Solutions of Sample questions

3. MARKS QUESTIONS

1. Without expansion, illustrate that $\begin{vmatrix} 6 & 1 & 3 & 2 \\ -2 & 0 & 1 & 4 \\ 3 & 6 & 1 & 2 \\ -4 & 0 & 2 & 8 \end{vmatrix} = 0.$

$$\begin{vmatrix} 6 & 1 & 3 & 2 \\ -2 & 0 & 1 & 4 \\ 3 & 6 & 1 & 2 \\ -4 & 0 & 2 & 8 \end{vmatrix} \rightarrow \begin{vmatrix} 6 & 1 & 3 & 2 \\ -2 & 0 & 1 & 4 \\ 3 & 6 & 1 & 2 \\ 0 & 0 & 0 & 0 \end{vmatrix} [R_4' = R_4 - 2R_2]$$

$$Since\ the\ determinant\ has\ one\ row\ whose\ every\ element\ is\ zero$$

Therefore, the given determinant is zero.

2. If det$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$=5 , evaluate the determinant of the matrix $\begin{bmatrix} 2a_1 & a_2 & a_3 \\ 6b_1 & 3b_2 & b3_3 \\ 2c_1 & c_2 & c_3 \end{bmatrix}$.

If det$\begin{bmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{bmatrix}$=5 , evaluate the determinant of the matrix $\begin{bmatrix} 2a_1 & a_2 & a_3 \\ 6b_1 & 3b_2 & 3b_3 \\ 2c_1 & c_2 & c_3 \end{bmatrix}$.

$$\begin{vmatrix} 2a_1 & a_2 & a_3 \\ 6b_1 & 3b_2 & 3b_3 \\ 2c_1 & c_2 & c_3 \end{vmatrix} = 2\begin{vmatrix} a_1 & a_2 & a_3 \\ 3b_1 & 3b_2 & 3b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 2.3\begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = 6.5 = 30$$

Long: 4:: Use Cramer‟s rule to solve the system – 4x + 2y – 9z = 2, 3x + 4y + z = 5, x – 3y + 2z = 8.

The equations can be expressed as

$4x - 2y + 9z + 2 = 0$

$3x + 4y + z - 5 = 0$

$x - 3y + 2z - 8 = 0$

Use Cramer's Rule to find the values of x, y, z.

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{z}{D_z} = \frac{-1}{D}$$

$$D_x = \begin{vmatrix} -2 & 9 & 2 \\ 4 & 1 & -5 \\ -3 & 2 & -8 \end{vmatrix}$$

$$= -2 \times \begin{vmatrix} 1 & -5 \\ 2 & -8 \end{vmatrix} - 9 \times \begin{vmatrix} 4 & -5 \\ -3 & -8 \end{vmatrix} + 2 \times \begin{vmatrix} 4 & 1 \\ -3 & 2 \end{vmatrix}$$

$$= -2 \times (1 \times (-8) - (-5) \times 2) - 9 \times (4 \times (-8) - (-5) \times (-3)) + 2 \times (4 \times 2 - 1 \times (-3))$$

$$= -2 \times (-8 + 10) - 9 \times (-32 - 15) + 2 \times (8 + 3)$$

$$= -2 \times (2) - 9 \times (-47) + 2 \times (11)$$

$= -4+423+22=441$

$$D_y = \begin{vmatrix} 4 & 9 & 2 \\ 3 & 1 & -5 \\ 1 & 2 & -8 \end{vmatrix}$$

$$= 4 \times \begin{vmatrix} 1 & -5 \\ 2 & -8 \end{vmatrix} -9 \times \begin{vmatrix} 3 & -5 \\ 1 & -8 \end{vmatrix} +2 \times \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix}$$

$$= 4 \times (1 \times (-8) - (-5) \times 2) - 9 \times (3 \times (-8) - (-5) \times 1) + 2 \times (3 \times 2 - 1 \times 1)$$

