d	In the last displayed equation of the proof, $q$ should be $r$ .

Location	Fixed in	Change
Theorem 7.1.10 §7.3	101-g713f48c 480-gdc84050	The base case in the proof is just Lemma 3.11.4. The third paragraph is wrong: in contrast to Remark 6.7.1, it <i>would</i> actually work to define $\ A\ _n$ omitting the hub point.
Theorem 7.2.2	1131-gc1748fa	In the second paragraph of the first proof, the codomain of the function $f(x, x)$ should be $x =_X x$ , not $x =_X y$ .
Lemma 7.2.4	644-g627c0a8	In the proof of the lemma, “If $x$ is $\text{inr}(f)$ ” should be “If $x$ is $\text{inr}(t)$ ”.
Theorem 7.3.12	412-gb9582fc	In the proof, encode and decode should be switched.
Lemma 7.5.12	801-g01922a8	The converse direction is false unless $Q$ is fiberwise merely inhabited. Also, the occurrences of $f(p)$ and $f(\text{pr}_2 w)$ in the proof should be just $p$ and $\text{pr}_2 w$ , respectively.
Lemma 7.5.14	367-g1c8c07e	In the proof that the first composite is the identity, all occurrences of $y$ should be $f(x)$ .
Theorem 7.7.4	658-g016f3a4	In the second paragraph of the proof, the first two occurrences of $\text{pr}_2$ (but not the third) should be $\text{pr}_1$ .
Exercise 7.2	101-ga366be2	“entires” should be “entirely”.
Exercise 7.2	683-g8941e50	This exercise needs more precise definitions of “diagram” and “colimit”.
Exercise 7.8	1074-gcd42187	$\text{AC}_{\infty, \infty}$ is not Theorem 2.15.7, but the identity function.
Exercise 7.8	603-ge113e08	The penultimate sentence should ask “Is $\text{AC}_{n, m}$ consistent with univalence for any $m \geq 0$ and any $n$ ?”.
Lemma 8.1.8	535-g0a9abfe	The proof by induction on $n : \mathbb{Z}$ is justified by Lemma 6.10.12, not Corollary 6.10.13.
Lemma 8.1.12	535-g0a9abfe	The clauses defining $q_z$ should use $:=$ rather than $:\equiv$ (see the erratum for Lemma 6.10.12).
Theorem 8.2.1	1062-gf3bfeae	In the proof, $E$ is not $(n + 1)$ -connected but $(n + 1)$ -truncated.
Lemma 8.4.4	1181-g3e51973	In the proof, $(x : A)$ should be $(x : X)$ .
Corollary 8.4.8	1023-gf188aeb	The proof requires a separate argument for $k = 0$ .
Theorem 8.5.1	256-g9e6fcb8	The phrase “whose fibers are $S^1$ ” should be “whose fiber over the basepoint is $S^1$ ”. The same change should be made in Exercises 8.8 and 8.9.
Lemma 8.5.3	1062-gf3bfeae	In the definition of $E^{\text{tot}}$ in the proof, $e_C$ should be $e_X$ .
Lemma 8.6.1	396-g868335b	In the proof, the function $k$ should have type $\prod_{(a:A)} P(f(a))$ . It should also be named $\ell$ , to avoid confusion with the integer $k$ .
Definition 8.6.5	87-g3f977b2	In the second displayed equation in the proof, $\text{merid}(x_1)$ should be $\text{merid}(x_1)^{-1}$ .
Lemma 8.6.2	1203-g7464bf1	The type family $P$ defined in the proof should instead be called $Q$ , to avoid clashes with the type family $P$ assumed in the statement.
Lemma 8.6.2	399-g8897c94	In the last sentence of the proof, “ $(n - 1)$ -connected” should be “ $(n - 1)$ -truncated”.



Location	Fixed in	Change
Lemma 8.6.10	88-g0c0be67	The type of $m$ should be $a_1 = a_2$ , the second display should begin with $C(a_1, \text{transport}^B(m^{-1}, b))$ , and the proof should say “we may assume $a_2$ is $a_1$ and $m$ is $\text{refl}_{a_1}$ ”.
§8.6	165-gd5584c6	In (8.6.11), $r''$ should be $r'$ , the end point of $r$ should be $\text{transport}^B(\text{merid}(x_0)^{-1}, q)$ , and obtaining $r'$ requires also identifying this with $q \cdot \text{merid}(x_0)^{-1}$ . Similarly, in (8.6.12), the end point of $r$ should be $\text{transport}^B(\text{merid}(x_1)^{-1}, q)$ .