$$= 4 \times (-8 + 10) - 9 \times (-24 + 5) + 2 \times (6 - 1)$$

$$= 4 \times (2) - 9 \times (-19) + 2 \times (5)$$

$$= 8 + 171 + 10$$

$$= 189$$

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{z}{D_z} = \frac{-1}{D}$$

$$\therefore \frac{x}{441} = \frac{-y}{189} = \frac{z}{-252} = \frac{-1}{-63}$$

$$\therefore \frac{x}{441} = \frac{-1}{-63}, \frac{-y}{189} = \frac{-1}{-63}, \frac{z}{-252} = \frac{-1}{-63}$$

$$\therefore x = \frac{-441}{-63}, y = \frac{189}{-63}, z = \frac{252}{-63}$$

$$\therefore x = 7, y = -3, z = -4$$

3.Use Cramer‟s rule to solve the system x-y=2, x+4y=5.

The equations can be expressed as

$x - y - 2 = 0$

$x + 4y - 5 = 0$

Use Cramer's Rule to find the values of x, y, z.

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{1}{D}$$

$$D_x = \begin{vmatrix} -1 & -2 \\ 4 & -5 \end{vmatrix}$$

$= -1 \times (-5) - (-2) \times 4$

$= 5 + 8$

$= 13$

$$D_y = \begin{vmatrix} 1 & -2 \\ 1 & -5 \end{vmatrix}$$

$$= 1 \times (-5) - (-2) \times 1$$

$$= -5 + 2$$

$$= -3$$

$$D = \begin{vmatrix} 1 & -1 \\ 1 & 4 \end{vmatrix}$$

$$= 1 \times 4 - (-1) \times 1$$

$$= 4 + 1$$

$$= 5$$

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{1}{D}$$

$$\therefore \frac{x}{13} = \frac{-y}{-3} = \frac{1}{5}$$

$$\therefore \frac{x}{13} = \frac{1}{5}, \frac{-y}{-3} = \frac{1}{5}$$

4. Use Cramer‟s rule to solve the system x+y+z=0, 2x-y-4z=15, x-2y-z=7.

B.Tech CSE and 1st semsester

The equations can be expressed as

$x + y + z + 0 = 0$

$2x - y - 4z - 15 = 0$

$x - 2y - z - 7 = 0$

Use Cramer's Rule to find the values of x, y, z.

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{z}{D_z} = \frac{-1}{D}$$

$$D_x = \begin{vmatrix} 1 & 1 & 0 \\ -1 & -4 & -15 \\ -2 & -1 & -7 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} -4 & -15 \\ -1 & -7 \end{vmatrix} - 1 \times \begin{vmatrix} -1 & -15 \\ -2 & -7 \end{vmatrix} + 0 \times \begin{vmatrix} -1 & -4 \\ -2 & -1 \end{vmatrix}$$

$$= 1 \times (-4 \times (-7) - (-15) \times (-1)) - 1 \times (-1 \times (-7) - (-15) \times (-2)) + 0 \times (-1 \times (-1) - (-$$

$$= 1 \times (28 - 15) - 1 \times (7 - 30) + 0 \times (1 - 8)$$

$$= 1 \times (13) - 1 \times (-23) + 0 \times (-7)$$

$$= 13 + 23 + 0$$

$$D_y = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -4 & -15 \\ 1 & -1 & -7 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} -4 & -15 \\ -1 & -7 \end{vmatrix} - 1 \times \begin{vmatrix} 2 & -15 \\ 1 & -7 \end{vmatrix} + 0 \times \begin{vmatrix} 2 & -4 \\ 1 & -1 \end{vmatrix}$$

$$= 1 \times (-4 \times (-7) - (-15) \times (-1)) - 1 \times (2 \times (-7) - (-15) \times 1) + 0 \times (2 \times (-1) - (-4) \times 1)$$

$$= 1 \times (28 - 15) - 1 \times (-14 + 15) + 0 \times (-2 + 4)$$

$$= 1 \times (13) - 1 \times (1) + 0 \times (2)$$

$$= 13 - 1 + 0$$

$$= 12$$

$$D_z = \begin{vmatrix} 1 & 1 & 0 \\ 2 & -1 & -15 \\ 1 & -2 & -7 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} -1 & -15 \\ -2 & -7 \end{vmatrix} - 1 \times \begin{vmatrix} 2 & -15 \\ 1 & -7 \end{vmatrix} + 0 \times \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}$$

=(7-30)-(-14+15)+0
=-23-1
=-24

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 2 & -1 & -4 \\ 1 & -2 & -1 \end{vmatrix}$$

$$= 1 \times \begin{vmatrix} -1 & -4 \\ -2 & -1 \end{vmatrix} - 1 \times \begin{vmatrix} 2 & -4 \\ 1 & -1 \end{vmatrix} + 1 \times \begin{vmatrix} 2 & -1 \\ 1 & -2 \end{vmatrix}$$

$$= 1 \times (-1 \times (-1) - (-4) \times (-2)) - 1 \times (2 \times (-1) - (-4) \times 1) + 1 \times (2 \times (-2) - (-1) \times 1)$$

$$= 1 \times (1 - 8) - 1 \times (-2 + 4) + 1 \times (-4 + 1)$$

$$= 1 \times (-7) - 1 \times (2) + 1 \times (-3)$$

$$= -7 - 2 - 3$$

$$= -12$$

$$\frac{x}{D_x} = \frac{-y}{D_y} = \frac{z}{D_z} = \frac{-1}{D}$$

$$\therefore \frac{x}{36} = \frac{-y}{12} = \frac{z}{-24} = \frac{-1}{-12}$$

$$\therefore \frac{x}{36} = \frac{-1}{-12}, \frac{-y}{12} = \frac{-1}{-12}, \frac{z}{-24} = \frac{-1}{-12}$$

$$\therefore x = \frac{-36}{-12}, y = \frac{12}{-12}, z = \frac{24}{-12}$$

5. Calculate the inverse, if it exists, of the matrix $\begin{pmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{pmatrix}$.

$$= \begin{bmatrix} 0 & -1 & 1 \\ 2 & -3 & 2 \\ 3 & -3 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ -1 & -3 & -3 \\ 1 & 2 & 2 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|} \times Adj(A)$

$$= \frac{1}{-1} \times \begin{bmatrix} 0 & 2 & 3 \\ -1 & -3 & -3 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{vmatrix}$$

$$= 0 \times \begin{vmatrix} 3 & 3 \\ -2 & -2 \end{vmatrix} + 2 \times \begin{vmatrix} 1 & 3 \\ -1 & -2 \end{vmatrix} - 3 \times \begin{vmatrix} 1 & 3 \\ -1 & -2 \end{vmatrix}$$

$$= 0 \times (3 \times (-2) - 3 \times (-2)) + 2 \times (1 \times (-2) - 3 \times (-1)) - 3 \times (1 \times (-2) - 3 \times (-1))$$