§8.6	474-g5289470	$\pi_3(\mathbb{S}^2) = \mathbb{Z}$ should be stated as Corollary 8.6.19, following from Corollary 8.5.2 and Theorem 8.6.17.
Theorem 8.8.3	1092-ge3b8b71	After applying the induction hypothesis, it additionally needs to be checked that for every path $p : a = a$ the map $\pi_k(\text{ap}_f) : \pi_k(x = x, p) \rightarrow \pi_k(f(x) = f(x), \text{ap}_f(p))$ is a bijection.
§8.9	1154-g301662b	In the strengthening of condition (iii) from Lemma 8.9.1, the right side should read just “ $c$ ” instead of “ $c.a$ ”.
Definition 9.2.1	807-gebec78b	In Item (iv), it should read “ $\text{hom}_A(b, c)$ ” instead of “ $\text{hom}_B(b, c)$ ”.
§9.4	1218-gcb6ba30	Just before Definition 9.4.6, it should say “However, if $A$ is not a category” instead of “However, if $B$ is not a category”.
Theorem 9.5.4	971-g6096085	The sequence of equations at the end of the proof should begin with $\alpha_{a'}(f) = \alpha_{a'}(\gamma a_{a,a'}(f)(1_a))$ , and thereafter the subscripts should remain $a, a'$ rather than $a', a$ .
Definition 9.8.1	897-g94fb722	In (iv), “if $f : \text{hom}_X(x, y)$ ” should be “if $f : \text{hom}_X(x, y)$ and $g : \text{hom}_X(y, z)$ ”.
§9.8	1111-g3332a31	The type of objects $A_0$ of the precategory $A$ of $(P, H)$ -structures should be defined as $\sum_{(x:X_0)} Px$ , not $\sum_{(x:X)} Px$ .
Chapter 9	966-g04374f5	The first sentence after Theorem 9.9.4 should begin “Therefore, if a precategory $A$ admits a weak equivalence functor $A \rightarrow \hat{A}$ into a category...”.
Theorem 9.9.5	313-g8ee79db	In the second proof, the third constructor of $\hat{A}_0$ is unneeded; it follows from the fourth constructor and path induction. In the fifth constructor, $j(g) \cdot j(f)$ should be $j(f) \cdot j(g)$ , and similarly throughout the proof. Finally, for consistency, the 1-truncation constructor should be included explicitly (this was intended to be implied by “higher inductive 1-type”).
Chapter 9 Notes	379-ga57eab2	It should be mentioned that Hofmann and Streicher (1998) also considered this definition of category.
Theorem 10.3.20	140-g55de417	The second sentence of the proof should say “By well-founded induction on $A$ , suppose $A/b$ is accessible for all $b < a$ ”.
Lemma 10.3.22	140-gd7f8960	The statement should say $X : \mathcal{U}$ rather than $X : \mathcal{U}_{\mathcal{U}}$ .
Theorem 10.4.3	140-gcca0bcf	The penultimate sentence of the proof should say “if $a < b$ and $b < c$ ” rather than “if $a < b$ and $a < c$ ”.
Theorem 10.4.4	871-g85bcd11	The statement of (i) should end with $Y : \mathcal{P}_+(X)$ , not $Y : \mathcal{P}(X)$ .

Location	Fixed in	Change
§10.5	753-gc87ce23	The second clause in the induction principle for $V$ should say “Verify that if $f : A \rightarrow V$ and $g : B \rightarrow V$ satisfy (10.5.2), then $h(\text{set}(A, f)) =_q^P h(\text{set}(B, g))$ , where $q$ is the path arising from the second constructor of $V$ and (10.5.2), assuming inductively that $h(f(a)) =_p^P h(g(b))$ whenever $p : f(a) = g(b)$ .”
§10.5	706-ged2c765	The proof that membership is well-defined should end with “hence $x = g(b)$ and $x \in \text{set}(B, g)$ .”
§10.5	1056-g4060c2b	In the definition of $V\text{-set}$ , the notation $v \in V$ should be $v : V$ .
Theorem 10.5.8	708-g6f53189	In the pairing axiom, the pair class should be denoted $\{u, v\}$ , not $u \cup v$ .
Theorem 10.5.8	723-g9cf5b44	The replacement axiom should be given $x : V$ (not $a : V$ ) and the displayed class should be $\{y \mid \exists(z : V). z \in x \wedge y = r(z)\}$ . Its proof should begin “let $C$ denote the class in question.”
Theorem 10.5.8	706-ged2c765	In the proof of the function set axiom, “the types of elements $[u] \rightarrow V$ and $[u] \rightarrow V$ ” should be “the types of members $[u] \rightarrow V$ and $[v] \rightarrow V$ .”