$$= 0 \times (-6 + 6) + 2 \times (-2 + 3) - 3 \times (-2 + 3)$$

$$= 0 \times (0) + 2 \times (1) - 3 \times (1)$$

$$= 0 + 2 - 3$$

$$= -1$$

$$Adj(A) = \mathrm{Adj}\begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

$$Adj(A) = \text{Adj}\begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{vmatrix} 3 & 3 \\ -2 & -2 \end{vmatrix} & -\begin{vmatrix} 1 & 3 \\ -1 & -2 \end{vmatrix} & +\begin{vmatrix} 1 & 3 \\ -1 & -2 \end{vmatrix} \\ -\begin{vmatrix} -2 & -3 \\ -2 & -2 \end{vmatrix} & +\begin{vmatrix} 0 & -3 \\ -1 & -2 \end{vmatrix} & -\begin{vmatrix} 0 & -2 \\ -1 & -2 \end{vmatrix} \\ +\begin{vmatrix} -2 & -3 \\ 3 & 3 \end{vmatrix} & -\begin{vmatrix} 0 & -3 \\ 1 & 3 \end{vmatrix} & +\begin{vmatrix} 0 & -2 \\ 1 & 3 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} +(3\times(-2)-3\times(-2)) & -(1\times(-2)-3\times(-1)) & +(1\times(-2)-3\times(-1)) \\ -(-2\times(-2)-(-3)\times(-2)) & +(0\times(-2)-(-3)\times(-1)) & -(0\times(-2)-(-2)\times(-1)) \\ +(-2\times 3-(-3)\times 3) & -(0\times 3-(-3)\times 1) & +(0\times 3-(-2)\times 1) \end{bmatrix}^T$$

$$= \begin{bmatrix} +(-6+6) & -(-2+3) & +(-2+3) \\ -(4-6) & +(0-3) & -(0-2) \\ +(-6+9) & -(0+3) & +(0+2) \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & -1 & 1 \\ 2 & -3 & 2 \\ 3 & -3 & 2 \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 2 & 3 \\ -1 & -3 & -3 \\ 1 & 2 & 2 \end{bmatrix}$$

Now, $A^{-1} = \frac{1}{|A|} \times Adj(A)$

$$= \frac{1}{-1} \times \begin{bmatrix} 0 & 2 & 3 \\ -1 & -3 & -3 \\ 1 & 2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -2 & -3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix}$$

6.Without expanding illustrate that $\begin{vmatrix} 0 & b-a & c-a \\ a-b & o & c-b \\ a-c & b-c & 0 \end{vmatrix} = 0.$

Step 1: Identify a Common Factor

Each row consists of differences of variables a, b, c. Notice that:

- The sum of all columns is **zero**:

  $$(0 + (a - b) + (a - c)) = 0, \quad ((b - a) + 0 + (b - c)) = 0, \quad ((c - a) + (c - b) + 0) = 0$$

  This means the **columns are linearly dependent**, implying that the determinant must be **zero**.

Step 2: Row Transformations

Observe that each row is a **linear combination of the others**:

- $R_1 + R_2 + R_3 = 0$, meaning one row can be expressed as a sum of the other two.
- If any row is a linear combination of the others, the determinant is **zero**.

Conclusion

Since the matrix has **linearly dependent** rows, its determinant must be **zero**:

$$\det(A) = 0.$$

7.If x= - 4 is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, calculate the other roots.

$$\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$$

$$x(x^2 - 2) - 2(x - 3) + 3(2 - 3x) = 0$$

⇨ $x^3 - 2x - 2x + 6 + 6 - 9x = 0$

⇨ $x^3 - 13x + 12 = 0$

⇨ $x^2(x - 1) + x(x - 1) - 12(x - 1) = 0$

⇨ $(x - 1)(x^2 + x - 12) = 0$

⇨ $(x - 1)(x + 4)(x - 3) = 0$

⇨ $x = 1, x = -4, x = 3$

8.If $\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$ then calculate values of x.

$$\begin{vmatrix} 4-x & 4+x & 4+x \\ 4+x & 4-x & 4+x \\ 4+x & 4+x & 4-x \end{vmatrix} = 0$$

⇨ $\begin{vmatrix} -2x & 0 & 4+x \\ 0 & -2x & 4+x \\ -2x & -2x & 4-x \end{vmatrix} = 0 \, [C_1' = C_1 - C_3,\ C_2' = C_2 - C_3]$

⇨ $\begin{vmatrix} -2x & 0 & 4+x \\ 0 & -2x & 4+x \\ 0 & 0 & -4-3x \end{vmatrix} = 0 \, [R_3' = R_3 - (R_1 + R_2)]$

⇨ $(-2x)(-2x)(-4-3x) = 0$

⇨ $-16x^2 - 12x^3 = 0$

⇨ $x^2(4+3x) = 0$

⇨ $x = 0, x = 0, x = -3/4$

9.If a+b+c $\neq 0$ and $\begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$ then illustrate that a = b = c.

$$\triangle = \begin{vmatrix} a & b & c \\ b & c & a \\ c & a & b \end{vmatrix} = 0$$

$$\Rightarrow C_1 = C_1 + C_2 + C_3$$

$$\Rightarrow \triangle = \begin{vmatrix} a+b+c & b & c \\ a+b+c & c & a \\ a+b+c & a & b \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & b & c \\ 1 & c & a \\ 1 & a & b \end{vmatrix} = 0$$

$$\Rightarrow R_2 = R_2 - R_1; R_3 = R_3 - R_1$$

$$\Rightarrow (a+b+c)\begin{vmatrix} 1 & b & c \\ 0 & c-b & a-c \\ 0 & a-b & b-c \end{vmatrix} = 0$$

expanding along C_1

$$\Rightarrow (a+b+c) \times 1 \begin{vmatrix} c-b & a-c \\ a-b & b-c \end{vmatrix} = (a+b+c)\,[(c-b)(b-c)-(a-b)(a-c)]$$

$$\Rightarrow (a+b+c)\,[bc - b^2 + bc - c^2 - a^2 + ab - c^2 - bc + ac] = 0$$

Rewriting the equation:

$$a^2 + b^2 + c^2 - ab - bc - ca = 0$$

we can rearrange it as:

$$a^2 + b^2 + c^2 = ab + bc + ca.$$

Step 2: Rewrite as a Sum of Squares

The identity:

$$\frac{1}{2}\left((a-b)^2 + (b-c)^2 + (c-a)^2\right) = a^2 + b^2 + c^2 - ab - bc - ca$$

substitutes into our equation, giving:

$$\frac{1}{2}\left((a-b)^2 + (b-c)^2 + (c-a)^2\right) = 0.$$

Step 3: Solve for Equal Values

Since a sum of squares is zero if and only if each squared term is zero, we get:

$$(a-b)^2 = 0, \quad (b-c)^2 = 0, \quad (c-a)^2 = 0.$$

This implies:

$$a - b = 0 \Rightarrow a = b, \quad b - c = 0 \Rightarrow b = c, \quad c - a = 0 \Rightarrow c = a.$$

Thus, we conclude:

$$a = b = c.$$

10.If A= $\begin{pmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{pmatrix}$, then calculate the value of λ for which A^{-1} exists.

If A= $\begin{pmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{pmatrix}$, then calculate the value of λ for which A^{-1} exists.

$$\begin{vmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{vmatrix} = 2(6-5) - \lambda(0-5) - 3(0-2) = 2 + 5\lambda + 6 = 5\lambda + 8$$

Therefore for $\lambda \neq -8/5$ A^{-1} exists

5 marks: Long Question

1.Establish that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$.

LHS

$$\begin{vmatrix} 1 & a & -bc \\ 1 & b & -ca \\ 1 & c & -ab \end{vmatrix}$$

$R_1 \to R_1 - R_2$

$$\begin{vmatrix} 0 & a-b & c(a-b) \\ 1 & b & -ca \\ 1 & c & -ab \end{vmatrix}$$

$R_2 \to R_2 - R_3$

$$\begin{vmatrix} 0 & a-b & c(a-b) \\ 0 & b-c & a(b-c) \\ 1 & c & -ab \end{vmatrix}$$

$$(a-b)(b-c)\begin{vmatrix} 0 & 1 & c \\ 0 & 1 & a \\ 1 & c & -ab \end{vmatrix}$$

$$(a-b)(b-c)(a-c)$$

RHS

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & a-b & (a-b)(a+b) \\ 0 & b-c & (a-c)(b+c) \\ 1 & b & c^2 \end{vmatrix}$$