Exercise 10.12	1053-ge13dd65	Extra parentheses around $\forall(x \in v). \exists(y). R(x, y)$ are needed to make the formula unambiguous.
Exercise 10.13	1053-ge13dd65	Extra parentheses around $\forall(y \in x). \exists(z \in V). z \in y$ are needed to make the formula unambiguous.
Exercise 10.13	1056-g4060c2b	The notation $\in V$ should be $: V$ .
Lemma 11.2.2	165-gb002a64	The statement should say “For all $x : \mathbb{R}_d$ and $q : \mathbb{Q}$ , $L_x(q) \Leftrightarrow (q < x)$ and $U_x(q) \Leftrightarrow (x < q)$ ”.
Theorem 11.2.4	165-g179b359	In the proof, the sentence beginning “From $0 < ac$ it follows” should be replaced by “From $0 < ac$ and $0 < bc$ it follows that $a, b$ , and $c$ are either all positive or all negative. Hence either $0 < a < x$ or $x < b < 0$ , so that $x \# 0$ ”.
§11.2.2	832-g0cb658e	In the second paragraph, at “From this we get”, the universal quantification should be over $\delta$ as well.
§11.3.2	1209-g3e5ad94	In the statement of $(\mathbb{R}_c, \sim)$ -recursion, “ $f(x) : A$ ” should be “ $f(\lim(x)) : A$ ”.
Theorem 11.3.16	1069-g3b333d5	In the description of openness of $\approx$ , “ $\exists(\epsilon : \mathbb{Q}_+)$ .” should be “ $\exists(\delta : \mathbb{Q}_+)$ .”.
Lemma 11.4.1	87-g82b27c3	(11.4.2) should be $c : \prod_{(q,r:\mathbb{Q})} (q < r) \rightarrow (q < x) + (x < r)$ , and therefore the use of $c$ in the proof should be $c(s, t)$ rather than $c(x, s, t)$ .
Theorem 11.5.6	merge of ad72d4e	In the proof, $n : \mathbb{N}$ should be $k : \mathbb{N}$ . And the range of $i$ should be $0 \leq i \leq k$ . Also in the last equation, $r(\lim x) = \ell$ should be $\lim x = \ell$ .
§11.6	1189-ga9c35f0	The inductive case of $\iota_{Q_D}$ should be defined as $\iota_{Q_D}(a/2^n) := \{\iota_{Q_D}(a/2^n - 1/2^n) \mid \iota_{Q_D}(a/2^n + 1/2^n)\}$ .

Location	Fixed in	Change
Example 11.6.18	636-g827e7ea	In the first bullet point, to prove $x^L + z < x + z$ requires a No-induction on $z$ , since only when $z$ is defined by a cut can we say that $x^L + z$ is a left option of $x + z$ .
Exercise 11.13	222-g3453cf1	This is the intermediate value theorem, not the mean value theorem.
Example 11.6.18	980-ge9d0398	For the codomain of the outer recursion, the conditions should be $(x < y) \rightarrow (g(x) < g(y))$ and $(x \leq y) \rightarrow (g(x) \leq g(y))$ . In the first bullet of the verification that inequalities are preserved, the outer inductive hypotheses give non-strict inequalities $x^L + y \leq x^L + z$ and $x^R + y \leq x^R + z$ , and no additional No-induction on $z$ is required (it is already known to be defined by a cut).
Example 11.6.18	980-ge9d0398	The verification that Conway's definition of $x + y$ is a surreal number (i.e. all its left options are $<$ all its right options) was omitted. This requires turning the inner recursion into an inner induction with codomain a varying subset of No, as in Theorem 11.6.7.
Appendix A	165-g76db618	After the introduction of the judgment " $\Gamma \text{ ctx}$ " in the Preliminaries, the sentence beginning "Therefore, if $\Gamma \vdash a : A, \dots$ " should say instead "In particular, therefore, if $\Gamma \vdash a : A, \dots$ ".
Appendix A.2.1	64-g7c2312e	Clarify the distinction between typing judgments and context well-formedness judgments, and remove the $\vdash$ from the notation for the latter.
Appendix A.2.5	26-gcd691e8	In $\Sigma$ -COMP and the following paragraph, $y.C$ should be $z.C$ , and "we bind $\dots y$ in $C$ " should likewise say $z$ .
Appendix A.2.8	338-g4e1c688	The $c$ argument in the eliminator for <b>1</b> (in the <b>1</b> -ELIM and <b>1</b> -COMP rules) should not bind a variable of type <b>1</b> .
Appendix A.2.10	578-ga4b94a5	The unbased eliminator for the identity type should be named $\text{ind}_{=A}$ , not $\text{ind}'_{=A}$ .