Taking out common $(a-b)\,\&\,(b-c)$ from R_1 and R_2 respevtively

$$= (a-b)(a-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expanding along c_1

$$= (a-b)(b-c)[b+c-a-b]$$

$$= (a-b)(b-c)(c-a)$$

2. Show that $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (b-c)(c-a)(a-b).$

B.Tech CSE and 1st semsester

$$\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & a-b & a^2-b^2 \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & a-b & (a-b)(a+b) \\ 0 & b-c & (a-c)(b+c) \\ 1 & b & c^2 \end{vmatrix}$$

Taking out common $(a-b)\,\&\,(b-c)$ from R_1 and R_2 respevtively

$$= (a-b)(a-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expanding along c_1

$$= (a-b)(b-c)[b+c-a-b]$$

$$= (a-b)(b-c)(c-a)$$

3. Recall that a square matrix A is said to be skew-symmetric if $A^T = -A$. If A is a 5 ×5 skew-symmetric matrix, show that det(A) = 0.

sol:

Proof. Let $A = (a_{ij})$ be a skew symmetric matrix of order n and n is odd. Since (a_{ij}) is skew symmetric, $a_{ij} = -a_{ji}$.

$$\det A = \begin{vmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \cdots & \cdots & & \\ a_{n1} & a_{n2} & \cdots & a_{nn} \end{vmatrix} = \begin{vmatrix} -a_{11} & -a_{21} & \cdots & -a_{n1} \\ -a_{12} & -a_{22} & \cdots & -a_{n2} \\ & \cdots & \cdots & \\ -a_{1n} & -a_{2n} & \cdots & -a_{nn} \end{vmatrix}.$$

$$= (-1)^n \begin{vmatrix} a_{11} & a_{21} & \cdots & a_{n1} \\ a_{12} & a_{22} & \cdots & a_{n2} \\ & \cdots & \cdots & \\ a_{1n} & a_{2n} & \cdots & a_{nn} \end{vmatrix} = (-1)^n \det A^t$$

$= -\det A$, since n is odd.

Therefore $2 \det A = 0$ and this implies $\det A = 0$.

4. *See in the short question part.*

5. Calculate the inverse of the matrix $\begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}$

$$|A| = \begin{vmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{vmatrix}$$

$$= 2 \times \begin{vmatrix} 11 & -7 \\ 3 & -2 \end{vmatrix} + 17 \times \begin{vmatrix} -1 & -7 \\ 0 & -2 \end{vmatrix} + 11 \times \begin{vmatrix} -1 & 11 \\ 0 & 3 \end{vmatrix}$$

$$= 2 \times (11 \times (-2) - (-7) \times 3) + 17 \times (-1 \times (-2) - (-7) \times 0) + 11 \times (-1 \times 3 - 11 \times 0)$$

$$= 2 \times (-22 + 21) + 17 \times (2 + 0) + 11 \times (-3 + 0)$$

$$= 2 \times (-1) + 17 \times (2) + 11 \times (-3)$$

$$= -2 + 34 - 33$$

$$= -1$$

$$Adj(A) = \text{Adj} \begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}$$

$$Adj(A) = \text{Adj}\begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{vmatrix} 11 & -7 \\ 3 & -2 \end{vmatrix} & -\begin{vmatrix} -1 & -7 \\ 0 & -2 \end{vmatrix} & +\begin{vmatrix} -1 & 11 \\ 0 & 3 \end{vmatrix} \\ -\begin{vmatrix} -17 & 11 \\ 3 & -2 \end{vmatrix} & +\begin{vmatrix} 2 & 11 \\ 0 & -2 \end{vmatrix} & -\begin{vmatrix} 2 & -17 \\ 0 & 3 \end{vmatrix} \\ +\begin{vmatrix} -17 & 11 \\ 11 & -7 \end{vmatrix} & -\begin{vmatrix} 2 & 11 \\ -1 & -7 \end{vmatrix} & +\begin{vmatrix} 2 & -17 \\ -1 & 11 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} +(11 \times (-2) - (-7) \times 3) & -(-1 \times (-2) - (-7) \times 0) & +(-1 \times 3 - 11 \times 0) \\ -(-17 \times (-2) - 11 \times 3) & +(2 \times (-2) - 11 \times 0) & -(2 \times 3 - (-17) \times 0) \\ +(-17 \times (-7) - 11 \times 11) & -(2 \times (-7) - 11 \times (-1)) & +(2 \times 11 - (-17) \times (-1)) \end{bmatrix}^T$$

$$= \begin{bmatrix} +(-22 + 21) & -(2 + 0) & +(-3 + 0) \\ -(34 - 33) & +(-4 + 0) & -(6 + 0) \\ +(119 - 121) & -(-14 + 11) & +(22 - 17) \end{bmatrix}^T$$

$$Adj(A) = \text{Adj}\begin{bmatrix} 2 & -17 & 11 \\ -1 & 11 & -7 \\ 0 & 3 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} +\begin{vmatrix} 11 & -7 \\ 3 & -2 \end{vmatrix} & -\begin{vmatrix} -1 & -7 \\ 0 & -2 \end{vmatrix} & +\begin{vmatrix} -1 & 11 \\ 0 & 3 \end{vmatrix} \\ -\begin{vmatrix} -17 & 11 \\ 3 & -2 \end{vmatrix} & +\begin{vmatrix} 2 & 11 \\ 0 & -2 \end{vmatrix} & -\begin{vmatrix} 2 & -17 \\ 0 & 3 \end{vmatrix} \\ +\begin{vmatrix} -17 & 11 \\ 11 & -7 \end{vmatrix} & -\begin{vmatrix} 2 & 11 \\ -1 & -7 \end{vmatrix} & +\begin{vmatrix} 2 & -17 \\ -1 & 11 \end{vmatrix} \end{bmatrix}^T$$

$$= \begin{bmatrix} +(11 \times (-2) - (-7) \times 3) & -(-1 \times (-2) - (-7) \times 0) & +(-1 \times 3 - 11 \times 0) \\ -(-17 \times (-2) - 11 \times 3) & +(2 \times (-2) - 11 \times 0) & -(2 \times 3 - (-17) \times 0) \\ +(-17 \times (-7) - 11 \times 11) & -(2 \times (-7) - 11 \times (-1)) & +(2 \times 11 - (-17) \times (-1)) \end{bmatrix}^T$$

$$= \begin{bmatrix} +(-22 + 21) & -(2 + 0) & +(-3 + 0) \\ -(34 - 33) & +(-4 + 0) & -(6 + 0) \\ +(119 - 121) & -(-14 + 11) & +(22 - 17) \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & -2 & -3 \\ -1 & -4 & -6 \\ -2 & 3 & 5 \end{bmatrix}^T$$

$$= \begin{bmatrix} -1 & -1 & -2 \\ -2 & -4 & 3 \\ -3 & -6 & 5 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{1}{|A|} \times Adj(A)$

$$= \frac{1}{-1} \times \begin{bmatrix} -1 & -1 & -2 \\ -2 & -4 & 3 \\ -3 & -6 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 & 2 \\ 2 & 4 & -3 \\ 3 & 6 & -5 \end{bmatrix}$$

6. Evaluate the rank of the matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$.

Step 1: Convert to Row Echelon Form

Perform row operations to simplify the matrix.

1. Subtract Row 1 from Row 2:

$$R_2 \to R_2 - R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 2 & 6 & 5 \end{bmatrix}$$

2. Subtract 2 × Row 1 from Row 3:

$$R_3 \to R_3 - 2R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 2 & -1 \end{bmatrix}$$

3. Subtract Row 2 from Row 3:

$$R_3 \to R_3 - R_2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Step 2: Count the Number of Non-Zero Rows

- The matrix is now in row echelon form.
- The first two rows are nonzero.
- The last row is zero.

Step 3: Determine the Rank

The number of **nonzero** rows in the row echelon form is **2**, so the **rank of the matrix is** 2.

7. Show that every square matrix can be uniquely expressed as sum of symmetric and skew symmetric matrix.

Let, A be any square matrix.

Now,

$$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A') \;=\; say,\; P + Q \; where$$

$$P = \frac{1}{2}(A + A') \;\; and \;\; Q = \frac{1}{2}(A - A')$$

$$To\ prove:\; P\ is\ symmetric,\ i.e.\ P' \;= P\ and\ Q\ is\ skew - symmetric,\ i.e.\ Q'$$

$$\therefore P' = \frac{1}{2}\left(A + A'\right)' \;=\; \frac{1}{2}\left(A' + (A')'\right) \;=\; \frac{1}{2}(A' + A) \;=\; P$$

$$\therefore P\ is\ symmetric$$

$$\therefore Q' = \frac{1}{2}\left(A - A'\right)' \;=\; \frac{1}{2}\left(A' - (A')'\right) \;=\; \frac{1}{2}(A' - A) \;=\; -Q$$

$$\therefore Q\ is\ skew - symmetric$$

8. If $\begin{vmatrix} x^3 + 3x & x - 1 & x + 3 \\ x + 1 & 1 - 2x & x - 4 \\ x - 2 & x + 4 & 3x \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$ be an identity in x where a, b, c, d, e are constants, then calculate the value of e.

$$\begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}.\det\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}.\det\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix}$$

$$+ a_{13}.\det\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

$$\Rightarrow \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

$$= a_{11}(a_{22} \times a_{33} - a_{23} \times a_{32}) - a_{12}(a_{21} \times a_{33} - a_{23} \times a_{31})$$
$$+ a_{13}(a_{21} \times a_{32} - a_{22} \times a_{31})$$

So,

$$\begin{vmatrix} x^2 + 3x & x - 1 & x + 3 \\ x + 1 & -2x & x - 4 \\ x - 3 & x + 4 & 3x \end{vmatrix} = ax^4 + bx^3 + cx^2 + dx + e$$

$$\Rightarrow (x^2 + 3x).\det\begin{bmatrix} -2x & x - 4 \\ x + 4 & 3x \end{bmatrix}$$

$$- (x - 1).\det\begin{bmatrix} x + 1 & x - 4 \\ x - 3 & 3x \end{bmatrix}$$

$$+ (x + 3).\det\begin{bmatrix} x + 1 & -2x \\ x - 3 & x + 4 \end{bmatrix}$$

$$= ax^4 + bx^3 + cx^2 + dx + e$$

10. Using the Gauss-Jordan method, compute the inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$.

$$\left(\begin{array}{ccc|ccc}1 & 2 & 3 & 1 & 0 & 0\\ 0 & 1 & 2 & 0 & 1 & 0\\ 0 & 0 & 1 & 0 & 0 & 1\end{array}\right)$$

$R_1 \to R_1 - 2R_3, R_2 \to R_2 - 2R_3$

$$\left(\begin{array}{ccc|ccc}1 & 2 & 0 & 1 & 0 & -3\\ 0 & 1 & 0 & 0 & 1 & -2\\ 0 & 0 & 1 & 0 & 0 & 1\end{array}\right)$$

$R_1 \to R_1 - 2R_2$

$$\left(\begin{array}{ccc|ccc}1 & 2 & 0 & 1 & -2 & 1\\ 0 & 1 & 0 & 0 & 1 & -2\\ 0 & 0 & 1 & 0 & 0 & 1\end{array}\right)$$

Therefore, by Gauss-Jordan method the inverse is

$$\begin{pmatrix}1 & -2 & 1\\ 0 & 1 & -2\\ 0 & 0 & 1\end{pmatrix}$$